

Gluino Contribution to the 3-loop Quark Mass Anomalous Dimension in the Minimal Supersymmetric Standard Model

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Abstract

We deduce the gluino contribution to the three-loop QCD quark mass anomalous dimension function within the minimal supersymmetric Standard Model (MSSM) from its standard QCD expression. This work is a continuation of the program of computation of MSSM renormalization group functions.

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The renormalization group method [1] is a powerful tool for the study of many physically interesting quantities within the Standard Model and beyond. Although experimental measurements at the highest available energy are consistent with the standard model [2], the observed relationship of the strong coupling constant at the Z and the weak angle as well as the value of the m_b/m_τ ratio vis-a-vis the top quark mass remain strong indications of a supersymmetric (SUSY) grand unification above $10^{16} GeV$ and a SUSY threshold for squarks and sleptons in the 0.1 to 1 TeV region. To use the renormalization group method to study the above and other quantities one needs to know the renormalization group functions - β - function and quark mass anomalous dimension γ_m . In our previous work we have calculated the three-loop QCD β function with gluino contribution included [3]. In the present work we deduce the three-loop quark mass anomalous dimension function including the gluino contribution. Here we use the known result for the standard QCD expression of the three-loop quark mass anomalous dimension [4]. The two-loop contributions in the ratio m_b/m_τ turned out to be 20% of the leading contribution [6]. For this reason a full calculation of the three-loop contribution would be useful.

The quark mass anomalous dimension is defined as usual

$$-\mu^2 \frac{\partial \ln m}{\partial \mu^2} = \gamma_m(\alpha_s), \quad (1)$$

The renormalization of a quark mass within the MS type framework [7,8] has the following form

$$m^B = Z_m m = m \left[1 + \sum_i \frac{a_i(\alpha_s)}{\varepsilon^i} \right] \quad (2)$$

where B indicates the “bare” mass. The anomalous dimension function is determined by the lowest order pole term in the quark mass renormalization constant within the MS framework. That is,

$$\gamma_m(\alpha_s) = \alpha_s \frac{\partial a_1(\alpha_s)}{\partial \alpha_s} = \gamma_1 \frac{\alpha_s}{4\pi} + \gamma_2 \left(\frac{\alpha_s}{4\pi} \right)^2 + \gamma_3 \left(\frac{\alpha_s}{4\pi} \right)^3 - \dots \quad (3)$$

The a_i coefficients are related via a renormalization group equation that serves as a powerful check of the calculation.

$$\left(\gamma_m(\alpha_s) - \beta(\alpha_s) \frac{\partial}{\partial \alpha_s} \right) a_i(\alpha_s) = -\alpha_s \frac{\partial}{\partial \alpha_s} a_{i+1}(\alpha_s), \quad (4)$$

where the three-loop QCD β function including the gluino contribution and ignoring squarks is defined as follows [3].

$$\mu^2 \frac{\partial \alpha_s}{\partial \mu^2} = \alpha_s \beta(\alpha_s), \quad (5)$$

where

$$\beta(\alpha_s) = \beta_1 \frac{\alpha_s}{4\pi} + \beta_2 \left(\frac{\alpha_s}{4\pi} \right)^2 + \beta_3 \left(\frac{\alpha_s}{4\pi} \right)^3 + \dots \quad (6)$$

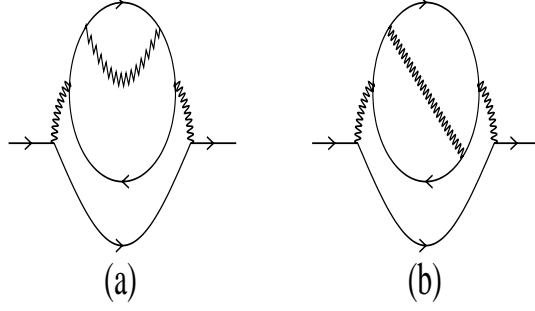


FIG. 1. Three-loop graphs giving nontrivial contribution to the quark mass anomalous dimension function with the gluino included. Wavy lines denote gluons and the solid loop corresponds to quark or gluino.

