Constraints on Neutrino Mixing from r-process Nucleosynthesis in Supernovae

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Abstract

In this paper we note that for neutrino mass squared differences in the range $10^{-4} - 10^{-1} eV^2$, vacuum neutrino oscillations can take place between the neutrino-sphere and the weak freeze-out radius in a type II supernova. Requiring that such oscillations are consistent with the r-process nucleosynthesis from supernovae one can constrain the mixing of ν_e with ν_μ (or ν_τ) down to $10^{-4} eV^2$. We first do a two-flavor study and find that the neutron rich condition $Y_e < 0.5$ is satisfied for all values of mixing angles. However if we take the criterion $Y_e < 0.45$ for a successful r-process and assume $\nu_\mu - \nu_e$ oscillations to be operative then this mode as a possible solution to the atmospheric neutrino anomaly is ruled out in accordance with the recent CHOOZ result. Furthermore since we can probe mass ranges lower than CHOOZ the narrow range that was allowed by the CHOOZ data at 99% C.L. is also ruled out. Next we do a three-generation analysis keeping $\Delta m^2_{12} \sim 10^{-5} eV^2$ or $10^{-11} eV^2$ (solar neutrino range) and $\Delta m^2_{13} \approx \Delta m^2_{23} \sim 10^{-4} - 10^{-1} eV^2$ (atmospheric neutrino range) and use the condition $Y_e < 0.45$ to give bounds on the mixing parameters and compare our results with the CHOOZ bound. We also calculate the increase in the shock-reheating obtained by such oscillations.

1 Introduction

Heavy neutron rich nuclei beyond the iron group are predominantly made by the rapid neutron capture process or the r-process. Over the last four decades different astrophysical environments have been proposed as possible sites for r-process nucleosynthesis [1]. The very high neutron number densities $> 10^{20} cm^{-3}$, temperatures $\sim 2 - 3 \times 10^{9^0} K$ [2] and time scales $\sim 1 s$, prompted investigators to suggest supernova as a plausible site for r-process nucleosynthesis [3]. But where exactly in the supernova does the r-process actually take place is a question which is still to be answered. A putative r-process site suggested in the recent years is the neutrino heated ejecta from the post core bounce environment of a type II supernova or the "hot bubble" [4, 5]. The major advantage which the "hot bubble" has over other proposed sites is that it correctly predicts that only $10^{-4}M_{\odot}$ of r-process nuclei are ejected per supernova [6]. The conditions of high temperatures and low density in the "hot bubble" makes it conducive for synthesising the right amount of r-process nuclei.

Near the neutrino-sphere the temperature is high so that nuclear statistical equilibrum persists and the mass fraction of the nuclei are determined by the Saha equation. Since the photon to baryon ratio is very high in the "hot bubble", at equilibrium we have nucleons and alpha particles as the most abundant species. It is on these nucleons that the ν_e and $\bar{\nu}_e$ capture takes place which determines the electron fraction Y_e. As the temperature drops below about 0.5 MeV, the expansion rate becomes faster than the nuclear reaction rates and one has the nuclear freeze-out. Below about 0.26 MeV the charge particle reactions freeze and beyond this point temperatures become so low that one can have only neutron captures on the heavy seed nuclei or the r-process.

For r-process to be possible in the supernova, the conditions should be neutron rich. The parameter which determines this is the electron fraction Y_e at the radius where the absorption of ν_e and $\bar{\nu}_e$ on free nucleons freeze out. This is called the weak freeze-out radius ($r_{\rm WFO}$) and is found to very close to the nuclear freeze-out radius in most supernova models. Since $\langle E_{\nu_e}\rangle < \langle E_{\bar{\nu}_e}\rangle < \langle E_{\nu_\mu}\rangle$ ($\langle E_{\nu_e}\rangle, \langle E_{\bar{\nu}_e}\rangle$ and $\langle E_{\nu_\mu}\rangle$ are the average energies of the $\nu_e, \bar{\nu}_e$ and ν_μ respectively and the energy spectrum of the $\nu_\mu, \bar{\nu_\mu}, \nu_\tau$ and $\bar{\nu}_\tau$ are identical) and since Y_e is determined by the properties of the ν_e and $\bar{\nu}_e$ fluxes we expect that neutrino flavor oscillations between the neutrino-sphere and the weak freeze-out radius will change the value of the electron fraction.

Such calculations were done by Qian et al. [7]. They made a two flavor analysis of the matter-enhanced level crossing between ν_e and ν_τ or ν_μ. They considered a mass spectrum in which $m_{\nu_{\tau,\mu}} > m_{\nu_e}$ so that there is resonance between the neutrinos only and not between the antineutrinos. As a result the more energetic $\nu_{\mu,\tau}$ ($< E_{\nu_\mu} > = < E_{\nu_\tau} > \sim 25 MeV$) get converted to ν_e ($< E_{\nu_e} > \sim 11 MeV$) increasing the average energy of the electron neutrinos.

