# Formation of color-singlet gluon-clusters and inelastic diffractive scattering

Part II. Derivation of the  $t$ - and  $M_x^2/s$ -dependence of cross-sections in the

SOC-approach

T. Meng, R. Rittel, K. Tabelow, and Y. Zhang \*

Institut für Theoretische Physik, Freie Universität Berlin, 14195 Berlin, Germany meng@physik.fu-berlin.de

# Abstract

The *t*-dependence and the  $(M_x^2/s)$ -dependence of the double differential crosssections for inelastic diffractive scattering off proton-target are discussed. Here t stands for the four-momentum-transfer squared,  $M_x$  for the missing mass, and  $\sqrt{s}$  for the total c.m.s. energy. It is shown, that the space-time properties of the color-singlet gluon-clusters due to SOC, discussed in Part I, lead to simple analytical formulae for  $d^2\sigma/dt\, d(M_x^2/s)$  and for  $d\sigma/dt,$  and that the obtained results are in good agreement with the existing data. Further experiments are suggested.

<sup>∗</sup>Present address: Chinese Academy of Sciences, Institute of Theoretical Physics, POB 2735, Beijing 100080, China.

#### 1. Introduction together with a brief summary of Part I

It is suggested in the preceding paper<sup>[1](#page-18-0)</sup> (hereafter referred to as Part I) that concepts and methods of Statistical Physics for open dynamical complex systems far from equilibrium can be used in describing the phenomena associated with large rapidity gap (LRG) events in deep-inelastic electron-proton scattering<sup>[2](#page-18-0),[3](#page-18-0)</sup> and other inelastic diffractive scattering processes $4^{-7}$  $4^{-7}$  $4^{-7}$  $4^{-7}$ . This approach is motivated by the following observations:

First, LRG events in deep-inelastic electron-proton scattering have been observed<sup>[2](#page-18-0),[3](#page-18-0)</sup> in the small- $x_B$  region  $(x_B<10^{-2}, \text{ say})$  — a kinematic region in which low-energy (soft) gluons dominate<sup>[8](#page-18-0)</sup>. This piece of experimental facts indicates that the occurrence of LRG events and the observed gluon-domination in this region are very much related to each other.

Second, the characteristic properties of the gluons — in particular the local gluon-gluon coupling prescribed by the QCD-Lagrangian, the confinement, and the non-conservation of gluon-numbers — strongly suggest that systems of interacting soft gluons should be considered as open, dynamical, complex systems with many degrees of freedom, and that such systems are in general far from equilibrium.

Third, it has been proposed by Bak, Tang and Wiesenfeld  $(BTW)^9$  $(BTW)^9$  some time ago, that a wide class of open dynamical complex systems far from equilibrium may evolve in a selforganized manner to critical states, where perturbations caused by local interactions can initiate long-range-correlations through "domino effects" which lead to spatial and temporal fluctuations extending over all length and time scales — in form of avalanches (which we call BTW-avalanches in Part I and hereafter). The size-distributions and the lifetimedistributions of such avalanches exhibit *power-law behaviors*. These behaviors are *universal* and *robust*. In fact these behaviors are considered as the fingerprints of self-organized criticality (SOC). In the macroscopic world, there are many open dynamical complex systems which show this kind of power-law behaviors<sup>[9](#page-18-0),[10](#page-18-0)</sup>: sandpiles, earthquakes, wood-fire, evolution, traffic jam, stock market etc.

Having these observations in mind, we are automatically led to the questions: Can self-

organized criticality also exist in the microscopic world — at the level of gluons and quarks? In particular, can there be BTW-avalanches in systems of interacting soft gluons? Is it possible to probe the existence and the properties of such avalanches experimentally — by examing reactions in which the interactions of soft gluons play a significant role?

To answer these questions, it is useful to recall the following: Inelastic diffractive scattering processes are characterized (see e.g. Gallo in Ref.[\[11](#page-15-0)] and the papers cited therein) by large rapidity gaps in the final state and the existence of such gaps has been interpreted as the consequence of "the exchange of colorless objects." These objects can be, and have  $been<sup>12</sup>$  $been<sup>12</sup>$  $been<sup>12</sup>$ , associated with color-singlet systems/clusters of interacting soft gluons. Such colorless objects can exist inside and/or outside the proton, and the interactions between such color-singlets, as well as those between such objects and "the mother proton", should be of Van der Waals type. Hence it is expected that such colorless objects can be easily separated from the other color-singlets including "the mother proton" in peripheral collision processes, in which (in contrast to "hard scattering") not much transfer of momentum is needed. Furthermore, since the process of "carring away" or "knocking-out" of such a colorless object from the proton is comparable with the knocking-out of a nucleon off a nucleus by energetic beam-particles, it is also expected that the characteristic features (which does not include the absolute values of the cross-sections) of such inelastic diffractive processes should be *independent* of the incident energy, and *independent* of the quantum-numbers of the projectile.

Based on these knowledge and expectations, we performed a systematic analysis<sup>[1](#page-18-0)</sup> of the existing data for inelastic diffractive scattering in electron-proton scattering processes $^{2,3}$  $^{2,3}$  $^{2,3}$  $^{2,3}$  $^{2,3}$ , in photoproduction<sup>[4](#page-18-0)</sup>, and in proton-proton and proton-antiproton collisions<sup>[5](#page-18-0)–[7](#page-18-0)</sup>. The obtained results (which are presented in Part I) can be summarized as follows:

The data<sup>[2](#page-18-0)-[7](#page-18-0)</sup> show that the above-mentioned characteristic features for SOC *indeed exist*, and that the relevant exponents in such power-laws are approximately the *same* for different reactions. The observed features imply that a color-singlet gluon-cluster is a BTWavalanche, and as such, it can have neither a typical size, nor a typical lifetime, nor a given

static structure. In fact, it has much in common with an earthquake, or an avalanche of snow (see Sections 2, 4 and 5 of Part I). By examing the data for inelastic diffractive scattering processes performed at different incident energies and/or in which different kinds of beam-particles are used, we are able to extract information about the colorless objects without specifying their structures. Furthermore, since the obtained results are approximatly independent of the incident energy and independent of the quantum numbers of the projectile, we can conclude that the extracted knowledge about the color-singlet gluon-clusters are universal. Hence, the following picture emerges:

Viewed from the beam-particle, the target-proton in diffractive scattering appears as a "cloud" of colorless objects which in general exist partly inside and partly outside the confinement region of the proton. Such objects are color-singlet gluon-clusters which exhibit the characteristic features of BTW-avalanches as a consequence of SOC. In (geometrically speaking) more peripheral scattering processes, such as inelastic diffractive scattering, the beam-particle will encounter one of these color-singlet gluon-clusters and "carry it away", because the interaction between the struck object and any other neighboring color-singlets (including the "mother proton") are expected to be of Van der Waals' type.

