

Perturbative QCD description of multiparticle correlations in quark and gluon jets

Sergio Lupia*

*Max-Planck-Institut für Physik (Werner-Heisenberg-Institut),
Föhringer Ring 6, 80805 München, Germany*

Abstract

The QCD evolution equations in Modified Leading Log Approximation for the factorial moments of the multiplicity distribution in quark and gluon jets are numerically solved with initial conditions at threshold by fully taking into account the energy conservation law. After applying Local Parton Hadron Duality as hadronization prescription, a consistent quantitative description of available experimental data for factorial cumulants and factorial moments of arbitrary order and for their ratio both in quark and gluon jets and in e^+e^- annihilation is achieved.

Keywords: Perturbative QCD, jets, multiparticle dynamics, hadronization, e^+e^- annihilation, multiplicity moments.

PACS Numbers: 12.38.Bx, 13.65.+i.

*E-mail address: lupia@mppmu.mpg.de

1 Introduction

The analytical perturbative approach to multiparticle production in jets [1] is widely used to describe inclusive observables in e^+e^- annihilation. In this framework, a parton cascade is evolved down to small scales of a few hundred MeV for the transverse momentum cutoff Q_0 , and partonic predictions are directly compared to hadronic observables, according to the notion of local parton hadron duality (LPHD) [2]. The description of the parton cascade is given within the so-called modified leading logarithmic approximation (MLLA), which takes into account the leading double logarithmic and next to leading single logarithmic terms; in this approximation, QCD coherence is included by means of angular ordering, and the running of the coupling α_s at one-loop level and energy conservation effects are included as well.

This approach has been applied to a large variety of experimental data and found to be rather successful (see [3] for a recent review). In particular, by considering predictions which fully take into account the effects of the energy conservation law, a quantitative description of available experimental data for single particle observables, like for instance the momentum spectrum [4] and both the jet and the particle multiplicity [5], has been achieved. However, in the case of observables related to genuine multiparticle correlations, like factorial cumulants and factorial moments of the multiplicity distributions, the available theoretical predictions [6], which include only partially the energy conservation effects, do not quantitatively reproduce the experimental data [7].

In this letter we extend the results of [5] for the average multiplicity and we present a numerical solution of the complete MLLA evolution equation of QCD for factorial moments of any order with the full inclusion of energy conservation effects. In this way, we can quantitatively describe within the purely perturbative approach recent experimental data on multiparticle correlations both in single quark and gluon jets and in e^+e^- annihilation, thus lending further support to the analytical perturbative approach to multiparticle production.

2 The theoretical framework

In order to study genuine multiparticle correlations in a quark or gluon jet, it is convenient to study the properties of the (unnormalized) factorial moments of order q , given in terms of the multiplicity distribution P_n by

$$\tilde{F}^{(q)} = \sum_{n=q}^{\infty} n(n-1)\dots(n-q+1)P_n, \quad (1)$$

and the (unnormalized) factorial cumulants of order q , related to the factorial moments by a cluster expansion:

$$\tilde{K}^{(q)} = \tilde{F}^{(q)} - \sum_{i=1}^{q-1} \binom{q-1}{i} \tilde{K}^{(q-i)} \tilde{F}^{(i)}. \quad (2)$$

Factorial moments and factorial cumulants are integrals of q -particle inclusive distributions and q -particle correlations functions, respectively. Normalized observables are obtained by using the average multiplicity $\bar{n} = F^{(1)} = K^{(1)}$:

$$F^{(q)} = \frac{\tilde{F}^{(q)}}{[\bar{n}]^q}, \quad K^{(q)} = \frac{\tilde{K}^{(q)}}{[\bar{n}]^q} \quad (3)$$

Also the ratio of factorial cumulants over factorial moments

$$H_q = K_q / F_q \quad (4)$$

has been widely studied.

One usually also considers the generating functions of the multiparton final states in quark and gluon jets, $Z_a(u)$, from which the factorial moments can be obtained via a Taylor expansion around $u = 1$ (here $a = q, g$ for quark and gluon jet respectively):

$$\tilde{F}_a^{(q)} = \frac{d^q Z_a(u)}{du^q} \Big|_{u=1}, \quad (5)$$

and similarly for the cumulant moments

$$\tilde{K}_a^{(q)} = \frac{d^q \ln Z_a(u)}{du^q} \Big|_{u=1}. \quad (6)$$

The evolution equations for the generating functions $Z_a(u)$ have been derived within a probabilistic description of the parton splitting processes $A \rightarrow B + C$, with the inclusion of angular ordering, energy conservation and the running coupling at the one-loop order[1,8,9] (we explicitly show the energy dependence and drop the label u everywhere for clarity):

