Production of e^+e^- pairs to all orders in $Z\alpha$ for collisions of high-energy muons with heavy nuclei

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Abstract

The e^+e^- pair production in collisions of muon with atoms is considered to all orders in the parameter $Z\alpha$. We obtain energy distribution of e^+ and e^- as well as energy loss of muon passing through matter with heavy atoms. The found corrections to the Born contribution do not depend on the details of the target properties except of a simple factor. For the considered example of Pb target the muon energy loss corrections vary from -65 % to -10 % depending on the pair energy.

1 Introduction

The production of e^+e^- pairs in collisions of high energy muons with nuclei and atoms is important for a number of problems. In particular, this process is dominant for energy losses of muons passing through matter. An precise knowledge of these losses is necessary for the construction of detectors and $\mu^+\mu^-$ colliders and an estimation of shielding at high energy colliders.

In Born approximation various cross sections for the process under discussion (A denotes an atom or a nucleus with charge number Z)

$$\mu A \to \mu A \ e^+ e^- \tag{1}$$

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have been calculated in Refs. [1]-[5]. A recent short review on the muon energy loss at high energy can be found in Sect. 23.9 of [6]. Some useful approximate formulae and figures are given in [7].

In all mentioned papers effects of high order corrections in the parameter

$$\nu = Z\alpha \approx \frac{Z}{137} \tag{2}$$

have not been taken into account. However, this parameter is of the order of 1 for heavy nuclei ($\nu = 0.6$ for Pb) and, therefore, the whole series in ν has to be summed to achieve an exact result for process (1).



Figure 1: Amplitude M_n with n exchanged photons for reaction $\mu A \to \mu A e^+ e^-$

Let M_n be the amplitude of the discussed process with n exchanged photons (Fig. 1). We present the cross section in the form

$$d\sigma = d\sigma_{\text{Born}} + d\sigma_{\text{Coul}},$$

$$d\sigma_{\text{Born}} \propto |M_{\text{Born}}|^2 = |M_1|^2, \qquad d\sigma_{\text{Coul}} \propto |\sum_{n=1}^{\infty} M_n|^2 - |M_1|^2$$
(3)

where $M_{\text{Born}} = M_1$ denotes the Born amplitude. We call the Coulomb correction (CC) the difference $d\sigma_{\text{Coul}}$ between the exact result and the Born approximation.

Such kind of CC is well known in the photoproduction of e^+e^- pairs on atoms (see [8], §32.2 of [9] and §98 of [10]). In case of the total cross section the corrections are negative and decrease the Born contribution by about 10 % for Pb.

In the present paper we calculate CC for reaction (1) neglecting only pieces of the order of

$$\frac{m_{\mu}^2}{E_{\mu}^2}, \quad \frac{m_e}{\varepsilon_{\pm}}.$$
(4)

Therefore, our results are valid for ultrarelativistic leptons. In (4) m_e and m_{μ} are the lepton masses, ε_{\pm} and E_{μ} denote the lepton energies.

The discussed process for $\varepsilon_{\pm} \ll E_{\mu}$ has a close relation to $A'A \to A'Ae^+e^-$ where A' is a fast nucleus with relatively small charge $Z'\alpha \ll 1$ and A is a heavy atom or nucleus with $Z\alpha \sim 1$. The latter process was considered in Refs. [11] and [12] assuming that the lepton energies are much smaller than the energy of the projectile nucleus A'. In [11, 12] the same complicated method has been used as in Ref. [8] which basically uses approximated relativistic wave functions of e^+ and e^- in the Coulomb field of the nucleus A.

Our approach is more simple and transparent. It is based on cross sections for the virtual process $\gamma^*A \to e^+e^-A$ (where γ^* denotes the virtual photon with 4-momentum squared $q^2 < 0$) which has been recently obtained in Ref. [13] by a direct summation of the corresponding Feynman diagrams. For $\varepsilon_{\pm} \ll E_{\mu}$ our Eqs. (20) and (30) coincide with Eqs. (38) and (39) of [11], respectively, while our Eq. (14) coincides with the corresponding equation of [12] only in the main logarithmic approximation. (It should be noted, however, that the results of [12] for the discussed process are also presented in logarithmic accuracy.)

