Cherenkov radiation of magnon and phonon by the slow magnetic monopole.

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The Cherenkov radiation of magnons at passage of the heavy slow magnetic monopole through an ordered magnetic matter is considered. Also the Cherenkov radiation of phonons at monopole movement in medium is discussed.

1 Introduction

The concept of magnetic monopole has been entered into modern physics in 1931 by Paul Dirac [1]. He supposed the existence of isolated magnetic charge g — $ge = \frac{n}{2}\hbar c$, where e — electrical charge, $\hbar$ — Planck constant, c — the light speed , $n = \pm 1, 2...$ — integer. Numerous and unsuccessful attempts of experimental search for magnetic monopole on accelerators [2, 3] and in cosmic rays [4, 5] were done since then.

The new interest to this problem has arised in 1974, when Polyakov [6] and 't Hooft [7] have shown, that such objects exist as solutions in a wide class of models with spontaneously broken symmetry.

The registration of Dirac's monopole or evaluation of their flux limit will the essential contribution to construction of Grand Unified Theory, as well as give a pulse to the decision of many problems in astrophysics. Therefore the study of various mechanisms of the Dirac's monopole interaction with medium is important as from fundamental, as from applied point of view. We consider here the Cherenkov radiation of magnons at passage of heavy slow moving magnetic monopole through an ordered magnetic matter and discuss also the Cherenkov radiation of phonons at monopole movement in matter.

2 Excitation of spin wave Cherenkov radiation by the heavy magnetic monopole.

As well known, the slowly moving heavy monopole cannot emit usual Cherenkov radiation in ferromagnetic media, because the phase speed of electromagnetic waves is of the order $c/10$ and always much faster than the monopole speed.

We shall consider the slow monopole passage through an ordered magnetic matter. In such case a main mechanism of kinetic energy loss is the Cherenkov radiation of magnons. This is because the magnon phase velocity reaches zero and the coupling of monopole to magnons is linear and large [8].

For definiteness we shall consider a ferromagnet, but the evaluations below are of more general character.

Magnon's Hamiltonian in presence of magnetic field of a moving monopole can be written in the form

$$H = \sum_{\mathbf{k}} \hbar\omega_{\mathbf{k}} a^{\dagger}_{\mathbf{k}} a_{\mathbf{k}} + \sum_{\mathbf{k}} \left(f_{\mathbf{k}} e^{-i\Omega_{\mathbf{k}} t} a^{\dagger}_{\mathbf{k}} + c.c \right) \ , \qquad (1)$$

where $a^{\dagger}_{\mathbf{k}}$ — operator of a magnon birth with a wave vector $\mathbf{k}$, $\omega_{\mathbf{k}}$—his dispersion law, $\Omega_{\mathbf{k}} = \mathbf{kv}$, $\mathbf{v}$—vector of a monopole speed and $f_{\mathbf{k}}$ — coupling factor of a monopole magnetic field $\mathbf{B} = g\nabla\frac{1}{r}$ with magnon.

The magnon energy, radiated in a unit of time, is

$$\epsilon = \frac{2\pi}{\hbar} \sum_{\mathbf{k}} \omega_{\mathbf{k}} |f_{\mathbf{k}}|^2 \delta(\Omega_{\mathbf{k}} - \omega_{\mathbf{k}}) \ . \qquad (2)$$

Let the monopole speed $\mathbf{v}$ be directed along the direction of the spontaneous magnetization, along Z-axis. Z [1]. Then

$$F_{\mathbf{k}} = \frac{4\pi g \mu_B}{a^{3/2}\sqrt{V}} \sqrt{\frac{S}{2}} \frac{k_x - ik_y}{k^2} \ , \qquad (3)$$

here a is the lattice constant, V is the sample volume, S is the spin size on the node and μ_B is the Bohr magneton.

Taking into consideration (3) the equation for ϵ can be written as

$$\epsilon = \frac{2g^2\mu_B^2 S}{a^3\hbar} \int d^3\mathbf{k}\,\omega_{\mathbf{k}} \frac{k_x^2 + k_y^2}{k^4} \delta(k_z v - \omega_{\mathbf{k}}) \ . \qquad (4)$$

[1] the general case is investigated absolutely similarly and the result differs only by a factor close to 1.

The integration in (4) is performed on the first Brillouin zone.

If $v \geq u$, where u — magnon speed near a border of Brillouin zone, then the magnons with large $\mathbf{k}$ are essential. Then

$$\epsilon \simeq \frac{\bar{\omega} g^2 \omega_M}{v} , \tag{5}$$

where the frequency $\omega_M = \frac{4\pi\mu_B^2 S}{\hbar a^3}$ characterizes magnetization of media [9],

$$\bar{\omega} = \frac{1}{2\pi} \int \frac{d^2\mathbf{k}_\perp}{k_\perp^2} \omega_{k_\perp} , \tag{6}$$

here $\mathbf{k}_\perp = (\mathbf{k_x}, \mathbf{k_y})$, and $\bar{\omega}$ has the value about maximal frequency of magnons.

For $g^2 \simeq 4700 \cdot e^2$ we obtain

$$\epsilon \simeq 10^3 \cdot Ry \cdot \omega_M(\bar{\omega}\tau) , \tag{7}$$

where $\tau = a/v$ is the characteristic time of interaction.

The typical values for magneto-ordered dielectrics are such: $\bar{\omega} \simeq 10^{-13} s^{-1}$, $\omega_M \simeq 10^{-11} s^{-1}$ and for $v/c \simeq 10^{-4}$ — $\epsilon \simeq 10^{14}$eV/s, that corresponds to losses per unit of length:

$$\frac{dE}{dl} \simeq 10^8 \; eV/cm$$

From (5) it is clear, that the losses ϵ and dE/dl grow with slowing down of monopole. When the speed v becomes $v < u$, the main contribution to losses contribute the magnons "from the bottom" of the spectrum. For them, $\omega_{\mathbf{k}} = \omega_{ex}(ak)^2$, where ω_{ex} is the frequency, characterizing the exchange interaction [9, 10] , and the expressions for losses acquire the shape:

$$\epsilon = g^2 \frac{\omega_M v}{4\omega_{ex} a^2} ; \tag{8}$$

$$\frac{dE}{dl} = \frac{\epsilon}{v} = g^2 \frac{\omega_M}{4\omega_{ex} a^2} . \tag{9}$$

As one can see, the energy losses per unit of a length approach a constant with reduction of the monopole speed. The characteristic values will be $\omega_M/\omega_{ex} \simeq 10^{-2}$, and for $v/c \simeq 10^{-4}$ have

$$a \simeq 10^{-8} \; cm ;$$

$$\frac{dE}{dl} \simeq 10^8 \ eV/cm \ .$$

We'd like to specially note, that the square-law of magnon dispersion leads to non-trivial spatial structure of Cherenkov radiation field of spin waves. As it is, usually, the structure of a radiation field is similar to a shock wave and advancing the charge radiation is away. For the square-law dispersion of the radiation field advances charge and is not equally to zero before charge. It is due to that for the square-law dispersion the group velocity of a wave is more then phase (and more than velocity of a charge movement).

From these evaluations it is clear, that a level of energy losses of a slow magnetic monopole in magneto-ordered matter could be compared to ionization losses of a fast monopole. This opens new opportunities for construction of detectors of monopoles in the range of $v/c < 10^{-4}$. The conversion of spin waves to electromagnetic [10] permits to detect a monopole passing through a magnetic layer by traditional means.

3 Excitation of Cherenkov acoustic (phonon) radiation by heavy magnetic monopole.

For valuation of energy losses by radiation of sound waves (excitation of phonons) by the monopole moving in the isotropic matter, we shall write the Hamiltonian of an elastic system in an external field as follows

$$H = \sum_n \frac{\mathbf{p}_n^2}{2M} + \frac{a}{2} \sum_{n,\Delta} (\mathbf{x}_n - \mathbf{x}_{n+\Delta})^2 + \sum_n \mathbf{F}_n(t)\mathbf{x}_n \ . \tag{10}$$