$$\left\{ \begin{array}{l} \frac{dZ_g(\eta)}{d\eta} = \int_{z_c}^{1-z_c} dz \frac{\alpha_s(\tilde{k}_\perp)}{2\pi} [\Phi_{gg}^{asy}(z) \{Z_g(\eta + \ln z) Z_g(\eta + \ln(1-z)) - Z_g(\eta)\} + \\ \quad n_f \Phi_{gq}(z) \{Z_q(\eta + \ln z) Z_q(\eta + \ln(1-z)) - Z_q(\eta)\}] \\ \frac{dZ_q(\eta)}{d\eta} = \int_{z_c}^{1-z_c} dz \frac{\alpha_s(\tilde{k}_\perp)}{2\pi} \Phi_{qg}(z) \{Z_g(\eta + \ln z) Z_q(\eta + \ln(1-z)) - Z_q(\eta)\} \end{array} \right. \quad (7)$$

Here the evolution variable $\eta = \ln \frac{\kappa}{Q_0}$ is related to the jet virtuality $\kappa = Q \sin \frac{\Theta}{2}$, where $Q = 2E$ is the hard scale involved in the process, and E is in this case the jet energy, Θ denotes the maximum angle between the outgoing partons B and C and Q_0 is the infrared cutoff in transverse momentum.

The splitting functions Φ_{AB} for parton splittings $A \rightarrow B$ [10] are taken with normalization as in [1,5]; here $\Phi_{gg}^{asy}(z) = (1-z)\Phi_{gg}(z)$ can replace $\frac{1}{2}\Phi_{gg}(z)$ thanks to symmetry properties of the kernel [9]. N_C and n_f denote the number of colours and flavours. The coupling is given by $\alpha_s^{(n_f)}(\tilde{k}_\perp) = 2\pi/(b \ln(\tilde{k}_\perp/\Lambda))$ with $b = (11N_C - 2n_f)/3$. $\alpha_s(\tilde{k}_\perp)^{-1}$ evolves with \tilde{k}_\perp with a smooth treatment of the heavy quark thresholds at the effective mass $m_i^* = e^{5/6}m_i/2 \simeq 1.15m_i$ as in [5]. The boundaries of the integrals are determined by the lower cutoff in the transverse momentum measure \tilde{k}_\perp defined according to the Durham jet finder algorithm[11] $\tilde{k}_\perp = \min(z, 1-z)\kappa \geq Q_c$:

$$z_c = \frac{Q_c \sqrt{2}}{Q} = \sqrt{2y_c} = e^{-\eta}. \quad (8)$$

By taking the derivatives now in eq. (7) according to eq. (5), one then obtains a system of coupled evolution equations for the unnormalized factorial moments for a single quark or gluon jet:

$$\left\{ \begin{array}{l} \frac{d\mathcal{N}_g(\eta)}{d\eta} = \int_{z_c}^{1-z_c} dz \frac{\alpha_s(\tilde{k}_\perp)}{2\pi} [\Phi_{gg}^{asy}(z) \{ \mathcal{N}_g(\eta + \ln z) + \mathcal{N}_g(\eta + \ln(1-z)) - \mathcal{N}_g(\eta) \} + \\ \quad n_f \Phi_{gq}(z) \{ \mathcal{N}_q(\eta + \ln z) + \mathcal{N}_q(\eta + \ln(1-z)) - \mathcal{N}_g(\eta) \}] \\ \frac{d\mathcal{N}_q(\eta)}{d\eta} = \int_{z_c}^{1-z_c} dz \frac{\alpha_s(\tilde{k}_\perp)}{2\pi} \Phi_{qg}(z) \{ \mathcal{N}_g(\eta + \ln z) + \mathcal{N}_q(\eta + \ln(1-z)) - \mathcal{N}_q(\eta) \} \\ \quad \vdots \\ \frac{d\tilde{F}_g^{(q)}(\eta)}{d\eta} = \int_{z_c}^{1-z_c} dz \frac{\alpha_s(\tilde{k}_\perp)}{2\pi} [\Phi_{gg}^{asy}(z) \times \\ \quad \{ \sum_{m=0}^q \binom{q}{m} \tilde{F}_g^{(m)}(\eta + \ln z) \tilde{F}_g^{(q-m)}(\eta + \ln(1-z)) - \tilde{F}_g^{(q)}(\eta) \} + \\ \quad n_f \Phi_{gq}(z) \{ \sum_{m=0}^q \binom{q}{m} \tilde{F}_q^{(m)}(\eta + \ln z) \tilde{F}_q^{(q-m)}(\eta + \ln(1-z)) - \tilde{F}_g^{(q)}(\eta) \}] \\ \frac{d\tilde{F}_q^{(q)}(\eta)}{d\eta} = \int_{z_c}^{1-z_c} dz \frac{\alpha_s(\tilde{k}_\perp)}{2\pi} \Phi_{qg}(z) \times \\ \quad \{ \sum_{m=0}^q \binom{q}{m} \tilde{F}_g^{(q-m)}(\eta + \ln z) \tilde{F}_q^{(q-m)}(\eta + \ln(1-z)) - \tilde{F}_q^{(q)}(\eta) \} \end{array} \right. \quad (9)$$