with

$$\beta_1 = -\frac{11}{3}C_A + \frac{4}{3}\left(N_f T + \frac{n_{\tilde{g}}}{2}C_A\right), \quad (7)$$

$$\beta_2 = -\frac{34}{3}C_A^2 + \frac{20}{3}\left(N_f T C_A + \frac{n_{\tilde{g}}}{2}C_A^2\right) + 4\left(N_f T C_F + \frac{n_{\tilde{g}}}{2}C_A^2\right), \quad (8)$$

$$\begin{aligned} \beta_3 = & -\frac{2857}{54}C_A^3 - N_f T(2C_F^2 - \frac{205}{9}C_F C_A - \frac{1415}{27}C_A^2) - (N_f T)^2\left(\frac{44}{9}C_F + \frac{158}{27}C_A\right) \\ & + \frac{988}{27}n_{\tilde{g}}C_A^3 - n_{\tilde{g}}N_f T\left(\frac{224}{27}C_A^2 + \frac{22}{9}C_A C_F\right) - \frac{145}{54}n_{\tilde{g}}^2C_A^3. \end{aligned} \quad (9)$$

The MS renormalization of quark mass can be expressed as the following multiplicative renormalization

$$Z_m = Z_{\overline{\psi}\psi} \times Z_2^{-1} \quad (10)$$

Where Z_2 is the quark propagator renormalization constant and $Z_{\overline{\psi}\psi}$ renormalizes the quark propagator with $\int \overline{\psi}(x)\psi(x)dx$ operator insertion. The gluino contributions to the above renormalization constants are in one-to-one correspondence with quark loop graphs and differ from them only by color and symmetry factors. Our procedure, as used to determine the gluino contributions to Z decay at four-loop level [5] and to the β -function at three-loop level, is to decompose the known QCD results into contributions from separate graphs. Then one can determine the color factors that relate each graph with an internal quark loop to the corresponding graph with a gluino loop. For instance, a subgraph consisting a simple quark loop has the color factor

$$N_f \text{Tr}(T^a T^b) = N_f T \delta^{ab}$$

while the color factor for a simple gluino loop has the color (and symmetry) factor

$$\frac{n_{\tilde{g}}}{2}\text{Tr}(F^a F^b) = \frac{n_{\tilde{g}}}{2}C_A\delta^{ab}$$

The relative factor 1/2 is due to the Majorana nature of the gluino. Here T^a and F^a are the gauge group generators in the fundamental and adjoint representations respectively. For the gauge group SU(N) they satisfy

$$\text{Tr}(T^a T^b) = T\delta^{ab}$$

$$\text{Tr}(F^a F^b) = N\delta^{ab}$$

Thus for graphs with a simple fermion loop subgraph one obtains the gluino contribution by making the substitution

$$N_f T \rightarrow N_f T + \frac{n_{\tilde{g}}}{2}C_A$$

Here $n_{\tilde{g}} = 0$ is the Standard Model limit and $n_{\tilde{g}} = 1$ corresponds to the minimal SUSY extension with one octet of gluinos.

The full set of three-loop graphs contributing in the standard QCD Z_2 and $Z_{\overline{\psi}\psi}$ were given in ref. [4]. We reanalyse all of the graphs up to and including three-loop level and added graphs with gluino propagators. The calculation of SU(N) group factors revealed that all but two graphs with the gluino contribution can easily be obtained from the known results by simply replacing N_f to $N_f + n_{\tilde{g}}C_A$. However, for the graph of Fig.1a we found that in order to take the particular topology into account one needs to make the following replacement in the standard QCD result for this graph

$$N_f \rightarrow N_f + \frac{C_A^2}{2TC_F} = N_f + \frac{27}{4} \quad (11)$$

The replacement for the graph in Fig.1b looks like

$$N_f \rightarrow N_f + \frac{C_A^2}{4T(C_F - C_A/2)} = N_f - 27. \quad (12)$$

If the gluino lies above the squark, the current calculation provides the contribution from gluinos alone up to three-loop order. This is a gauge invariant subset of the full SUSY three-loop graphs and leaves a vastly reduced number of graphs (those with one or more internal squark lines) still to be calculated at this order. If the gluino lies lower in mass than the squarks, the current calculation produces the full SUSY anomalous dimension of quark mass up to and including the three loop order in the region up to the squark mass scale.