Here $n + \Delta$ are the numbers the closest neighbors to the node n,

$$\mathbf{F}_n(t) = \mathbf{F}(\mathbf{r}_n - \mathbf{v}t) \ . \tag{11}$$

We shall estimate the strength of the force $\mathbf{F}(\mathbf{r}_n)$, acting from the monopole to the given node, as follows. First, we shall assume that this force is located on one node (it's short-range nature allows this):

$$\mathbf{F}(\mathbf{r_n}) = \mathbf{F}\delta_{n0} \ . \tag{12}$$

Secondly, at rest this force causes deformation, and the affected node is shifted by

$$\delta a \sim \frac{F}{A} \ , \tag{13}$$

and, assuming the deformation energy to be $\epsilon_{def} \sim A\delta a^2$, we have the force as

$$\mathbf{F} \sim A\sqrt{\frac{\epsilon_{def}}{A}} \sim \sqrt{\epsilon_{def} A}\ . \tag{14}$$

The Hamiltonian in (10) can be expressed completely similar to (1):

$$H = \sum_{\mathbf{k}} \hbar\omega_{\mathbf{k}} a^{\dagger}_{\mathbf{k}} a_{\mathbf{k}} + \sum_{\mathbf{k}} \left(f_{\mathbf{k}} e^{-i\Omega_{\mathbf{k}} t} a^{\dagger}_{\mathbf{k}} + c.c \right)\ , \tag{15}$$

but now: $a^{\dagger}_{\mathbf{k}}$ is the operator of phonon creation with the wave vector $\mathbf{k}$, $\omega_{\mathbf{k}}$ is it's dispersion, $\Omega_{\mathbf{k}} = \mathbf{kv}$, $\mathbf{v}$ is the vector of monopole speed, and $f_{\mathbf{k}}$ is the coupling factor of the magnetic monopole field $\mathbf{B} = g\nabla\frac{1}{r}$ with the phonon. We shall write the following expressions for them:

$$F_{\mathbf{k}} = \frac{1}{2i}\hbar^{1/2}\frac{1}{\sqrt{N}}\frac{F}{(D_{\mathbf{k}}M)^{1/4}}\ , \tag{16}$$

$$\omega_{\mathbf{k}} = \sqrt{\frac{2D_{\mathbf{k}}}{M}} \tag{17}$$

$$D_{\mathbf{k}} = \frac{A}{2}\sum_{\mathbf{\Delta}} \left| 1 - e^{i\mathbf{k\Delta}} \right|\ . \tag{18}$$

Accordingly, the energy of phonons, radiated in a unit of time, is equal to

$$\epsilon = \frac{a^3}{4(2\pi)^2}\int d^3\mathbf{k}\frac{F^2}{(D_k M)^{1/2}}\omega_{\mathbf{k}}\delta(k_z v - \omega_{\mathbf{k}})\ , \tag{19}$$

where a is the constant of the lattice $a^3 = V/N$, V is the sample volume.

For the essentially supersound monopoles, integration over dk_z gives the factor $1/v$, and the integral (19) results to:

$$\epsilon = \frac{a^3}{4(2\pi)^2}\frac{1}{v}\int d^2\mathbf{k}_{\perp}\frac{F^2}{(D_{\mathbf{k}_{\perp}} M)^{1/2}}\omega_{\mathbf{k}_{\perp}} = \frac{a}{2\sqrt{2}v}\frac{F^2}{M}\ . \tag{20}$$

Using the evaluation from Eq.(14), for F we shall obtain:

$$\epsilon \simeq \epsilon_{def}\frac{a}{v}\frac{A}{M}\ . \tag{21}$$

Now from decomposition (18) at small $\mathbf{k}$ and using Eq. (17) it is possible to express A/M through speed of sound. As a result (21) acquires the shape

$$\epsilon \simeq \epsilon_{def}\frac{1}{Z}\frac{c_s}{v}\frac{c_s}{a} \simeq \epsilon_{def}\frac{c_s}{v}\bar{\omega}_a\ , \tag{22}$$

where $\bar{\omega}_a$ is the cut-off frequency for phonons, Z is the number of nearest neighbors.

If $\epsilon_{def} \sim Ry$, $c_s/v \sim 0.1$ and $\bar{\omega}_a \sim 10^{13}\ s^{-1}$ ($\bar{\omega}_a$ is about the Debye temperature in energy units), then:

$$\epsilon \simeq 10^{13} eV/s,$$

$$\frac{dE}{dl} \simeq 10^{7} eV/cm,$$

which is a little less than loss by radiation of magnons.

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