Since $z_c = e^{-\eta} \leq \frac{1}{2}$, the system of differential equations (9) is defined for $\eta \geq \ln 2$ only. Its initial conditions for $0 \leq \eta \leq \ln 2$ read

$$\begin{aligned} \mathcal{N}_g = \mathcal{N}_q &= 1 \\ \tilde{F}_g^{(q)} = \tilde{F}_q^{(q)} &= 0 \quad \text{for } q > 1 \end{aligned} \quad (10)$$

No analytical solution of the complete equations (9) with the boundary conditions (10) has been so far obtained. A numerical solution of the two coupled equations for the average multiplicity has been given in [5]. For high order factorial moments, both solutions within high energy approximations at leading[12] and next-to-leading order[13] and a solution of yet higher order with partially takes into account the energy conservation law[6] have been obtained. Notice that these solutions do not fulfill the absolute normalization at threshold, where they reach unphysical negative values for the moments.

Here we numerically solve the system of equations (9) with boundary conditions at threshold (10) for the first 20 factorial moments.

2.1 From one to two hemispheres

We have so far considered particle production inside a single quark or gluon jet. In this framework, a single quark jet is defined in an inclusive sense, i.e., it experimentally corresponds to a single hemisphere in a e^+e^- annihilation event. At low cms energies this factorization of the two hemispheres does not hold anymore and nonlogarithmic corrections become important. At large cms energies, however, the behaviour of a whole e^+e^- annihilation event is controlled by the generating function

$$Z_{two-hem}(u) = [Z_q(u)]^2 \quad \Leftrightarrow \quad \ln Z_{two-hem}(u) = 2 \ln[Z_q(u)] \quad (11)$$

It is then easy to relate the moments for the whole event to the moments for one hemisphere that one can calculate according to the evolution equation (9). One obtains indeed for the factorial cumulants:

$$\bar{n}_{two-hem} = 2\bar{n}_q \quad , \quad \tilde{K}_{two-hem}^{(q)} = 2\tilde{K}_q^{(q)} \quad (12)$$

and for normalized cumulants

$$K_{two-hem}^{(q)} = \frac{K_q^{(q)}}{2^{q-1}} \quad (13)$$

Factorial moments can then be calculated from the factorial cumulants by inverting eq. (2). For instance, for the second order factorial moment, one

q	$F_q^{(q)}$ Quark jet		$F_g^{(q)}$ Gluon jet	
	Exp[7]	Theory	Exp[7]	Theory
2	$1.0820 \pm 0.0006 \pm 0.0046$	1.080	$1.023 \pm 0.008 \pm 0.011$	1.026
3	$1.275 \pm 0.002 \pm 0.017$	1.265	$1.071 \pm 0.026 \pm 0.034$	1.078
4	$1.627 \pm 0.005 \pm 0.042$	1.600	$1.146 \pm 0.059 \pm 0.074$	1.157
5	$2.274 \pm 0.014 \pm 0.093$	2.168	$1.25 \pm 0.11 \pm 0.13$	1.268

Table 1: Experimental data for the normalized factorial moments of order $q = 2, \dots, 5$ in single quark and gluon jets of 45.6 and 41.8 GeV respectively [7] are compared with our theoretical predictions with parameters given in (15).

gets:

$$\begin{aligned}
\tilde{F}_{two-hem}^{(2)} &= [\bar{n}_{two-hem}]^2 + 2[\tilde{F}_q^{(2)} - \bar{n}_q^2] \\
&= 2[\bar{n}_q^2 + \tilde{F}_q^{(2)}]
\end{aligned} \tag{14}$$

The simple treatment of the whole e^+e^- event in terms of a superposition of two independent single quark jets have been improved for the average multiplicity by explicitly including the $\mathcal{O}(\alpha_s)$ matrix element for $e^+e^- \rightarrow 3$ partons[14,15]. In this way, an improved description of data in the low cms energy region could be achieved[4]. Since we are here mainly interested in the high energy regime of the moments, where the correction is negligible, we will use in the following the aforementioned simple ansatz.

