PQCD ANALYSIS OF INCLUSIVE SEMILEPTONIC DECAYS OF A POLARIZED Λ_b BARYON

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Abstract

We investigate the Λ_b polarization problem in the inclusive semileptonic decays of a polarized Λ_b baryon, using the modified perturbative QCD formalism which includes Sudakov suppression. According to HQEFT, we show that, at the leading order in the $1/M_b$ expansion, the polarized and unpolarized distribution functions become one single universal distribution function. To explore the mechanisms which determine the spin properties of a polarized Λ_b baryon, we construct four formalisms which are the naive quark model (QM), the modified quark model (MQM) , the naive parton model (PM), and the modified parton model (MPM), and calculate their corresponding Λ_b polarizations, denoted as P's. The modified quark and parton models are with Sudakov suppression. The resulting P's are -0.23 (QM), -0.94 (MQM), -0.37 (PM), and -0.68 (MPM), respectively. We note that $\mathrm{P}_{\mathrm{MQM}}$ (equal to -0.94) is very close to the b quark polarization asymmetry, $A_{RL} = -0.94$, calculated at the Z vertex in $Z \to b\bar{b}$ process, and that $\mathrm{P}_{\mathrm{MPM}}$ (equal to -0.68) is also very close to the Λ_b baryon polarization (equal to -0.68) which was estimated from the fragmentation processes under the heavy quark limit. Based on our analysis, there exists no any paradox in the theoretical explanations of the Λ_b polarization for the experimental data.

1 Introduction

A recent measurement by ALEPH Collaboration [1] indicated that the Λ_b polarization was -0.23. This deviated largely from the Standard Model expectation that the Λ_b polarization should be -0.94 [2, 3, 4]. It was, therefore, argued that the spin-spin interactions between the b quark and the light quarks and gluons produced from the vacuum as the b quark undergoing its fragmentation, should bring about large spin-flip of the b quark spin. A heavy quark effective field theory (HQEFT) calculation showed that the final Λ_b polarization was -0.68[5].

In this paper, we would like to re-interpret the measurement by the ALEPH Collaboration. The Λ_b polarization, denoted as P, could be related to the experimental measured quantity, $R = 1.12 \pm 0.10$, as

$$R = \frac{< E_\nu^* > (< E_l^* > + < P_l^*(\mathrm{P}) >)}{< E_l^* > (< E_\nu^* > + < P_\nu^*(\mathrm{P}) >)} \quad . \tag{1}$$

Based on the naive quark model, the averaged quantities, $< E_l^* >$, $< E_\nu^* >$, $< P_l^* >$ and $< P_\nu^* >$ were calculated in the Λ_b rest frame . The resulting P was equal to -0.23 [1]. Since the spectra are still not available, the explanation for the Λ_b polarization is therefore theoretical dependent. For the purpose to explore the mechanisms which control the spin properties of a polarized Λ_b baryon, we shall construct four formalisms to investigate, which are the naive quark model (QM), the modified quark model (MQM), the naive parton model (PM), and the modified parton model (MPM). The modified quark model and modified parton model are with Sudakov suppression.

We emphasize the importance of the transverse degrees of freedom of partons inside a Λ_b baryon in our analysis. First of all, the transverse momenta regularize the divergences when the outgoing q quark is approaching the end point [7]. And second, the transverse momenta enhance the contributions from the longitudinal component of the Λ_b baryon spin vector. These effects indicate that the intrinsic b dependence of the distribution function, *e.g.*, $f(z, b)$, is nonignorable. The form of the b dependence of the distribution function could be determined by exploiting the infrared (IR) renormalon contributions [9, 10], such that one can parameterize $f(z, b)$ as $\exp[-tM^2b^2]f(z)$. The parameter t will be determined from the charged lepton spectra.

The arrangement of our paper is as follows. In the next section, we develop a power expansion scheme which is appropriate for heavy quark system. By employing the HQEFT, we generalize the naive collinear expansion scheme [6] to include heavy massive quark partons and to apply to decay processes. Using this generalized collinear expansion scheme, we then show that, in the heavy quark limit, there exists an universal distribution function which respects both the unpolarized and polarized matrix elements. By taking into account the radiative corrections, the factorization formula is expressed as the convolution of a hard scattering amplitude with a jet and a universal soft function. In Section 3, we construct the four formalisms based on the factorization formula. Section 4 is the numerical result and Section 5 the conclusion. An Appendix is presented for those details skipped by the main text.

2 Factorization Formula

The quadruple differential decay rate for polarized $\Lambda_b \to X_q \ell \bar{\nu}$ is expressed as

$$\frac{d^4\Gamma}{dE_l dq^2 dq_0 d\cos\theta_l} = \frac{|V_{qb}|^2 G_F^2}{256\pi^4 M} L^{\mu\nu} W_{\mu\nu} \ , \tag{2}$$

where M is the Λ_b baryon mass, $L^{\mu\nu}$ is the leptonic tensor and $W_{\mu\nu}$ is the hadronic tensor.

The kinematically independent variables E_l, q, q_0 and $\cos\theta_l$ are expressed as follows. We set our working frame in the Λ_b baryon rest frame and specify relevant momenta as

$$P = \frac{M}{\sqrt{2}}(1,1,\mathbf{0}), p_l = (p_l^+, 0, \mathbf{0}), p_{\bar{\nu}} = (p_{\bar{\nu}}^+, p_{\bar{\nu}}^-, \mathbf{p}_{\bar{\nu}\perp}) \ . \tag{3}$$

E_l, q, and q_0 are expressed as $E_l = p_l^+/\sqrt{2}$, $q^2 = 2p_l^+ p_{\bar{\nu}}^-$, and $q_0 = (p_l^+ + p_{\bar{\nu}}^+ + p_{\bar{\nu}}^-)/\sqrt{2}$, respectively. We let $P_b = P - l$ whose square is set as $P_b^2 \approx M_b^2$, M_b the b quark mass. l is the momentum of the light degree of freedom inside the Λ_b baryon, and has a large plus component and small transverse components $\mathbf{l}_\perp$. The final state quark momentum is $P_q = P - l - q$. θ_l is the angle between the third component of p_l and that of the b quark polarization vector, $S_b = (S_b^+, S_b^-, \mathbf{S}_{b\perp})$.

It is convenient to use dimensionless variables $x = 2E_l/M$, $y = q^2/M^2$, and $y_0 = 2q_0/M$. The integration regions for x, y and y_0 are

$$0 \le x \le 1, \qquad 0 \le y \le x, \qquad \frac{y}{x} + x \le y_0 \le 1 + y. \tag{4}$$

Note that we have chosen M as scale variable and have set $m_q = 0$ for simplicity. The cases for $m_q \neq 0$ will be considered in our future publish.

By optical theorem, $W^{\mu\nu}$ relates to the forward matrix element $T^{\mu\nu}$ as

$$W^{\mu\nu} = -\frac{\mathrm{Im}(T^{\mu\nu})}{\pi}. \tag{5}$$

The lowest order of $T^{\mu\nu}$ is defined as

$$\begin{aligned} T^{\mu\nu}(P,q,S) &= -i \int d^4y e^{iq\cdot y} \langle \Lambda_b(P,S)|\mathcal{T}[J^{\dagger\mu}(0), J^\nu(y)]|\Lambda_b(P,S)\rangle \\ &= -i \int \frac{d^4P_b}{(2\pi)^4} S^{\mu\nu}(P_b - q) T(P,S,P_b), \end{aligned} \tag{6}$$

where $S^{\mu\nu}(P_b - q)$ describes the short distance $b \to Wq$ decay subprocess and $T(P,S,P_b)$ relates the corresponding long distance matrix element

$$T(P,S,P_b) = \int d^4y e^{iP_b \cdot y} \langle \Lambda_b(P,S)|\bar{b}(0)b(y)|\Lambda_b(P,S)\rangle. \tag{7}$$

$J^\mu = \bar{q}\gamma^\mu(1-\gamma_5)b$ is the V $-$ A current. Because the momentum P_b in $S^{\mu\nu}(P_b)$ has non-collinear component which would give high order power contributions [6], it is then necessary to investigate the collinear expansion for $T^{\mu\nu}$. The main procedures are demonstrated as follows.

For a Λ_b baryon which carries momentum P and mass M, we specify P to be

$$P^\mu = p^\mu + \frac{M^2}{2p\cdot n} n^\mu, \tag{8}$$

where $p^2 = n^2 = 0$, $p \cdot n = P \cdot n$. The b quark, inside the Λ_b baryon, carries momentum P_b chosen as

$$\begin{aligned} P_b^\mu &= zp^\mu + \frac{P_b^2 + P_{g\perp}^2}{2P_b \cdot n} n^\mu + P_{b\perp}^\mu & (9) \\ &= \hat{P}_b^\mu + \frac{P_b^2 - M_b^2}{2P_b\cdot n} n^\mu, & (10) \end{aligned}$$

where $\hat{P}_b^2 = M_b^2$ is the on-shell part of P_b and the momentum fraction z defined by $z = P_b^+/P^+ = 1 - l^+/P^+$. By the parameterization of P_b, the b quark propagator is then expressed as

$$\frac{i}{\not{P}_b - M_b + i\epsilon} = \frac{i(\hat{\not{P}}_b + M_b)}{\not{P}_b - M_b + i\epsilon} + \frac{i\not{n}}{2P_b\cdot n} \quad . \tag{11}$$

The second part of eq. (11) is called special propagator introduced by Qiu [6]. The special propagator describes the short distance nature of the loop propagator in a Feynman diagram. To generalize the naive collinear expansion scheme to include heavy massive parton, we employ the technology of the HQEFT to rescale the b quark field, $b(x)$, as $b_v(x) = \exp(iM_b v\cdot x)\frac{1+\not{v}}{2}b(x)$. In this way, P_b is parameterized as $P_b = M_b v + k$, with k the residual momentum. The rescaled b quark field, $b_v(x)$, carries the residual momentum k and has a small effective mass $\bar{\Lambda}$, with $\bar{\Lambda} \equiv M - M_b$. Since k is of order $O(\Lambda_{QCD})$ and $\bar{\Lambda}$ much smaller than M_b, it is thus expected that the whole program of Qiu's collinear expansion for massless parton would be applicable for $b_v(x)$. We now demonstrate the main procedure of this generalized collinear expansion. First, one expands k as $k = \xi p + (k - \xi p)$, where $\xi p = (z-1)p + \bar{\Lambda}/Mp$. Secondly, one performs a Taylor expansion of $S^{\mu\nu}(k)$ around $S^{\mu\nu}(\xi p)$ as

$$S^{\mu\nu}(k) = S^{\mu\nu}(\xi p) + \frac{\partial S^{\mu\nu}}{\partial k^\alpha}|_{k=\xi p}(k-\xi p)^\alpha + \cdots \ . \tag{12}$$

The high order terms eq. (12) are to replace the higher order gluon field operators in the higher order components of $T^{\mu\nu}$ by the relating covariant operators. In the same way, the non-collinear component of k in $T(k)$ would yield high order contributions. It happens when $T(k)$ is contracted with $\not p$ (or $\not p\gamma_5$). As a result, at leading order, we could write $T^{\mu\nu}$ in the form

$$T^{\mu\nu}(P,q,S) \approx -i\int \frac{d^4k}{(2\pi)^4}\Big\{[S^{\mu\nu}(k=\xi p,q)\not P_b][T(P,S,k=\xi p)\frac{\not n}{4P_b\cdot n}] - [S^{\mu\nu}(k=\xi p,q)\not S_b\gamma_5][T(P,S,k=\xi p)\frac{\not n\gamma_5}{4S_b\cdot n}]\Big\} \quad . \quad (13)$$

(The details of this proof will be described in the Appendix A.)

