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Do heavy sfermions decouple from low energy Standard Model?

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We explore analytically how does the Standard Model emerge as the quantum low energy effective theory of the Minimal Supersymmetric Standard Model (MSSM) in the decoupling limit where the sparticles are much heavier than the electroweak scale. In this work we integrate the sfermions to one-loop and compute their contributions to the effective action for standard electroweak gauge bosons. A proof of decoupling of sfermions is performed by analyzing the resulting effective action in the asymptotic limit $m_{\tilde{f}} \gg m_z$. A discussion on how the decoupling takes place in terms of both the sparticle physical masses and the non-physical MSSM parameters is included.

1 Introduction

The Standard Model (SM) is a pillar of success as an effective theory. Experimental measurements agree with SM radiative corrections to a precision of greater than 0.1%. However, SM contains nagging theoretical problems which cannot be solved without the introduction of some new physics. In this sense, supersymmetry (SUSY) is the favorite of many theorists. The simplest model of this type is called the Minimal Supersymmetric Standard Model (MSSM) [1], which is the one we have chosen to work with in this paper.

One interesting aspect that arises in these softly broken SUSY theories, and in particular in the MSSM, is the question of decoupling of heavy sparticles from the low energy SM and how does it really occurs if it occurs at all. We will concentrate our attention in this subject. In general, perturbative considerations [2] lead us to believe that heavy particles can be decoupled from low energy degrees of freedom. We expect that the lagrangian describing the low energy degrees of freedom is affected by heavy particles only through renormalization effects and higher dimension operators which become negligible as the particles are made infinitely massive.

At present, there are indications that when the spectrum of supersymmetric particles at the MSSM is considered much heavier than the low energy electroweak scale they decouple from the low energy physics, even at the quantum level, and the resulting low energy effective theory is the SM itself. However, a rigorous proof of decoupling is still lacking. On one hand there are numerical studies of observables that measure electroweak radiative corrections, like Δr and $\Delta\rho$ [3], or the S, T and U parameters [4] as well as in the Z boson, top quark and Higgs decays [5], which indicate that the one loop corrections from supersymmetric particles decrease up to negligible values in the limit of very heavy sparticle masses. Decoupling of SUSY particles is also found in some analytical studies of these and related observables [3-7].

It has been known for some time that there are some exceptions where the Decoupling Theorem [2] does not apply. Particularly interesting are the cases of the Higgs particle and the top quark in the SM which are known not to decouple from low energy physics [8,9,10]. The question whether the Decoupling Theorem applies or not in the case of heavy sparticles in MSSM is not obvious at all, in our opinion. The MSSM is a gauge theory which incorporates the spontaneous symmetry breaking $SU(2)_{\rm L} \times U(1)_{\rm Y} \to U(1)_{\rm em}$ and chiral fermions as the SM and therefore, the direct application of this theorem should, in the principle, be questioned [11].

In our opinion, a formal proof of decoupling must involve the explicit computation of the effective action by integrating out one by one all the sparticles in the MSSM to all orders in perturbation theory, and by considering the heavy sparticle masses limit. The proof will be conclusive if the remaining effective action, to be valid at energies much lower than the supersymmetric particle masses, turns out to be that of the SM with all the SUSY effects being absorbed into a redefinition of the SM parameters or else they are suppressed by inverse powers of the SUSY particle masses and vanish in the infinite masses limit.

In this work we discuss part of the effective action which results by integrating out the sfermions of the MSSM at the one loop level. This is a reduced version of the more complete papers to which we refer the reader for a more detailed discussion [12,13]. Here, we have devoted our attention on the derivation of the two, three and four-point functions with external $W^{\pm}$, Z and γ gauge bosons. In order to keep our computation of the heavy SUSY particle quantum effects in a general form we have chosen to work with the masses themselves. Nevertheless, a discussion on how the decoupling takes places in terms of both the physical sparticle masses and the non-physical MSSM parameters, as the μ-parameter or the soft-SUSY-breaking parameters, $M_{\tilde{Q}}, M_{\tilde{U}}, M_{\tilde{D}}, M_{\tilde{L}}, M_{\tilde{E}}$, is included.

It is important to remark that we have considered the physically plausible situation where all the sparticle masses are large as compared to the electroweak scale but they are allowed, in principle, to be different from each other. We will explore the interesting question of to what extent the usual hypothesis of SUSY masses being generated by soft-SUSY-breaking terms and the universality of the mass parameters do or do not play a relevant role in getting decoupling. In fact, we will show in this paper, that the basic requirement of $SU(2)_{\rm L} \times U(1)_{\rm Y}$ gauge invariance on the SUSY breaking terms is sufficient to obtain decoupling in the MSSM.

Finally, we would like to point out that in order to evaluate analytically the large SUSY masses limit of the Green functions we have applied the so-called m-Theorem [14], which provides a rigorous technique to compute Feynman integrals with both large and small masses in the asymptotic regime of the large masses being very heavy. This theorem will enable us not only to disregard integrals that do not contribute to the limit of large masses without having to compute them, but also to evaluate exactly the non-decoupling contributions.

The paper is organized as follows: In section 2 we present a brief discussion about the large mass limit for all the sfermions at the MSSM. The third section is devoted to present the effective action for the electroweak gauge bosons $W^{\pm}$, Z and γ in the MSSM that results by integrating out, in the path integral, sfermions to one-loop. The asymptotic results in the large SUSY masses limit for the two, three and four-points functions are included and analyzed in section 3. Finally, the conclusions are summarized in section 4.

2 The large supersymmetric masses limit.

As we point out before, in the present work we are interested in the Green functions with external electroweak gauge bosons and in the large mass limit of the SUSY particles, which means the situation where all the sparticle masses are much larger than the electroweak scale and the external momenta. In particular this could be the case if the sparticle masses are well above m_Z, m_W and m_t but still below the few TeV upper bound that is imposed by the standard solution of the hierarchy problem. Furthermore, unless we are in a particular model, the masses of the various sparticles are, in general, different and independent. Therefore, we must take these masses to be large as compared to the external gauge boson masses and external momenta, but we must specify, in addition, how they compare to each other. More specifically, we assume here the most plausible situation where all the sparticle masses are large but close to each other; namely $\tilde{m}_i^2, \tilde{m}_j^2 \gg M_{EW}^2, k^2$ and $|\tilde{m}_i^2 - \tilde{m}_j^2| \ll |\tilde{m}_i^2 + \tilde{m}_j^2|$, where M_{EW} denotes any of the electroweak masses involved $(m_Z, m_W, m_t, \ldots)$ and k denotes any external momentum. Notice that this includes the case that has been the most studied in the literature where universality of sparticle masses is assumed.

In principle, our asymptotic limit is on the physical masses, which implies, of course, some conditions over the parameters of the model. In other words, our masses hypothesis, together with the requirement that all the sparticles must be heavier than their corresponding partners, imply some constraints on the SUSY parameters. In particular, in the squarks sector, if we ignore mixing between different generations to avoid unacceptable large flavor changing neutral currents and if we use the notation of the third family for the mass eigenstates $\tilde{t}_1, \tilde{t}_2, \tilde{b}_1, \tilde{b}_2$ and the corresponding mass squared eigenvalues by $\tilde{m}^2_{t_{1,2}}$, $\tilde{m}^2_{b_{1,2}}$, it can be shown the following constraints on the soft SUSY breaking and μ parameters hold[12]:

$$
\begin{aligned}
& M_{\tilde{Q}}^2, M_{\tilde{U}}^2 \gg m_t^2, m_Z^2 \quad , \quad |M_{\tilde{Q}}^2 - M_{\tilde{U}}^2| \ll |M_{\tilde{Q}}^2 + M_{\tilde{U}}^2| \\
& m_t^2 (A_t - \mu \cot\beta)^2 < M_{\tilde{Q}}^2 M_{\tilde{U}}^2 \, . \qquad (1)
\end{aligned}
$$

Here A_t is the trilinear coupling and $\cot\beta \equiv v_1/v_2$. The first condition implies, in turn, the limiting behaviour $\tilde{m}^2_{t_1} \to M^2_{\tilde{Q}}$, $\tilde{m}^2_{t_2} \to M^2_{\tilde{U}}$. The second condition means that $M_{\tilde{Q}}$ and $M_{\tilde{U}}$ must be close to each other and the third one means that the mixing can never be arbitrarily large. Similar conclusions can be obtained for the sbottoms.

In summary, in other to get large stop and sbottom masses one needs large values of the SUSY breaking masses $M_{\tilde{Q}}, M_{\tilde{U}}$ and $M_{\tilde{D}}$ as compared to the electroweak scale and, in order not to get a too large mixing, the trilinear couplings A_t, A_b and the μ parameter must be constrained from above by the previous inequalities. Notice that an arbitrarily large μ or A_t, A_b with $M_{\tilde{Q}}, M_{\tilde{U}}, M_{\tilde{D}}$ fixed is not allowed.

We would like to mention that the asymptotic limit considered here is not the unique possibility to study decoupling. Other possibilities are now under study [12].

3 Effective action for the electroweak gauge bosons to one-loop.

This section is devoted to present the computation of the part of the effective action that contains the two, three and four-point Green's functions with external gauge bosons, $A, Z, W^{\pm}$, which results by integrating out all the sfermions particles of the MSSM at the one loop level. Details of the computation, including the integration of neutralinos $\tilde{\chi}^o$ and charginos $\tilde{\chi}^+$ can be found in [12,13]. The computation has been performed using dimensional regularization.

We start by writing, in a general and compact form, the effective action for the standard particles, $\Gamma_{eff}[\phi]$, that is defined through functional integration of all the sparticles of the MSSM,

$$\mathrm{e}^{i\Gamma_{eff}[\phi]} = \int [\mathrm{d}\tilde{\phi}]\, \mathrm{e}^{i\Gamma_{\mathrm{MSSM}}[\phi,\tilde{\phi}]} \ ,$$
$$\Gamma_{\mathrm{MSSM}}[\phi,\tilde{\phi}] \equiv \int dx \mathcal{L}_{\mathrm{MSSM}}(\phi,\tilde{\phi}) \ ; \ \mathrm{d}x \equiv \mathrm{d}^4x \ , \quad (2)$$

where $\phi = l, q, A, W^{\pm}, Z, g, H$ are the SM particles, $\tilde{\phi} = \tilde{l}, \tilde{q}, \tilde{A}, \tilde{W}^{\pm}, \tilde{Z}, \tilde{g}, \tilde{H}$ their supersymmetric partners, and $\mathcal{L}_{\mathrm{MSSM}}$ is the lagrangian of the MSSM [12]. In the following we will use the notation of ref. [12].

As we have already said, in the present work we are interested only in the sfermions contribution. The corresponding part of the effective action can be written as:

$$e^{i\Gamma^{\tilde{f}}_{eff}[V]} = \int [d\tilde{f}][d\tilde{f}^*] e^{i\Gamma_{\tilde{f}}[V,\tilde{f}]} \ , \quad (3)$$

where $\tilde{f} = \tilde{q}, \tilde{l}$; $V = W^{\pm}, Z, A$ and $\Gamma_{\tilde{f}}[V, \tilde{f}]$ is the action for the sfermions.

Notice that the effective action as a function of the n-point Green functions, $\Gamma^{V_1 V_2 \ldots V_n}_{\mu\,\nu \ldots\, \rho}$, can be written in the following form:

$$\Gamma^{\tilde{f}}_{eff}[V] = \sum_n \frac{1}{m!} \int \mathrm{d}^4x_1 \ldots \mathrm{d}^4x_n$$
$$\Gamma^{V_1 V_2 \ldots V_n}_{\mu\,\nu \ldots\, \rho}(x_1\, x_2 \ldots x_n) V_1^{\mu}\, V_2^{\nu} \ldots V_n^{\rho} \ , \quad (4)$$

with $V_i\ (i = 1 \ldots n)$ being the external gauge bosons and m denotes the number of these bosons which are identical.

In order to perform the functional integration, it is convenient to write the classical action in terms of operators. We have computed $\Gamma^{\tilde{f}}_{eff}[V]$ by using the standard path integral techniques. The details of the computation can be found in [12]. It is also worth mentioning that we have worked in the momentum space, which simplify the calculation considerably.

The total resulting effective action for two, three and four-point functions, which are generated from sfermions can be summarized in the following expression:

$$\begin{aligned}
\Gamma^{\tilde{f}}_{eff}[V] = & \ i\mathrm{Tr}(A^{(0)^{-1}}_{\tilde{f}} A^{(2)}_{\tilde{f}}) - \frac{i}{2}\mathrm{Tr}(A^{(0)^{-1}}_{\tilde{f}} A^{(1)}_{\tilde{f}})^2 \\
& - i\mathrm{Tr}(A^{(0)^{-1}}_{\tilde{f}} A^{(1)}_{\tilde{f}} A^{(0)^{-1}}_{\tilde{f}} A^{(2)}_{\tilde{f}}) \\
& + \frac{i}{3}\mathrm{Tr}(A^{(0)^{-1}}_{\tilde{f}} A^{(1)}_{\tilde{f}})^3 - \frac{i}{2}\mathrm{Tr}(A^{(0)^{-1}}_{\tilde{f}} A^{(2)}_{\tilde{f}})^2 \\
& + i\mathrm{Tr}(A^{(0)^{-1}}_{\tilde{f}} A^{(1)}_{\tilde{f}} A^{(0)^{-1}}_{\tilde{f}} A^{(1)}_{\tilde{f}} A^{(0)^{-1}}_{\tilde{f}} A^{(2)}_{\tilde{f}}) \\
& - \frac{i}{4}\mathrm{Tr}(A^{(0)^{-1}}_{\tilde{f}} A^{(1)}_{\tilde{f}})^4 + O(V^5) \ , \quad (5)
\end{aligned}$$

where the operators are,

$$\begin{aligned}
A^{(0)}_{\tilde{f}kp} \equiv & \ (2\pi)^4 \delta(p+k)\, (k^2 - \tilde{M}^2_{\tilde{f}}) \ , \\
A^{(1)}_{\tilde{f}kp} \equiv & -(2\pi)^4 \int \mathrm{d}\tilde{\mathrm{q}}\, \delta(p+k+q)(q_{\mu} + 2p_{\mu}) \\
& \left\{ eA^{\mu}_q \hat{Q}_f + \frac{g}{c_w} Z^{\mu}_q \hat{G}_f + \frac{g}{\sqrt{2}} W^{+\mu}_q \Sigma^{tb}_f + \mathrm{h.c.} \right\} \ , \\
A^{(2)}_{\tilde{f}kp} \equiv & \ (2\pi)^4 \int \mathrm{d}\tilde{\mathrm{q}}\, \mathrm{d}\tilde{\mathrm{r}}\, \delta(p+k+q+r) \left\{ e^2 \hat{Q}^2_f A_{\mu\, q} A^{\mu}_r \right. \\
& + \frac{2\, g\, e}{c_w} A_{\mu\, q} Z^{\mu}_r \hat{Q}_f \hat{G}_f + \frac{g^2}{c^2_w} \hat{G}^2_f Z_{\mu\, q} Z^{\mu}_r \\
& + \frac{g^2}{2} \Sigma_f W^+_{\mu\, q} W^{\mu -}_r + \frac{eg}{\sqrt{2}} Y_{\tilde{f}} A_{\mu\, q} \left(W^+_{\mu\, r} \Sigma^{tb}_f + W^-_{\mu\, r} \Sigma^{bt}_f \right) \\
& \left. - \frac{g^2}{\sqrt{2}} Y_{\tilde{f}} \frac{s^2_w}{c_w} Z_{\mu\, q} \left(W^+_{\mu\, r} \Sigma^{tb}_f + W^-_{\mu\, r} \Sigma^{bt}_f \right) \right\} \ . \quad (6)
\end{aligned}$$