Since there is no transformation between the antineutrinos, the energy of the electron antineutrino remains the same($< E_{\bar{\nu}_e} >\sim 16MeV$). The mass of the ν_τ (or ν_μ) required to undergo MSW resonance in the relevant region was shown to be between 1 and 100 eV which is the right range for neutrinos to be the hot dark matter of the Universe. Finally they used the condition $Y_e < 0.5$ to put constraints on the mixing angle.

There are several other hints of non-zero neutrino mass and mixing coming from the solar neutrino experiments, the observation of atmospheric neutrinos and from the recent LSND data.

Four experiments (Homestake, Kamiokande, SAGE, Gallex) measuring the solar neutrino flux observed event rates significantly smaller compared to the predictions of the Standard Solar Models. This contradiction constitutes the solar neutrino problem. This can be explained by neutrino oscillations in vacuum for $\Delta m^2 \sim 0.615 \times 10^{-10} eV^2$ and $\sin^2 2\theta \sim 0.864$ [8] or by the Mikheyev-Smirnov-Wolfenstein (MSW) resonant flavor conversions [9] for $\Delta m^2 \sim 5.4 \times 10^{-6} eV^2$ and $\sin^2 2\theta \sim 7.9 \times 10^{-3}$ (non-adiabatic solution) and $\Delta m^2 \sim 1.7 \times 10^{-5} eV^2$ and $\sin^2 2\theta \sim 0.69$ (large-angle solution) [10]. The preliminary results from the Super-Kamiokande confirm this deficit of solar neutrinos and favor the long wavelength vacuum oscillation solution [11].

The atmospheric neutrino anomaly is the discrepancy in the measured and expected values of the ratio of contained μ-like and e-like events in the Kamiokande, IMB and Soudan experiments. This can be explained by $\nu_\mu - \nu_\tau$ or $\nu_\mu - \nu_e$ oscillations for $\Delta m^2 \sim 10^{-2} eV^2$ and $\sin^2 2\theta \sim 1.0$ [12]. A preliminary Super-Kamiokande data has confirmed the atmospheric neutrino problem for both sub-GeV and multi-GeV neutrinos [13]. Combining their result with the results from other experiments, the allowed range for the $\nu_\mu - \nu_\tau$ mode is $4 \times 10^{-4} \leq \Delta m^2 \leq 5 \times 10^{-3} eV^2$ at 90% C.L. with $\sin^2 2\theta$ = 0.8 -1 [14]. For the $\nu_\mu - \nu_e$ channel at 90% C.L. the allowed range is $10^{-3} - 7 \times 10^{-3} eV^2$ and $\sin^2 2\theta = 0.65 - 1$. If one combines the result of the CHOOZ experiment [15], then the $\nu_\mu - \nu_e$ oscillation solution is ruled out at 90% C.L. A narrow range $6 \times 10^{-4} - 10^{-3} eV^2$ is allowed at 99% C.L..[14].

A third indication for a non-zero Δm^2 comes from the recent LSND data which gives the first evidence for $\bar{\nu}_\mu - \bar{\nu}_e$ [16] and $\nu_e - \nu_\mu$ oscillations using a laboratory neutrino source. This along with the non-observation of neutrino oscillations in E776 at BNL [17] and in the reactor experiment Bugey [18] indicates Δm^2 in the range 0.2 - 3 eV^2.

In this paper we observe that for neutrino mass squared differences in the range $10^{-4} - 10^{-1} eV^2$ vacuum neutrino oscillations can take place between the neutrino-sphere and weak freeze-out radius. Requiring that such oscillations are consistent with heavy element nucleosynthesis from supernovae one can constrain the mixing of ν_e with ν_μ (or ν_τ) in this range. Since this is the relevant range for $\nu_\mu - \nu_e$ oscillation of atmospheric neutrinos, assuming such oscillations to be operative in supernovae one can compare the allowed values of parameters. We first do a two-generation analysis keeping Δm^2 in this range. We find that the condition $Y_e < 0.5$ is satisfied for all values of mixing angles. However using the condition $Y_e < 0.45$ one can rule out the $\nu_\mu - \nu_e$ solution to the atmospheric neutrino anomaly in agreement with the CHOOZ result. Moreover since using the r-process constraint we can probe down to $\Delta m^2 = 10^{-4} eV^2$, the narrow range that was consistent with the CHOOZ data can also be ruled out. If on the other hand we assume two-generation $\nu_e - \nu_\tau$ oscillations to be operative then this provides the only bound on such mixing in this mass range. The earlier bounds on $sin^2 2\theta_{e\tau}$ were for $\Delta m^2 > 80 - 100 eV^2$ [19].

Next we go to the more realistic three-flavor picture. We consider the scenario where $\Delta m^2_{12} \sim 10^{-5} eV^2$ or $10^{-11} eV^2$ and $\Delta m^2_{13} \sim \Delta m^2_{23} = \Delta m^2$ in the range $10^{-4} - 10^{-1} eV^2$. This mass spectrum can simultaneously explain the solar and atmospheric neutrino data and has received considerable interest in the recent past [20]. We find that this scenario together with the CHOOZ constraint [15] is consistent with $Y_e < 0.5$. However if we use the more stringent condition $Y_e < 0.45$ then bounds can be given on the mixing parameters. For $\Delta m^2 > 2 \times 10^{-3} eV^2$ these bounds are weaker than the CHOOZ bound but we can probe Δm^2 down to $10^{-4} eV^2$ which is not probed by any terrestrial experiment so far. The three-generation scenario considered does not explain the LSND data and to explain it as well one needs to introduce a sterile neutrino [21].

Another place in supernovae, where neutrino oscillations can have important effect is in reviving the shock during the reheating epoch of the supernova [22]. The result of neutrino oscillation is to enhance the rate of energy deposition by the neutrinos and hence produce a more energetic shock. We study the effect of vacuum oscillation of neutrinos in the delayed neutrino heating phase of type II supernovae.

The plan of the paper is as follows. In section 2 we discuss the r-process in the "hot bubble" and give the expression of Y_e. In section 3 we first give the two-generation vacuum oscillation formula and discuss the implications of such oscillations on the value of Y_e at the r-process epoch. We next give the three-generation oscillation analysis. In the following section we discuss the effects of these oscillations on shock-reheating. Finally we present the conclusions.

2 Supernova r-process Nucleosynthesis

Above the neutrino-sphere, the electron fraction Y_e which is the second most important factor for r-process abundance calculations after the entropy per baryon, is determined by the competition between the rate of the reactions

$$\nu_e + n \rightleftharpoons p + e^- \tag{1}$$

$$\bar{\nu}_e + p \rightleftharpoons n + e^+ \tag{2}$$

The expression for the value of Y_e at freeze out is given by Qian et al. [7] as

$$Y_e \approx \frac{1}{1 + \lambda_{\bar{\nu}_e p}/\lambda_{\nu_e n}} \tag{3}$$

Where $\lambda_{\nu_e n}$ and $\lambda_{\bar{\nu}_e p}$ are the reaction rates in (1) and (2). The reaction rate $\lambda_{\nu N}$, where N can be either p or n is given by

$$\lambda_{\nu N} \approx \frac{L_\nu}{4\pi r^2} \frac{\int_0^\infty \sigma_{\nu N}(E) f_\nu(E) dE}{\int_0^\infty E f_\nu(E) dE} \tag{4}$$

where L_ν is the neutrino luminosity (we consider identical luminosity for all the neutrino species), $\sigma_{\nu N}$ is the reaction cross-section and $f_\nu(E)$ is the normalised Fermi-Dirac distribution function with zero chemical potential

$$f_\nu(E) = \frac{1}{1.803 {T_\nu}^3} \frac{E^2}{exp(E/T_\nu) + 1} \tag{5}$$

where T_ν is the temperature of the particular neutrino concerned. The cross section is approximately given by [22]

$$\sigma_{\nu N} \approx 9.23 \times 10^{-44} (E/MeV)^2 cm^2 \tag{6}$$

If we calculate $\lambda_{\nu_e n}$ and $\lambda_{\bar{\nu}_e p}$ using eq (4) then the expression for Y_e becomes

$$Y_e \approx \frac{1}{1 + T_{\bar{\nu}_e}/T_{\nu_e}} \tag{7}$$

Typical values for the neutrino temperatures when r-process is operative are [7], $T_{\nu_e} = 3.49$ MeV, $T_{\bar{\nu}_e} = 5.08$ MeV and T_{ν_μ} =7.94 MeV so that $Y_e \approx 0.41$. This being less than 0.5 neutron rich conditions are obtained in the hot bubble and r-process is possible.

3 Neutrino Oscillations in Vacuum

In the standard model of particle physics, neutrinos are assumed to be massless. But there is no compelling reason for this assumption. If the neutrinos are indeed massive then in general the mass eigenstates will be different from the flavor eigenstates. The neutrinos are produced in the flavor eigenstates but propagate in their mass eigenstates. Because these bases are in general not identical, there will be mixing and neutrino flavor will not be conserved.

3.1 Two Generation Analysis

The two-generation conversion probability of an initial neutrino flavor ν_α to a flavor ν_β after traveling a distance L in vacuum is given by

$$P_{\nu_\alpha \nu_\beta} = \frac{1}{2} sin^2 2\theta (1 - cos 2\pi L/\lambda) \tag{8}$$

where λ denotes the oscillation wavelength and can be expressed as

$$\lambda = 2.5 \times 10^{-3} km \frac{E}{MeV} \frac{eV^2}{\Delta m^2} \tag{9}$$

The various limits of the equation (8) are as follows,

- $\lambda >> L, P_{\nu_\alpha \nu_\beta} \to 0$
- $\lambda << L, P_{\nu_\alpha \nu_\beta} \to \frac{1}{2} \sin^2 2\theta$ when averaged over the source or detector distances or energy.
- $\lambda \sim L/2, P_{\nu_\alpha \nu_\beta} = sin^2 2\theta$

Oscillation effects are observable for $\lambda \sim L$. Now for us, as we will discuss later in this section, $L \sim 50$ -100 km. Then, assuming an average value of $E \sim 10 MeV$ one gets the condition $\Delta m^2 \sim 10^{-4} eV^2$. For higher values of Δm^2 the oscillation effects are averaged out and there is a constant conversion depending on the mixing angle. Beyond 1 eV^2 the MSW resonance being within the weak freeze-out radius [7] matter enhanced resonant flavor conversions take place.

As a result of flavor oscillations the neutrino energy distribution function itself will change to (assuming two flavors)

$$f^{osc}_{\nu_e}(E) = P_{\nu_e\nu_e} f_{\nu_e}(E) + P_{\nu_\mu\nu_e} f_{\nu_\mu}(E) \quad (10)$$

$$f^{osc}_{\bar{\nu}_e}(E) = P_{\bar{\nu}_e\bar{\nu}_e} f_{\bar{\nu}_e}(E) + P_{\bar{\nu_\mu}\bar{\nu}_e} f_{\bar{\nu_\mu}}(E) \quad (11)$$

For the probabilities we use the expression given in (8). The L here corresponds to the distance between the neutrino-sphere and the week freeze-out radius. For the values of L relevant for r-process we use the results of the 20 $M_\odot$ supernova model given in ref [5]. According to this the weak freeze-out occurs at about 0.5 MeV while the r-process sets in much later at temperatures below 0.26 MeV between $t_{pb} \approx 3$ to 15s. From the temperature vs. radius curves given in [5] we get the following values for the radii at two different times relevant for supernova r-process nucleosynthesis at T=0.5 MeV :

$$\text{For } t_{pb} = 3s, r_{\text{WFO}} \approx 100km$$
$$t_{pb} = 12s, r_{\text{WFO}} \approx 55km.$$

The position of the neutrino-sphere is $\sim 10km$. Thus the L relevant for us is $\sim$ 90 km at $t_{pb} = 3s$ and $\sim$ 45 km at $t_{pb} = 12s$.

We find that for Δm^2 in the range from $10^{-4} - 10^{-1}eV^2$ the value of Y_e stays less than 0.5 for all values of mixing angles. For r-process to be possible the conditions should be neutron rich, that is, $Y_e < 0.5$. But it is seen that for most r-process nuclei, the calculated abundances matches their solar abundances only for $Y_e < 0.45$ [5, 23]. In Fig. 1 we give the contour plot for $Y_e = 0.45$ for two different values of L. The region to the left of the lines are allowed by r-process (see figure caption for details). The oscillation channel relevant is $\nu_\mu - \nu_e$ (or $\nu_\tau - \nu_e$) and the mass range that we study is $10^{-4} - 10^{-1}eV^2$. For comparison we also plot in the same figure the exclusion plot given by the CHOOZ experiment [15] as well as the region allowed from the atmospheric neutrino data for the $\nu_\mu - \nu_e$ mode [14]. For $\Delta m^2 \geq 2 \times 10^{-3}eV^2$ we get the constraint $sin^2 2\theta_{e\mu} \leq 0.26$. This is weaker than the CHOOZ result but nevertheless rules out the $\nu_\mu - \nu_e$ solution to the atmospheric neutrino problem. Furthermore using r-process we can give bounds on the mixing angle upto $\Delta m^2 = 10^{-4} eV^2$ and our results show that even in the mass range $10^{-4} - 10^{-3} eV^2$ the condition $Y_e < 0.45$ is inconsistent with the $\nu_\mu - \nu_e$ solution of atmospheric neutrino anomaly. Thus the narrow range which was allowed by the CHOOZ data at 99% C.L. is ruled out by r-process. The above conclusions are based on the assumption that the relevant oscillation mode is $\nu_\mu - \nu_e$. If on the other hand we take $\nu_e - \nu_\tau$ oscillations to be operative then this bounds will apply to $sin^2 2\theta_{e\tau}$. In that case this provides the only bound on $\nu_e - \nu_\tau$ mixing in this mass range [24] .

The equation (8) is derived assuming that neutrinos can be represented by plane waves. But a correct quantum mechanical treatment of neutrino oscillation should describe them in terms of wave packets [25]. Then for extremely relativistic neutrinos the two-flavor probability (8) gets modified as [25]

$$P_{\nu_\alpha \nu_\beta} = \frac{1}{2} sin^2 2\theta (1 - cos \frac{2\pi L}{\lambda} e^{-L^2/L^2_{coh}}) \tag{12}$$

where

$$L_{coh} = 4\sqrt{2}\sigma_x (E/\Delta m^2) \tag{13}$$

where σ_x denotes the spread of the wave packet. If $L << L_{coh}$, $e^{(-L^2/L^2{}_{coh})} \rightarrow 1$ one obtains eq. (8). If on the other hand $L >> L_{coh}$ $P_{\nu_\alpha \nu_\beta} = 0.5 sin^2 2\theta$. Thus in this case oscillations don't take place but there is a constant conversion probability similar to the $\lambda << L$ case of the plane wave approximation. The important feature which comes out of the wave packet treatment is the coherence length L_{coh}. Beyond this the separation between the wave packets associated with the propagating mass eigenstates becomes large enough to cause any interference. Thus the oscillation can take place only if the distance $L \leq L_{coh}$. Otherwise there is a constant conversion depending on the mixing angle. For supernova neutrinos emitted from the neutrino-sphere $\sigma_x \approx 10^{-9}$ cm [25] so that for Δm^2 in the range $10^{-4} - 10^{-1} eV^2$ the coherence length varies from 5.656 $\times 10^4 - 56.56$ km for $E \sim 10\ MeV$. Thus in the range $10^{-4} - 10^{-2} eV^2$, $L_{coh} >> L$ and the plane wave approximation is valid. For $\Delta m^2 = 0.1 eV^2$ at low energies (upto 10 MeV) $L \lesssim L_{coh}$ and the wave packet corrections are important in principle. However because of the nature of the Fermi-Dirac distribution function very few neutrinos of low energies are present and thus this does not make any difference in practice. We have checked numerically that use of the wave packet formula (12) does not change our results.

3.2 Three Generation Analysis

In this section we study the impact of three-generation vacuum oscillation of neutrinos on the freeze-out value of Y_e. We fix one of the mass squared differences $\Delta m_{12}^2 \sim 10^{-5}$ or $10^{-11}eV^2$, corresponding to solar neutrinos and take the other two equal to each other i.e. $\Delta m_{23}^2 \approx \Delta m_{13}^2$, each being in the range $10^{-4} - 10^{-1}eV^2$, suitable for atmospheric neutrino oscillations. The general expression for the probability that an initial ν_α of energy E gets converted to a ν_β after traveling a distance L in vacuum is,

$$P_{\nu_\alpha \nu_\beta} = \delta_{\alpha\beta} - 4\sum_{j>i} U_{\alpha i}U_{\beta i}U_{\alpha j}U_{\beta j} Sin^2 \frac{\pi L}{\lambda_{ij}} \tag{14}$$

where, $\alpha = e, \mu, \tau$ and $i, j = 1, 2, 3$

- $\lambda_{ij} = 2.5 \times 10^{-3} km \frac{E}{MeV} \frac{eV^2}{\Delta m^2{}_{ij}}$
- $\Delta m_{ij}^2 = m_j^2 - m_i^2$

We neglect CP violation in the lepton sector so that probabilities of neutrinos and antineutrinos are same. For the mass spectrum under consideration $\Delta m_{12}^2 \sim 10^{-5}eV^2$ or $10^{-11}eV^2$, so that $sin^2 \frac{\pi L}{\lambda_{12}} \to 0$ and $\Delta m_{13}^2 \approx \Delta m_{23}^2$. Thus one mass scale dominance (OMSD) applies. Using the orthogonality of the mixing matrix U the various probabilities relevant for us are

$$P_{\nu_e \nu_e} = P_{\bar{\nu}_e \bar{\nu}_e} = 1 - 4(|U_{e3}|^2(1 - |U_{e3}|^2)) sin^2 \frac{\pi L}{\lambda_{13}} \tag{15}$$

$$\begin{aligned} P_{\nu_\mu \nu_e} + P_{\nu_\tau \nu_e} &= P_{\bar{\nu_\mu} \bar{\nu}_e} + P_{\bar{\nu_\tau} \bar{\nu}_e} \\ &= 4(|U_{e3}|^2(1 - |U_{e3}|^2)) sin^2 \frac{\pi L}{\lambda_{13}} \end{aligned} \tag{16}$$

We note that since the one mass scale dominance holds good the probabilities are functions of only one mass squared difference. Secondly since the energy spectra of the ν_μ and ν_τ are identical, the conversion probablities to ν_μ and

ν_τ always appear as $P_{\nu_\mu\nu_e} + P_{\nu_\mu\nu_\tau}$ and the probabilities depend on only U_{e3}. For 3 flavor neutrino oscillations the neutrino distribution function becomes

$$f_{\nu_e}^{osc}(E) = P_{\nu_e\nu_e} f_{\nu_e}(E) + (P_{\nu_\mu\nu_e} + P_{\nu_\tau\nu_e}) f_{\nu_\mu}(E) \quad (17)$$

$$f_{\bar{\nu}_e}^{osc}(E) = P_{\bar{\nu}_e\bar{\nu}_e} f_{\bar{\nu}_e}(E) + (P_{\bar{\nu_\mu}\bar{\nu}_e} + P_{\bar{\nu}_\tau\bar{\nu}_e}) f_{\bar{\nu_\mu}}(E) \quad (18)$$

From expression (15) we see that in the limit of OMSD 3 flavor oscillation reduces effectively to 2 flavor case with $sin^2 2\theta$ for the 2 generation case replaced by the corresponding factor $4|U_{e3}|^2(1 - |U_{e3}|^2)$. Therefore just as in the case of 2 flavor oscillations $Y_e < 0.5$ constraint is never violated in our model and hence if this is the only constraint on the value of Y_e then r-process is compatible with the solar neutrino data, the terrestrial accelerator-reactor data (excluding LSND) as well as data coming from atmospheric neutrino anomaly. However if we use $Y_e < 0.45$ we will get a contour in the mass-mixing angle plane same as Fig. 1 with the relevant mass and mixing parameters being Δm_{13}^2 and $4|U_{e3}|^2(1 - |U_{e3}|^2)$ instead of Δm^2 and $sin^2 2\theta$ respectively. It turns out that in the three generation framework that we have chosen, these are also the parameters relevant for the CHOOZ experiment [26]. We have compared the CHOOZ result with our result in Fig. 1. In the range $2 \times 10^{-3} - 10^{-2} eV^2$ CHOOZ gives a slightly stronger constraint on U_{e3} than r-process but there is an overall agreement between the two. Our results like CHOOZ [26] also imply a decoupling of solar and atmospheric neutrino oscillations into separate two-genaration pictures for $\Delta m_{13}^2 > 10^{-3} eV^2$. In the range $10^{-4} \leq \Delta m_{13}^2 \leq 10^{-3} eV^2$ only r-process can give constraints on the mixing parameter U_{e3}.

4 Shock Reheating

We finally study the effect of vacuum neutrino oscillations on supernova dynamics. The prompt shock generally stalls at a radius of ~ 100 km mainly due to energy loss from the shock radiated in neutrinos, energy loss due to dissociation of iron-group nuclei and due to accretion. In the delayed neutrino heating scenario [27] the matter behind the shock is heated by the energy deposited by the neutrinos through the capture processes (1) and (2), leading to an explosion.

If neutrinos have mass they will oscillate leading to the conversion of more energetic muon and tau neutrinos and antineutrinos to electron type neutrinos and antineutrinos, which results in enhanced neutrino energy deposition resulting in a more energetic shock. The effect of MSW conversion [22], resonant spin flavor precession [28] and resonant conversion of massless neutrinos due to flavor-changing neutral current interaction [23] have been considered before. In this work we examine the effect of vacuum neutrino oscillations on the rate of shock reheating and compare it to the corresponding values obtained earlier.

The rate at which energy is deposited by the neutrino capture on nucleons is given by [27]

$$\dot{E} \approx \frac{1}{4\pi R_m^2}[K_n(T_{\nu_e})L_{\nu_e} + K_p(T_{\bar{\nu}_e})L_{\bar{\nu}_e}] \tag{19}$$

where L_{ν_e} and $L_{\bar{\nu}_e}$ are the total ν_e and $\bar{\nu}_e$ liminosities, R_m is the radius and K_n and K_p are the neutrino absorption coefficients due to the reactions (1) and (2)

$$K_\alpha = N_A Y_\alpha \langle\sigma(E_\nu)\rangle, \alpha = p, n \tag{20}$$

where N_A is the Avagadro's number, Y_α is the nucleon number per baryon and $\langle\sigma(E_\nu)\rangle$ is the reaction cross-section [22] averaged over the neutrino spectrum, e.i.

$$\langle\sigma(E_\nu)\rangle = (9.23 \times 10^{-44}) \int_0^\infty f_\nu(E_\nu){E_\nu}^2 dE_\nu \tag{21}$$

In the expression (19) we have neglected the neutrino energy loss term from $e^\pm$ capture for simplicity [22].

The luminosities of the different neutrino species are found to be almost equal in supernova models. For simplicity we consider identical energy spectrum for the ν_e and $\bar{\nu}_e$ and the matter to be composed entirely of free nucleons. Then the heating rate becomes

$$\dot{E} \approx \frac{L_{\nu_e}}{4\pi R_m^2} N_A (9.23 \times 10^{-44}) \int_0^\infty f_{\nu_e}(E_\nu){E_\nu}^2 dE_\nu \tag{22}$$

If one takes into account the neutrino flavor oscillations then the rate of heating is increased by

$$\frac{\dot{E_{osc}}}{\dot{E}} = \frac{\int\limits_0^\infty f_{\nu_e}^{osc}(E_\nu){E_\nu}^2 dE_\nu}{\int\limits_0^\infty f_{\nu_e}(E_\nu){E_\nu}^2 dE_\nu} \tag{23}$$

where $f_{\nu_e}^{osc}$ is given by (17).