We note, in Part I of this paper, we simply adopted the currently popular definition of "inelastic diffractive scattering processes". That is, when we talked about "inelastic diffractive scattering" we were always referring to processes in which "colorless objects" are "exchanged". In other words, in that part of the paper, the following question has not been asked: Are the above-mentioned "inelastic diffractive scattering processes" indeed comparable with *diffraction* in Optics, in the sense that the beam particles should be considered as waves, and the target-proton together with the associated (in whatever manner) colorless objects can indeed be viewed as a "scattering screen"?

In Part II of this paper, we discuss in detail this question, and we examine in particular the existing data<sup>[5](#page-18-0),[6](#page-18-0)</sup> for the double differential cross-section  $d^2\sigma/dt d(M_x^2/s)$  for proton-proton and antiproton-proton collisions (where t is the 4-momentum-transfer squared,  $M_x$  is the missing-mass, and  $\sqrt{s}$  is the total c.m.s. energy). To be more precise, what we wish to

find out in this connection is: "Can the observed t-dependence and the  $(M_x^2/s)$ -dependence of  $d^2\sigma/dt d(M_x^2/s)$  in the given kinematic range  $(0.2 \text{ GeV}^2 \leq |t| \leq 3.25 \text{ GeV}^2, 16 \text{ GeV} \leq \sqrt{s} \leq$ 630 GeV, and  $M_x^2/s \leq 0.1$ ) be understood in terms of the well-known concept of diffraction in Optics?

The answer to this question is of particular interest for several reasons:

(a) High-energy proton-proton and proton-antiproton scattering at small momentum transfer has played, and is still playing a very special role in understanding diffraction and/or diffractive dissociation in lepton-, photon- and hadron-induced reactions<sup> $2-7,13,11,14,15$  $2-7,13,11,14,15$  $2-7,13,11,14,15$  $2-7,13,11,14,15$  $2-7,13,11,14,15$  $2-7,13,11,14,15$  $2-7,13,11,14,15$  $2-7,13,11,14,15$  $2-7,13,11,14,15$  $2-7,13,11,14,15$  $2-7,13,11,14,15$ </sup>. Many experiments have been performed at various incident energies for elastic and inelastic diffractive scattering processes. It is known that the double differential cross section  $d^2\sigma/dt d(M_x^2/s)$  is a quantity which can yield much information on the reaction mechanism(s) and/or on the structure of the participating colliding objects. In the past, the  $t$ -,  $M_x$ - and s-dependence of the differential cross-sections for inelastic diffractive scattering processes has been presented in different forms, where a number of interesting features have been observed<sup>[5](#page-18-0),[6](#page-18-0),[13](#page-19-0)</sup>. For example, it is seen that, the t-dependence of  $d^2\sigma/dt dM_x^2$  at fixed s depends very much on  $M_x$ ; the  $M_x^2$ -dependence of  $d^2\sigma/dt dM_x^2$  at fixed t depends on s. But, when  $d^2\sigma/dt d(M_x^2/s)$  is plotted as function of  $M_x^2/s$  at given t -values (in the range  $0.2 \,\text{GeV}^2 \leq |t| \leq 3.25 \,\text{GeV}^2$  they are approximately independent of s! What do these observed striking features tell us? The first precision measurement of this quan-tity was published more than twenty years ago<sup>[5](#page-18-0)</sup>. Can this, as well as the more recent  $d^2\sigma/dt d(M_x^2/s)$ -data<sup>[6](#page-18-0)</sup> be understood theoretically?

(b) The idea of using optical and/or geometrical analogies to describe high-energy hadron-nucleus and hadron-hadron collisions at small scattering angles has been discussed by many authors<sup>[14](#page-19-0),[13](#page-19-0)</sup> many years ago. It is shown in particular that this approach is very useful and successful in describing elastic scattering. However, it seems that, until now, no attempt has been made to describe the data<sup>[5](#page-18-0),[6](#page-18-0)</sup> by performing quantitative calculations for  $d^2\sigma/dt d(M_x^2/s)$  by using optical geometrical analogies. It seems worthwhile to make such an attempt. This is because, it has been pointed out<sup>[17](#page-19-0)</sup> very recently, that the above-mentioned <span id="page-5-0"></span>analogy can be made to understand the observed t-dependence in  $d\sigma/dt$ .

(c) Since inelastic diffractive  $pp$ - and  $\bar{p}p$ -scattering belongs to those soft processes in which the initial and final states are well-known hadrons, it is expected that also they can be described in the well-known Regge-pole approach<sup>[15](#page-19-0),[13](#page-19-0),[11](#page-18-0)</sup>. The basic idea of this approach is that colorless objects in form of Regge trajectories (Pomerons, Reggions etc.) are exchanged during the collision, and such trajectories are responsible for the dynamics of the scattering processes. In this approach, it is the t-dependence of the Regge trajectories, the t-dependence of the corresponding Regge residue functions, the properties of the coupling of the contributing trajectories (e.g. triple Pomeron or Pomeron-Reggion-Pomeron coupling), and the number of contributing Regge trajectories which determine the experimentally observed t- and  $M_x$ -dependence of  $d^2\sigma/dt d(M_x^2/s)$ . A number of Regge-pole models<sup>[11](#page-18-0),[15](#page-19-0)</sup> have been proposed, and there exist good  $fits^{11,15}$  $fits^{11,15}$  $fits^{11,15}$  $fits^{11,15}$  $fits^{11,15}$  to the data. What remains to be understood in this approach is the dynamical origin of the Regge-trajectories on the one hand, and the physical meaning of the unknown functions (for example the t-dependence of any one of the Regge-residue functions) on the other. It has been pointed out<sup>[16](#page-19-0),[17](#page-19-0)</sup>, that there may be an overlap between the "Partons in Pomeron and Reggeons" picture and the SOC-picture<sup>[17](#page-19-0)</sup>, and that one way to study the possible relationship between the two approaches is to take a closer look at the double differential cross-section  $d^2\sigma/dt d(M_x^2/s)$ .