3 Phenomenology of high order correlations

3.1 Correlations in single quark and gluon jets

Both the evolution equation (7) and eq. (9) refer to parton production inside a single quark or gluon jet in a fully inclusive configuration. This is a very important point for the phenomenological application of these predictions. It is indeed not possible to directly compare our predictions with data extracted in different configurations, like for instance data on single quark and gluon jets from 3-jet events with the Mercedes configuration, where topology has been shown to play an important role [16,17]. However, it is possible to study a particular subsample of 3-jet events, with a very energetic gluon-jet practically

q	1	2	3	4	5	6
$F_q^{(q)}$	1	0.07431	0.01922	0.00135	-0.00211	-0.00150
$F_g^{(q)}$	1	0.028737	0.000330	-0.000197	-0.000125	-0.000024
q	7	8	9	10	11	12
$F_q^{(q)}$	-0.00018	0.00056	0.00052	0.00003	-0.00041	-0.00043
$F_g^{(q)}$	0.000033	0.000004	-0.000005	-0.000005	0.000007	-0.000009
q	13	14	15	16	17	18
$F_q^{(q)}$	0.00004	0.00060	0.00066	-0.00023	-0.00155	-0.00169
$F_g^{(q)}$	-0.000015	-0.000018	0.000012	0.000001	-0.000024	0.000067

Table 2: Predictions for the ratios H_q in single quark and gluon jets of 45.6 and 41.8 GeV respectively with parameters given in (15).

back-to-back to the quark-antiquark pair, where the experimental configuration is comparable to the theoretical one[18]. This analysis has been performed by the OPAL Collaboration both for the average multiplicity[19] and for higher order factorial moments[7] for quark jets of 45.6 GeV and gluon jets of 41.8 GeV. Table (1) compares the experimental data for the first four factorial moments inside a single quark and gluon jet with the theoretical predictions obtained by numerical solution of eq. (9) with the boundary conditions (10). The free parameters of the theory, i.e., the infrared cutoff Q_0 , the QCD scale Λ and the normalization parameter K_{all} , have been kept equal to the values fixed in [5] to describe the jet and particle multiplicities:

$$K_{all} = 1, \quad \Lambda = 500 \text{ MeV} \quad Q_0 = 0.507 \text{ MeV} \quad (15)$$

A very good quantitative agreement between data and the predictions of the complete theory is achieved, contrary to previous comparisons with approximate predictions[7].

To be more explicit, Fig. 1 compares the data for factorial cumulants of order $q = 2$ up to 5 in single quark jets of 45.6 GeV and single gluon jets of 41.8 GeV with the predictions computed in this paper and with the predictions of [6]. It can be seen that the full inclusion of energy conservation is very important for these observables and that, by including this effect, the perturbative approach can quantitatively describe experimental data in this configuration.

It is also possible to obtain predictions for high order moments in single quark and gluon jets. Predictions for the ratio H_q inside single quark and gluon jets up to order 18 are shown in Table 2. It is particularly interesting to notice that oscillations of the H_q moments are up to two orders of magnitude larger in quark jets than in gluon jets.

3.2 Correlations in e^+e^- annihilation

Fig. 2 shows the dependence on $Y = \ln(\sqrt{s}/Q_0)$ of the second order normalized factorial moment $F^{(2)}$. Experimental data[20] are compared with the leading[12] and next-to-leading predictions[13] and with our predictions, where the result for the full event has been obtained from the prediction for a single quark-jet according to eq. (13) in the following way:

$$F_{e^+e^-}^{(2)} = 1 + \frac{K_q^{(2)}}{2} \quad (16)$$

Also in this case, our improved description is in quantitative agreement with the experimental data and reaches an accuracy comparable to the widely used Monte Carlo generators.

In order to investigate whether our predictions give a good description of correlations of yet higher order as well, we have also compared them with the data for the ratio of factorial cumulants over factorial moments, H_q , as a function of the order q . It has been shown in [21] that an important part of the oscillatory behaviour of the H_q moments actually comes from the superposition of two different samples of two-jet and multi-jets events, an effect which is not taken into account in our simple treatment of correlations between the two hemispheres according to (13). At the moment, we cannot then aim at a description of data for the whole e^+e^- annihilation events [22], which requires an improved treatment of the composition of the two hemispheres, but we should look at data for two-jet events only. We have therefore compared in Fig. 3 our predictions with data for the H_q moments extracted as in [21] from experimental data[23] on multiplicity distributions in three samples of two-jet events, defined according to the JADE jet finder algorithm with three different values of the jet resolution parameter y_{cut} . One can easily see that the theory can describe the gross features of the data for correlations of arbitrary order as well. In this case, a more quantitative comparison should not be pursued, since data have been extracted by using a different jet finder algorithm than the one used in the theory.