We now follow [7] to derive the factorization formula for the inclusive semileptonic decay $\Lambda_b \to X_q\ell\nu$. After including the radiative corrections, the formula is written as

$$\frac{1}{\Gamma^{(0)}}\frac{d^3\Gamma}{dxdydy_0d\cos\theta_l} = \frac{M^2}{2}\int_{z_{\rm min}}^{z_{\rm max}} dz\int d^2\mathbf{l}_\perp \times S(z,\mathbf{l}_\perp,\mu)J(z,P_q^-,\mathbf{l}_\perp,\mu)H(z,P_q^-,\mu) \;, \quad (14)$$

with $\Gamma^{(0)} = \frac{G_F^2}{16\pi^3}|V_{qb}|^2M^5$ and μ the renormalization and factorization scale. The transverse momentum $\mathbf{l}_\perp$ has been introduced for the regularization of the end point singularities [7]. In order to make factorization of the intrinsic transverse momentum from the radiative transverse momentum of J, we transform eq. (14) into the impact $\mathbf{b}$ space

$$\frac{1}{\Gamma^{(0)}}\frac{d^3\Gamma}{dxdydy_0d\cos\theta_l} = \frac{M^2}{2}\int_x^1 dz\int \frac{d^2\mathbf{b}}{(2\pi)^2} \times\tilde{S}(z,\mathbf{b},\mu)\tilde{J}(z,P_q^-,\mathbf{b},\mu)H(z,P_q^-,\mu) \;. \quad (15)$$

To deal with the collinear and soft divergences resulting from the radiative corrections for massless parton inside the jet, the resummation technique is necessary and these divergences could be resummed into a Sudakov form factor [7]. The jet function is then re-expressed in the form

$$\tilde{J}(z,P_q^-,b,\mu) = \exp\left[-2s(P_q^-,b)\right]\tilde{J}(z,b,\mu), \quad (16)$$

where $\exp\left[-2s(P_q^-,b)\right]$ is the Sudakov form factor.

The scale invariance of the differential decay rate in eq. (15) and the Sudakov form factor in eq. (16) requires the functions $\tilde{J}$, $\tilde{S}$, and H to obey the following RG equations:

$$\begin{aligned}\mathcal{D}\tilde{J}(b,\mu) &= -2\gamma_q\tilde{J}(b,\mu) \;,\\ \mathcal{D}\tilde{S}(b,\mu) &= -\gamma_S\tilde{S}(b,\mu) \;,\\ \mathcal{D}H(P_q^-,\mu) &= (2\gamma_q+\gamma_S)H(P_q^-,\mu) \;,\end{aligned} \quad (17)$$

with

$$\mathcal{D} = \mu\frac{\partial}{\partial\mu} + \beta(g)\frac{\partial}{\partial g} \ . \tag{18}$$

$\gamma_q = -\alpha_s/\pi$ is the quark anomalous dimension in axial gauge, and $\gamma_S = -(\alpha_s/\pi)C_F$ is the anomalous dimension of $\tilde{S}$. After solving eq. (17), we obtain the evolution of all the convolution factors in eq. (15),

$$\begin{aligned}
\tilde{J}(z, P_q^-, b, \mu) &= \exp\left[-2s(P_q^-, b) - 2\int_{1/b}^{\mu}\frac{d\bar{\mu}}{\bar{\mu}}\gamma_q(\alpha_s(\bar{\mu}))\right]\tilde{J}(z, b, 1/b) \ , \\
\tilde{S}(z, b, \mu) &= \exp\left[-\int_{1/b}^{\mu}\frac{d\bar{\mu}}{\bar{\mu}}\gamma_S(\alpha_s(\bar{\mu}))\right] f(z, b, 1/b) \ , \\
H(z, P_q^-, \mu) &= \exp\left[-\int_{\mu}^{P_q^-}\frac{d\bar{\mu}}{\bar{\mu}}[2\gamma_q(\alpha_s(\bar{\mu})) + \gamma_S(\alpha_s(\bar{\mu}))]\right] H(z, P_q^-, P_q^-) \ .
\end{aligned} \tag{19}$$

In the above solutions, we set the $1/b$ as an IR cut-off for single logarithism evolution. However, the intrinsic b dependence of $f(z, b)$ gives more nonperturbative higher order contributions which could be determined by exploiting the IR renormalon contributions. We employ a minima setting for $f(z, b)$ as

$$f(z, b) = f(z)e^{-\Sigma(b)} \ . \tag{20}$$

The IR renormalon contributions arising from a real soft gluon attaching the two valence b quarks as one calculating the Sudakov form factors in eq. (16). As a result, one could simply parameterize $\exp[-\Sigma(b)]$ in the form

$$e^{-\Sigma(b)} = e^{-tM^2b^2} \tag{21}$$

[9]. A few comments are needed. For the end point regime where the Sudakov suppression dominates, we employ the approximation

$$f(z, b) = f(z) \tag{22}$$

, while for other regimes which are not under the control of the Sudakov suppression, we take into account the intrinsic b dependence of $f(z, b)$ to give more suppressions

$$f(z, b) = e^{-tM^2b^2}f(z) \ . \tag{23}$$

We make further approximations such that $f(z, b, 1/b) = f(z, b)$, $\tilde{J}(z, b, 1/b) = \tilde{J}^{(0)}(z, b)$, and $H(z, P_q^-, P_q^-) = H^{(0)}(z, P_q^-)$.

Combining the above results, we write factorization formula for the inclusive semileptonic Λ_b baryon decay as

$$\begin{aligned}
\frac{1}{\Gamma^{(0)}}\frac{d^4\Gamma}{dxdydy_0d\cos\theta_l} &= M^2\int_x^1 dz\int_0^\infty\frac{bdb}{4\pi}\tilde{J}^{(0)}(z, b)H^{(0)}(z, P_q^-)e^{-S(P_q^-, b)} \\
&\times\begin{cases} f(z) & \text{for } x \text{ in end point regimes} \ , \\ \exp[-tM^2b^2]f(z) & \text{for } x \text{ in other regimes} \ , \end{cases}
\end{aligned} \tag{24}$$

where

$$S(P_q^-, b) = 2s(P_q^-, b) - \int_{1/b}^{P_q^-} \frac{d\bar{\mu}}{\bar{\mu}} [2\gamma_q(\bar{\mu}) + \gamma_S(\bar{\mu})] \; . \tag{25}$$

The parameter t will be determined by interpolating from the end point regimes for the relevant charged lepton spectrum.