In the above expressions and in the following, $\tilde{f}$ is a four-entries column vector including the four mass eigenstates per generation, i.e $(\tilde{t}_1, \tilde{t}_2, \tilde{b}_1, \tilde{b}_2)$ for squarks and $(\tilde{\nu}, 0, \tilde{\tau}_1, \tilde{\tau}_2)$ for sleptons. The sum $\sum_{\tilde{f}}$ is over the three generations and, in the case of squarks, it runs also over the N_c color indexes. The coupling matrices $\hat{Q}_f, \hat{G}_f, \Sigma^{tb}_f$ and Σ_f can be found in [12]. The parameter $Y_{\tilde{f}}$ and the corresponding mass matrices are given by:

$$\begin{aligned}
Y_{\tilde{f}} = \frac{1}{3} \ \ if \ \ \tilde{f} = \tilde{q} \ \ or \ \ Y_{\tilde{f}} = -1 \ \ if \ \ \tilde{f} = \tilde{l} \ , \\
\tilde{M}^2_{\tilde{f}} = \mathrm{diag}(\tilde{m}^2_{t_1}, \tilde{m}^2_{t_2}, \tilde{m}^2_{b_1}, \tilde{m}^2_{b_2}) \quad if \quad \tilde{f} = \tilde{q} \ , \\
\tilde{M}^2_{\tilde{f}} = \mathrm{diag}(\tilde{m}^2_{\nu}, 0, \tilde{m}^2_{\tau_1}, \tilde{m}^2_{\tau_2}) \quad if \quad \tilde{f} = \tilde{l} \ . \quad (7)
\end{aligned}$$

Clearly, we can identify the first and second terms in eq. (5) with the one-loop contributions to the two-point functions, the third and fourth terms with the contributions to the three-point function and the last three terms are the corresponding contributions to the four-point functions.

In order to get the explicit expressions for the two-point functions one must work out the traces in the above formulae. Basically one must substitute all the operators and compute all the appearing Dirac traces. The traces also involve to perform the sum in the corresponding matrix indexes, the sum over the various types of sfermions and the sum in color indexes in the case of squarks. We have done this computation, in addition, by diagrammatical methods and we have found the same results.

In the following we will present the results of the two, three and four-point functions with external gauge bosons at one loop.

3.1 *Effective action for the two-point functions.*

We present here the contributions to the two-point function in momentum space, $\Gamma^{V_1 V_2}_{\mu\,\nu}$. They are given in terms of the coupling matrices and the one-loop integrals:

$$\Gamma^{A\,A}_{\mu\,\nu}(k) = \Gamma_0{}^{A\,A}_{\mu\,\nu}(k) + ie^2 \sum_{\tilde{f}} \left\{ 2 \sum_a (\hat{Q}_f^2)_{aa} I_0\, g_{\mu\nu} - \sum_{ab} (\hat{Q}_f)_{ab} (\hat{Q}_f)_{ba} I^{ab}_{f_{\mu\nu}} \right\} \quad (8)$$

$$\Gamma^{Z\,Z}_{\mu\,\nu}(k) = \Gamma_0{}^{Z\,Z}_{\mu\,\nu}(k) + i\frac{g^2}{c_W^2} \sum_{\tilde{f}} \left\{ 2 \sum_a (\hat{G}_f^2)_{aa} I_0\, g_{\mu\nu} - \sum_{ab} (\hat{G}_f)_{ab} (\hat{G}_f)_{ba} I^{ab}_{f_{\mu\nu}} \right\} \quad (9)$$

$$\Gamma^{A\,Z}_{\mu\,\nu}(k) = \Gamma^{Z\,A}_{\mu\,\nu}(k) = \frac{ige}{c_W} \sum_{\tilde{f}} \left\{ 2 \sum_a (\widehat{Q}_f \widehat{G}_f)_{a\,a} I_0\, g_{\mu\,\nu} - \sum_{ab} (\widehat{Q}_f)_{a\,b} (\widehat{G}_f)_{b\,a} I^{a\,b}_{f_{\mu\,\nu}} \right\} \quad (10)$$

$$\Gamma^{W\,W}_{\mu\,\nu}(k) = \Gamma_0{}^{W\,W}_{\mu\,\nu}(k) + \frac{ig^2}{2} \sum_{\tilde{f}} \left\{ \sum_a (\Sigma_f)_{a\,a} I_0\, g_{\mu\,\nu} - \sum_{a,b} (\Sigma_f^{t\,b})_{a\,b} (\Sigma_f^{t\,b})_{a\,b} I^{a\,b}_{f_{\mu\,\nu}} \right\} \quad (11)$$

In the above formulae the indexes a and b run from one to four, corresponding to the four entries of the column vector $\tilde{f}$. $\Gamma_0{}^{V\,V}_{\mu\,\nu}$ $(V = Z, W)$ and $\Gamma_0{}^{A\,A}_{\mu\,\nu}$ are the two-point functions at tree level, which are defined by:

$$\Gamma_0{}^{V\,V}_{\mu\,\nu}(k) = (M_V - k^2) g_{\mu\,\nu} + \left(1 - \frac{1}{\xi_V}\right) k_\mu k_\nu \;;$$
$$\Gamma_0{}^{A\,A}_{\mu\,\nu} = -k^2 g_{\mu\,\nu} + \left(1 - \frac{1}{\xi_A}\right) k_\mu k_\nu \;, \quad (12)$$

The one-loop integrals $I_0(\tilde{m}^2_{f_a})\,, I^{a\,b}_{f_{\mu\,\nu}}(k, \tilde{m}_{f_a}, \tilde{m}_{f_b})$ are defined in Appendix A.

3.2 *Three and four-point functions.*

For simplicity, we will show here the results for the three and four-point functions in the more general and compact form. The corresponding effective action for the three and four-point functions, $\Gamma^{\tilde{f}}_{eff}[V]_{[3]}$ and $\Gamma^{\tilde{f}}_{eff}[V]_{[4]}$ can be expressed as,

$$\Gamma^{\tilde{f}}_{eff}[V]_{[3]} = i(2\pi)^4 \int \mathrm{d\tilde{p}}\, \mathrm{d\tilde{q}}\, \mathrm{d\tilde{k}}\; \delta(p+k+q) \sum_{\tilde{f}} \left(\sum_{a,b} (\hat{O}^{1\,\mu}_p)_{ab} (\hat{O}^{2\,\nu\sigma}_{qk})_{ba} T^{a\,b}_\mu\, g_{\nu\sigma} - \frac{1}{3} \sum_{a,b,c} (\hat{O}^{1\,\mu}_p)_{ab} (\hat{O}^{1\,\nu}_q)_{bc} (\hat{O}^{1\,\sigma}_k)_{ca}\; T^{a\,b\,c}_{\mu\,\nu\,\sigma} \right), \quad (13)$$

$$\Gamma^{\tilde{f}}_{eff}[V]_{[4]} = -i(2\pi)^4 \int \mathrm{d\tilde{p}}\, \mathrm{d\tilde{q}}\, \mathrm{d\tilde{k}}\, \mathrm{d\tilde{r}}\; \delta(p+k+q+r) \sum_{\tilde{f}} \left(\frac{1}{2} \sum_{a,b} (\hat{O}'^{2\,\mu\nu}_{pq})_{ab} (\hat{O}'^{2\,\sigma\lambda}_{qk})_{ba}\, g_{\mu\,\nu} g_{\sigma\lambda}\, J^{a\,b} - \sum_{a,b,c} (\hat{O}^{1\,\mu}_p)_{ab} (\hat{O}^{1\,\nu}_q)_{bc} (\hat{O}'^{2\,\sigma\lambda}_{qk})_{ca}\, g_{\sigma\lambda}\; J^{a\,b\,c}_{\mu\,\nu} + \frac{1}{4} \sum_{a,b,c,d} (\hat{O}^{1\,\mu}_p)_{ab} (\hat{O}^{1\,\nu}_q)_{bc} (\hat{O}^{1\,\sigma}_k)_{cd} (\hat{O}^{1\,\lambda}_r)_{da}\; J^{a\,b\,c\,d}_{\mu\,\nu\,\sigma\,\lambda} \right), \quad (14)$$

where, similarly to the two-point functions, the indexes a, b, c and d run from one to four and the "operators" $\hat{O}^{1\,\mu}_p, \hat{O}^{2\,\mu\nu}_{pq}$ and $\hat{O}'^{2\,\mu\nu}_{pq}$ can be summarized by,

$$\begin{aligned}
\hat{O}^{1\,\mu}_p &= \left\{ eA^\mu_p \hat{Q}_f + \frac{g}{c_w} Z^\mu_p \hat{G}_f + \frac{g}{\sqrt{2}} W^{+\mu}_p \Sigma^{tb}_f + \text{h.c.} \right\}\;, \\
\hat{O}^{2\,\mu\nu}_{pq} &= \left\{ e^2 \hat{Q}_f^2 A_{\mu\,p} A^\nu_q + \frac{2\,g\,e}{c_w} A_{\mu\,p} Z^\nu_q \hat{Q}_f \hat{G}_f + \frac{g^2}{c_w^2} \hat{G}_f^2 Z_{\mu\,p} Z^\nu_q + \frac{g^2}{2} \Sigma_f W^+_{\mu\,p} W^{\nu-}_q \right\}\;, \\
\hat{O}'^{2\,\mu\nu}_{pq} &= \left\{ \hat{O}^{2\,\mu\nu}_{pq} + \frac{eg}{\sqrt{2}} Y_{\tilde{f}} A_{\mu\,p} \left(W^+_{\nu\,q} \Sigma^{tb}_f + W^-_{\nu\,q} \Sigma^{bt}_f \right) - \frac{g^2}{\sqrt{2}} Y_{\tilde{f}} \frac{s^2_w}{c_w} Z_{\mu\,p} \left(W^+_{\nu\,q} \Sigma^{tb}_f + W^-_{\nu\,q} \Sigma^{bt}_f \right) \right\}\;. \quad (15)
\end{aligned}$$

$T^{a\,b}_\mu$, $T^{a\,b\,c}_{\mu\,\nu\,\sigma}$, $J^{a\,b}$, $J^{a\,b\,c}_{\mu\,\nu}$ and $J^{a\,b\,c\,d}_{\mu\,\nu\,\sigma\,\lambda}$ are the one-loop integrals, which are given in Appendix A.

It is important to emphasize that all these formulae are exact to one loop.

4 The Green's functions in the large mass limit.

Since we are interested in the large mass limit of the SUSY particles we need to have at hand not just the exact results of the above mentioned integrals but their asymptotic expressions to be valid in that limit. We have analyzed the integrals by means of the so-called m-Theorem [14]. This theorem provides a powerful technique to study the asymptotic behaviour of Feynman integrals in the limit where some of the masses are large. Notice that this is not trivial since some of these integrals are divergent and the interchange of the integral with the limit is not allowed. Thus, one should first compute the integrals in dimensional regularization and at the end take the large mass limit. Instead of this direct way it is also possible to proceed as follows: First, in order to decrease the ultraviolet divergent degree, one rearranges the integrand through algebraic manipulations up to separate the Feynman integral into a divergent part, which can be evaluated exactly using the standard techniques of dimensional regularization, and a convergent part that satisfies the requirements demanded by the m-Theorem and therefore, goes to zero in the infinite mass limit. By means of this procedure the correct asymptotic behaviour of the integrals is guaranteed. Some examples of the computation of the Feynman integrals by means of the m-Theorem as well as details of this theorem are given in [12]. The results for the above one loop integrals in the large masses limit are presented also in the Appendix A of this paper.

We now proceed to present the asymptotic expressions for the Green's functions in the large sfermions masses limit. Making use of the results of the one-loop integrals given in eqs.(A.9-A.10) and by using the formulae of eqs.(8-11), (13) and (14), we find the results summarized in the next subsections.

All the results presented in the following are valid for:

$$\begin{aligned}
&\tilde{m}^2_{t_1}, \tilde{m}^2_{t_2}, \tilde{m}^2_{b_1}, \tilde{m}^2_{b_2} \gg k^2 \\
&|\tilde{m}^2_{t_1} - \tilde{m}^2_{t_2}| \ll |\tilde{m}^2_{t_1} + \tilde{m}^2_{t_2}| \\
&|\tilde{m}^2_{b_1} - \tilde{m}^2_{b_2}| \ll |\tilde{m}^2_{b_1} + \tilde{m}^2_{b_2}| \text{ and} \\
&|\tilde{m}^2_{t_i} - \tilde{m}^2_{b_j}| \ll |\tilde{m}^2_{t_i} + \tilde{m}^2_{b_j}| \ \ (i,j=1,2).
\end{aligned} \tag{16}$$

4.1 Two-points functions in the asymptotic limit:

In order to present our results we write the functions $\Gamma^{V_1 V_2}_{\mu\nu}(k)$ as,

$$\Gamma^{V_1 V_2}_{\mu\nu} = \Gamma_{0\,\mu\nu}^{\,V_1 V_2} + \Delta\Gamma^{V_1 V_2}_{\mu\nu} \ , \tag{17}$$

where the tree level functions $\Gamma_{0\,\mu\nu}^{\,V_1 V_2}$ are given in eq.(12) and the contributions from sfermions $\Delta\Gamma^{V_1 V_2}_{\mu\nu}$ are defined by,

$$\Delta\Gamma^{V_1 V_2}_{\mu\nu}(k) = \Sigma^{V_1 V_2}(k) g_{\mu\nu} + R^{V_1 V_2}(k) k_\mu k_\nu \ . \tag{18}$$

We omit to write the explicit formulae for the $\Sigma^{XY}(k)$ and $R^{XY}(k)$ functions for brevity. The complete results can be found in [12]. The results for the $R^{XY}(k)$ functions can, generically, be written as:

$$R^{XY}(k) = -\left[\ k^2 \text{ term of } \Sigma^{XY}(k)\right]/k^2 \tag{19}$$

As can be seen from our work in ref.[12], the asymptotic results in the large SUSY masses limit are of the generic form:

$$\begin{aligned}
\Sigma^{V_1 V_2}(k) &= \Sigma^{V_1 V_2}_{(0)} + \Sigma^{V_1 V_2}_{(1)} k^2 + H\left[O\left(\frac{k^2}{\tilde{m}^2_i}, \frac{\tilde{m}^2_i - \tilde{m}^2_j}{\tilde{m}^2_i + \tilde{m}^2_j}\right)\right], \\
R^{V_1 V_2}(k) &= R^{V_1 V_2}_{(0)} + J\left[O\left(\frac{k^2}{\tilde{m}^2_i}, \frac{\tilde{m}^2_i - \tilde{m}^2_j}{\tilde{m}^2_i + \tilde{m}^2_j}\right)\right] \ ,
\end{aligned} \tag{20}$$

where $\Sigma^{V_1 V_2}_{(1)}$ and $R^{V_1 V_2}_{(0)}$ contain the divergent $O(1/\epsilon)$ contribution of dimensional regularization and are functions of the large SUSY masses but are k independent. $\Sigma^{V_1 V_2}_{(0)}$ is also a finite and k independent function, but not contains divergent contribution. It goes to zero in our asymptotic behaviour. H and J are finite functions which vanish in the large masses limit.

In order to clarify the before comments, we present here the results for $\Sigma^{V_1 V_2}_{(1)}$,

$$\begin{aligned}
\Sigma^{AA}_{(1)\ \tilde{q}}(k) = -\frac{e^2}{16\pi^2}\frac{N_c}{27}\sum_{\tilde{q}}\Bigg\{&10\Delta_\epsilon - 4\log\frac{\tilde{m}^2_{t_1}}{\mu^2_o} \\
&-4\log\frac{\tilde{m}^2_{t_2}}{\mu^2_o} - \log\frac{\tilde{m}^2_{b_1}}{\mu^2_o} - \log\frac{\tilde{m}^2_{b_2}}{\mu^2_o}\Bigg\},
\end{aligned} \tag{21}$$

$$\begin{aligned}
\Sigma^{AZ}_{(1)\ \tilde{q}}(k) = -\frac{e^2}{16\pi^2}\frac{N_c}{9 s_W c_W}\sum_{\tilde{q}}\Bigg\{&\left(\frac{3}{2} - \frac{10}{3}s^2_W\right)\Delta_\epsilon \\
&-\left(c^2_t - \frac{4}{3}s^2_W\right)\log\frac{\tilde{m}^2_{t_1}}{\mu^2_o} - \left(s^2_t - \frac{4}{3}s^2_W\right)\log\frac{\tilde{m}^2_{t_2}}{\mu^2_o} \\
&-\left(\frac{1}{2}c^2_b - \frac{1}{3}s^2_W\right)\log\frac{\tilde{m}^2_{b_1}}{\mu^2_o} - \left(\frac{1}{2}s^2_b - \frac{1}{3}s^2_W\right)\log\frac{\tilde{m}^2_{b_2}}{\mu^2_o}\Bigg\},
\end{aligned} \tag{22}$$

$$\begin{aligned}
\Sigma^{ZZ}_{(1)\ \tilde{q}}(k) = -\frac{e^2}{16\pi^2}\frac{N_c}{3 s^2_W c^2_W}\sum_{\tilde{q}}\Bigg\{&\frac{1}{18}(32 s^4_W - 24 s^2_W + 9)\Delta_\epsilon \\
&-\left(\frac{c^2_t}{2} - \frac{2s^2_W}{3}\right)^2\log\frac{\tilde{m}^2_{t_1}}{\mu^2_o} - \left(\frac{s^2_t}{2} - \frac{2s^2_W}{3}\right)^2\log\frac{\tilde{m}^2_{t_2}}{\mu^2_o} \\
&-\left(-\frac{c^2_b}{2} + \frac{s^2_W}{3}\right)^2\log\frac{\tilde{m}^2_{b_1}}{\mu^2_o} - \left(-\frac{s^2_b}{2} + \frac{s^2_W}{3}\right)^2\log\frac{\tilde{m}^2_{b_2}}{\mu^2_o} \\
&-\frac{1}{2}s^2_t c^2_t\log\frac{\tilde{m}^2_{t_1} + \tilde{m}^2_{t_2}}{2\mu^2_o} - \frac{1}{2}s^2_b c^2_b\log\frac{\tilde{m}^2_{b_1} + \tilde{m}^2_{b_2}}{2\mu^2_o}\Bigg\},
\end{aligned} \tag{23}$$

$$\Sigma^{WW}_{(1)\;\tilde q}(k) = -\frac{e^2}{16\pi^2}\frac{N_c}{6s_W^2}\sum_{\tilde q}\left\{\Delta_\epsilon - c_t^2c_b^2\log\frac{\tilde m^2_{t_1}+\tilde m^2_{b_1}}{2\mu_o^2} - c_t^2s_b^2\log\frac{\tilde m^2_{t_1}+\tilde m^2_{b_2}}{2\mu_o^2} - s_t^2c_b^2\log\frac{\tilde m^2_{t_2}+\tilde m^2_{b_1}}{2\mu_o^2} - s_t^2s_b^2\log\frac{\tilde m^2_{t_2}+\tilde m^2_{b_2}}{2\mu_o^2}\right\}\;, \tag{24}$$

where $s_W^2 = sin^2\theta_W$ and $c_f = cos\theta_f, s_f = sin\theta_f$, with θ_f being the mixing angle in the f-sector.

Here and from now on,

$$\Delta_\epsilon = \frac{2}{\epsilon} - \gamma_\epsilon + \log(4\pi)\;\;,\;\;\epsilon = 4 - D\,, \tag{25}$$

and μ_o is the usual mass scale of dimensional regularization.

As can be easily shown, it implies that all non-decoupling effects in the two-point functions are contained in $\Sigma^{V_1\,V_2}_{(1)}$ and $R^{V_1\,V_2}_{(0)}$ and, therefore, they can be absorbed into a redefinition of the SM relevant parameters, m_W, m_Z and e and the gauge bosons wave functions. In consequence, the decoupling of squarks in the two point functions does indeed occur.

4.2 Three-points functions in the asymptotic limit:

The result for the effective action under the mass conditions given in section 4 can be written as,

$$\Gamma^{\tilde f}_{eff}[V]_{[3]} = \frac{1}{9\pi^2}\int \mathrm{d}\tilde{\mathrm{p}}\;\mathrm{d}\tilde{\mathrm{q}}\;\mathrm{d}\tilde{\mathrm{k}}\;\delta(p+k+q) \sum_{\tilde f}\left\{\sum_{a,b,c}(\hat O^{1\,\mu}_p)_{ab}(\hat O^{1\,\nu}_q)_{bc}(\hat O^{1\,\sigma}_k)_{ca} \left(\Delta_\epsilon - \log\frac{\tilde m^2_{fa}+\tilde m^2_{fb}+\tilde m^2_{fc}}{3\mu_o^2}\right)\text{Ł}_{\mu\,\nu\,\sigma}\right\}\;, \tag{26}$$

where $\text{Ł}_{\mu\,\nu\,\sigma}$ denotes the tree level operator defined by,

$$\text{Ł}_{\mu\,\nu\,\sigma} \equiv [(p-q)_\sigma g_{\mu\,\nu} + (k-p)_\nu g_{\mu\,\sigma} + (q-k)_\mu g_{\nu\,\sigma}]\;. \tag{27}$$

Notice that this asymptotic result is proportional to the tree level operator $\text{Ł}_{\mu\,\nu\,\sigma}$ and therefore, we can at this point already conclude that the sfermions decouple in the three-point functions. We find interesting anyway to give explicitly also each contribution different from zero to the three-point Green's functions with specific external gauge bosons, $\Gamma^{V_1V_2V_3}_{\mu\,\nu\,\sigma}$. We present the results in the following form:

$$\Gamma^{V_1\,V_2\,V_3}_{\mu\,\nu\,\sigma} = \Gamma_0{}^{V_1\,V_2\,V_3}_{\mu\,\nu\,\sigma} + \Delta\Gamma^{V_1\,V_2\,V_3}_{\mu\,\nu\,\sigma}\,, \tag{28}$$

where the contributions at tree level are:

$$\Gamma_0{}^{AW^+W^-}_{\mu\,\nu\,\sigma} = -e\,\text{Ł}_{\mu\nu\sigma}\;\;,\;\;\Gamma_0{}^{ZW^+W^-}_{\mu\,\nu\,\sigma} = -gc_W\,\text{Ł}_{\mu\nu\sigma}\,. \tag{29}$$

In order to get the sfermions contributions, one must substitute all the "operators" that appear in eq.(26), perform the corresponding sums and after rather lengthy calculation, the following results, written in a compact form, are obtained:

$$\Delta\Gamma^{AW^+W^-}_{\mu\,\nu\,\sigma\;\;\tilde q} = \frac{eg^2}{16\pi^2}\frac{N_c}{9}\,\text{Ł}_{\mu\,\nu\,\sigma}\sum_{\tilde q}\frac{1}{2}\left\{(\Delta_\epsilon + \log\mu_o^2) + f_1(\tilde m^2_{t_1},\tilde m^2_{t_2},\tilde m^2_{b_1},\tilde m^2_{b_2})\right\} + F_{1\,\mu\,\nu\,\sigma}\left[O\left(\frac{p2}{\tilde m^2},\frac{\tilde m_i^2-\tilde m_j^2}{\tilde m_i^2+\tilde m_j^2}\right)\right]\;, \tag{30}$$

$$\Delta\Gamma^{ZW^+W^-}_{\mu\,\nu\,\sigma\;\;\tilde q} = -\frac{g^3}{16\pi^2}\frac{N_c}{6c_W}\,\text{Ł}_{\mu\,\nu\,\sigma}\sum_{\tilde q}\frac{1}{3}s_W^2\left\{(\Delta_\epsilon + \log\mu_o^2) + f_2(\tilde m^2_{t_1},\tilde m^2_{t_2},\tilde m^2_{b_1},\tilde m^2_{b_2})\right\} + F_{2\,\mu\,\nu\,\sigma}\left[O\left(\frac{p2}{\tilde m^2},\frac{\tilde m_i^2-\tilde m_j^2}{\tilde m_i^2+\tilde m_j^2}\right)\right]\;, \tag{31}$$

where the functions $F_{i\,\mu\,\nu\,\sigma}\,(i = 1, 2)$ are finite and we have proved explicitly that they go to zero in the limit of $\tilde m \to \infty$ with $|\tilde m_i^2 - \tilde m_j^2| \ll |\tilde m_i^2 + \tilde m_j^2|$.

In the above two expressions, the functions $f_i(\tilde m^2_{t_1},\tilde m^2_{t_2},\tilde m^2_{b_1},\tilde m^2_{b_2})\,(i = 1, 2)$ are finite and different from zero in the large masses limit. Therefore, they contain all the potentially non-decoupling effects of the three-point functions. Their explicit expressions can be found in [13]. In principle, the dependence on the various sfermions masses of each of these functions are different from each other. However, in order to implement the large supersymmetric masses limit one must choose a proper combination of masses such that there is just one large mass parameter. We choose here, suitably, a sum of three masses as the large parameter. The remaining mass parameters can be expressed in terms of the differences of masses which in our approximation are small as compared to the sum. In terms of these mass combination, we get:

$$f_1(\tilde m^2_{t_1},\tilde m^2_{t_2},\tilde m^2_{b_1},\tilde m^2_{b_2}) = -\log\hat M_1^2 + O\left(\frac{\tilde m_i^2-\tilde m_j^2}{\hat M^2}\right)\;,$$
$$f_2(\tilde m^2_{t_1},\tilde m^2_{t_2},\tilde m^2_{b_1},\tilde m^2_{b_2}) = -\log\hat M_2^2 + O\left(\frac{\tilde m_i^2-\tilde m_j^2}{\hat M^2}\right)\;, \tag{32}$$

being $\hat M_1^2 = \frac{1}{3}\left(\tilde m^2_{t_2} + 2\tilde m^2_{b_1}\right)$ and $\hat M_2^2 = \frac{1}{3}\left(2\tilde m^2_{t_2} + \tilde m^2_{b_1}\right)$.

As we have mentioned above, the corrections $\Delta\Gamma$ are proportional to the tree level, $Ł_{\mu\,\nu\,\sigma}$, and therefore the potentially non-decoupling effects in the three-point functions can be absorbed into redefinitions of the coupling constants and wave functions.

4.3 Four-points functions in the asymptotic limit:

Analogously to the previous section, we write the results for the four-point functions as,

$$\Gamma^{V_1\,V_2\,V_3\,V_4}_{\mu\,\nu\,\sigma\,\lambda} = \Gamma_0{}^{V_1\,V_2\,V_3\,V_4}_{\mu\,\nu\,\sigma\,\lambda} + \Delta\Gamma^{V_1\,V_2\,V_3\,V_4}_{\mu\,\nu\,\sigma\,\lambda}\,, \qquad (33)$$

where the different contributions to the effective action at tree level are defined by,

$$\Gamma^{AAW^+W^-}_{0\;\mu\,\nu\,\sigma\,\lambda} = -e^2 ß_{\mu\nu\sigma\lambda}\,,\ \Gamma^{AZW^+W^-}_{0\;\mu\,\nu\,\sigma\,\lambda} = -g^2 s_w c_w ß_{\mu\nu\sigma\lambda},$$

$$\Gamma^{ZZW^+W^-}_{0\;\mu\,\nu\,\sigma\,\lambda} = -g^2 c_W ß_{\mu\nu\sigma\lambda}\,,\ \Gamma^{W^+W^-W^+W^-}_{0\;\mu\,\nu\,\sigma\,\lambda} = g^2 ß_{\mu\nu\sigma\lambda}\,, \qquad (34)$$

with,

$$ß_{\mu\,\nu\,\sigma\,\lambda} \equiv \left[2g_{\mu\,\nu}g_{\sigma\,\lambda} - g_{\mu\,\sigma}g_{\nu\,\lambda} - g_{\mu\,\lambda}g_{\nu\,\sigma}\right]. \qquad (35)$$

Similar expressions to the eqs.(30) and (31) are obtained for the squarks contributions to the four-point functions,

$$\Delta\Gamma^{AAW^+W^-}_{\mu\,\nu\,\sigma\,\lambda\;\tilde q} = -\frac{N_c}{6}\frac{e^2g^2}{16\pi^2} ß_{\mu\nu\sigma\lambda} \sum_{\tilde q}\left\{\left(\Delta_\epsilon + \log\mu_o^2\right)\right.$$
$$\left. +\ g_1(\tilde m^2_{t_1}, \tilde m^2_{t_2}, \tilde m^2_{b_1}, \tilde m^2_{b_2})\right\}$$
$$+\ G_{1\mu\,\nu\,\sigma\,\lambda}\left[O\left(\frac{p2}{\tilde m^2}, \frac{\tilde m_i^2 - \tilde m_j^2}{\tilde m_i^2 + \tilde m_j^2}\right)\right]\,, \qquad (36)$$

$$\Delta\Gamma^{AZW^+W^-}_{\mu\,\nu\,\sigma\,\lambda\;\tilde q} = -\frac{N_c}{6}\frac{eg^3}{16\pi^2} c_W\, ß_{\mu\nu\sigma\lambda} \sum_{\tilde q}\left\{\left(\Delta_\epsilon + \log\mu_o^2\right)\right.$$
$$\left. +\ g_2(\tilde m^2_{t_1}, \tilde m^2_{t_2}, \tilde m^2_{b_1}, \tilde m^2_{b_2})\right\}$$
$$+\ G_{2\mu\,\nu\,\sigma\,\lambda}\left[O\left(\frac{p2}{\tilde m^2}, \frac{\tilde m_i^2 - \tilde m_j^2}{\tilde m_i^2 + \tilde m_j^2}\right)\right]\,, \qquad (37)$$

$$\Delta\Gamma^{ZZW^+W^-}_{\mu\,\nu\,\sigma\,\lambda\;\tilde q} = -\frac{N_c}{6}\frac{g^4}{16\pi^2} c_W^2\, ß_{\mu\nu\sigma\lambda} \sum_{\tilde q}\left\{\left(\Delta_\epsilon + \log\mu_o^2\right)\right.$$
$$\left. +\ g_3(\tilde m^2_{t_1}, \tilde m^2_{t_2}, \tilde m^2_{b_1}, \tilde m^2_{b_2})\right\}$$
$$+\ G_{3\mu\,\nu\,\sigma\,\lambda}\left[O\left(\frac{p2}{\tilde m^2}, \frac{\tilde m_i^2 - \tilde m_j^2}{\tilde m_i^2 + \tilde m_j^2}\right)\right]\,, \qquad (38)$$

$$\Delta\Gamma^{W^+W^-W^+W^-}_{\mu\,\nu\,\sigma\,\lambda\;\tilde q} = -\frac{N_c}{3}\frac{g^4}{16\pi^2} ß_{\mu\nu\sigma\lambda} \sum_{\tilde q}\left\{\left(\Delta_\epsilon + \log\mu_o^2\right)\right.$$
$$\left. +\ g_4(\tilde m^2_{t_1}, \tilde m^2_{t_2}, \tilde m^2_{b_1}, \tilde m^2_{b_2})\right\}$$
$$+\ G_{4\mu\,\nu\,\sigma\,\lambda}\left[O\left(\frac{p2}{\tilde m^2}, \frac{\tilde m_i^2 - \tilde m_j^2}{\tilde m_i^2 + \tilde m_j^2}\right)\right]\,, \qquad (39)$$

It is important to point out that the functions $G_{k\mu\,\nu\,\sigma\,\lambda}$ and $g_k(\tilde m^2_{t_1}, \tilde m^2_{t_2}, \tilde m^2_{b_1}, \tilde m^2_{b_2})\,(k = 1, \ldots 4)$ are both finite, but the first ones go to zero in our asymptotic limit whereas the second ones are different from zero in this limit, and therefore these latter contain all the potentially non-decoupling effects of the four-point functions. For brevity, we will not present here the explicit formulae of g_k functions. However, we would like to have noticed that if one takes the sum of all the masses as the large parameter in the expansion of the logarithm's coefficients one get,

$$g_k(\tilde m^2_{t_1}, \tilde m^2_{t_2}, \tilde m^2_{b_1}, \tilde m^2_{b_2}) = -\log\hat M^2 + O\left(\frac{\tilde m_i^2 - \tilde m_j^2}{\hat M^2}\right)\,, \qquad (40)$$

where $\hat M^2 = \frac{1}{4}(\tilde m^2_{t_1} + \tilde m^2_{t_2} + \tilde m^2_{b_1} + \tilde m^2_{b_2})$.

From eqs.(36) through (39), it can be seen that all the corrections $\Delta\Gamma$ are proportional to the tree level, $ß_{\mu\nu\sigma\lambda}$, and therefore the potentially non-decoupling effects in the four-point functions can be also absorbed into redefinitions of the coupling constants and wave functions.

Similar results are obtained for the sleptons sector for Green's functions doing the corresponding replacements: $\tilde q \to \tilde l$, $N_c \to 1$, $\tilde m_{t_1} \to \tilde m_\nu$, $\tilde m_{b_1} \to \tilde m_{\tau_1}$, $\tilde m_{b_2} \to \tilde m_{\tau_2}$, $c_t \to 1, s_t \to 0, c_b \to c_\tau$ and $s_b \to s_\tau$.

In summary, from our results it is clear that there is indeed decoupling in the two, three and four-point electroweak gauge boson functions:
All SUSY effects can be absorbed into redefinitions of m_Z, m_W, e and the wave functions of the gauge bosons $W^\pm, Z, A$, or else they are suppressed by inverse powers of the heavy SUSY particles masses.

5 Conclusions

The computation of the effective action for the standard particles which results by integrating out all the heavy supersymmetric particles will provide the answer to the question whether the decoupling of heavy supersymmetric particles in the MSSM occurs leading to the SM as the remaining low energy effective theory. In this work we have shown that all the contributions from the heavy sfermions to the two, three and four-point functions of the electroweak gauge bosons can be absorbed into redefinitions of the SM parameters or they are suppressed by inverse powers of the heavy sparticles masses. Therefore we have proved analytically that the decoupling of heavy sfermions at one loop level does occur.

We have considered the asymptotic limit where the sfermion masses are all large as compared to the $W^{\pm}$ and Z masses and the external momenta and we have always worked under the assumption that the differences of their squared masses are much smaller than their sums. Notice that we have not assumed exact universality of the masses.

Our results for these Green functions in the large SUSY masses limit have been presented analytically and given in terms of the sparticle masses. They do not depend on the particular choice for the soft-breaking terms and therefore they are general results. In our opinion, it is more convenient for the analysis of the phenomenon of decoupling to use the physical sparticle masses as, being the proper parameters, rather than some other possible mass parameters of the MSSM as, for instance, the μ-parameter or the soft-SUSY breaking parameters.

Finally, we have explored to what extent the hypothesis of generation of SUSY masses by soft-SUSY breaking terms is relevant for decoupling and we have found instead that the requirement of $SU(3)_{\rm c}\times SU(2)_{\rm L}\times U(1)_{\rm Y}$ gauge invariance of the explicit mass terms by itself is sufficient to get it.

A complete proof of decoupling of supersymmetric particles will include the integration of all the heavy supersymmetric spectrum. Here we only discuss the squarks and sleptons sectors. The analysis of charginos and neutralinos have been done in [12,13]. The integration of the Higgs sector will be considered in a forthcoming work.[15] The complete proof would lead eventually to the conclusion that the SM is indeed the low energy effective of the MSSM in the large SUSY masses limit.

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Appendix A.

In this appendix we give the definition of the one-loop integrals $I_0\,,I^{a\,b}_{f_{\mu\,\nu}}\,,T^{a\,b}_{\mu}\,,T^{a\,b\,c}_{\mu\,\nu\,\sigma}\,,J^{a\,b}\,,J^{a\,b\,c}_{\mu\,\nu}$ and $J^{a\,b\,c\,d}_{\mu\,\nu\,\sigma\,\lambda}$ that have been used in the computation of the two, three and four-point functions and their results in the large masses limit. We start by giving the definition of the integrals in dimensional regularization.