4 Conclusions

We have numerically solved the QCD evolution equations in MLLA for factorial moments of any order in quark and gluon jets, by fully taking into account the effects of energy conservation law. By keeping the same parameters previously fixed in order to describe the mean jet and particle multiplicities, a quantitative description of existing data on factorial moments, factorial cumulants and their ratio both in single quark and gluon jets and in the two-jet sample of e^+e^- annihilation has been achieved. The analytical perturbative approach, based on perturbative QCD description of parton evolution assuming Local Parton Hadron Duality, is then shown to be quantitatively successful in describing genuine multiparticle correlations inside jets.

Acknowledgements

I thank Wolfgang Ochs for many useful discussions and suggestions and a careful reading of the manuscript.

References

- [1] Yu. L. Dokshitzer, V. A. Khoze, A. H. Mueller and S. I. Troyan, Rev. Mod. Phys. **60** (1988) 373; “Basics of Perturbative QCD”, ed. J. Tran Thanh Van, Editions Frontières, Gif-sur-Yvette, 1991.
- [2] Ya. I. Azimov, Yu. L. Dokshitzer, V. A. Khoze and S. I. Troyan, Z. Phys. **C27** (1985) 65 and **C31** (1986) 213.
- [3] V.A. Khoze and W. Ochs, Int. J. Mod. Phys. **A12** (1997) 2949.
- [4] S. Lupia and W. Ochs, Eur. Phys. J. **C2** (1998) 307.
- [5] S. Lupia and W. Ochs, Phys. Lett. **B418** (1998) 214.
- [6] I. M. Dremin, B. B. Levtchenko and V. A. Nechitailo, Sov. J. Nucl. Phys. **59** (1994) 1091.
- [7] OPAL Coll., K. Ackerstaff et al., Eur. Phys. Journ. **C1** (1998) 479.
- [8] Yu. L. Dokshitzer and S. I. Troyan, Proc. 19th Winter School of the LNPI, Vol. 1, p.144; Leningrad preprint LNPI-922 (1984).
- [9] Yu. L. Dokshitzer and M. Olsson, Nucl. Phys. **B396** (1993) 137.

- [10] G. Altarelli and G. Parisi, Nucl. Phys. **B126** (1977) 298;
Yu. L. Dokshitzer, Sov. Phys. JETP, **46** (1977) 641.
- [11] Report of the Hard QCD Working Group: W. J. Stirling, J. Phys. **G17**
(1991) 1567.
- [12] Yu. L. Dokshitzer, V. S. Fadin and V. A. Khoze, Z. Phys. **C18** (1983) 37.
- [13] E. D. Malaza, B. R. Webber, Phys. Lett. **B149** (1984) 501; Nucl. Phys.
B267 (1986) 70.
- [14] N. Brown and W. J. Stirling, Phys. Lett. **B252** (1990) 657; Z. Phys. **C53**
(1992) 629.
- [15] S. Catani, Yu. L. Dokshitzer, F. Fiorani and B. R. Webber, Nucl. Phys.
B377 (1992) 445.
- [16] ALEPH Coll., R. Barate et al., Z. Phys. **C76** (1997) 191.
- [17] P. Eden, preprint LU TP 98-11, hep-ph/9805228, May 1998, subm. to
JHEP
- [18] J. W. Gary, Phys. Rev. **D49** (1994) 4503.
- [19] OPAL Coll., G. Alexander et al., Phys. Lett. **B388** (1996) 659.
- [20] M. Schmelling, Physica Scripta **51** (1995) 683.
- [21] A. Giovannini, S. Lupia and R. Ugoccioni, Phys. Lett. **B374** (1996) 231.
- [22] SLD Coll., K. Abe et al., Phys. Lett. **B371** (1996) 149.
- [23] DELPHI Coll., P. Abreu et al., Z. Phys. **C56** (1992) 63.

Figure Captions

Fig. 1: **a** Data on the first 4 normalized factorial cumulants for single quark-jets of 45.6 GeV as measured by the OPAL collaboration[7] (diamonds) compared with predictions by Dremin et al. [6] (triangle) and with our predictions (square); **b**: same as in **a**, but for single gluon jets of 41.8 GeV.