The outline of our paper is as follows: In Sect. 2 we obtain the energy distributions for electrons and positrons. The next Sect. is devoted to the muon energy loss. In Sect. 4 we summarise our results and compare the obtained Coulomb corrections with the Born contributions. Additionally we discuss CC for similar processes with other charged projectiles $(e, \pi, p \text{ instead of } \mu)$.

Our main notations are collected in Fig. 1: q and ω are the 4-momentum and energy of the virtual photon generated by the projectile muon and k_1, \ldots, k_n are the 4-momenta of the photons exchanged with the nucleus. Besides we use

$$x_{\pm} = \frac{\varepsilon_{\pm}}{\omega}, \quad x_{+} + x_{-} = 1, \quad y = \frac{\omega}{E_{\mu}}, \quad Q^{2} = -q^{2} > 0.$$
 (5)

Throughout the paper the well known function

$$f(\nu) = \nu^2 \sum_{n=1}^{\infty} \frac{1}{n(n^2 + \nu^2)}$$
(6)

and the abbreviation

$$\sigma_0 = \frac{4}{3\pi} \frac{Z^2 \alpha^4}{m_e^2} \tag{7}$$

are used.

2 Energy distribution of e^+ and e^-

It is well known [6] that the cross section for process (1) as well as for electroproduction can be exactly written in terms of two structure functions or two cross sections $\sigma^T(\omega, Q^2)$ and $\sigma^S(\omega, Q^2)$ for the virtual processes $\gamma_T^* A \to e^+ e^- A$ and $\gamma_S^* A \to e^+ e^- A$ (where γ_T^* and γ_S^* denote the transverse and scalar/longitudinal photons with helicity $\lambda_T = \pm 1$ and $\lambda_S = 0$, respectively):

$$d\sigma = \sigma^T(\omega, Q^2) \, dn_T(\omega, Q^2) + \sigma^S(\omega, Q^2) \, dn_S(\omega, Q^2) \,. \tag{8}$$

Here the coefficients dn_T and dn_S are called the number of transverse and scalar virtual photons (generated by the muon) with energy ω and virtuality Q^2 , respectively. The cross

sections σ^T and σ^S have been calculated recently in Ref. [13]¹:

$$d\sigma^{T} = d\sigma_{1}^{T} + d\sigma_{2}^{T} = \frac{4}{3} \frac{Z^{2} \alpha^{3}}{m_{e}^{2}} \left[L - f(\nu) \right] \left[\frac{m_{e}^{4}}{(m_{e}^{2} + Q^{2}x_{+}x_{-})^{2}} + \frac{2(x_{+}^{2} + x_{-}^{2})m_{e}^{2}}{m_{e}^{2} + Q^{2}x_{+}x_{-}} \right] dx_{+} ,$$

$$d\sigma^{S} = d\sigma_{1}^{S} + d\sigma_{2}^{S} = \frac{4}{3} \frac{Z^{2} \alpha^{3}}{m_{e}^{2}} \left[L - f(\nu) \right] \frac{4m_{e}^{2}Q^{2}x_{+}^{2}x_{-}^{2}}{(m_{e}^{2} + Q^{2}x_{+}x_{-})^{2}} dx_{+}$$
(9)

with

$$L = \ln \frac{2\omega x_+ x_-}{m_e} - \frac{1}{2} \ln \frac{m_e^2 + Q^2 x_+ x_-}{m_e^2} - \frac{1}{2}$$
(10)

and the function $f(\nu)$ is given in Eq. (6). The cross sections $d\sigma_1^{T,S} \propto L$ correspond to the Born contributions and $d\sigma_2^{T,S} \propto -f(\nu)$ to CC. The accuracy of cross sections (9) is determined omitting only pieces of the order of

$$\frac{m_e}{\omega}, \quad \frac{Q}{\omega}.$$
 (11)