Let's now discuss how to parameterize $T(k)$ defined in eq. (7). As discussed in previous paragraphs that, at leading order, $T(k)$ should not be contracted with $\not{p}$ (or $\not{p}\gamma_5$). So, we recast $T(k)$ in the form

$$T(k) = \frac{1}{4} P_b \cdot n f(z) \not{\bar{n}} - \frac{1}{4} S_b \cdot n g(z) \not{\bar{n}} \gamma_5 \tag{26}$$

where $\bar{n}^\mu = p^\mu / |p^\mu|$. The unpolarized and polarized distribution functions, $f(z)$ and $g(z)$, are defined as

$$f(z) \quad = \quad \frac{1}{P_b \cdot n} \int \frac{d\lambda}{2\pi} e^{-i\xi\lambda n} \langle P, S | \bar{b}_v(0) \not{n} b_v(\frac{\lambda n}{P \cdot n}) | P, S \rangle \tag{27}$$

and

$$g(z) \quad = \quad \frac{1}{S_b \cdot n} \int \frac{d\lambda}{2\pi} e^{-i\xi\lambda n} \langle P, S | \bar{b}_v(0) \not{n} \gamma_5 b_v(\frac{\lambda n}{P \cdot n}) | P, S \rangle \; . \tag{28}$$

It is easy to show that $f(z)$ and $g(z)$, in the heavy quark limit, share a common matrix element which could be described by an universal distribution function, $f_{\Lambda_b}(z)$. This just reflects the heavy quark spin symmetry. We adopt the distribution function proposed in [7] in the form

$$f_{\Lambda_b}(z) = \frac{N z^2 (1-z)^2}{((z-a)^2 + \epsilon z)^2} \theta(1-z) \; . \tag{29}$$

The parameters N, a and ϵ are fixed by first three moments

$$\begin{aligned} &\int_0^1 f_{\Lambda_b}(z) dz = 1 \; , \\ &\int_0^1 dz (1-z) f_{\Lambda_b}(z) = \bar{\Lambda}/M + \mathcal{O}(\Lambda_{\rm QCD}^2/M^2) \; , \\ &\int_0^1 dz (1-z)^2 f_{\Lambda_b}(z) = \frac{\bar{\Lambda}^2}{M^2} + \frac{2}{3} K_b + \mathcal{O}(\Lambda_{\rm QCD}^3/M^3) \; . \end{aligned} \tag{30}$$

By substituting these constants,

$$M = 5.641 \text{GeV} \; , \quad M_b = 4.776 \text{GeV} \; , \quad K_b = 0.012 \pm 0.0026 \; , \tag{31}$$

into eq. (30), we determine the parameters N, a and ϵ to be

$$N = 0.10615 \; , \quad a = 1 \; , \quad \epsilon = 0.00413 \; . \tag{32}$$

For simplicity we will omit the subscript of $f_{\Lambda_b}(z)$ in the following text.

3 Differential Decay Rates

In this section, we employ the factorization formula eq. (24) to construct four formalisms which are the naive quark model (QM), the modified quark model (MQM), the naive parton model (PM), and the modified parton model (MPM). The charged lepton spectrum for the decay $\Lambda_b \to X_q \ell \nu$ from the naive quark model are expressed as

$$\frac{1}{\Gamma^{(0)}}\frac{d^2\Gamma^{\rm T}_{\rm QM}}{dxd\cos\theta_\ell} = \frac{1}{\Gamma^{(0)}}\frac{d^2\Gamma^{\rm U}_{\rm QM}}{dxd\cos\theta_\ell} + {\rm P}_{\rm QM}\cos\theta_\ell \frac{1}{\Gamma^{(0)}}\frac{d^2\Gamma^{\rm S}_{\rm QM}}{dxd\cos\theta_\ell} , \tag{33}$$

with

$$\frac{1}{\Gamma^{(0)}}\frac{d^2\Gamma^{\rm U}_{\rm QM}}{dxd\cos\theta_\ell} = \frac{x^2}{6}(3-2x) , \tag{34}$$

and

$$\frac{1}{\Gamma^{(0)}}\frac{d^2\Gamma^{\rm S}_{\rm QM}}{dxd\cos\theta_\ell} = \frac{x^2}{6}(1-2x) . \tag{35}$$

We draw the unpolarized, polarized and total charged leptonic spectra of QM in Fig.1 and denote them as curve 1.

By taking into account Sudakov suppression from the resummation of large radiative corrections, and substituting $f(z,b) = \delta(1-z)\exp[-tM^2b^2]$, $H^{(0)} = (x-y)[(y_0-x)+{\rm P}_{\rm MQM}\cos\theta_\ell({\rm y}_0-{\rm x}-2{\rm y}/{\rm x})]$ and the Fourier transform of $J^{(0)} = \delta(P_q^2)$ with $P_q^2 = M^2(1-y_0+y-p_\perp^2/M_B^2)$ into eq. (24), we derive the modified quark model spectrum. This spectrum is, after integrating eq. (24) over z and y_0, described by

$$\frac{1}{\Gamma^{(0)}}\frac{d^2\Gamma^{\rm T}_{\rm MQM}}{dxd\cos\theta_\ell} = \frac{1}{\Gamma^{(0)}}\frac{d^2\Gamma^{\rm U}_{\rm MQM}}{dxd\cos\theta_\ell} + {\rm P}_{\rm MQM}\cos\theta_\ell \frac{1}{\Gamma^{(0)}}\frac{d^2\Gamma^{\rm S}_{\rm MQM}}{dxd\cos\theta_\ell} , \tag{36}$$

$$\begin{aligned}\frac{1}{\Gamma^{(0)}}\frac{d^2\Gamma^{\rm U}_{\rm MQM}}{dxd\cos\theta_\ell} &= M\int_0^x dy\int_0^{1/\Lambda} dbe^{[-t^{\rm U}M^2b^2]}e^{-S(P_q^-,b)}(x-y)\eta \\ &\times\left[(1+y-x)J_1(\eta Mb) - \frac{2}{Mb}\eta J_2(\eta Mb) + \eta^2 J_3(\eta Mb)\right] ,\end{aligned} \tag{37}$$

$$\begin{aligned}\frac{1}{\Gamma^{(0)}}\frac{d^2\Gamma^{\rm S}_{\rm MQM}}{dxd\cos\theta_\ell} &= M\int_0^x dy\int_0^{1/\Lambda} dbe^{[-t^{\rm S}M^2b^2]}e^{-S(P_q^-,b)}(x-y)\eta \\ &\times\left[(1+y-x-2\frac{y}{x})J_1(\eta Mb) - \frac{2}{Mb}\eta J_2(\eta Mb) + \eta^2 J_3(\eta Mb)\right] ,\end{aligned} \tag{38}$$

where $P_q^- = (1-y/x)M/\sqrt{2}$, $\eta = \sqrt{(x-y)(1/x-1)}$ and J_1,J_2,J_3 are the Bessel functions of order 1, 2 and 3, respectively. The parameters $t^{\rm U}$ and $t^{\rm S}$ are chosen as $t^{\rm U} = 0.005$ and $t^{\rm S} = 0.0423$. Note that we have made an approximation by substituting $f(z)\exp[-t^{\rm U,S}M^2b^2]$ for the end point regimes. The spectra of MQM are drawn in Fig.1 and denoted as curve 2. One observes that the polarized and total spectra of MQM deviate largely from those of QM. The profile of the total spectrum of MQM is broader than that of QM and the peak position sifts to near $x = 1$ end point. These differences between the MQM spectra and the QM spectra indicate the resulting Λ_b polarizations to be largely different, which are equal to -0.94 and -0.23 for $\rm P_{MQM}$ and $\rm P_{QM}$ respectively.

The naive parton model spectra are obtained by adopting $H^{(0)} = (x-y)[(y_0-x-(1-z)y/x)+{\rm P_{PM}}\cos\theta_\ell(y_0-x-(1+z)y/x)]$ and $P_q^2 = M^2[1-y_0+y-(1-z)(1-y/x)]$. With integration over y_0, we derive

$$\frac{1}{\Gamma^{(0)}}\frac{d^2\Gamma_{\rm PM}}{dx d\cos\theta_\ell} = \int_0^x dy\int_x^1 dz f(z)(x-y)[(y+z-x)+{\rm P_{PM}}\cos\theta_\ell(y+z-x-2z\frac{y}{x})]\ . \tag{39}$$

The spectra are shown as curve 3 in Fig.1.

We finally come to the charged lepton spectra of the modified parton model that takes into account both large perturbative and nonperturbative corrections,

$$\frac{1}{\Gamma^{(0)}}\frac{d^2\Gamma^{\rm T}_{\rm MPM}}{dx d\cos\theta_\ell} = \frac{1}{\Gamma^{(0)}}\frac{d^2\Gamma^{\rm U}_{\rm MPM}}{dx d\cos\theta_\ell} + {\rm P_{MPM}}\cos\theta_\ell\frac{1}{\Gamma^{(0)}}\frac{d^2\Gamma^{\rm S}_{\rm MPM}}{dx d\cos\theta_\ell}\ , \tag{40}$$

$$\begin{aligned}\frac{1}{\Gamma^{(0)}}\frac{d^2\Gamma^{\rm U}_{\rm MPM}}{dx d\cos\theta_\ell} &= M\int_0^x dy\int_0^{1/\Lambda} db e^{-S(P_q^-,b)}e^{-t^{\rm U}M^2b^2}f(z)(x-y)\eta \\ &\times\left[(1+y-x)J_1(\eta Mb)-\frac{2}{Mb}\eta J_2(\eta Mb)+\eta^2J_3(\eta Mb)\right]\ ,\end{aligned} \tag{41}$$

$$\begin{aligned}\frac{1}{\Gamma^{(0)}}\frac{d^2\Gamma^{\rm S}_{\rm MPM}}{dx d\cos\theta_\ell} &= M\int_0^x dy\int_0^{1/\Lambda} db e^{-S(P_q^-,b)}e^{-t^{\rm S}M^2b^2}f(z)(x-y)\eta \\ &\times\left[(z+y-x-2z\frac{y}{x})J_1(\eta Mb)-\frac{2}{Mb}\eta J_2(\eta Mb)+\eta^2J_3(\eta Mb)\right],\end{aligned} \tag{42}$$

with $\eta = \sqrt{(x-y)(z/x-1)}$. The parameters $t^{\rm U,S}$ are set to be $t^{\rm U} = 0.003$ and $t^{\rm S} = 0.024$. The spectra of MPM are shown in Fig.1 and denoted as curve 4. The resulting $\rm P_{MPM}$ with value -0.68 is different from the $\rm P_{PM}$ with value -0.37.

4 Numerical Result

A recent experimental measurement of Λ_b polarization, P, is determined through the variable y proposed in [8]

$$y = \frac{< E_l >}{< E_\nu >}\ , \tag{43}$$

and the experimental measured quantity

$$R = \frac{y(\mathrm{P})}{y(0)} \quad . \tag{44}$$

y could be expressed in terms of Λ_b rest frame measurements in the form

$$y = \frac{< E_l^* > + < P_l^*(\mathrm{P}) >}{< E_\nu^* > + < P_\nu^*(\mathrm{P}) >}. \tag{45}$$

Since there are still no spectra available, the averaged quantities are then theoretical dependent. We calculate the P's from the four formalisms and list their values in Table.1. We note that $\mathrm{P_{MQM}}$ with value -0.94 is very close to the b quark polarization asymmetry, $\mathrm{A_{RL}}$, which was calculated at the Z vertex in $Z \to b\bar{b}$ [2] and expressed in the form

$$\mathrm{A_{RL}} = -\frac{2v_b a_b}{v_b^2 + a_b^2} = -0.936 \quad , \tag{46}$$

where v_b and a_b are the vector and axial vector couplings of the b quark to the Z boson, respectively. As shown in Table.2, the ratio of $< P_l^* > / < P_\nu^* >$ of MQM is about one half of that of QM. This leads to $\mathrm{P_{MQM}}$ larger by 75% than $\mathrm{P_{QM}}$.

The value of $\mathrm{P_{MPM}}$, -0.68, in Table.1 is very close to the Λ_b polarization, -0.68 estimated in [5]. The authors of [5] used the HQEFT to estimate the polarization retention of the Λ_b baryon produced from the b quark fragmentation processes. Another similar result of Λ_b polarization was calculated by employing the spectator diquark fragmentation model [12]. Since the identification between the fragmentation function and the distribution function could be made in the infinite momentum frame [11], which could be accessed under the heavy quark limit. There is, therefore, no surprise that our $\mathrm{P_{MPM}}$ matches the Λ_b polarization determined from the fragmentation processes.

5 Conclusion

In this paper we have constructed four formalisms based on the factorization formula for $\Lambda_b \to X_q l \bar{\nu}$. We used the four formalisms to calculate their corresponding Λ_b polarizations denoted as $\mathrm{P_{QM}}$, $\mathrm{P_{MQM}}$, $\mathrm{P_{PM}}$ and $\mathrm{P_{MPM}}$, respectively. The resulting $\mathrm{P_{MQM}}$ with value -0.94 is very close to the b quark polarization asymmetry with value -0.936. The $\mathrm{P_{MPM}}$ with value -0.68 is also close to the Λ_b polarization with value -0.68. Our result is consistent with those estimates from the Standard Model calculation of the b quark polarization asymmetry and the polarization retention of the Λ_b baryon produced from the b quark fragmentation processes.

We have also developed an power expansion scheme which generalizes the naive collinear expansion scheme to include heavy massive quark partons and to apply to decay processes.

Finally, we emphasize the correctness of the measurement by the ALEPH Collaboration [1]. Of course, there should need measuring the spectra to make the measured Λ_b polarization more consistent.

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A The collinear expansion

We now derive the collinear expansion skipped by the text. For the amplitude

$$M_a = \int \frac{d^4k}{(2\pi)^4} S^{\mu\nu}(k) T(k) \ , \tag{47}$$

we apply the expansion $k = \xi p + (k - \xi p)$ to $S^{\mu\nu}(k)$ as in eq. (12) and recast M_a in the form

$$M_a \approx \int \frac{d^4k}{(2\pi)^4} S^{\mu\nu}(k = \xi p) T(k) \ . \tag{48}$$

It is easy to factorize M_a as

$$\begin{aligned} M_a \ = \ & \int \frac{d^4k}{(2\pi)^4} \Big\{ [S^{\mu\nu}(k=\xi p) \not{P}_b][T(k) \frac{\not{n}}{4P_b \cdot n}] \\ & + [S^{\mu\nu}(k=\xi p) \not{P}_b][T(k) \frac{\not{p}}{4P_b \cdot p}] \\ & - [S^{\mu\nu}(k=\xi p) \not{S}_b \gamma_5][T(k) \frac{\not{n}\gamma_5}{4S_b \cdot n}] \\ & - [S^{\mu\nu}(k=\xi p) \not{S}_b \gamma_5][T(k) \frac{\not{p}\gamma_5}{4S_b \cdot p}] \Big\} \ , \end{aligned} \tag{49}$$

where the bracket denotes the trace over Dirac indices. We now prove that those terms involving the contraction of $T(k)$ with $\not{p}$ (or $\not{p}\gamma_5$) will lead to high order contributions. Recall that the b quark propagator could be expanded in the form

$$\frac{i}{\not{P}_b - M_b + i\epsilon} = i \frac{\hat{\not{P}}_b + M_b}{\not{P}_b - M_b + i\epsilon} + \frac{i\not{n}}{2P_b \cdot n} \ . \tag{50}$$

The contraction of $T(k)$ with $\not{p}$ is equivalent to the contraction of the b quark propagator with $\not{p}$. This leads to two effects: (1) The contraction of the first term in eq. (50) with $\not{p}$ leads to the form

$$i \frac{\hat{\not{P}}_b + M_b}{\not{P}_b - M_b + i\epsilon} \not{p} = \frac{i}{\not{P}_b - M_b + i\epsilon} [i(\not{k} - \xi \not{p}) - iM_b] \frac{i\not{n}}{2P_b \cdot n} \not{p} \ . \tag{51}$$

(2) For the contraction of the special propagator with $\not p$, we have the substitution

$$\frac{i\not n}{2P_b\cdot n}\not p \longrightarrow \frac{i}{\not P_b - M_b + i\epsilon}(i\gamma_\alpha)\frac{i\not n}{2P_b\cdot n}\not p \ . \tag{52}$$

By Fourier transforming $k-\xi p$ into $\omega^\alpha_{\alpha'} i\partial^{\alpha'}$, the effect (1) leads to the expression

$$S^{\mu\nu}_\alpha(k,k)T^\alpha_1(k) \tag{53}$$

with

$$T^\alpha_1(k) = \omega^\alpha_{\alpha'}\int d^4x e^{ikx}\langle\Lambda_b|\bar b_v(0)i\partial^{\alpha'} b_v(x)|\Lambda_b\rangle \ , \tag{54}$$

and the effect (2) induces the form

$$S^{\mu\nu}_\alpha(k,k_1)T^\alpha_2(k,k1) \tag{55}$$

with

$$T^\alpha_2(k,k1) = \omega^\alpha_{\alpha'}\int d^4x\int d^4y e^{i(k-k_1)x}e^{ik_1y}\langle\Lambda_b|\bar b_v(-gA^{\alpha'}_a T^a)(x)b_v(y)|\Lambda_b\rangle \ , \tag{56}$$

where $\omega^\alpha_{\alpha'} = g^\alpha_{\alpha'} - \bar n^\alpha n_{\alpha'}$ and $n\cdot A = 0$ gauge has been used. Summing the possible contributions, we express $S^{\mu\nu}_\alpha(k,k_1)$ and $S^{\mu\nu}_\alpha(k,k)$ as

$$\begin{aligned} S^{\mu\nu}_\alpha(k,k) = S^{\mu\nu}_\alpha(k,k_1) &= \mathrm{Disc}[(i\gamma_\alpha)\frac{i\not n}{2P_b\cdot n}\Gamma^\mu\frac{i\not P_q}{P_q^2+i\epsilon}\Gamma^\nu] \\ &+\mathrm{Disc}[\Gamma^\mu\frac{i\not P_q}{P_q^2+i\epsilon}\Gamma^\nu\frac{-i\not n}{2P_b\cdot n}(-i\gamma_\alpha)] \ . \end{aligned} \tag{57}$$

The mass $-iM_b$ involved term in eq. (51) does not contribute because of the $V-A$ structure of the Standard Model. The cases for the contraction of $T(k)$ with $\not p\gamma_5$ may be considered in the similar way. As a result we arrive at

$$\begin{aligned} M_a &= \int\frac{d^4k}{(2\pi)^4}\left\{[S^{\mu\nu}(k)\not P_b][T(k)\frac{\not n}{4P_b\cdot n}]\right. \\ &\qquad \left. -[S^{\mu\nu}(k)\not S_b\gamma_5][T(k)\frac{\not n\gamma_5}{4S_b\cdot n}]\right\} \\ &+\int\frac{d^4k}{(2\pi)^4}\int\frac{d^4k_1}{(2\pi)^4}\left\{[S^{\mu\nu}_\alpha(k,k_1)\not P_b][T^\alpha(k,k_1)\frac{\not p}{4P_b\cdot p}]\right. \\ &\qquad \left. -[S^{\mu\nu}_\alpha(k,k_1)\not S_b\gamma_5][T^\alpha(k,k_1)\frac{\not p\gamma_5}{4S_b\cdot p}]\right\} \ , \end{aligned} \tag{58}$$

where

$$T^\alpha(k,k1) \equiv T^\alpha_1(k,k)+T^\alpha_2(k,k_1) \tag{59}$$

$$= \omega^\alpha_{\alpha'}\int d^4x\int d^4y e^{i(k-k_1)x}e^{ik_1y}\langle\Lambda_b|\bar b_v(0)iD^{\alpha'}(x)(x)b_v(y)|\Lambda_b\rangle \tag{60}$$

with $iD^\alpha = i\partial^\alpha - gA^\alpha_a T^a$.

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Table.1 The values of the Λ_b polarization are determined from the quark model, the modified quark model, the parton model and the modified parton model.

P_{QM}	P_{MQM}	P_{PM}	P_{MPM}
-0.23	-0.94	-0.37	-0.68

Table.2 The values of the $< E_l^* >$, $< E_\nu^* >$, $< P_l^* > /\mathrm{P}$ and $< P_\nu^* > /\mathrm{P}$ are calculated from the quark model, the modified quark model, the parton model and the modified parton model. The units are in 1 GeV.

Model	$< E_l^* >$	$< E_\nu^* >$	$< P_l^* > /\mathrm{P}$	$< P_\nu^* > /\mathrm{P}$
QM	0.3290	0.2820	-0.0470	0.0940
MQM	0.2731	0.2379	-0.0153	0.0153
PM	0.1696	0.1570	-0.0242	0.0238
MPM	0.1502	0.1296	-0.0105	0.0124

Figure Captions

Fig.1(a):
Unpolarized charged lepton spectra of the $\Lambda_b \to X_q \ell \nu$ decay are shown as curve (1) for the quark model, curve (2) for the modified quark model, curve (3) for the parton model, and curve (4) for the modified parton model.

Fig.1(b):
Polarized charged lepton spectra of the $\Lambda_b \to X_q \ell \nu$ decay are shown as curve (1) for the quark model, curve (2) for the modified quark model, curve (3) for the parton model, and curve (4) for the modified parton model.

Fig.1(c):
Total charged lepton spectra of the $\Lambda_b \to X_q \ell \nu$ decay are shown as curve (1) for the quark model, curve (2) for the modified quark model, curve (3) for the parton model, and curve (4) for the modified parton model. The $\mathrm{P} \cos\theta_l = 1$ condition has been used.

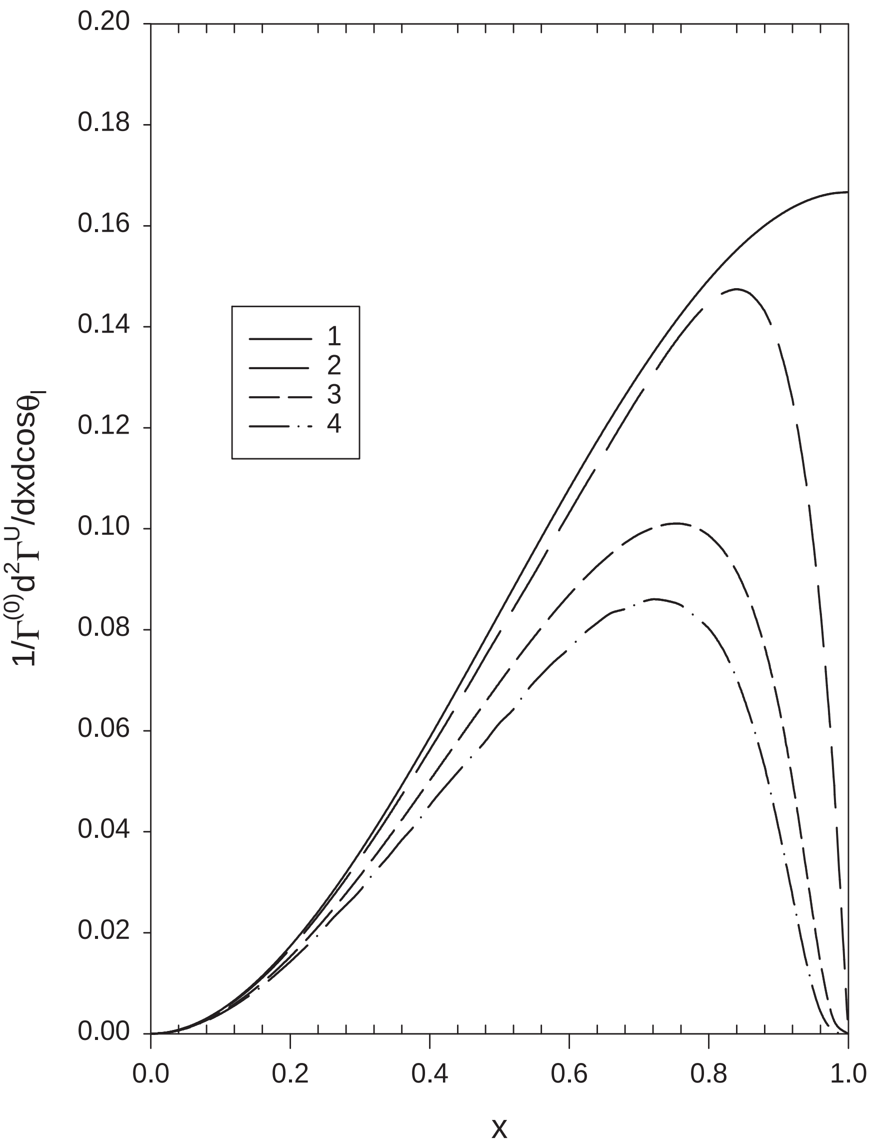

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Fig.1(a)

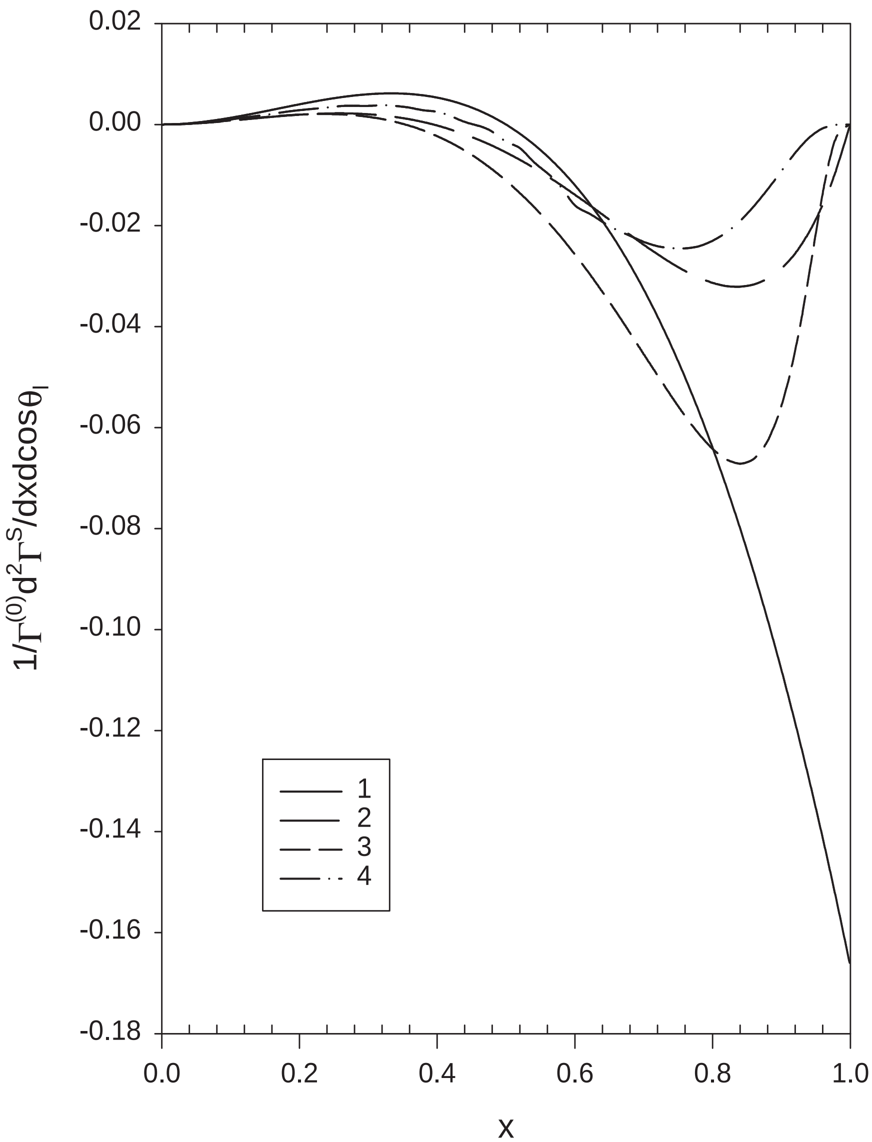

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Fig.1(b)

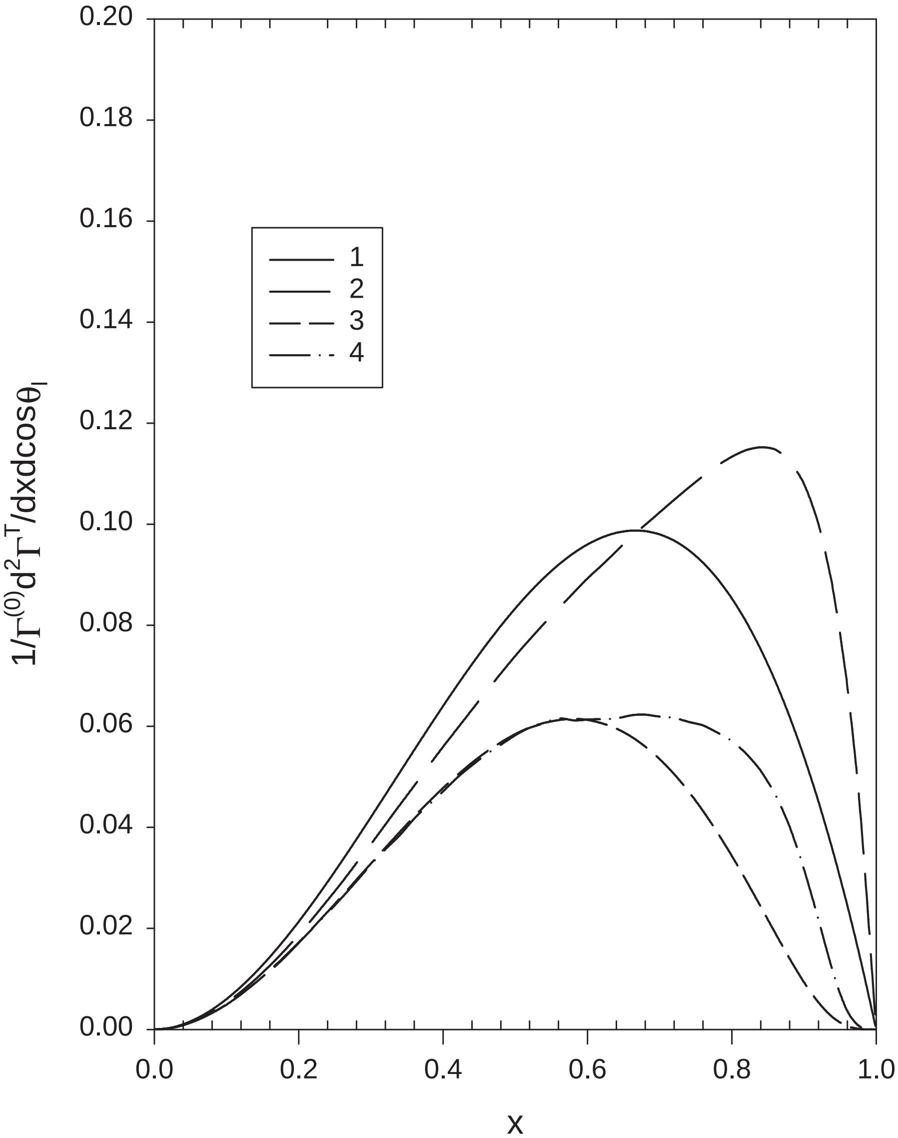
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Fig.1(c)