We perform our calculations for $\langle E_{\nu_e}\rangle = \langle E_{\bar{\nu}_e}\rangle \approx 15$ MeV and $\langle E_{\nu_\mu}\rangle \approx 21$ MeV [23] and with $L \sim 100$ km. We do our calculations for $\Delta m^2_{12} \sim 10^{-5}$ or $10^{-11} eV^2$ and $\Delta m^2_{23} \approx \Delta m^2_{13} \sim 10^{-4} - 10^{-1} eV^2$. For Δm^2_{13} in the range $10^{-3} - 10^{-1} eV^2$ we use the CHOOZ limiting value of 0.18, which is also consistent with r-process constraints, for the mixing parameter $4|U_{e3}|^2(1 - |U_{e3}|^2)$. For $\Delta m^2_{13} = 10^{-4} eV^2$ there is no constraint from CHOOZ and for this we use the maximum possible value of the mixing parameter allowed by r-process. The value of $\dot{\mathrm{E}}_{\mathrm{osc}}/\dot{\mathrm{E}}$ obtained is presented in Table 1. Since above $2 \times 10^{-3} eV^2$ one gets average oscillations the heating rate is independent of the value of Δm^2. It is seen that the increase in heating obtained is not much. The reason for such low value of $\dot{E_{osc}}/\dot{E}$ can be traced to the very small value of U_{e3} allowed by CHOOZ and r-process. For $\Delta m^2 = 10^{-4} eV^2$ although the mixing parameter can be large the factor $\sin^2 \pi L/\lambda$ itself becomes very small and there is very little effect of oscillations on the heating rate.

5 Conclusion

In this paper we have studied the effect of vacuum neutrino oscillations on supernova nucleosynthesis and dynamics. The mass range that we can probe is from $10^{-4} - 10^{-1} eV^2$. The position of the MSW resonance for such mass squared values is beyond the weak freeze-out radius. We have studied the effect of such oscillations on the freeze-out value of Y_e during the r-process nucleosynthesis epoch and use the condition $Y_e < 0.45$ as the criterion for a successful r-process [23, 5] to give constraints on the mixing between ν_e and ν_μ (or ν_τ) from a two generation analysis. If we assume two-generation $\nu_e - \nu_\mu$ oscillations to be operative then r-process rules out this mode as a solution to the atmospheric neutrino anomaly, a result consistent with CHOOZ. Even the small range that is allowed by CHOOZ at 99% C.L. is not allowed by r-process. On the other hand if we assume two-genaration $\nu_e - \nu_\tau$ oscillations to be operative then this provides the only bound on $\sin^2 2\theta_{e\tau}$ in this mass range.

We next go to the more realistic three-generation picture and take a scenario of mass and mixing angles which simultaneously explains (i) the solar neutrino problem and (ii) the atmospheric neutrino puzzle. The length scales involved are such that the one-mass scale dominance limit applies and the probabilities are functions of only one Δm^2 and one gets an effective two-generation picture with Δm^2 replaced by $\Delta m_{13} \approx \Delta m_{23}$ and $sin^2 2\theta$ replaced by $4U_{e3}^2(1 - U_{e3}^2)$. In the three-generation scheme under consideration these are also the parameters relevant for CHOOZ and a comparison of the two shows that our constraints on the mixing angles for the mass range $2 \times 10^{-3} - 10^{-1} eV^2$ are somewhat weaker. But for the mass range $10^{-4} - 10^{-3} eV^2$ only r-process can give bounds on the mixing parameters $\theta_{e\mu}(\theta_{e\tau})$ in two- generations and U_{e3} in three-generations.

We also calculate the increase in the shock-reheating obtained via vacuum neutrino oscillations. In three-generation framework with the CHOOZ constraint we obtain $\dot{E_{osc}}/\dot{E} \approx 1.1$. Since U_{e3} is very low the mixing is very small resulting in very little change in the heating rate.

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Table 1. The ratio of the heating rate with and without oscillations for different mass and mixing parameters.

Δm^2_{13}	$4\|\mathrm{U_{e3}}\|^2(1-\|\mathrm{U_{e3}}\|^2)$	$\dot{\mathrm{E}}_{\mathrm{osc}}/\dot{\mathrm{E}}$
$10^{-1}eV^2$	0.18	1.087
$10^{-2}eV^2$	0.18	1.087
$10^{-3}eV^2$	0.18	1.073
$10^{-4}eV^2$	0.90	1.067

Figure Caption

Fig 1. The $Y_e = 0.45$ contours at two different post bounce times 3s (L=90 km) and 12s (L=45 km). The region to the left of the lines is allowed by r-process. The mass and mixing parameters relevant for three-flavor mixing are given within brackets. Also shown are the allowed parameters of the $\nu_\mu - \nu_e$ oscillation mode for the atmospheric neutrino data for all experiments combined at 90% and 99% C.L. and the exclusion plot from the CHOOZ experiment.

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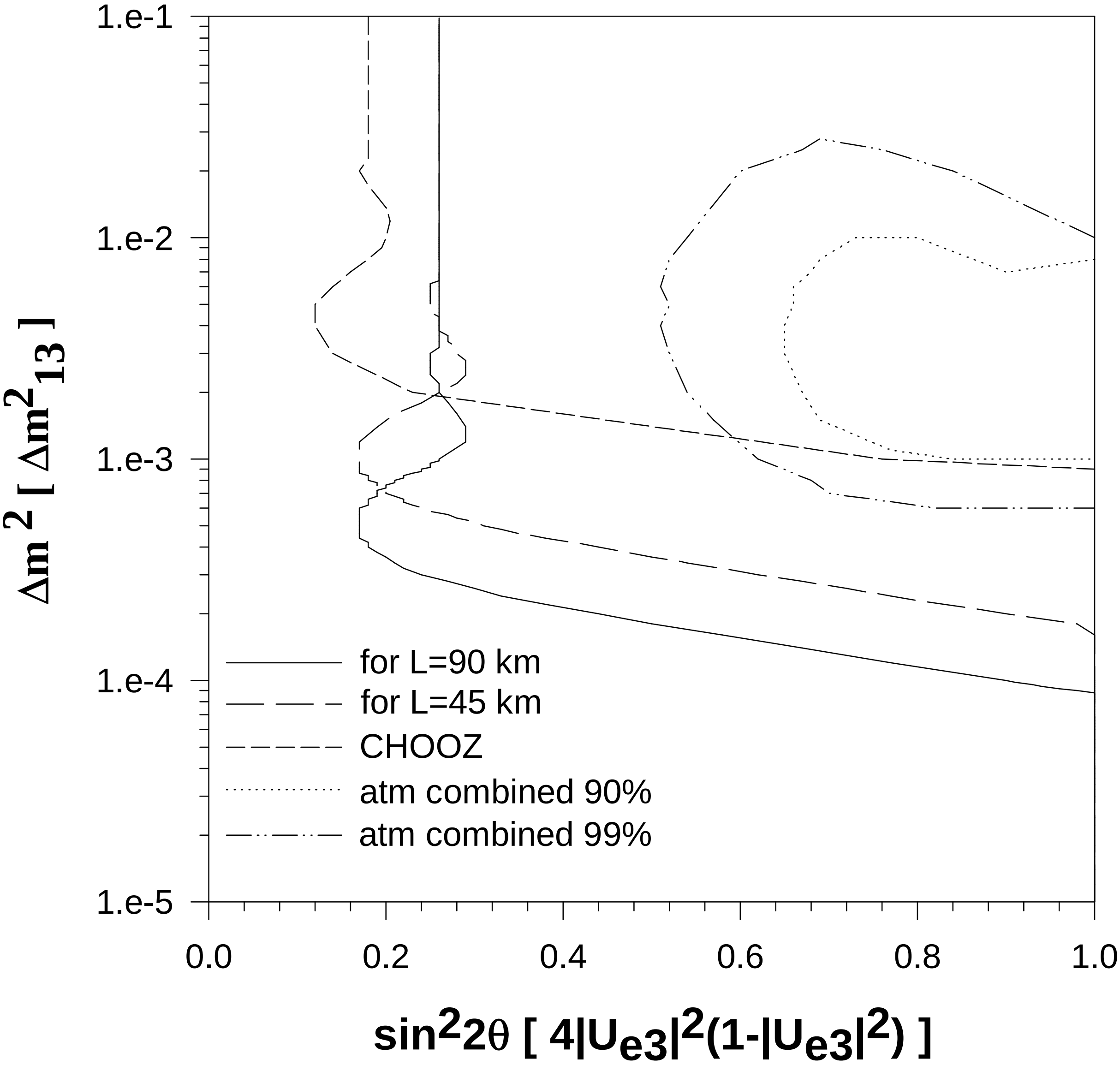

1.e-1
1.e-2
1.e-3
1.e-4
1.e-5
0.0
0.2
0.4
0.6
0.8
1.0
Δm 2 [Δm2 13]
sin22θ [4|Ue3|2(1-|Ue3|2)]
for L=90 km
for L=45 km
CHOOZ
atm combined 90%
atm combined 99%

Fig. 1