The number of photons can be found in Sect. 6 and App. D of review [14] (with accuracy $\mathcal{O}(m_{\mu}^2/E_{\mu}^2), \mathcal{O}(Q^2/\omega^2)$)

$$dn_T = \frac{\alpha}{\pi} (1-y) \left[\left(1 - \frac{Q_{\min}^2}{Q^2} \right) D + \lambda C \right] \frac{d\omega}{\omega} \frac{dQ^2}{Q^2},$$

$$dn_S = \frac{\alpha}{\pi} (1-y) \left[\left(1 + \frac{\lambda}{2} \right) D - \frac{\lambda}{2} C \right] \frac{d\omega}{\omega} \frac{dQ^2}{Q^2}$$
(12)

where

$$\lambda = \frac{1}{2} \frac{y^2}{1 - y}, \quad Q_{\min}^2 = \frac{y^2}{1 - y} m_{\mu}^2, \quad y = \frac{\omega}{E_{\mu}}.$$
 (13)

For the considered case of muon projectile C = D = 1, other particles are discussed in Sect. 4. Eqs. (8)-(13) are the basis for our following calculations.

Integrating Eq. (8) over Q^2 from Q^2_{\min} to infinity (the upper limit can be set to infinity due to the fast convergence of the integral) we obtain the known Born contribution and the new expression for CC:

$$d\sigma_{\text{Coul}} = -\sigma_0 f(\nu) \ F(x, y) \frac{d\omega}{\omega} dx_+$$
(14)

with

$$F(x,y) = (1-y) \left\{ \left[(1+\lambda+\xi) \ a-1-\lambda \right] \ln \left(1+\frac{1}{\xi} \right) - a + \frac{4-a-\lambda}{1+\xi} \right\}, \quad (15)$$
$$a = 2(1+x_{+}^{2}+x_{-}^{2}), \quad \xi = \frac{m_{\mu}^{2}}{m_{e}^{2}} \frac{y^{2}}{1-y} x_{+} x_{-}.$$

¹In Eqs. (46), (49) and (59) of [13] a factor x_+x_- is missing in the integrands of quantities σ_1^S and σ_2^S .

Since the integration variables can be transformed as follows

$$\frac{d\omega}{\omega}dx_{+} = \frac{d\varepsilon_{+}d\varepsilon_{-}}{\omega^{2}} \tag{16}$$

Eq. (14) describes the energy distribution of e^+ and e^- in CC. In the limit $\xi \ll 1$ (or $y \ll m_e/m_{\mu}$) the function F(x, y) is approximated by

$$F(x,y) = (1 + 2x_{+}^{2} + 2x_{-}^{2})\ln\frac{1}{\xi} - 4(x_{+}^{2} + x_{-}^{2}).$$
(17)

At $\xi \gg 1$ we obtain

$$F(x,y) = \frac{1}{\xi} [1 - y + y^2 + 2(1 - y - y^2)x_+x_-].$$
(18)

It is easy to see that the main contribution to σ_{Coul} arises from the region

$$m_e^2 \ll \varepsilon_+ \varepsilon_- \ll \left(m_e \frac{E_\mu}{m_\mu} \right)^2 .$$
 (19)

Strictly speaking, the cross sections (9) are valid for pair production processes on nuclei. In the collisions of virtual photons with atoms, an atomic screening effect has to be taken into account. For high energy photons the screening effect changes considerably the differential and total cross section as well as the energy loss for the Born contribution. The reason is that the region of small transverse momenta $k_{1\perp} \leq 1/a \sim m_e \alpha Z^{1/3}$ (a denotes the atomic radius) significantly contributes to the cross sections. As a consequence, the function in the Born contribution equivalent to our F(x, y) becomes very complicated and not universal for different atoms (see [5]). On the contrary, the region mainly contributing to CC, is determined by the condition $k_{1\perp}, \ldots, k_{n\perp} \sim m_e \ll 1/a$. Therefore, the atomic screening effect is negligible in CC and the function F(x, y) as well as some other distributions are universal and do not depend on atomic properties.

However, if one is interested in very high energy pairs effects of the nucleus form factor have to be taken into account both in the Born contributions as well as in the Coulomb corrections. This happens in the case when the characteristic squared momentum transferred to the nucleus $\sim m_e^2 + Q_{\min}^2 x_+ x_-$ becomes comparable with $(1/R_A)^2$ where R_A is the radius of the nucleus. From this condition it follows that the just mentioned universal behaviour is spoiled for y > 0.5 where this pair production is strongly suppressed.

3 Muon energy loss

The Coulomb correction to the spectrum of the muon energy loss can be obtained from Eq. (14) after integrating over x_+ :

$$d\sigma_{\text{Coul}} = -\sigma_0 f(\nu) F(y) \frac{dy}{y}, \quad F(y) = (1-y)F_1(z) + y^2 F_2(z), \quad z = \frac{m_\mu^2}{m_e^2} \frac{y^2}{1-y}$$
(20)

where

$$F_{1}(z) = \frac{44}{15z} - \frac{16}{15} - \left(\frac{7}{3} + \frac{8z}{15}\right) \ln z + \\ + \left(-\frac{44}{15z} + \frac{4}{4+z} + \frac{38}{15} + \frac{16z}{15}\right) \sqrt{1 + \frac{4}{z}} \ln\left(\sqrt{1 + \frac{z}{4}} + \sqrt{\frac{z}{4}}\right), \\ F_{2}(z) = -\frac{4}{3z} - \frac{7}{6} \ln z + \\ + \left(-\frac{2}{3z} + \frac{8}{z(4+z)} + \frac{7}{3}\right) \sqrt{1 + \frac{4}{z}} \ln\left(\sqrt{1 + \frac{z}{4}} + \sqrt{\frac{z}{4}}\right).$$
(21)

The function F(y) is presented in Fig. 2. At small $z \ll 1$ (where $y \ll m_e/m_{\mu}$) the



Figure 2: Function F(y) defined in Eq. (20) vs. the fractional energy loss of the muon

spectrum is logarithmically enhanced:

$$F(y) = \left(\frac{7}{3} + \frac{8z}{15}\right) \ln \frac{1}{z} + \frac{20}{9} + \frac{511z}{450} + \dots$$
(22)

whereas at large $z \gg 1$ it is powerlike suppressed:

$$F(y) = \frac{1}{z} \left\{ (1-y) \left[\left(2 - \frac{22}{3z}\right) \ln z + 6 + \frac{5}{9z} \right] + y^2 \left[\left(2 + \frac{1}{z}\right) \ln z + 1 + \frac{1}{2z} \right] \right\} + \dots \quad (23)$$

The approximate expressions (22-23) agree with the exact spectrum (20) within 1 % accuracy everywhere except in the region $y = 0.004 \div 0.02$.

From the experimental point of view of special interest is the relative mean rate of muon energy loss due to pair production (or stopping power) on unit length in matter. This quantity can be calculated as

$$-\frac{1}{E}\frac{dE}{dx} = n \int_{2m_e/E_{\mu}}^{1} y \frac{d\sigma}{dy} dy = n \sigma_0 \left(S_{\text{Born}} + S_{\text{Coul}}\right)$$
(24)

where n is the number of atoms per unit volume. Formulae and tables for the Born contribution S_{Born} are given in Ref. [4]. In particular, for the case without screening

$$S_{\rm Born} = S_0 \left[(1 - \delta_1) \ln \frac{E_{\mu}}{4m_{\mu}} - 1.771 \right]$$
(25)

and for complete screening

$$S_{\text{Born}} = S_0 \left[(1 - \delta_1) \ln \frac{189}{Z^{1/3}} + 0.604 \right]$$
(26)

where

$$S_0 = \frac{19\pi^2}{12} \frac{m_e}{m_\mu}, \quad \delta_1 = \frac{48}{19\pi^2} \frac{m_e}{m_\mu} \left(\ln \frac{m_\mu}{m_e} \right)^2 = 0.0352.$$
 (27)

From Eqs. (20) and (24) we derive the Coulomb correction

$$S_{\text{Coul}} = -f(\nu) \int_{0}^{1} F(y) dy = -(1-\delta_1) f(\nu) S_0.$$
(28)

In the integration we have used as lower limit zero, since the region near the threshold $y_{\min} = 2m_e/E_{\mu}$ can be safely neglected.

4 Discussion

In this paper we have presented the Coulomb correction to pair production of high energy muons on atoms. Differently to the Born contributions of various distributions, these corrections do not depend on the target properties, except of a simple factor $\sigma_0 f(\nu)$ (if we do not consider the exceptional case of pair energies close to the muon energy).

To demonstrate the relative importance of CC we discuss two simple examples: Firstly, we present in Fig. 3 the ratio of the spectral distribution $d\sigma_{\text{Coul}}/dy$ to the corresponding Born cross section

$$\frac{d\sigma_{\rm Coul}/dy}{d\sigma_{\rm Born}/dy} = -\frac{f(\nu)F(y)}{12F_a(y,E_\mu)}$$
(29)

where the universal function F(y) is given in Eq. (20), and values for $F_a(y, E_{\mu})$ are taken from Table I of [4] for collisions of muons with energy $E_{\mu} = 86.4$ GeV on lead target, $f(\nu) = 0.331$. The presented ratio varies from about -65 % to roughly -10 % in the considered interval of pair energies.

Secondly, we compare the stopping power S_{Coul} with the Born term in the two limiting cases of Eqs. (25) and (26). For muon scattering on a Pb target $S_{\text{Coul}}/S_{\text{Born}} = -15$ % at $E_{\mu} = 25$ GeV without screening and -7.7 % for the case of complete screening.



Figure 3: Ratio of Coulomb to Born energy distribution vs. energy fraction y for muon collisions on Pb target at $E_{\mu} = 86.4 \text{ GeV}$

It might be useful to present a simple expression for the contribution to σ_{Coul} above some cut $\omega > \omega_{\text{cut}}$ where this cut is restricted to the region $2m_e \ll \omega_{\text{cut}} \ll m_e E_{\mu}/m_{\mu}$. From Eq. (20) we obtain

$$\sigma_{\rm Coul}(\omega_{\rm cut}) = -\frac{7}{3}\sigma_0 f(\nu) \left(l^2 + \frac{20}{21}l + \frac{101}{63} \right), \quad l = \ln \frac{m_e E_\mu}{m_\mu \omega_{\rm cut}}.$$
 (30)

The expression (20) does not remain valid close to the threshold $\omega_{\min} = 2m_e$. Therefore, from Eq. (30) the Coulomb correction to the total pair production cross section can be obtained only in leading logarithmic approximation choosing $\omega_{\text{cut}} = 2m_e$:

$$\sigma_{\rm Coul} = -\frac{28}{9\pi} \frac{Z^2 \alpha^4}{m_e^2} f(\nu) \left(\ln \frac{E_\mu}{2m_\mu} \right)^2 \,. \tag{31}$$

Finally, let us discuss the case when the muon projectile is replaced by other charged projectiles such as electron, pion or proton. For an electron projectile the distributions (14) and (20) remain valid changing $m_{\mu} \rightarrow m_{e}$. However, in these distributions as well as in the Born contributions one has to take into account the effect of the identity of the final state electrons and the bremsstrahlung mechanism of the e^+e^- -pair production (according to [15] this changes the result only slightly).

For pion and proton projectiles in the basic formulae the number of photons should be changed (besides the trivial mass replacements). The numbers of photons are given by Eqs. (12) with C = 0, $D = F_{\pi}^2(Q^2)$ for pion and $C = G_M^2(Q^2)$, $D = [4m_p^2 G_E^2(Q^2) + Q^2 G_M^2(Q^2)]/(4m_p^2 + Q^2)$ for proton. Here F_{π} , G_E and G_M are the pion, proton electric and proton magnetic form factors, respectively, m_p is the proton mass. For the pion case these changes are essential only for y close to 1 where we should take into account the nucleus form factor, too. For the proton case the nucleus form factor becomes important for somewhat smaller y where the influence of the proton form factors is still small.

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