We obtain the following result for three-loop quark mass anomalous dimension function with gluino included.

$$\gamma_1 = 3C_F \quad (13)$$

$$\gamma_2 = C_F \left[\frac{3}{2}C_F + \frac{97}{6}C_A - \frac{10}{3} \left(TN_f + \frac{n_{\tilde{g}}}{2}C_A \right) \right] \quad (14)$$

$$\begin{aligned} \gamma_3 = C_F \left[\frac{129}{2} C_F^2 - \frac{129}{4} C_F C_A - C_F T N_f \left(46 + 48 \zeta(3) (C_A - C_F) \right) + \frac{11413}{108} C_A^2 - \frac{556}{27} C_A T N_f \right. \\ \left. - \frac{140}{27} \left(T N_f + \frac{n_{\tilde{g}}}{2} C_A \right)^2 \right] - n_{\tilde{g}} C_F \left[\frac{1}{2} C_F C_A + \frac{1771}{54} C_A^2 \right] \end{aligned} \quad (15)$$

The eigenvalues of the Casimir operators for the adjoint ($N_A = 8$) and the fundamental ($N_F = 3$) representations of $SU_c(3)$ are

$$C_A = 3, \quad C_F = 4/3, \quad \text{and} \quad T = 1/2. \quad (16)$$

We obtain the following values for the above perturbative coefficients of the γ -function.

$$\gamma_1 = 4 \quad (17)$$

$$\gamma_2 = \frac{202}{3} - \frac{20}{9} N_f - \frac{20}{3} n_{\tilde{g}} \quad (18)$$

$$\gamma_3 = 1249 - \left(\frac{2216}{27} + \frac{160}{3} \zeta(3) \right) N_f - \frac{140}{81} N_f^2 - n_{\tilde{g}} \left(\frac{3566}{9} + \frac{280}{27} N_f \right) - \frac{140}{9} n_{\tilde{g}}^2 \quad (19)$$

We see that the gluino gives a substantial contribution to two- and especially three-loop levels. Indeed, the gluino contribution reduces the two-loop coefficient by about 10% and reduces the three-loop contribution by about 50%. Such a large contribution might ultimately be important in phenomenological applications.

The anomalous dimension of quark mass along with the QCD β -function determines the running of the quark mass. Indeed, at the three-loop level, for the running quark mass one has (see, e.g., [9])

$$\frac{m_f(\mu_1)}{m_f(\mu_2)} = \frac{\phi(\alpha_s(\mu_1))}{\phi(\alpha_s(\mu_2))}, \quad (20)$$

where,

$$\begin{aligned} \phi(\alpha_s(\mu)) = \left(-2\beta_1 \frac{\alpha_s(\mu)}{4\pi} \right)^{-\frac{\gamma_1}{\beta_1}} \left\{ 1 - \left(\frac{\gamma_2}{\beta_1} - \frac{\beta_2 \gamma_1}{\beta_1^2} \right) \frac{\alpha_s(\mu)}{4\pi} \right. \\ \left. + \frac{1}{2} \left[\left(\frac{\gamma_2}{\beta_1} - \frac{\beta_2 \gamma_1}{\beta_1^2} \right)^2 - \frac{\gamma_3}{\beta_1} + \frac{\beta_2 \gamma_2}{\beta_1^2} + \frac{\beta_3 \gamma_1}{\beta_1^2} - \frac{\beta_2^2 \gamma_1}{\beta_1^3} \right] \left(\frac{\alpha_s(\mu)}{4\pi} \right)^2 \right\} \end{aligned} \quad (21)$$

The above equation indicates that there will be a substantial shift in the running of a quark mass due to the gluino.

To verify this, we will need the following equations. The running coupling is parametrized as follows:

$$\frac{\alpha_s(\mu)}{4\pi} = -\frac{1}{\beta_1 L} - \frac{\beta_2 \log L}{\beta_1^3 L^2} - \frac{1}{\beta_1^5 L^3} (\beta_2^2 \log^2 L - \beta_2^2 \log L + \beta_3 \beta_1 - \beta_2^2) + O(L^{-4}), \quad (22)$$

where $L = \log(\mu^2/\Lambda_{\overline{MS}}^2)$.

The general evolution equation for the running coupling to $O(\alpha_s^3)$ [9] has the form

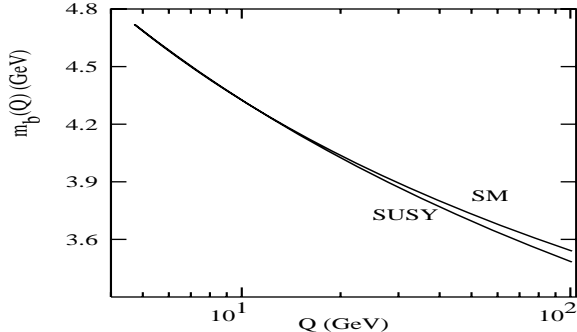


FIG. 2. The gluino effect on the running of the b quark mass

$$\begin{aligned}
\frac{\alpha_s^{(n)}(\mu)}{4\pi} &= \frac{\alpha_s^{(N)}(M)}{4\pi} - \left(\frac{\alpha_s^{(N)}(M)}{4\pi}\right)^2 \left(\beta_1^{(N)} \log \frac{M^2}{\mu^2} - \frac{2}{3} \sum_l \log \frac{m_l^2}{\mu^2}\right) \\
&\quad - \left(\frac{\alpha_s^{(N)}(M)}{4\pi}\right)^3 \left[\beta_2^{(N)} \log \frac{M^2}{\mu^2} - \frac{38}{3} \sum_l \log \frac{m_l^2}{\mu^2}\right. \\
&\quad \left. + \left(\beta_1^{(N)} \log \frac{M^2}{\mu^2} - \frac{2}{3} \sum_l \log \frac{m_l^2}{\mu^2}\right)^2 + \frac{50}{9}(N - n)\right]
\end{aligned} \tag{23}$$

where the superscript n (N) indicates that the corresponding quantity is evaluated for n (N) numbers of participating quark flavors. Conventionally, n (N) is specified to be the number of quark flavors with mass $\leq \mu$ ($\leq M$). However, the eq.(23) is relevant for any $n \leq N$ and arbitrary μ and M , regardless of the conventional specification of the number of quark flavors. The $\log m_l/\mu$ terms are due to the “quark threshold” crossing effects and the constant coefficients $2/3 = \beta_1^{(k-1)} - \beta_1^{(k)}$, $38/3 = \beta_2^{(k-1)} - \beta_2^{(k)}$ represent the contributions of the quark loop in the β -function. The sum runs over $N - n$ quark flavors (e.g., $l = b$ if $n = 4$ and $N = 5$). Note that m_l is the pole mass of the quark with flavor l . Quark masses can be estimated from QCD sum rules. For instance, the b quark pole mass is $m_b = 4.72\text{GeV}$ [10].

In fig.2 we show the evolution of $m_b(\mu)$ from $\mu=4.72\text{GeV}$ to M_Z . We see that the gluino effect is a few percent at M_Z which could ultimately be important for grand unification studies. If the gluino is not light but is nevertheless below the squark mass, our current result will still have a region of relevance. Ultimately, of course, it will be desirable to have the full SUSY three-loop effect including squark contributions.

Analyzing numerical results, we see that the third-loop effect is only a few MeV. Based on this, we conclude that the three-loop anomalous dimension with the gluino included is a good approximation for phenomenological applications and the error in the perturbative evaluation of this quantity can be estimated at few MeV.

An observable quantity R is invariant under the renormalization group transformations and obeys the homogeneous renormalization group equation:

$$\left(\mu^2 \frac{\partial}{\partial \mu^2} + \beta(\alpha_s) \alpha_s \frac{\partial}{\partial \alpha_s} - \gamma_m(\alpha_s) \sum_f m_f \frac{\partial}{\partial m_f}\right) R(\mu, \alpha_s, m_f) = 0 \tag{24}$$

The quantity R may denote various cross sections and decay rates that are usually calculated using the perturbation theory method. In our previous works we have calculated the

gluino contribution to the QCD β -function at the three-loop level [3] and to the hadronic decay rate of the Z boson to the four-loop level [5]. Thus, the present work completes the evaluation of gluino contributions necessary for $O(\alpha_s^3)$ renormalization group analysis for the above quantity. These results can be used in the renormalization group analysis for other quantities, for instance, hadronic decay rates of the τ -lepton and various Higgs bosons.

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