## 2. Optical diffraction off dynamical complex systems

Let us begin our discussion on the above-mentioned questions by recalling that the concept of "diffraction" or "diffractive scattering" has its origin in Optics, and Optics is part of Electrodynamics, which is not only the classical limit, but also the basis of Quantum Electrodynamics (QED). Here, it is useful to recall in particular the following: Optical diffraction is associated with departure from geometrical optics caused by the finite wavelength of light. Frauenhofer diffraction can be observed by placing a scatterer (which can in general be a scattering screen with more than one aperture or a system of scattering objects) in the path of propagation of light (the wavelength of which is less than the linear dimension of the scatterer) where not only the light-source, but also the detecting device, are very far away from the scatterer. The parallel incident light-rays can be considered as plane waves (characterized by a set of constants  $\vec{k}, w \equiv |\vec{k}|$ , and u say, which denote the wave vector, the frequency and the amplitude of a component of the electromagnetic field respectively in the laboratory frame). After the scattering, the scattered field can be written in accordance with Huygens' principle as

$$
u_P = \frac{e^{i|\vec{k}'|R}}{R} f(\vec{k}, \vec{k}'). \tag{1}
$$

Here,  $u_P$  stands for a component of the field originating from the scatterer,  $\vec{k}'$  is the wave vector of the scattered light in the direction of observation,  $|\vec{k'}| \equiv \omega'$  is the corresponding frequency, R is the distance between the scatterer and the observation point P, and  $f(\vec{k}, \vec{k}')$ is the (unnormalized) scattering amplitude which describes the change of the wave vector in the scattering process. By choosing a coordinate system in which the z-axis coincides with the incident wave vector  $\vec{k}$ , the scattering amplitude can be expressed as follows<sup>[18](#page-19-0),[14](#page-19-0),[13](#page-19-0)</sup>

$$
f(\vec{q}) = \frac{1}{(2\pi)^2} \iint d^2 \vec{b} \,\alpha(\vec{b}) \, e^{-i\vec{q}\cdot\vec{b}} \,. \tag{2}
$$

Here,  $\vec{q} \equiv \vec{k}' - \vec{k}$  determines the change in wave vector due to diffraction;  $\vec{b}$  is the impact parameter which indicates the position of an infinitesimal surface element on the wave-front "immediately behind the scatterer" where the incident wave would reach in the limit of geometrical optics, and  $\alpha(\vec{b})$  is the corresponding amplitude (associated with the boundary conditions which the scattered field should satisfy) in the two-dimensional impact-parameterspace (which is here the xy-plane), and the integration extends over the region  $\Sigma$  in which  $\alpha(\vec{b})$  is different from zero. In those cases in which the scatterer is symmetric with respect to the scattering axis (here the z-axis),  $Eq.(2)$  can be expressed, by using an integral representation for  $J_0$ , as

$$
f(q) = \frac{1}{2\pi} \int_{0}^{\infty} b \, db \, \alpha(b) J_0(qb) \tag{3}
$$

where q and b are the magnitudes of  $\vec{q}$  and  $\vec{b}$  respectively.

The following should be mentioned in connection with Eqs.(2) and (3): Many of the well-known phenomena related to Frauenhofer diffraction have been deduced<sup>[18](#page-19-0)</sup> from these equations under the additional condition (which is directly related to the boundary conditions imposed on the scattered field)  $|\vec{k}'| = |\vec{k}| = \omega' = \omega$ , that is,  $\vec{k}'$  differs from  $\vec{k}$  only in direction. In other words, the outgoing light wave has exactly the same frequency, and exactly the same magnitude of wave-vector as those for the incoming wave. (This means, quantum mechanically speaking, the outgoing photons are also on-shell photons, the energies of which are the same as the incoming ones.) In such cases, it is possible to envisage that  $\vec{q}$  is approximately perpendicular to  $\vec{k}$  and to  $\vec{k'}$ , that is,  $\vec{q}$  is approximately perpendicular to the chosen z-axis, and thus in the above-mentioned xy-plane (that is  $\vec{q} \approx \vec{q}_{\perp}$ ). While the scattering angle distribution in such processes (which are considered as the characteristic features of elastic diffractive scattering) plays a significant role in understanding the observed diffraction phenomena, it is of considerable importance to note the following:

(A) Eqs.(2) and (3) can be used to describe diffractive scattering with, or without, this additional condition, provided that the difference of  $\vec{k'}$  and  $\vec{k}$  in the longitudinal direction (i.e. in the direction of  $\vec{k}$ ) is small compared to  $q_{\perp} \equiv |\vec{q}_{\perp}|$  so that  $\vec{q}_{\perp}$  can be approximated by  $\vec{q}$ . In fact, Eqs.(2) and (3) are strictly valid when  $\vec{q}$  is a vector in the above-mentioned xy-plane, that is when we write  $\vec{q}_\perp$  instead of  $\vec{q}$ . Now, since Eqs.(2) and (3) in such a form (that is when the replacement  $\vec{q} \rightarrow \vec{q}_{\perp}$  is made) are valid without the condition  $\vec{q}$ should approximately be equal to  $\vec{q}_\perp$  and in particular without the additional condition  $|\vec{k}'| = |\vec{k}| = \omega' = \omega$ , it is clear that they are also valid for *inelastic* scattering processes. In other words, Eqs.(2) and (3) can also be used to describe inelastic diffractive scattering (that is, processes in which  $\omega' \neq \omega$ ,  $|\vec{k'}| \neq |\vec{k}|$ ) provided that the following replacements are made.  $\text{In Eq. (2), } \vec{q} \to \vec{q}_{\perp}, f(\vec{q}) \to f_{\text{inel.}}(\vec{q}_{\perp}), \alpha(\vec{b}) \to \alpha_{\text{inel.}}(\vec{b}); \text{ and in Eq. (3) } q \to q_{\perp}, f(q) \to f_{\text{inel.}}(q_{\perp}),$  $\alpha(b) \to \alpha_{\rm inel.}(b)$ . Hereafter, we shall call Eqs.(2) and (3) with these replacements Eq.(2') and Eq.(3') respectively. We note, in order to specify the dependence of  $f_{\text{inel.}}$  on  $\omega'$  and  $k'_{\parallel}$ 

(that is on  $\omega' - \omega$  and  $k_{\parallel}' - k_{\parallel}$ ), further information on energy-momentum transfer in such scattering processes is needed. This point will be discussed in more detail in Section [3](#page-9-0).

(B) In scattering processes at large momentum-transfer where the magnitude of  $\vec{q}_\perp$  is large ( $|\vec{q}_{\perp}|^2 \gg 0.05 \,\text{GeV}^2$ , say), it is less probable to find diffractive scattering events in which the additional condition  $|\vec{k}'| = |\vec{k}|$  and  $\omega' = \omega$  can be satisfied. This means, it is expected that most of the diffraction-phenomena observed in such processes are associated with inelastic diffractive scattering.

(C) Change in angle but no change in magnitude of wave-vectors or frequencies is likely to occur in processes in which neither absorption nor emission of light takes place. Hence, it is not difficult to imagine, that the above-mentioned condition can be readily satisfied in cases where the scattering systems are time-independent macroscopic apertures or objects. But, in this connection, we are also forced to the question: "How large is the chance for a incident wave not to change the magnitude of its wave-vector in processes in which the scatterers are *open dynamical complex systems*, where energy- and momentum-exchange take place at anytime and everywhere?!"

The picture for inelastic diffractive scattering has two basic ingredients:

First, having the well-known phenomena associated with Frauenhofer's diffraction and the properties of de Broglie's matter waves in mind, the beam particles  $(\gamma^*, \gamma, \overline{p})$  or p shown in Fig.8 of Part I) in these scattering processes are considered as high-frequency waves passing through a medium. Since, in general, energy- and momentum-transfer take place during the passage through the medium, the wave-vector of the outgoing wave differs, in general, from the incoming one, not only in direction, but also in magnitude. For the same reason, the frequency and the longitudinal component of the wave-vector of the outgoing wave (that is the energy, and/or the invariant mass, as well as the longitudinal momentum of the outgoing particles) can be different from their incoming counterparts.

Second, according to the results obtained in Part I (and summarized in the Introduction of Part II) of this paper, the medium is a system of color-singlet gluon-clusters which are in general partly inside and partly outside the proton  $-$  in form of a "cluster cloud"<sup>[1](#page-18-0)</sup>. <span id="page-9-0"></span>Since the average binding energy between such color-singlet aggregates are of Van der Waals type<sup>[19](#page-20-0)</sup>, and thus it is negligibly small compared with the corresponding binding energy between colored objects, we expect to see that, even at relatively small values of momentumtransfer ( $|t|$ <1 GeV<sup>2</sup>, say), the struck colorless clusters can unify with (be absorbed by) the beam-particle, and "be carried away" by the latter, similar to the process of "knocking out nucleons" from nuclear targets in high-energy hadron-nucleus collisions. It should, however, be emphasized that, in contrast to the nucleons in nucleus, the colorless gluon-clusters which can exist inside or outside the confinement-region of the proton are *not* hadron-like (See Sections 3 - 6 of Part I for more details). They are BTW-avalanches which have neither a typical size, nor a typical lifetime, nor a given static structure. Their size- and lifetimedistributions obey simple power-laws as consequence of SOC. This means, in the diffraction processes discussed here, the size of the scatterer(s), and thus the size of the carried-away colorless gluon-cluster(s), is in general different in every scattering event. It should also be emphasized that these characteristic features of the scatterer are consequences of the basic properties of the gluons.

### 3. Can such scattering systems be modeled quantitatively?

To model the proposed picture quantitatively, it is convenient to consider the scattering system in the rest frame of the proton target. Here, we choose a right-handed Cartesian coordinate with its origin  $O$  at the center of the target-proton, and the z-axis in the direction of the incident beam. The  $xy$ -plane in this coordinate system coincides with the two-dimensional impact-parameter space mentioned in connection with  $Eqs.(2')$  and  $(3')$ which are respectively Eq.(2) and Eq.(3) after the replacements mentioned in  $(A)$  below Eq.(3), while the yz-plane is the scattering plane. We note, since we are dealing with inelastic scattering (where the momentum transfer, including its component in the longitudinal direction, can be large; in accordance with the uncertainty principle) it is possible to envisage that (the c.m.s. of) the incident particle in the beam meets colorless gluon-clusters

at one point  $B \equiv (0, b, z)$ . where the projection of  $\overline{OB}$  along the y-axis characterizes the corresponding impact parameter  $\vec{b}$ . We recall that such clusters are avalanches initiated by local perturbations (caused by local gluon-interactions associated with absorption or emission of one or more gluons; see Part I for details) of SOC states in systems of interacting soft gluons. Since gluons carry color, the interactions which lead to the formation of colorless gluon-clusters must take place inside the confinement region of the proton. This means, while a considerable part of such colorless clusters in the cloud can be outside the proton, the location A, where such an avalanche is initiated, must be inside the proton. That is, in terms of  $\overline{OA} \equiv r$ ,  $\overline{AB} \equiv R_A(b)$ , and proton's radius  $r_p$ , we have  $r \leq r_p$  and  $[R_A(b)]^2 = b^2 + z^2 + r^2 - 2(b^2 + z^2)^{1/2}r \cos \angle BOA$ . For a given impact parameter b, it is useful to know the distance  $R_A(b)$  between B and A, as well as "the average squared distance"  $\langle R_A^2(b) \rangle = b^2 + z^2 + a^2, a^2 \equiv 3/5 r_p^2$  $_p^2$ , which is obtained by averaging over all allowed locations of A in the confinement region. That is, we can model the effect of confinement in cluster-formation by picturing that all the avalanches, in particular those which contribute to scattering events characterized by a given  $b$  and a given  $z$  are initiated from an "effective initial point"  $\langle A_B \rangle$ , because only the mean distance between A and B plays a role. (We note, since we are dealing with a complex system with many degrees of freedom, in which  $B$  as well as  $A$  are randomly chosen points in space, we can compare the mean distance between  $B$  and  $A$  with the mean free path in a gas mixture of two kinds of gas molecules — "Species"  $B$ " and "Species  $A$ " say, where those of the latter kind are confined inside a subspace called "region p". For a given mean distance, and a given point  $B$ , there is in general a set of A's inside the "region  $p$ ", such that their distance to B is *equal* to the given mean value. Hence it is useful to introduce a *representative* point  $\langle A_B \rangle$ , such that the distance between  $\langle A_B \rangle$  and B is equal to the given mean distance.) Furthermore, since an avalanche is a dynamical object, it may propagate within its lifetime in any one of the  $4\pi$  directions away from  $\langle A_B \rangle$ . (Note: avalanches of the same size may have different lifetimes, different structures, as well as different shapes. The location of an avalanche in space-time is referred to <span id="page-11-0"></span>its center-of-mass.) Having seen how SOC and confinement can be implemented in describing the properties and the dynamics of colorless gluon-clusters, which are nothing else but BTW-avalanches in systems of interacting soft gluons, let us now go one step further, and discuss how these results can be used to obtain the amplitudes in impact-parameter-space that leads, via Eq.(3), to the scattering amplitudes.

In contrast to the usual cases, where the scatterer in the optical geometrical picture of a diffractive scattering process is an aperture, or an object, with a given static structure, the scatterer in the proposed picture is an open dynamical complex system of colorless gluonclusters in form of BTW-avalanches. This implies in particular: The object(s), which the beam particle hits, has (have) neither a typical size, nor a typical lifetime, nor a given static structure.

With these in mind, let us now come back to our discussion on the double differential cross section  $d^2\sigma/dt d(M_x^2/s)$ . Here, we need to determine the corresponding amplitude  $\alpha_{\rm inel.}(b)$  in Eq.(3') [see the discussion in (A) below Eq.(3)]. What we wish to do now, is to focus our attention on those scattered matter-waves whose de Broglie wavelengths are determined by the energy-momentum of the scattered object, whose invariant mass is  $M_x$ . For this purpose, we characterize the corresponding  $\alpha_{\text{inel}}(b)$  by considering it as a function of  $M_x$ , or  $M_x^2/s$ , or  $x_P$ . We recall in this connection that, for inelastic diffractive scattering processes in hadron-hadron collisions, the quantity  $M_x^2/s$  is approximately equal to  $x_P$ , which is the momentum fraction carried by the struck colorless gluon-cluster with respect to the incident beam (see Fig.8 of Part I for more details; note however that  $q_c$ , k and  $p_x$  in Fig.8 of part I correspond respectively to  $q$ ,  $k$  and  $k'$  in the discussions here.). Hence, we shall write hereafter  $\alpha(b|M_x^2/s)$  or  $\alpha(b|x_P)$  instead of the general expression  $\alpha_{\text{inel.}}(b)$ . This, together with Eq.(3'), leads to the corresponding scattering amplitude  $f(q_{\perp}|x_P)$ , and thus to the corresponding double differential cross-section  $d^2\sigma/dt dx_P$ , in terms of the variables  $|t| \approx |\vec{q}_\perp|^2$  and  $x_P \approx M_x^2/s$  in the kinematical region:  $|t| \ll M_x^2 \ll s$ .

#### 4. The role played by the space-time properties of the gluon-clusters

For the determination of  $\alpha(b|x_p)$ , it is of considerable importance to recall the following space-time properties of the color-singlet gluon-clusters which are BTW-avalanches due to SOC:

(i) SOC dictates, that there are BTW-avalanches of all sizes (which we denote by different S values), and that the probability amplitude of finding an avalanche of size S can be obtained from the size-distribution  $D_S(S) = S^{-\mu}$  where the experimental results presented in Part I show:  $\mu \approx 2$ . This means,  $D_S(S)$  contributes a factor  $S^{-1}$ , thus a factor  $x_P^{-1}$  to the scattering amplitude  $\alpha(b|x_P)$ . Here, as well as in (ii), we take into account (see Sections 4 and 5 of Part I for details) that the size  $S$  of a colorless gluon-cluster is directly proportional to the total amount of the energy the cluster carries; the amount of energy is  $x_P P^0$ , where  $P^0$  is the total energy of the proton, and  $x_P$  is the energy fraction carried by the cluster.

(ii) QCD implies<sup>[19](#page-20-0)</sup> that the interactions between two arbitrarily chosen colored constituents of a BTW-avalanche (which is a colorless gluon cluster) is stronger than those between two color-singlet BTW-avalanches, because the latter should be interactions of Van der Waals type. This means, the struck avalanche can unify with the beam-particle (maybe by absorbing each other), and viewed from any Lorentz frame in which the beam-particle has a larger momentum than that of the colorless gluon-cluster, the latter is "carried away" by the beam particle. Geometrically, the chance for the beam-particle to hit an avalanche of size S ( on the plane perpendicular to the incident axis) is proportional to the area that can be struck by the  $(c.m.s.)$  of the beam particle. The area is the  $2/3$ -power of the volume S,  $S^{2/3}$ , and thus it is proportional to  $x_P^{2/3}$  $\overset{2}{P}$ .

(iii) Based on the above-mentioned picture in which the BTW-avalanches propagate isotropically from  $\langle A_B \rangle$ , the relative number-densities at different b-values can be readily evaluated. Since for a given b, the distance in space between  $\langle A_B \rangle$  and  $B \equiv (0, b, z)$  is simply  $(b^2 + z^2 + a^2)^{1/2}$ , the number of avalanches which pass a unit area on the shell of radius  $(b^2 + z^2 + a^2)^{1/2}$  centered at  $\langle A_B \rangle$  is proportional to  $(b^2 + z^2 + a^2)^{-1}$ , provided that

(because of causality) the lifetimes (T's) of these avalanches are not shorter than  $\tau_{\min}(b)$ . The latter is the time interval for an avalanche to travel from  $\langle A_B \rangle$  to B. This means, because of the space-time properties of such avalanches it is of considerable importance to note: First, only avalanches having lifetimes  $T \geq \tau_{\min}(b)$  can contribute to such a collision event. Second, during the propagation from  $\langle A_B \rangle$  to B, the motion of such an avalanche has to be considered as Brownian. In fact, the continual, and more or less random, impacts received from the neighboring objects on its path leads us to the well known<sup>[20](#page-20-0)</sup> result that the time elapsed is proportional to the mean-square displacement. That is:  $\tau_{\min}(b) \propto b^2 + z^2 + a^2$ . Furthermore, we recall that avalanches are due to SOC, and thus the chance for an avalanche of lifetime T to exist is  $D_T(T) \propto T^{-\nu}$  where the experimental value<sup>[1](#page-18-0)</sup> for  $\nu$  is  $\nu \approx 2$ . Hence, by integrating  $T^{-2}$  over T from  $\tau_{\min}(b)$  to infinity, we obtain the fraction associated with all those whose lifetimes satisfy  $T \geq \tau_{\min}(b)$ : This fraction is  $\tau_{\min}(b)^{-1}$ , and thus a constant times  $(b^2 + z^2 + a^2)^{-1}$ .

The amplitude  $\alpha(b|x_P)$  can now be obtained from the probability amplitude for avalanche-creation mentioned in (i), by taking the weighting factors mentioned in (ii) and (iii) into account, and by integrating over  $z^{21}$  $z^{21}$  $z^{21}$ . The result is:

$$
\alpha(b|x_P) = \text{const.} x_P^{-1/3} (b^2 + a^2)^{-3/2} \,. \tag{4}
$$

By inserting this probability amplitude in impact-parameter-space, for the beam particle to encounter colorless gluon-clusters (avalanches in the BTW-theory) which carries a fraction  $x_P$  of the proton's total energy, in Eq.(3') [which is Eq.(3) with the folowing replacements:  $q \to q_{\perp}, f(q) \to f_{\text{inel.}}(q|x_P)$  and  $\alpha(b) \to \alpha_{\text{inel.}}(b|x_P)$  we obtain the corresponding probability amplitude  $f(q|x_P)$  in momentum-space:

$$
f(q_{\perp}|x_P) = \text{const.} \int_0^\infty b \, db \, x_P^{-1/3} (b^2 + a^2)^{-3/2} J_0(q_{\perp}b), \tag{5}
$$

where  $q_{\perp} = |\vec{q}_{\perp}| \approx \sqrt{|t|}$  (in the small  $x_P$ -region,  $x_P < 0.1$ , say) is the corresponding momentum-transfer. The integration can be carried out analytically<sup>[22](#page-20-0)</sup>, and the result is

$$
f(q_{\perp}|x_P) = \text{const.} x_P^{-1/3} \exp(-aq_\perp). \tag{6}
$$

Hence, the corresponding double differential cross-section  $d^2\sigma/dtdx_P$  can approximately be written as

$$
\frac{1}{\pi} \frac{d^2 \sigma}{dt \, dx_P} = N x_P^{-2/3} \exp(-2a\sqrt{|t|}),\tag{7}
$$

where  $N$  is an unknown normalization constant. Because of the kinematical relationship  $x_P \approx M_x^2/s$  for single diffractive scattering in proton-proton and proton-antiproton collisions (see Fig.8 of Part I for more details), this can be, and should be, compared with the measured double differential cross-sections  $d^2\sigma/dt d(M_x^2/s)$  at different t- and s-values and for different missing masses  $M_x$  in the region  $M_x^2/s \ll 1$  where  $q_\perp$  is approximately  $\sqrt{|t|}$ . The comparison is shown in Fig.1. Here, we made use of the fact that  $a^2 \equiv 3/5 r_p^2$  $_p^2$ , where  $r_p$  is the proton radius, and calculated a by setting  $r_n^2$  $_p^2$  to be the well-known<sup>[23](#page-20-0)</sup> mean square proton charge radius, the value of which is  $r_p^2 = (0.81 \text{ fm})^2$ . The result we obtained is:  $a = 3.2 \text{ GeV}^{-1}$ . The unknown normalization constant is determined by inserting this calculated value for a in Eq.(7), and by comparing the right-hand-side of this equation with the  $d^2\sigma/dt d(M_x^2/s)$ data taken at  $|t| = 0.2 \,\text{GeV}^2$ . The value is  $N = 31.1 \,\text{mb GeV}^{-2}$ . All the curves shown in Fig.1 are obtained by inserting these values for  $a$  and  $N$  in Eq.(7).

While the quality of the obtained result, namely the expression given on the right-handside of Eq.(7) together with the above-mentioned values for  $a$  and  $N$ , can be readily judged by comparing it with the data, or by counting the unknown parameters, or both, it seems worthwhile to recall the following: The two basic ingredients of the proposed picture which have been used to derive this simple analytical expression are: first, the well-known optical analogy, and second, the properties of the dynamical scattering system. The latter is what we have learned through the data-analysis presented in Part I of this paper.

Based on the theoretical arguments and experimental indications for the observation (see Ref.[\[17\]](#page-15-0) and Part I of this paper for details) that the characteristic features of inelastic diffractive scattering processes are approximately independent of the incident energy and independent of the quantum-numbers of the beam-particles, the following results are expected: The explicit formula for the double differential cross-section as shown in Eq.(7) should also

<span id="page-15-0"></span>be valid for the reactions  $\gamma p \to Xp$  and  $\gamma^* p \to Xp$ . While the normalization constant N (which should in particular depend on the geometry of the beam particle) is expected to be different for different reactions, everything else – especially the "slope" as well as the power of  $x_P$  should be exactly the same as in  $pp$ - and  $p\bar{p}$ -collisions. In this sense, Eq.(7) with  $a^2 = 3/5r_F^2$  $P_P^2$  (r<sub>P</sub> is the proton radius) is our prediction for  $\gamma p \to Xp$  and  $\gamma^* p \to Xp$ which can be measured at HERA.

Furthermore, in order to obtain the integrated differential cross-section  $d\sigma/dt$ , which has also been measured for different reactions at different incident energies, we only need to sum/integrate over  $x_P$  in the given kinematic range  $(x_P < 0.1, \text{say})$ . The result is

$$
\frac{d\sigma}{dt}(t) = C \exp(-2a\sqrt{|t|}),\tag{8}
$$

where  $C$  is an unknown normalization constant. While this observation has already been briefly discussed in a previous note<sup>[17](#page-19-0)</sup>, we now show the result of a further test of its universality: In Fig. 2, we plot

$$
-\frac{1}{2\sqrt{|t|}}\log[\frac{1}{C}\frac{d\sigma}{dt}(t)]\,\,\text{vs.}\,\,t\tag{9}
$$

for different reactions at different incident energies in the range  $0.2 \,\text{GeV}^2 \leq |t| \leq 4 \,\text{GeV}^2$ . Here we see in particular that, measurements of  $d\sigma/dt$  for  $\gamma^{\star}p$  and  $\gamma p$  reactions at larger |t|-values would be very useful.

# 5. Concluding remarks

Based on the characteristic properties of the gluons — in particular the local gluon-gluon coupling prescribed by the QCD-Lagrangian, the confinement, and the non-conservation of gluon-numbers, we suggest that a system of interacting soft gluons should be considered as an open dynamical complex system which is in general far away from equilibrium. Taken together with the observations made by Bak, Tang and Wiesenfeld  $(BTW)^{9,10}$  $(BTW)^{9,10}$  $(BTW)^{9,10}$  $(BTW)^{9,10}$  $(BTW)^{9,10}$ , we are led to the conclusion<sup>[1](#page-18-0)</sup>, that self-organized criticality (SOC) and thus BTW-avalanches exist in such systems, and that such avalanches manifest themselves in form of color-singlet gluon-clusters in inelastic diffractive scattering processes.

In order to test this proposal, we performed a systematic data-analysis, the result of which is presented in Part I of this paper: It is shown that the size-distributions, and the lifetime-distributions, of such gluon-clusters indeed exhibit power-law behaviors which are known as the fingerprints of  $SOC<sup>9,10</sup>$  $SOC<sup>9,10</sup>$  $SOC<sup>9,10</sup>$  $SOC<sup>9,10</sup>$  $SOC<sup>9,10</sup>$ . Furthermore, it is found that such exponents are approximately the same for different reactions at different incident energies — indicating the expected universality and robustness of SOC. Hence, the following picture emerges: For the beam particle (which may be a virtual photon, or a real photon, or a proton, or an antiproton; see Fig.8 in Part I for more details) in an inelastic diffractive scattering process off proton (one may wish to view this from a "fast moving frame" such as the c.m.s. frame), the target proton appears as a cloud of colorless gluon-clusters which exist inside and outside the confinement region of the proton. The size  $(S)$  distribution  $D_S(S)$  and the lifetime distribution  $D_T(T)$  can be expressed as  $S^{-\mu}$  and  $T^{-\nu}$  respectively, where the empirical values for  $\mu$  and  $\nu$  are  $\mu \approx \nu \approx 2$ , independent of the incident energy, and independent of the quantum numbers of the beam particles.

What do we learn from this? Is this knowledge helpful in understanding hadronic structure and/or hadronic reactions in Particle Physics? In particular, can this knowledge be used to do *quantitative* calculations  $-$  especially those, the results of which could not be achieved otherwise?

In order to demonstrate how the obtained knowledge can be used to relate hadronstructure and hadronic reactions in general, and to perform quantitative calculations in particular, we discuss the following question — a question which has been with the highenergy physics community for many years:

"Can the measured double differential cross section  $d^2\sigma/dt d(M_x^2/s)$  for inelastic diffractive scattering in proton-proton and in antiproton-proton collisions, in the kinematical region given by  $0.2 \text{ GeV}^2 \le |t| \le 3.25 \text{ GeV}^2$ ,  $16 \text{ GeV} \le \sqrt{s} \le 630 \text{ GeV}$ , and  $M_x^2/s \le 0.1$ , be understood in terms of optical geometrical concepts?"

The answer to this question is "Yes!", and the details are presented in Sections [2,](#page-5-0) [3](#page-9-0) and [4](#page-11-0) of Part II where the following are explicitly shown: The characteristic features of the existing  $d^2\sigma/dt d(M_x^2/s)$ -data are very much the same as those in optical diffraction, provided that the high-energy beams are considered as high-frequency waves, and the scatterer is a system of colorless gluon-clusters described in Part I of this paper. Further measurements of double differential cross sections, especially in  $\gamma^* p$ - and  $\gamma p$ -reactions, will be helpful in testing the ideas presented here.

# Acknowledgement

We thank C.B. Yang and W. Zhu for helpful discussions, and FNK der FU Berlin for financial support. Y. Zhang thanks Alexander von Humboldt Stiftung for the fellowship granted to him.

# <span id="page-18-0"></span>REFERENCES

- 1. C. Boros, T. Meng, R. Rittel and Y. Zhang, Part I of this paper.
- 2. M. Derrick, et al., ZEUS Coll., Phys. Lett. B315, 481 (1993); T. Ahmed, et al., H1 Coll., Nucl. Phys. B429, 477 (1994).
- 3. M. Derrick, et al., ZEUS Coll., Phys. Lett. B345, 576 (1995); M. Derrick, et al., ZEUS Coll., Z. Phys. C68, 569 (1995); ibid, C70, 391 (1996); T. Ahmed, et al., H1 Coll., Phys. Lett. B348, 681 (1995); J.Breitweg, et al., ZEUS Coll., Eur. Phys. J. C1, 81 (1998).
- 4. C. Adloff, et al., H1 Coll., Z. Phys. C74, 221 (1997); J. Breitweg, et al., ZEUS Coll., Z. Phys. C75, 421 (1997); M. Derrick, et al., ZEUS Coll., Phys. Lett. B293, 465 (1992).
- 5. M.G. Albrow, et al., CHLM Coll., Nucl. Phys. B108, 1 (1976); R. D. Schamberger, et al., Phys. Rev. D17, 1268 (1978).
- 6. M. Bozzo, et al., UA4 Coll., Phys. Lett. B136, 217 (1984); D. Bernard, et al., UA4 Coll., Phys. Lett. B186, 227 (1987); A. Brandt, et al., UA8 Coll., Nucl. Phys. B514, 3 (1998).
- 7. R. L. Cool, et al., Phys. Rev. Lett. 47, 701 (1981).
- 8. See for example: M. Derrick, et al., ZEUS Coll., Phys. Lett. B345, 576 (1995); S. Aid, et al., H1 Coll., Phys. Lett. B354, (1995) 494; and the references cited therein.
- 9. P. Bak, C. Tang and K. Wiesenfeld, Phys. Rev. Lett. 59, 381 (1987); Phys. Rev. A38, 364 (1988).
- 10. See for example: P. Bak and M. Creutz, in Fractals and Self-organized Criticality, in Fractals in Science, edited by A. Bunde and S. Havlin (Springer, Berlin, Heidelberg 1994); P. Bak, How nature works (Springer, New York 1996); and the references therein.
- 11. See for example: J. Bartels, in Proceedings of the 17th Int. Symp. on Lepton-Photon

<span id="page-19-0"></span>Interactions, Beijing, 1995, edited by Zheng Zhi-peng and Chen He-sheng, (World Scientific, 1996), Vol. 2, p. 554; H. Abramowicz, J. Bartels, L. Frankfurt and H. Jung, in Proceedings of the Workshop on Future Physics at HERA, edited by G. Ingelman, A. De Roeck, R. Klanner, (DESY, 1996), Vol. 2, p. 535; E. Gallo, in Proceedings of the 18th Int. Symp. on Lepton - Photon Interactions, Hamburg, 1997, edited by A. DeRoeck and A. Wagner, (World Scientific, Singapore 1998); and the references therein.

- 12. F. E. Low, Phys. Rev. D12, 163 (1975); S. Nussinov, Phys. Rev. Lett. 34, 1286 (1975); C. Boros, Z. Liang and T. Meng, Phys. Rev. D54, 6658 (1996).
- 13. See for example: M. M. Block and R. N. Cahn, Rev. Mod. Phys., Vol. 57, No. 2, 563 (1985); J. Lee-Franzini, in AIP Conference Proceedings No. 15, (American Institute of Physics, New York 1973) p.147; K. Goulianos, Phys. Rep. 101 No.3, 169 (1983); and in Proceedings of DIS97, Chicago, 1997, edited by J. Repond and D. Krakauer, (AIP), p. 527; U. Amaldi, M. Jacob, and G. Matthiae, Ann. Rev. Nucl. Sci. 26, 385 (1976) and the references therein.
- 14. See for example: R. Serber, Rev. Mod. Phys. 36, 649 (1964); N. Byers and C. N. Yang, Phys. Rev. 142, 976 (1966); T. T. Chou and C. N. Yang, Phys. Rev. 170, 1591 (1968); 175, 1832 (1968); 22, 610 (1980); and the references therein.
- 15. See in particular: F. E. Low, Phys. Rev. D12, 163 (1975); S. Nussinov, Phys. Rev. Lett.34, 1286 (1975); G. Ingelman and P. Schlein, Phys. Lett. B152, 256 (1985); A. Donnachie and P.V. Landshoff, Phys. Lett. B191, 309 (1987); Y. A. Simonov, Phys. Lett. B249, 514 (1990); G. Ingelman and K. Janson-Prytz, Phys. Lett. B281, 325 (1992); G. Ingelman and K. Prytz, Z. Phys. C58, 285 (1993); and the references therein.
- 16. G. Sterman (private communication).
- 17. T. Meng, R. Rittel and Y. Zhang, Phys. Rev. Lett. (in press).
- 18. See for example: L. D. Landau and E. M. Lifshitz, The Classical Theory of Fields, 2nd

<span id="page-20-0"></span>rev. edn. (Pergamon Press, Oxford 1962), p. 177 and p. 165.

- 19. See for example: K. Gottfried and V. Weiskopf, Concepts in Particle Physics, (Oxford University Press, New York, and Clarendon Press, Oxford 1986), vol. II, p. 347.
- 20. See for example: R. K. Pathria, Statistical Mechanics, (Pergamon Press, Oxford 1972), p. 451.
- 21. To be more precise, by taking all the mentioned factors into account we obtain  $\alpha(b, z|x_P) \propto \text{const } x_P^{-1+2/3}$  $\int_{P}^{-1+2/3} (b^2 + z^2 + a^2)^{-1-1}$ . Note also that  $\int_{-\infty}^{\infty} (b^2 + z^2 + a^2)^{-2} dz \propto$  $(b^2 + a^2)^{-3/2}.$
- 22. See for example: I. S. Gradshteyn and I. M. Ryzhik, Table of Integrals, Series and Products, (Academic Press, New York 1980), p. 682.
- 23. See for example: F. Halzen and A. D. Martin, Quarks and Leptons: An Introductory Course in Modern Particle Physics, (John Wiley and Sons, 1984), p. 179.

## FIGURES

Fig. 1. The double differential cross section  $(1/\pi) d^2\sigma/dt d(M_x^2/s)$  for single diffractive pp and  $\bar{p}p$ reactions is shown as function of  $x_P$  at fixed values of t where  $0.15 \,\text{GeV}^2 \leq |t| \leq 3.25 \,\text{GeV}^2$ . The data are taken from Refs.[[5-7\]](#page-15-0). The solid curve is the result obtained from Eq.(7). The dashed curve stands for the result obtained from the same formula by using the t-value given in the bracket.

Fig. 2. The quantity  $\left(\frac{-1}{2\sqrt{|t|}}\right)$  log  $\left[\frac{1}{C}d\sigma/dt\right]$  is plotted versus  $\sqrt{|t|}$  for different single diffractive reactions in the range  $0.2 \text{ GeV}^2 \le |t| \le 4 \text{ GeV}^2$ . The data are taken from Refs. [\[2-7](#page-15-0)]. Here, C, the normalization constant is first determined by performing a two-parameter fit of the corresponding  $d\sigma/dt$ -data to Eq.(8).







24

