

**ON THE NUMERICAL CLOSENESS OF THE EFFECTIVE
PHENOMENOLOGICAL ELECTROWEAK MIXING ANGLE θ
AND THE $\overline{\text{MS}}$ PARAMETER $\hat{\theta}$**

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It happens that s^2 and \hat{s}^2 are equal with 0.1% accuracy, though they are split by radiative corrections and a natural estimate for their difference is 1%. This degeneracy occurs only for m_t value close to 170 GeV, so no deep physical reason can be attributed to it. However, another puzzle of the Standard Model, the degeneracy of s_{eff}^2 and s^2 , is not independent of the previous one since a good physical reason exists for s_{eff}^2 and \hat{s}^2 degeneracy. We present explicit formulas which relate these three angles.

Keywords: electroweak angle, radiative corrections.

1. Introduction

Nowadays, when almost all LEP I data are analyzed and published, one can finally tell that the Standard Model is absolutely adequate to the experimental data. The quality of fit of LEP I, SLC and other precision data is characterized by the value of $\chi^2/\text{d.o.f} = 14.4/14$ ¹, which cannot be better. What can be extracted from precision measurements for the future in addition to the bounds on the Higgs boson mass m_H (which unfortunately are rather weak^{1,2})? As everybody knows, it is the value of $\hat{s}_Z^2 \equiv \sin^2 \hat{\theta}_Z$ which is used to study gauge couplings unification in the framework of GUT models. The corresponding angle is calculated in the modified minimal subtraction scheme ($\overline{\text{MS}}$), with $\mu = m_Z$. From³ one can see that this quantity appears to be numerically very close to the phenomenological parameter $s^2 \equiv \sin^2 \theta$, which is defined by the best measured quantities G_F , m_Z and $\bar{\alpha} \equiv \alpha(m_Z)$:

$$c^2 s^2 \equiv \cos^2 \theta \sin^2 \theta = \frac{\pi \bar{\alpha}}{\sqrt{2} G_F m_Z^2}, \quad (1)$$

$$s^2 = 0.2311(2), \quad (2)$$

and which was used to describe electroweak precision data in a natural way (for

review and references see ⁴). This should be compared with ^{3,*}

$$\hat{s}_Z^2 = \bar{c}(m_t, m_H)s^2 = 1.002(1) \cdot s^2 = 0.2316(2). \quad (3)$$

The aim of the present paper is to present a formula which provides the relation between s^2 and $\hat{s}^2 \equiv \sin^2 \hat{\theta}$. Analyzing it we will see that this numerical coincidence occurs only for the top quark mass m_t close to 170 GeV, so it is really a coincidence without any physical explanation. At this point it is useful to remind that there is one more coincidence in the Standard Model: $s_{\text{eff}}^2 \equiv \sin^2 \theta_{\text{eff}}$, which describes asymmetries in Z boson decays, happens to be very close to s^2 . And also this occurs only for m_t close to 170 GeV. However, writing the expression for \hat{s}^2 through s_{eff}^2 we will see that these two angles are naturally close, and their coincidence does not depend on the top mass and has a straightforward physical explanation. In this way we will see that, instead of two accidental coincidences between three mixing angles, we have only one.

2. \hat{s}^2 versus s^2

To get necessary formulas we should start from the expression for the $\overline{\text{MS}}$ quantity \hat{s}^2 . By definition,

$$\hat{c} = \frac{\hat{g}_0}{\hat{g}_0}, \quad (4)$$

$$\hat{c}^2 + \hat{s}^2 = 1, \quad (5)$$

where \hat{g}_0 and \hat{g}_0 are W and Z boson bare coupling constants defined in $\overline{\text{MS}}$ renormalization scheme with $\mu = m_Z$. The simplest way to get the expression for \hat{s}^2 in terms of s^2 and the combination of polarization operators is to follow the procedure discussed in ⁶. That is, to write the expressions for G_F , m_Z and $\bar{\alpha}$ through bare parameters plus radiative corrections and to solve them for bare charges through $\cos \theta$, $\sin \theta$ and radiative corrections. At a certain stage, angle θ_0 was introduced in ⁶ ($c_0 \equiv \cos \theta_0 \equiv g_0/\bar{g}_0 = m_{W0}/m_{Z0}$), and the following expression for its cosine was obtained:

$$c_0 = c - \frac{cs^2}{2(c^2 - s^2)} \left(2\frac{s}{c}\Pi_{\gamma Z}(0) + \Pi_{\gamma}(m_Z^2) - \Pi_Z(m_Z^2) + \Pi_W(0) + D \right), \quad (6)$$

where D comes from the box and vertex radiative corrections to muon decay, and Π_i are the polarization operators. This angle θ_0 will coincide with $\hat{\theta}$ if D and Π_i are calculated in $\overline{\text{MS}}$ framework with $\mu = m_Z$. From (6) we easily get:

$$\hat{s}^2 = s^2 + \frac{c^2 s^2}{c^2 - s^2} \left(2\frac{s}{c}\hat{\Pi}_{\gamma Z}(0) + \hat{\Pi}_{\gamma}(m_Z^2) - \hat{\Pi}_Z(m_Z^2) + \hat{\Pi}_W(0) + \hat{D} \right), \quad (7)$$

where the quantities with a hat are calculated in $\overline{\text{MS}}$. Since the last equation is central for the present paper, let us give a different derivation of it. We start from

*According to the 1998 edition of RPP, $\bar{c} = 1.0003(7)$, and $\hat{s}_Z^2 = 0.23124(24)$.

the formulas for the vector boson masses which take place in $\overline{\text{MS}}$ renormalization scheme:

$$m_W^2 = \frac{\hat{g}_0^2 \hat{\eta}_0^2}{4} - \hat{\Sigma}_W(m_W^2), \quad (8)$$

$$m_Z^2 = \frac{\hat{g}_0^2 \hat{\eta}_0^2}{4} - \hat{\Sigma}_Z(m_Z^2), \quad (9)$$

where $\Sigma_i(q^2) \equiv \Pi_i(q^2)m_i^2$. From eqs. (4), (5), (8) and (9) we get (see also ⁵):

$$\begin{aligned} \hat{s}^2 &= 1 - \frac{\hat{g}_0^2}{\hat{g}_0^2} = 1 - \frac{m_W^2 + \hat{\Sigma}_W(m_W^2)}{m_Z^2 + \hat{\Sigma}_Z(m_Z^2)} \\ &= 1 - \frac{m_W^2}{m_Z^2} + c^2 \left[\hat{\Pi}_Z(m_Z^2) - \hat{\Pi}_W(m_W^2) \right], \end{aligned} \quad (10)$$

where in the last expression we have substituted $(m_W/m_Z)^2$ with c^2 in the factor which multiplies Π_i , which is correct at one-loop level. Now for the ratio m_W/m_Z in the last expression in (10) we should use a formula which takes radiative corrections into account. We follow a general approach to the electroweak radiative corrections presented partly in ⁶, so we use eq. (38) from that paper:

$$\begin{aligned} \frac{m_W^2}{m_Z^2} &= c^2 + \frac{c^2 s^2}{c^2 - s^2} \left(\frac{c^2}{s^2} \left[\Pi_Z(m_Z^2) - \Pi_W(m_W^2) \right] \right. \\ &\quad \left. + \Pi_W(m_W^2) - \Pi_W(0) - \Pi_\gamma(m_Z^2) - 2 \frac{s}{c} \Pi_{\gamma Z}(0) - D \right). \end{aligned} \quad (11)$$

Since both m_W/m_Z and c are finite, the expression for the radiative corrections is finite as well and we can use $\overline{\text{MS}}$ quantities in it:

$$\begin{aligned} \frac{m_W^2}{m_Z^2} &= c^2 + \frac{c^2 s^2}{c^2 - s^2} \left(\frac{c^2}{s^2} \left[\hat{\Pi}_Z(m_Z^2) - \hat{\Pi}_W(m_W^2) \right] \right. \\ &\quad \left. + \hat{\Pi}_W(m_W^2) - \hat{\Pi}_W(0) - \hat{\Pi}_\gamma(m_Z^2) - 2 \frac{s}{c} \hat{\Pi}_{\gamma Z}(0) - \hat{D} \right). \end{aligned} \quad (12)$$

Substituting the last equation in (10), we obtain:

$$\hat{s}^2 = s^2 + \frac{c^2 s^2}{c^2 - s^2} \left(2 \frac{s}{c} \hat{\Pi}_{\gamma Z}(0) + \hat{\Pi}_\gamma(m_Z^2) - \hat{\Pi}_Z(m_Z^2) + \hat{\Pi}_W(0) + \hat{D} \right), \quad (13)$$

which coincides with eq. (7). In figure 1 we show the $\hat{s}^2 - s^2$ dependence on m_H and m_t .[†] It is clear that \hat{s}^2 is close to s^2 only for m_t around 170 GeV, so one cannot find any physical reason for the closeness of these two angles. The fact that $\hat{s}^2 - s^2$ rapidly varies with m_t can be figured out from the large m_t approximation:

$$\hat{s}^2 - s^2 \Big|_{m_t \gg m_Z} \approx - \frac{3\bar{\alpha}}{16\pi(c^2 - s^2)} \left(\frac{m_t}{m_Z} \right)^2. \quad (14)$$

[†]In order to take into account top and W boson contributions to α , we used $s^2 + 0.00015$ instead of s^2 in (13) (see ⁷).

At this point we state that the numerical closeness of \hat{s}^2 and s^2 is a mere coincidence without any deep physical reason. However, the reason exists for the closeness of $\hat{\theta}$ and another electroweak mixing angle, θ_{eff} . On the other hand, θ_{eff} appeared to be numerically close to θ and this solves the puzzle (according to the last data fit, $\sin^2 \theta_{\text{eff}}^{\text{lept}} = 0.2315(2)$).

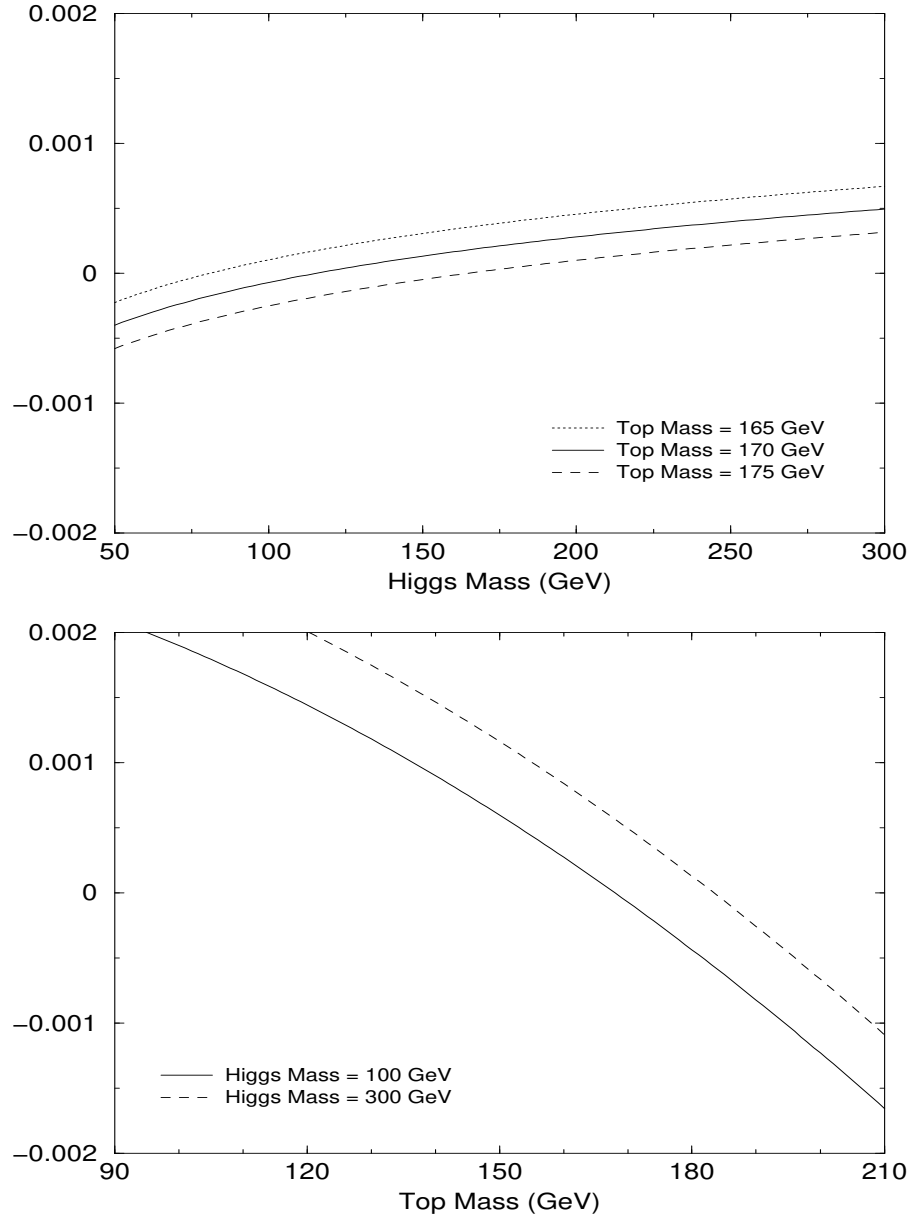


Figure 1: $\hat{s}^2 - s^2$ as a function of the Higgs mass m_H and of the top mass m_t .

3. \hat{s}^2 versus s_{eff}^2

The quantity s_{eff} describes the asymmetries in Z boson decay; $s_{\text{eff}}^{\text{lept}}$, $s_{\text{eff}}^{\text{up}}$ and $s_{\text{eff}}^{\text{down}}$ refers to decays into pairs of leptons, up-quarks and down-quarks, respectively. Let us discuss $s_{\text{eff}}^{\text{lept}}$. Using eq. (73) from ⁶, we easily obtain:

$$s_{\text{eff}}^{\text{lept}^2} = s^2 + cs \left[F_V^{Ze} - (1 - 4s^2) F_A^{Ze} \right] + cs \Pi_{\gamma Z}(m_Z^2) + \frac{c^2 s^2}{c^2 - s^2} \left[\Pi_{\gamma}(m_Z^2) - \Pi_Z(m_Z^2) + \Pi_W(0) + 2 \frac{s}{c} \Pi_{\gamma Z}(0) + D \right], \quad (15)$$

where F_A^{Ze} and F_V^{Ze} describe the radiative corrections to Zee axial and vector vertices.

Since both s_{eff}^2 and s^2 are finite, equality (15) will be correct if all radiative corrections are calculated in $\overline{\text{MS}}$ scheme as well. Comparing equations (7) and (15) we get (see also ⁸):

$$\hat{s}^2 = s_{\text{eff}}^{\text{lept}^2} - cs \left[\hat{F}_V^{Ze} - (1 - 4s^2) \hat{F}_A^{Ze} \right] - cs \hat{\Pi}_{\gamma Z}(m_Z^2). \quad (16)$$

The form of the last equation can be foreseen without any calculation. The point is that both $\hat{\theta}$ and θ_{eff} are defined by the ratio of bare gauge coupling constants; the difference between them arises since θ_{eff} describes $Z \rightarrow e^+e^-$ decays and in this case the additional vertex radiative corrections as well as $Z \rightarrow \gamma \rightarrow e^+e^-$ transition contribute to θ_{eff} . In (16) these additional terms are subtracted from $s_{\text{eff}}^{\text{lept}^2}$ in order to get \hat{s}^2 . The vertex term in (16) is a mere number, while $\hat{\Pi}_{\gamma Z}$ depends on m_t only logarithmically due to the non-decoupling property of $\overline{\text{MS}}$ scheme (since a diagonal vector current is conserved, there is no m_t^2 term in $\hat{\Pi}_{\gamma Z}$, that is why $\hat{\Pi}_{\gamma Z}$ is numerically small). There is also no m_H dependence in the difference $\hat{s}^2 - s_{\text{eff}}^2$. From the $\hat{\Pi}_{\gamma Z}(m_Z^2)$ term we get the following expression for the logarithmically enhanced contribution for $m_t \gg m_Z$:

$$\hat{s}^2 - s_{\text{eff}}^{\text{lept}^2} \Big|_{m_t \gg m_Z} \approx \frac{\bar{\alpha}}{\pi} \left(\frac{1}{6} - \frac{4}{9} s^2 \right) \ln \left(\frac{m_t}{m_Z} \right)^2. \quad (17)$$

Having all the necessary formulas in our disposal, we are ready to make numerical estimates. Using expressions (93), (94) from ⁴ and formulas from Appendix G of ⁶, we get:

$$\hat{F}_V^{Ze} = 0.00197 + \frac{\bar{\alpha}}{8\pi} \frac{c}{s^3} \ln \left(\frac{m_W}{m_Z} \right)^2 = 0.00133, \quad (18)$$

$$\hat{F}_A^{Ze} = 0.00186 + \frac{\bar{\alpha}}{8\pi} \frac{c}{s^3} \ln \left(\frac{m_W}{m_Z} \right)^2 = 0.00122, \quad (19)$$

where the logarithmic terms arise from the divergent parts of vertex functions after imposing $\overline{\text{MS}}$ renormalization conditions with $\mu = m_Z$. Note that in numerical calculations we substituted c^2 for $(m_W/m_Z)^2$.

To calculate $\hat{\Pi}_{\gamma Z}(m_Z^2)$ we use formulas from Appendices of paper ⁶, which take into account W^+W^- , light fermions and (t, b) doublet contributions. For $m_t = 170$ GeV we obtain:

$$\hat{\Pi}_{\gamma Z}(m_Z^2) = -0.00119. \quad (20)$$

Substituting (18), (19) and (20) into (16), we finally obtain:

$$\hat{s}^2 = s_{\text{eff}}^{\text{lept}^2} - 0.00052 + 0.00050 = s_{\text{eff}}^{\text{lept}^2} - 0.00002, \quad (21)$$

where an accidental cancellation between vertex and $\hat{\Pi}_{\gamma Z}$ contributions occurs (see also ⁸). This cancellation is peculiar to $s_{\text{eff}}^{\text{lept}^2}$ and does not occur for $s_{\text{eff}}^{2\text{up}}$ or $s_{\text{eff}}^{2\text{down}}$. As a consequence, for $s_{\text{eff}}^{\text{lept}^2}$ and \hat{s}^2 difference 2 loop contributions can be comparable or even larger than 1 loop.

Now we will calculate the leading two loop corrections. They are of the order of $\alpha\alpha_s$ and come from the insertion of a gluon into quark loops which contribute to $\hat{\Pi}_{\gamma Z}(m_Z^2)$. There are two types of one-loop diagrams: with light quarks (u, d, c, s, b) and with heavy top (t). We extract necessary 2-loop formulas from the Kniehl paper ¹⁰. However, in that article all calculations were made with ultraviolet cutoff Λ . To convert to $\overline{\text{MS}}$ we compare these formulas with calculations of Djouadi and Gambino ¹⁰. In this way we find the following replacement rule:

$$\ln\left(\frac{\Lambda^2}{m_Z^2}\right) \rightarrow \Delta_Z + \frac{55}{12} - 4\zeta(3) = \frac{55}{12} - 4\zeta(3) \approx -0.225, \quad (22)$$

where the last equality holds for $\mu = m_Z$.

For the case of light quarks contribution (u, d, c, s, b), we get:

$$\delta_{\text{light}}^{\alpha\alpha_s} \hat{\Pi}_{\gamma Z}(m_Z^2) = \frac{\hat{\alpha}_s(m_Z)}{\pi} \frac{\bar{\alpha}}{\pi c s} \left(\frac{7}{12} - \frac{11}{9} s^2 \right) \left[\frac{55}{12} - 4\zeta(3) \right] \approx -0.00002, \quad (23)$$

where we use $\hat{\alpha}_s(m_Z) = 0.12$ for numerical estimate. For the contribution of the top quark we obtain:

$$\begin{aligned} \delta_t^{\alpha\alpha_s} \hat{\Pi}_{\gamma Z}(m_Z^2) &= \frac{\hat{\alpha}_s(m_t)}{\pi} \frac{\bar{\alpha}}{\pi c s} \left(\frac{1}{6} - \frac{4}{9} s^2 \right) \left[\frac{55}{12} - 4\zeta(3) - \ln t + 4tV_1 \left(\frac{1}{4t} \right) \right] \\ &\approx 0.00004, \end{aligned} \quad (24)$$

where $t \equiv (m_t/m_Z)^2$ and ^{11,10}:

$$\hat{\alpha}_s(m_t) = \frac{\hat{\alpha}_s(m_Z)}{1 + \frac{23}{12\pi} \hat{\alpha}_s(m_Z) \ln t} \approx 0.11, \quad (25)$$

$$V_1(x) = \left[4\zeta(3) - \frac{5}{6} \right] x + \frac{328}{81} x^2 + \frac{1796}{675} x^3 + \dots, \quad (26)$$

$$\zeta(3) = 1.2020569\dots \quad (27)$$

Substituting (23) and (24) into (16), we find:

$$\hat{s}^2 = s_{\text{eff}}^{\text{lept}^2} - 0.00002 - 0.00001, \quad (28)$$

where the first number corresponds to the corrections of order α shown in (21), while the second to corrections of the order $\alpha\alpha_s$.

Since the leading $\sim \alpha$ correction cancel almost completely in (21), one start to worry about significance of two loop α^2 corrections. Enhanced $\alpha^2 t$ correction in (16) was calculated in ¹², where it is stated that it is numerically negligible; α^2 corrections are not calculated yet. However, according to ¹² there exist enhanced two-loop $\alpha^2\pi^2$ correction, which come from the interference of the imaginary parts of $\Pi_{\gamma Z}$ and Π_{γ} . Numerically it gives ¹²:

$$\delta_{\text{int}}^{\alpha^2} \left(\hat{s}^2 - s_{\text{eff}}^{\text{lept}^2} \right) = -0.00004. \quad (29)$$

Adding (29) to (28) we finally get:

$$\hat{s}^2 = s_{\text{eff}}^{\text{lept}^2} - 0.00007. \quad (30)$$

It is instructive to compare the last formula with the corresponding numbers in Tables 1 and 2 from ¹³ as well as the last formula in ¹².

In figure 2 the dependence of $\hat{s}^2 - s_{\text{eff}}^{\text{lept}^2}$ on m_t is presented. One can easily see that, unlike the case of $\hat{s}^2 - s^2$ difference, here the dependence on m_t is really small for large m_t values interval.

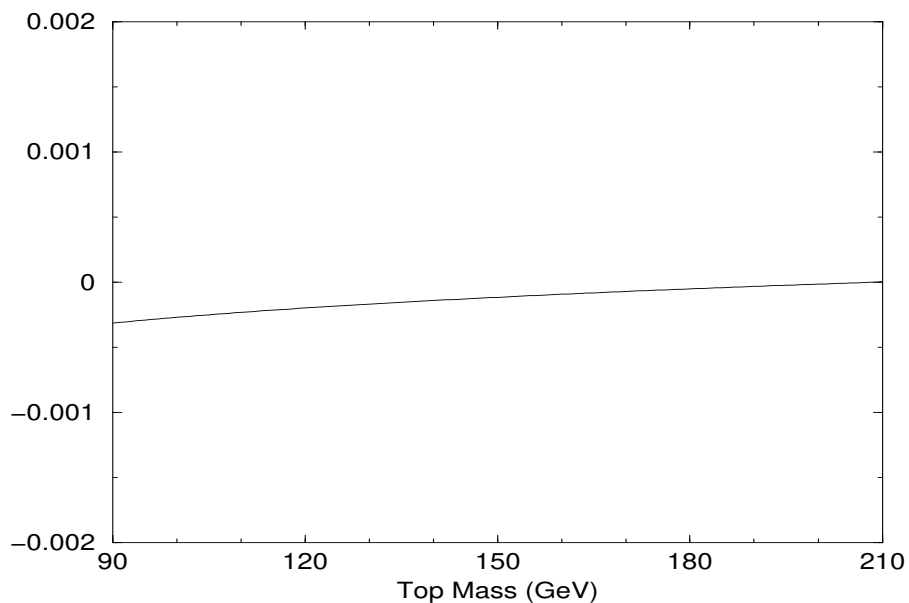


Figure 2: $\hat{s}^2 - s_{\text{eff}}^{\text{lept}^2}$ as a function of the top mass m_t . This quantity does not depend on the Higgs mass.

4. Conclusions

Coming back to the title of the present paper, we should study eq. (13) in more details. From this equation (or looking at fig. 1) one can see that, for $m_t = 170$ GeV and $m_H = 111$ GeV, \hat{s}^2 equals s^2 with high accuracy:

$$\hat{s}^2 - s^2 \Big|_{\substack{m_t=170 \text{ GeV} \\ m_H=111 \text{ GeV}}} = -0.00002. \quad (31)$$

Taken into account “theoretical” prediction:

$$s_{\text{eff}}^{\text{lept}^2} = 0.2315, \quad (32)$$

which is valid for $m_t = 170$ GeV, $m_H = 111$ GeV, and comparing (2), (31) with (30), we observe evident inconsistency. To overcome it small higher loop corrections in (13) should be accounted for, in analogy with what was done in eqs. (23) and (24). One can act straightforwardly, taking into account corrections to polarization operators entering (13). Another possible way is to take expression for \hat{s}^2 through $s_{\text{eff}}^{\text{lept}^2}$ (eq. (16)) and to use in it expression for $s_{\text{eff}}^{\text{lept}^2}$ through s^2 and higher order radiative corrections:

$$\begin{aligned} \delta(\hat{s}^2 - s^2) = & -cs\delta^{\alpha\alpha_s}\hat{\Pi}_{\gamma Z}(m_Z^2) \\ & - \frac{3\bar{\alpha}}{16\pi(c^2 - s^2)}(\delta_2 V_R + \delta_3 V_R + \delta_4 V_R + \delta'_4 V_R), \end{aligned} \quad (33)$$

where we take into account that in expression for $s_{\text{eff}}^{\text{lept}^2}$ through s^2 radiative corrections are finite, so $\overline{\text{MS}}$ subtraction should not be imposed; expressions for $\delta_i V_R$ can be found in ⁴ and ⁹:

$$\delta_2 V_R(t, h) = \frac{4}{3} \frac{\hat{\alpha}_s(m_t)}{\pi} \left[tA_1 \left(\frac{1}{4t} \right) - \frac{5}{3} tV_1 \left(\frac{1}{4t} \right) - 4tF_1(0) + \frac{1}{6} \ln t \right], \quad (34)$$

$$\delta_3 V_R(t, h) = -14.594 \frac{\hat{\alpha}_s^2(m_t)}{\pi^2} t, \quad (35)$$

$$\delta_4 V_R(t, h) = -\frac{\bar{\alpha}}{16\pi s^2 c^2} A \left(\frac{m_H}{m_t} \right) t^2, \quad (36)$$

$$\delta'_4 V_R(t, h) = -\frac{3\bar{\alpha}}{16\pi(c^2 - s^2)^2} t^2 \quad (37)$$

where the expression for V_1 is given in (26) and expressions for A_1 , F_1 and A can be found in ⁴ and ⁹.

Substituting eqs. (23), (24) and (34)-(37) into (33), taking into account eq. (29) and making numerical estimate, we get:

$$\delta(\hat{s}^2 - s^2) = 0.00042, \quad (38)$$

$$\hat{s}^2 = s^2 - 0.00002 + 0.00042 = 0.2315 \quad (39)$$

(-0.00002 comes from (31)), which is quite close to (30) (taking into account the numerical value of $s_{\text{eff}}^{\text{lept}^2}$ from (32)).

Let us mention that in generalizations of Standard Model a lot of new heavy particles occur, and all of them contribute to \hat{s}^2 due to the non-decoupling property of $\overline{\text{MS}}$ renormalization. To avoid this non-universality of $\overline{\text{MS}}$ quantities, it was suggested to subtract contributions of the particles with masses larger than μ from Π_γ and $\Pi_{\gamma Z}$, and in particular, to subtract the logarithmic term shown in (17) from the \hat{s}^2 value^{14,3}. According to the definition accepted in³, the quantity \hat{s}^2 which has been discussed up to now is called \hat{s}_{ND}^2 , while a new “decoupled” $\overline{\text{MS}}$ parameter \hat{s}_Z^2 is introduced:

$$\hat{s}_Z^2 = \hat{s}_{\text{ND}}^2 - \frac{\bar{\alpha}}{\pi} \left(\frac{1}{6} - \frac{4}{9}s^2 \right) \ln \left(\frac{m_t}{m_Z} \right)^2 = \hat{s}_{\text{ND}}^2 - 0.0002. \quad (40)$$

From (28) and (40), taken into account the latest precision data fit value $s_{\text{eff}}^{\text{lept}^2} = 0.2315 \pm 0.0002$, we get:

$$\hat{s}_Z^2 = 0.2312 \pm 0.0002, \quad (41)$$

where \hat{s}_Z^2 is uniquely defined both in the Standard Model and in its extensions (unlike \hat{s}^2).

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