- One loop integrals.

$$I_0=\int d\widehat{q}\,\frac{1}{\left[q^2-\tilde{m}^2_{f_a}\right]}\,, \qquad (A.1)$$

$$I^{a\,b}_{f_{\mu\,\nu}}=\int d\widehat{q}\,\frac{(2q+k)_\mu(2q+k)_\nu}{\left[(k+q)^2-\tilde{m}^2_{f_a}\right]\left[q^2-\tilde{m}^2_{f_b}\right]}\,, \qquad (A.2)$$

$$T^{a\,b}_{\mu}=\int d\widehat{p}\,\frac{(2p+q)_\mu}{\left[p^2-\tilde{m}^2_{f_a}\right]\left[(p+q)^2-\tilde{m}^2_{f_b}\right]}\,, \qquad (A.3)$$

$$\begin{aligned}T^{a\,b\,c}_{\mu\,\nu\,\sigma}&=8I^{a\,b\,c}_{\mu\,\nu\,\sigma}+4[k_\sigma I^{a\,b\,c}_{\mu\,\nu}+(k-p)_\nu I^{a\,b\,c}_{\mu\,\sigma}+p_\mu I^{a\,b\,c}_{\nu\,\sigma}]\\&+2[k_\sigma(k-p)_\nu I^{a\,b\,c}_{\mu}+k_\sigma p_\mu I^{a\,b\,c}_{\nu}-(k-p)_\nu p_\mu I^{a\,b\,c}_{\sigma}]\\&+p_\mu k_\sigma(k-p)_\nu I^{a\,b\,c}\,, \qquad (A.4)\end{aligned}$$

where,

$$\begin{aligned}I^{a\,b\,c}&=\int d\widehat{p}\,\frac{1}{D}\quad,\ I^{a\,b\,c}_{\mu}=\int d\widehat{p}\,\frac{p_\mu}{D}\,,\\I^{a\,b\,c}_{\mu\,\nu}&=\int d\widehat{p}\,\frac{p_\mu\,p_\nu}{D}\quad,\ I^{a\,b\,c}_{\mu\,\nu\,\sigma}=\int d\widehat{p}\,\frac{p_\mu\,p_\nu p_\sigma}{D}\,,\end{aligned} \qquad (A.5)$$

defining D as,

$$D=\left[p^2-\tilde{m}^2_{f_a}\right]\left[(p+q)^2-\tilde{m}^2_{f_b}\right]\left[(p+q+k)^2-\tilde{m}^2_{f_c}\right]\,.$$

And,

$$J^{a\,b}=\int d\widehat{p}\,\frac{1}{\left[p^2-\tilde{m}^2_{f_a}\right]\left[(p+q+k)^2-\tilde{m}^2_{f_b}\right]}\,, \qquad (A.6)$$

$$J^{a\,b\,c}_{\mu\,\nu}=\int d\widehat{p}\,\frac{P_{\mu\nu}}{E}\,, \qquad (A.7)$$

$$J^{a\,b\,c\,d}_{\mu\,\nu\,\sigma\,\lambda}=\int d\widehat{p}\,\frac{P_{\mu\nu}(2p+2q+2k+r)_\sigma(2p+q+k+r)_\lambda}{E\left[(p+q+k+r)^2-\tilde{m}^2_{f_d}\right]}\,, \qquad (A.8)$$

where,

$$E=\left[p^2-\tilde{m}^2_{f_a}\right]\left[(p+q)^2-\tilde{m}^2_{f_b}\right]\left[(p+q+k)^2-\tilde{m}^2_{f_c}\right]$$

$$P_{\mu\nu}=(2p+q)_\mu\,(2p+2q+k)_\nu$$

• Results in the asymptotic limit.

The results in the large masses limit have been obtained using the m-Theorem [14].

$$I_0 = \frac{i}{16\pi^2}\left(\Delta_\epsilon + 1 - \log\frac{\tilde m_{fa}^2}{\mu_o^2}\right)\tilde m_{fa}^2$$

$$\begin{aligned} I_f^{ab} = &\frac{i}{16\pi^2}\left\{(\tilde m_{fa}^2 + \tilde m_{fb}^2)\left(\Delta_\epsilon + 1 - \log\frac{\tilde m_{fa}^2+\tilde m_{fb}^2}{2\mu_o^2}\right)g_{\mu\nu}\right.\\ &-\frac{1}{3}k^2\left(\Delta_\epsilon - \log\frac{\tilde m_{fa}^2+\tilde m_{fb}^2}{2\mu_o^2}\right)g_{\mu\nu}\\ &\left.+\frac{1}{3}k_\mu k_\nu\left(\Delta_\epsilon - \log\frac{\tilde m_{fa}^2+\tilde m_{fb}^2}{2\mu_o^2}\right)\right\}\end{aligned}$$

$$T_\mu^{a\,b} = 0\ ,\ I^{a\,b\,c} = I_\mu^{a\,b\,c} = 0\,,$$

$$I_{\mu\,\nu}^{a\,b\,c} = \frac{i}{16\pi^2}\frac{1}{4}\left(\Delta_\epsilon - \log\frac{\tilde m_{fa}^2+\tilde m_{fb}^2+\tilde m_{fc}^2}{3\mu_o^2}\right)g_{\mu\,\nu}\,,$$

$$\begin{aligned} I_{\mu\,\nu\,\sigma}^{a\,b\,c} = &-\frac{i}{16\pi^2}\frac{1}{12}(2q+k)_\rho\left(\Delta_\epsilon - \log\frac{\tilde m_{fa}^2+\tilde m_{fb}^2+\tilde m_{fc}^2}{3\mu_o^2}\right)*\\ &[g_{\mu\,\sigma}g_{\nu\,\rho} + g_{\mu\,\rho}g_{\nu\,\sigma}]\ ,\end{aligned}$$

$$\begin{aligned} T_{\mu\,\nu\,\sigma}^{a\,b\,c} = &\frac{i}{16\pi^2}\frac{1}{3}\left(\Delta_\epsilon - \log\frac{\tilde m_{fa}^2+\tilde m_{fb}^2+\tilde m_{fc}^2}{3\mu_o^2}\right)*\\ &[(p-q)_\sigma g_{\mu\,\nu} + (k-p)_\nu g_{\mu\,\sigma} + (q-k)_\mu g_{\nu\,\sigma}]\end{aligned}$$

$$J^{a\,b\,c} = \frac{i}{16\pi^2}\left(\Delta_\epsilon - \log\frac{\tilde m_{fa}^2+\tilde m_{fb}^2}{2\mu_o^2}\right)\,,$$

$$J_{\mu\,\nu}^{a\,b\,c} = \frac{i}{16\pi^2}\left(\Delta_\epsilon - \log\frac{\tilde m_{fa}^2+\tilde m_{fb}^2+\tilde m_{fc}^2}{3\mu_o^2}\right)g_{\mu\,\nu}\,, \tag{A.9}$$

and finally,

$$\begin{aligned} J_{\mu\,\nu\,\sigma\,\lambda}^{a\,b\,c\,d} = &\frac{i}{16\pi^2}\frac{2}{3}\left(\Delta_\epsilon - \log\frac{\tilde m_{fa}^2+\tilde m_{fb}^2+\tilde m_{fc}^2+\tilde m_{fd}^2}{4\mu_o^2}\right)*\\ &[g_{\mu\,\nu}g_{\sigma\,\lambda} + g_{\mu\,\sigma}g_{\nu\,\lambda} + g_{\mu\,\lambda}g_{\nu\,\sigma}]\ .\end{aligned} \tag{A.10}$$

The corrections to these formulae are suppressed by inverse powers of the sums of the sfermion masses and vanish in the large masses limit.

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