Fig. 2: Second order normalized factorial moment $F^{(2)}$ as a function of $Y = \ln(\sqrt{s}/2Q_0)$, with $Q_0 = 0.507$ GeV; comparison of experimental data[20] with leading order predictions[12] (dashed line), next-to-leading order predictions[13] (dotted line) and our prediction (solid line).

Fig. 3: Ratio H_q of factorial cumulants over factorial moments as a function of the order q ; data for two-jet events as extracted in [21] from experimental data on MD's measured by DELPHI Collaboration[23] with three different values of jet resolution parameters within the JADE algorithm ($y_c = 0.01$ (diamond), 0.02 (triangle) and 0.04 (square)) are compared with our predictions (solid line).

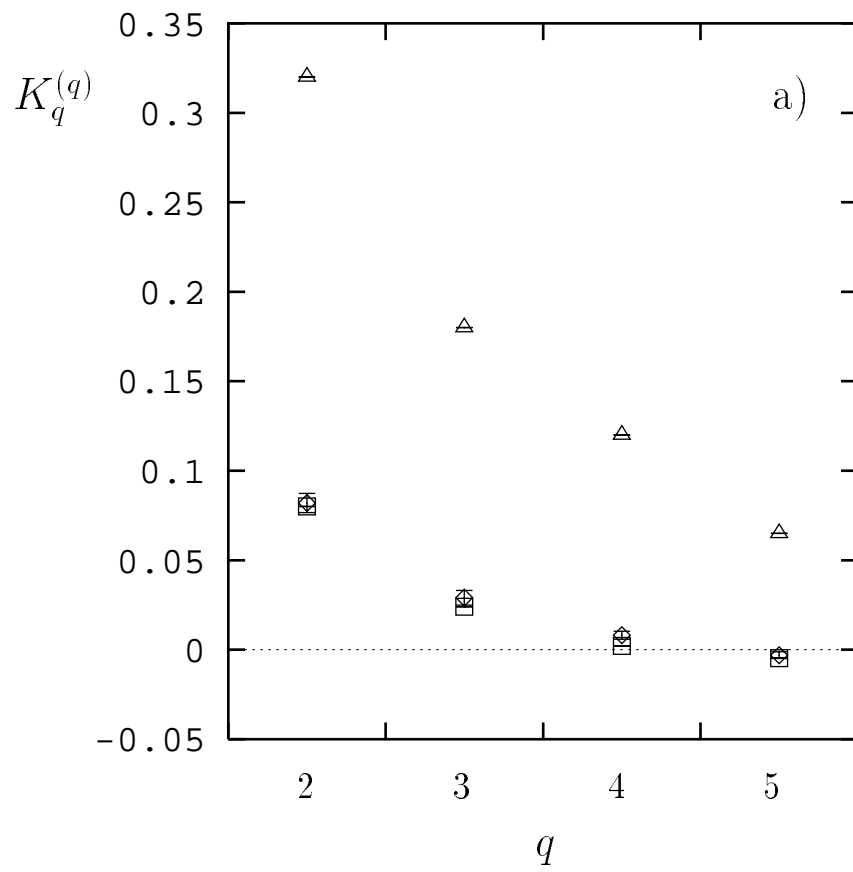


Figure 1: a

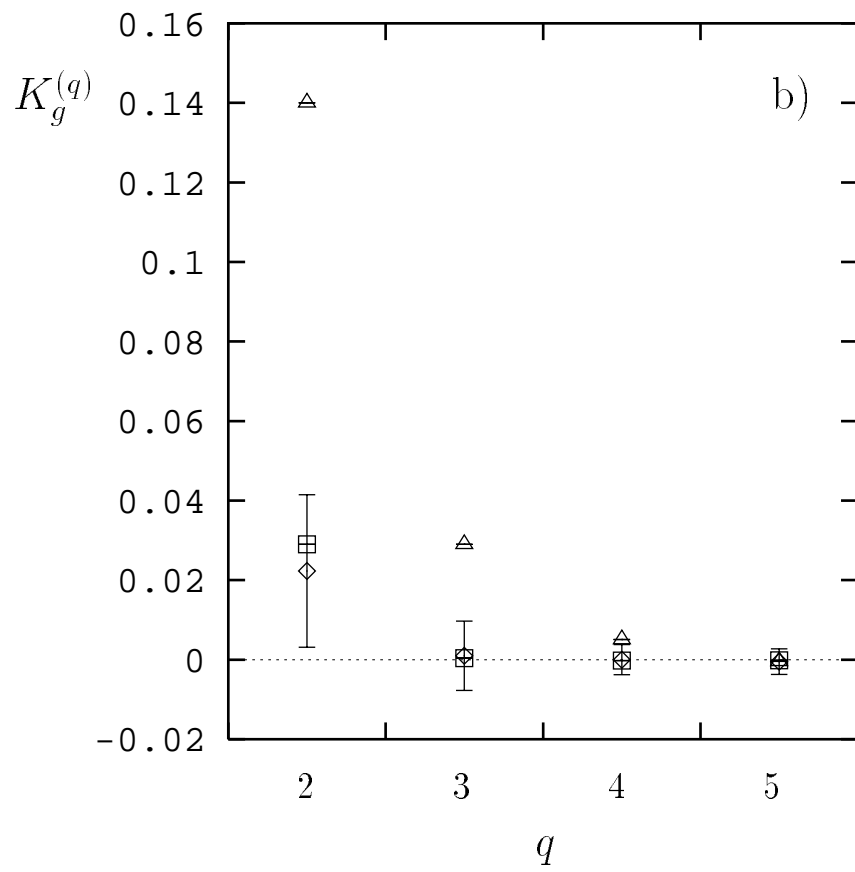


Figure 1: b

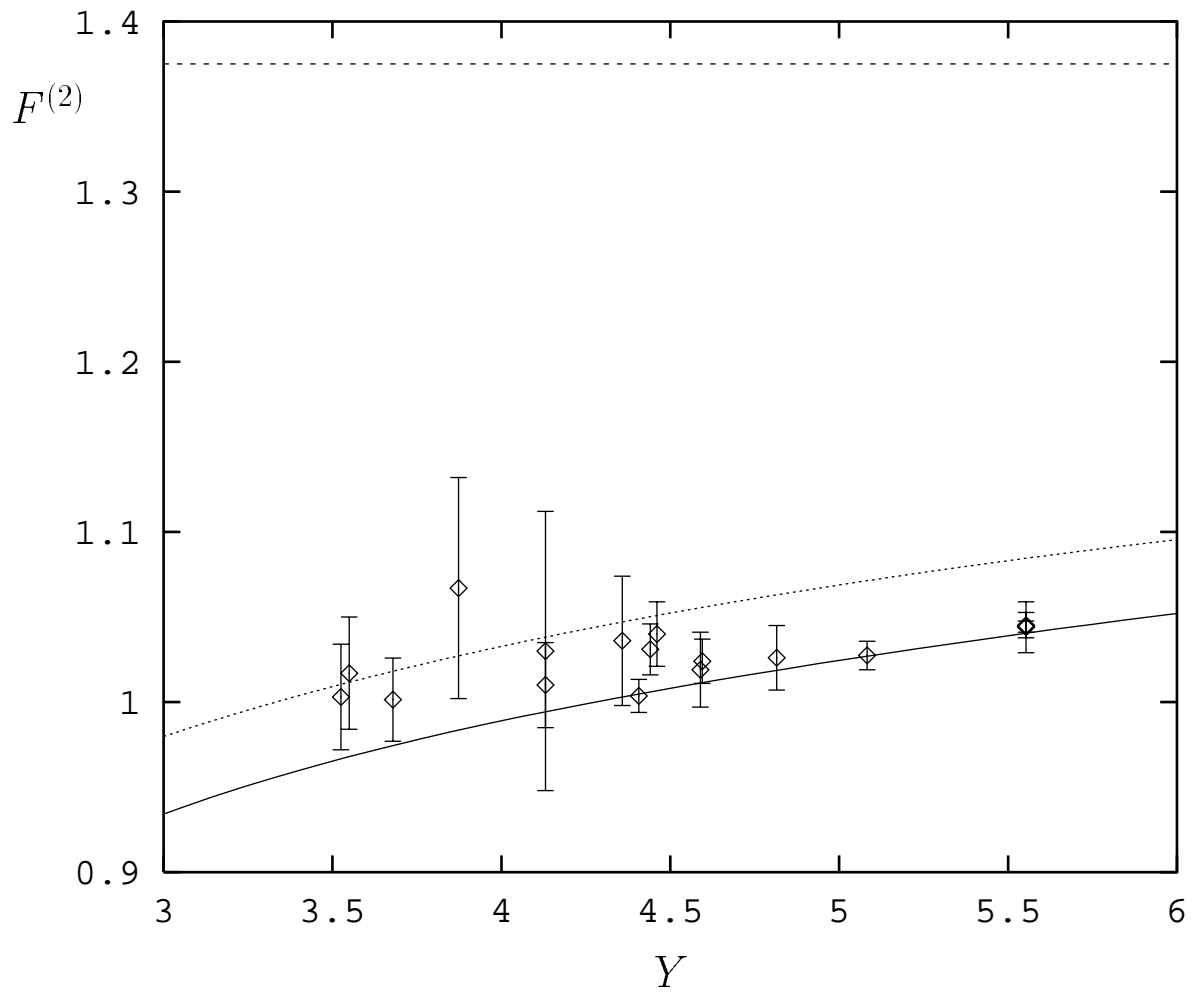


Figure 2:

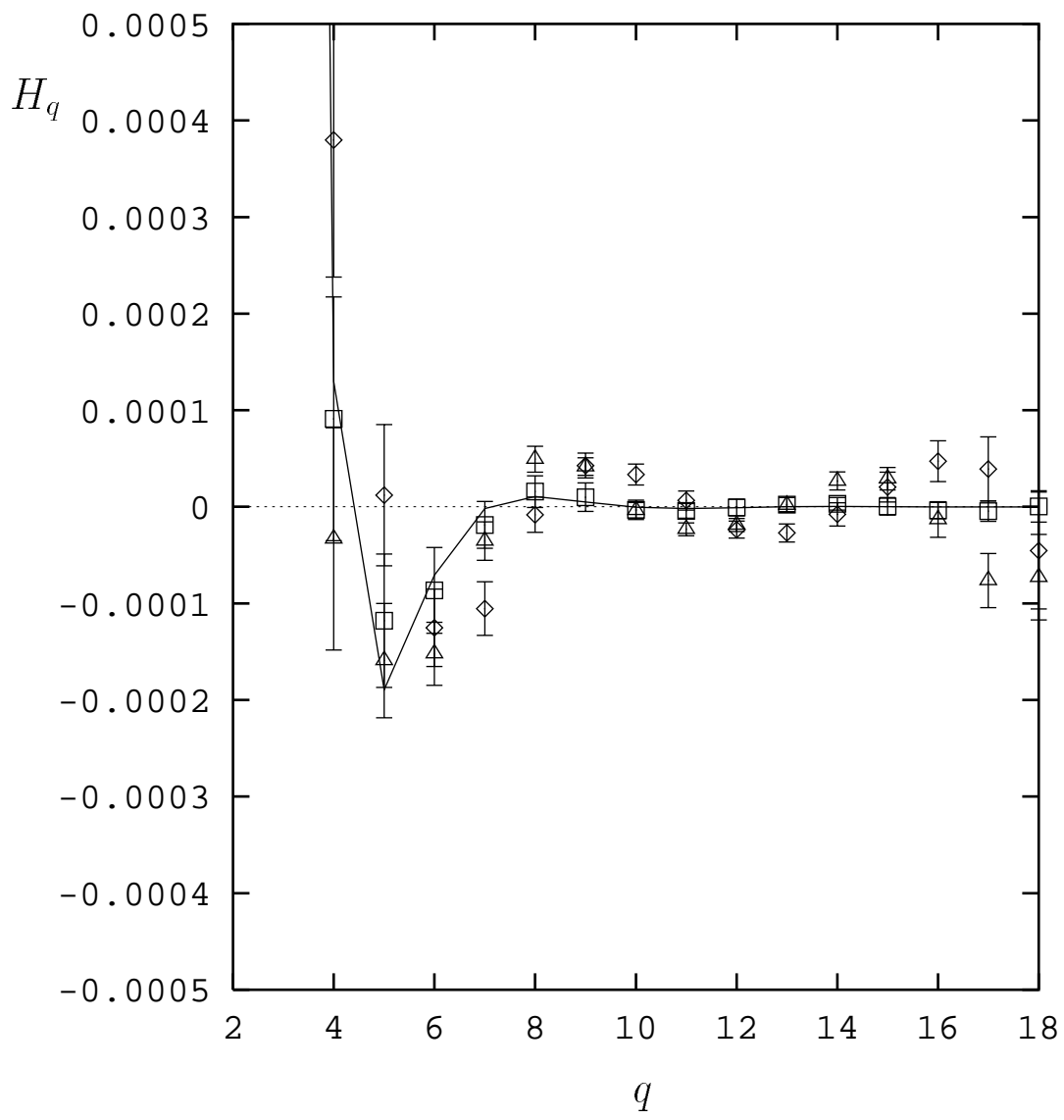


Figure 3: