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Anomalous Couplings in W Pair Production[1]

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Abstract

I present a short overview over W pair production and studies of angular differential cross-sections with and without initial state radiation applying semi-analytical methods and using the Fortran program `GENTLE`. The influence of anomalous couplings to this process is also discussed.

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1 Introduction

Since the formulation of the standard model of electroweak interactions [1], more and more precision tests have confirmed its validity. However, for a central part of the theory, the non-abelian structure of the gauge couplings, we have only poor experimental information. With the results of LEP2 [2] and potential future linear colliders [3] the situation will change. In processes with W production triple and quartic gauge boson vertices appear and may be measured. Of special interest in e^+e^- annihilation is the process

$$e^+e^- \to W^+W^-, \tag{1}$$

with contributions from γW^+W^- and ZW^+W^- vertices.

The current limits for anomalous couplings from combined results of LEP2 and D0 are [4]:

$$\begin{aligned} \alpha_{W\phi} &= -0.03^{+0.06}_{-0.06}, & (2)\\ \alpha_{W} &= -0.03^{+0.08}_{-0.08}, & (3)\\ \alpha_{B\phi} &= -0.05^{+0.22}_{-0.20}, & (4) \end{aligned}$$

where the α's are given by the identities:

$$\begin{aligned} \alpha_{W\phi} &= c_W s_W \delta_Z, & (5)\\ \alpha_{W} &= y_\gamma = \frac{s_W}{c_W} y_Z, & (6)\\ \alpha_{B\phi} &= x_\gamma - c_W s_W \delta_Z = -\frac{c_W}{s_W}\left(x_Z + s_W^2 \delta_Z\right). & (7) \end{aligned}$$

The anomalous parameters δ_Z, x_γ, x_Z, y_γ, y_Z, and z_Z are defined by the Lagrangian in eq. (18).

In section 2 I give an overview on W pair production and the studies of the differential cross-sections performed with `GENTLE` version 2 [5]. I discuss the influence of potential anomalous three gauge boson couplings to W pair production in section 3.

2 W Pair Production

The first calculations in the standard model for the process in (1) were done in the narrow width approximation [6], e.g. neglecting the finite width of the W bosons. At this time it was known, that the decay width Γ_W of the W will give rise to large corrections if the W is much heavier than the proton [7]. As a consequence the finite

W width must be considered. This can be done by convoluting the cross-section with Breit-Wigner factors [8]:

$$\sigma(s) = \int_0^s \mathsf{d}s_1\, \rho(s_1) \int_0^{(\sqrt{s}-\sqrt{s_1})^2} \mathsf{d}s_2\, \rho(s_2)\sigma_0(s, s_1, s_2) \tag{8}$$

with

$$\rho(s_i) = \frac{1}{\pi}\frac{\sqrt{s_i}\Gamma_W(s_i)}{(s_i - m_W^2)^2 + m_W^2\Gamma_W^2(s_i)} \times B(f), \tag{9}$$

where s is the center-of-mass energy squared and $B(f)$ the branching fraction for the W decaying in the fermion doublet f. The invariant masses of the decay products of the W bosons are denoted by s_1 and s_2.

Since the produced W bosons decay almost immediately, the production of 4 fermions

$$e^+e^- \to W^+W^- \to 4f \tag{10}$$

is observed. Additional diagrams, so called background diagrams, contribute to the same final states and should be taken into account, too [9]. The number of contributing diagrams depends on the final state fermions. Here, I will concentrate on the `CC11` class, since it includes the semi-leptonic final states. These final states are important in the measurement of the gauge couplings, because they offer the most complete kinematical information. The `CC11` class is defined by having two different weak doublets and no electrons nor neutrinos as final state fermions. Depending on the number of produced neutrinos, there are 9, 10, or 11 Feynman diagrams.

For the `CC11` class the σ_0 of (8) can be written as a sum over all interferences and combinations of coupling constants. For the differential cross-section one gets:

$$\frac{\mathsf{d}\sigma_0}{\mathsf{d}\cos\theta} = \frac{\sqrt{\lambda(s, s_1, s_2)}}{\pi s^2}\sum_k \mathcal{C}_k \cdot \mathcal{G}_k(s; s_1, s_2, \cos\theta) \tag{11}$$

The coefficient functions $\mathcal{C}_k$ are rather trivial and contain the coupling constants of the particles and the s-channel propagators. The kinematical functions $\mathcal{G}_k$ are more complicated and describe the non-trivial dependencies of s, s_1, and s_2 and other variables like the scattering angle $\cos\theta$. To express the total cross-section a smaller set of $\mathcal{C}$ and $\mathcal{G}$ functions as in eq. (11) is needed, since the parity violating contributions disappear after the integration over the scattering angle.

As a simple example I give the expressions of the $\mathcal{C}$ and $\mathcal{G}$ functions for the differential cross-section for the square of the t-channel diagram [10]:

$$\mathcal{C}^t = \frac{\left(G_\mu m_W^2\right)^2}{s_1 s_2}\rho_W(s_1)\rho_W(s_2), \tag{12}$$

and

$$\mathcal{G}^t = \frac{1}{8}\left[2s(s_1+s_2) + \frac{\lambda}{4}\sin^2\theta + \frac{\lambda s_1 s_2 \sin^2\theta}{t_\nu^2}\right], \tag{13}$$

where λ is the Källen-function

$$\lambda \equiv \lambda(s,s_1,s_2) = s^2 + s_1^2 + s_2^2 - 2ss_1 - 2ss_2 - 2s_1 s_2, \tag{14}$$

and t_ν is the neutrino propagator

$$t_\nu = \frac{1}{2}\left(s - s_1 - s_2 - \sqrt{\lambda}\cos\theta\right). \tag{15}$$

The interferences between signal diagrams and background diagrams are more complicated and I give only the the kinematical function for the interference between the t-channel diagram and the u_1-diagram as an example:

$$\begin{aligned}
&\mathcal{G}^{tu_1}(s,s_1,s_2) = \\
&\quad \frac{-1}{\lambda}\left\{\frac{3}{4}\frac{\cos\theta}{\sqrt{\lambda}}\; s^2 s_1 s_2^2(5\sin^2\theta - 2)\left[\frac{1}{t_\nu}(s+s_1-s_2) + 2s\mathcal{L}(s_1;s_2,s)\right]\right. \\
&\quad + \lambda\left[\frac{\sin^2\theta}{8t_\nu}[2s_1s_2(s_2-s_1) - 6s^2 s_2(s_1+s_2)\mathcal{L}(s_1;s_2,s) - 3ss_2(s+s_2)]\right. \\
&\quad \left. + \frac{\sin^2\theta}{16}[(s-s_1)^2 - s_2^2] + \frac{ss_1}{2}\right] + \frac{ss_1s_2}{t_\nu}\left[-\frac{3}{4}ss_2\mathcal{L}(s_1;s_2,s)(5s\sin^4\theta\right. \\
&\quad + 4(s_1+s_2)) - \frac{1}{8}(3s_2^2 - 2ss_1 + 4s_1s_2 - 7s_1^2 + 30ss_2 + 9s^2)\sin^2\theta \\
&\quad \left. - \frac{1}{2}(3s_2^2 - 2s_1^2 - s_1s_2 + 2ss_1)\right] \\
&\quad + \frac{3s^2s_2}{4}\mathcal{L}(s_1;s_2,s)([4s_1s_2 + s_1^2 + s_2^2 - s(s_1+s_2)]\sin^2\theta \\
&\quad - 4[s_1s_2 + s_1^2 + s_2^2 - s(s_1+s_2)]) + \frac{ss_2\sin^2\theta}{8}(2s_1s_2 - 5s_1^2 + 3s_2^2 \\
&\quad \left. - 14ss_1 - 3s^2) + \frac{s}{2}(5s_1^2s_2 - 2s_1s_2^2 - 3s_2^3 + 5ss_1s_2 + 3s^2s_2)\right\},
\end{aligned} \tag{16}$$

with

$$\mathcal{L}(s;s_1,s_2) = \frac{1}{\sqrt{\lambda}}\ln\frac{s-s_1-s_2+\sqrt{\lambda}}{s-s_1-s_2-\sqrt{\lambda}}. \tag{17}$$

The remaining coefficient and kinematical functions are presented in [11].

To make precise predictions for the cross-section, radiative corrections have to be taken into account [12]. To demonstrate the size of initial state radiation (ISR), I show in fig. 1 the difference between the ISR corrected cross-section and the Born cross-section in the case of signal diagrams (`CC03`) and for the complete semi-leptonic process (`CC10`). While the difference peaks in the region $\cos\theta > 0.8$ it is almost constant in the other parts. Especially in the region of $\cos\theta \to -1$ this leads to important effects, because the differential cross-section drops here significantly and the relative corrections amount to 30% for $\cos\theta = -1$ [11].

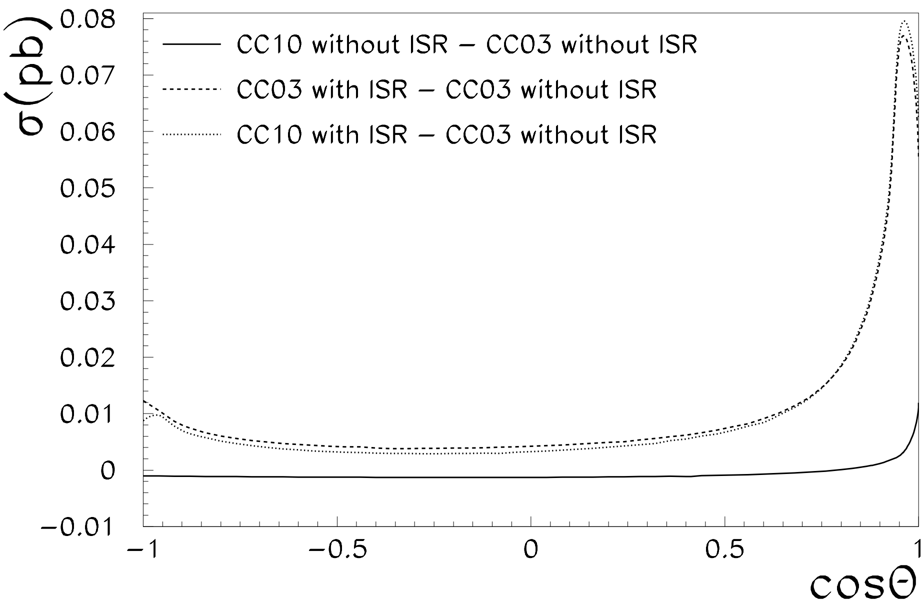

Figure 1: *Differences between Born cross-section and cross-section with initial state radiation.*

The numerical results of fig. 1 were produced with `GENTLE` and radiative corrections were treated as described in [5]. A short overview over `GENTLE` is given in appendix A.

3 Anomalous Couplings

The most general form for the γWW and ZWW vertices compatible with Lorentz invariance was first considered in [13], where 9 parameters were introduced for each vertex. In [14] it was shown that these parameters were not independent and the number could be reduced to 7.

The number of parameters can be further reduced by using a restricted set of anomalous couplings, which is invariant under $\mathcal{CP}$ transformations. The anomalous couplings are defined by the Lagrangian:

$$\begin{aligned}
\mathcal{L} &= -ie\left[A_\mu\left(W^{-\mu\nu}W^+_\nu - W^{+\mu\nu}W^-_\nu\right) + F_{\mu\nu}W^{+\mu}W^{-\nu}\right] \\
&\quad - ie\left(\cot\Theta_W + \delta_Z\right)\left[Z_\mu\left(W^{-\mu\nu}W^+_\nu - W^{+\mu\nu}W^-_\nu\right) + Z_{\mu\nu}W^{+\mu}W^{-\nu}\right] \\
&\quad - iex_\gamma F_{\mu\nu}W^{+\mu}W^{-\nu} - iex_Z Z_{\mu\nu}W^{+\mu}W^{-\nu} \\
&\quad + ie\frac{y_\gamma}{M_W^2}F^{\nu\lambda}W^-_{\lambda\mu}W^{+\mu}_\nu + ie\frac{y_Z}{M_W^2}Z^{\nu\lambda}W^-_{\lambda\mu}W^{+\mu}_\nu \\
&\quad + \frac{ez_Z}{M_W^2}\partial_\alpha\tilde{Z}_{\rho\sigma}\left(\partial^\rho W^{-\sigma}W^{+\alpha} - \partial^\rho W^{-\alpha}W^{+\sigma}\right. \\
&\quad \left.+\partial^\rho W^{+\sigma}W^{-\alpha} - \partial^\rho W^{+\alpha}W^{-\sigma}\right). \qquad (18)
\end{aligned}$$

In the standard model the anomalous parameters δ_Z, x_γ, x_Z, y_γ, y_Z, and z_Z are zero. The parameter z_Z violates both $\mathcal{C}$ and $\mathcal{P}$ symmetry, but is invariant under the product $\mathcal{CP}$. The parameters x_γ and y_γ contribute to the magnetic dipole moment μ_W and the electromagnetic quadrupole moment q_W of the W boson [15]:

$$\mu_W = \frac{e}{2m_W^2}\left(2 + x_\gamma + y_\gamma\right), \qquad (19)$$

$$q_W = -\frac{e}{m_W^2}\left(1 + x_\gamma - y_\gamma\right). \qquad (20)$$

With these additional parameters the cross-section for W pair production can be written as:

$$\begin{aligned}
\sigma^{\mathsf{ano}} &= \sigma^{\mathsf{SM}} \\
&\quad + x_\gamma \cdot \sigma^{x_\gamma} + x_Z \cdot \sigma^{x_Z} + \ldots \qquad (21) \\
&\quad + x_\gamma^2 \cdot \sigma^{x_\gamma x_\gamma} + x_\gamma x_Z \cdot \sigma^{x_\gamma x_Z} + x_Z^2 \cdot \sigma^{x_Z x_Z} + \ldots,
\end{aligned}$$

where the anomalous parameters appear at most bilinearly.

If one considers all anomalous couplings of eq. (18) 28 coefficients are needed to calculate σ^{ano}. Eq. (21) can also be applied to multi-differential cross-sections. In the search for anomalous couplings multi-differential cross-sections are used, since they contain more kinematical information [16].

`GENTLE` can be used to calculate the coefficients in eq. (21). This can be done for the differential cross-section and in the `CC03` process also for the bin-wise integrated differential cross-section. By setting the number of bins to 2, one gets predictions for forward ($\cos\theta > 0$) and backward ($\cos\theta < 0$) scattering. A study of the sensitivity of the forward-backward asymmetry to pairs of anomalous couplings was performed for the pairs (x_γ, δ_z) and (x_γ, z_Z) in [11] and for (x_γ, x_Z) and (x_Z, z_Z) in [17]. The forward-backward asymmetry proved to be useful for studies which include the parity violating parameter z_Z.

4 Conclusions

I gave a short report over the present state of GENTLE and the studies of differential cross-sections and anomalous couplings with it. It was shown that radiative corrections (initial state radiation) give sizeable effects to the differential cross-section at a center-of-mass energy of 500 GeV. This is especially important in the region of backward scattering, where the cross-section is small and the corrections are about 30%.

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I would like to thank the organizers of the conference for their hospitality and the very pleasant atmosphere they provided. Further, I am especially grateful to T. Riemann for the collaboration on this topic.

A GENTLE

GENTLE version 2 is a Fortran package to calculate cross-sections for 4 fermion production processes for charged currents (CC) and neutral currents (NC) with the semi-analytical method. Table 1 gives an overview over the different branches of GENTLE and the publications they are based on.

CC	QED ISR total cross-section	[18]
	Background total cross-section	[10]
	Anomalous couplings	[11]
	Differential cross-section	[11]
NC	QED ISR total cross-section	[19]
	Background total cross-section	[20]

Table 1: *Overview over the different branches of* GENTLE.

While in GENTLE version 2 the calculation of the differential cross-section and the effects of anomalous couplings were only available for the signal diagrams, in the newer version 2.01 these features were extended to the complete CC11 class.

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