## IMPLICATIONS OF THE SUPERKAMIOKANDE RESULT ON THE NATURE OF NEW PHYSICS<sup>1</sup>

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## Abstract

It is remarked that the SuperKamiokande (SK) discovery of  $\nu_{\mu}$  to  $\nu_{\tau}$  (or  $\nu_{\text{X}}$ )-oscillation, with a  $\delta m^2 \approx 10^{-2} - 10^{-3} eV^2$  and  $\sin^2 2\theta > 0.8$ , provides a clear need for the right-handed (RH) neutrinos. This in turn reinforces the ideas of the left-right symmetric gauge structure  $SU(2)_L \times SU(2)_R$  as well as  $SU(4)$ -color, for which the RH neutrinos are a compelling feature. It is noted that by assuming (a) that B-L and  $I_{3R}$ , contained in a string-derived  $G(224) = SU(2)_L \times SU(2)_R \times SU(4)^c$  or  $SO(10)$ , break near the GUT-scale, as opposed to an intermediate scale, (b) the see-saw mechanism, and (c) the  $SU(4)$ -color relation between the Dirac mass of the tau neutrino and  $m_{top}$ , one obtains a mass for  $\nu_L^{\tau}$  which is just about what is observed. This is assuming that the SK group is actually seeing  $\nu_L^{\mu}$  $L^{\mu} - \nu_L^{\tau}$  (rather than  $\nu_{\mathrm{L}}^{\mu}$  $L_{\rm L}^{\mu} - \nu_{\rm X}$ )-oscillation. Following a very recent work by Babu, Wilczek and myself, it is furthermore noted that by adopting familiar ideas of understanding Cabibbo-like mixing angles in the quark-sector, one can quite plausibly obtain a large  $\nu_L^{\mu}$  $\bar{\mu} - \nu_{\text{L}}^{\tau}$  oscillation angle, as observed, in spite of highly non-degenerate masses of the light neutrinos: e.g. with  $\mathrm{m}(\nu_{\mathrm{L}}^{\mu})$  $\binom{\mu}{L}$  /m( $\nu_L^{\tau}$ )  $\approx 1/10 - 1/20$ . Such non-degeneracy is of course natural to see-saw. In this case,  $\nu_{\rm L}^{\rm e} - \nu_{\rm L}^{\mu}$  $L<sup>\mu</sup>$  oscillation can be relevant to the small angle MSW explanation of the solar neutrino-puzzle. Implications of the mass of  $\nu_{\text{L}}^{\tau}$  suggested by the SK result, on proton decay are noted. Comments are made at the end on how the SuperKamiokande result supplements the LEP result in selecting out the route to higher unification.

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1. Introduction: The SuperKamiokande (SK) result, convincingly showing the oscillation of  $\nu_\mu$  to  $\nu_\tau$  (or  $\nu_\text{X}$ ), with a value of  $\delta m^2 \approx 10^{-2}$  to  $10^{-3}$  eV<sup>2</sup> and  $\sin^2 2\theta > 0.8$ [[1\]](#page-15-0), appears to be the *first* clear evidence for the existence of new physics beyond the standard model. The purpose of this note is to make two points regarding the implications of the SK result, which though simple, seem to be far-reaching. The first is the argument as to why one needs new physics beyond the standard model. The second is the remark that the SK result already tells us much about the nature of the new physics. In particular, it seems to suggest clearly the existence of right-handed neutrinos, a new form of matter, accompanying the observed left-handed ones. This in turn reinforces the twin ideas of the left-right symmetric gauge structure  $\rm SU(2)_L \times SU(2)_R$  and of  $\rm SU(4)$  color, which were proposed some time ago as a step towards higher unification[[2\]](#page-15-0). Either one of these symmetries require the existence of the right-hand neutrinos. I note that by assuming (a) that B-L and  $I_{3R}$ , contained in a string or a GUT-derived  $G(224) = SU(2)_L \times SU(2)_R \times SU(4)^c$ , break near the GUT-scale as opposed to an intermediate or a low-energy scale, (b) the see-saw mechanism [\[3\]](#page-15-0), and (c) the SU(4)-color relation between the Dirac mass of  $\nu_{\tau}$  and  $m_{top}$ , one obtains a mass for  $\nu_{\rm L}^{\tau}$  which is just about what is observed. This is presuming that the SK group is actually observing  $\nu_{\rm L}^{\mu} - \nu_{\rm L}^{\tau}$ , (rather than  $\nu_{\rm L}^{\mu} - \nu_{\rm X}$ ), oscillation and that the neutrino masses are hierarchical  $\rm (m(\nu_L^e) << m(\nu_L^\mu$  $\binom{\mu}{L}$  < m( $\nu_L^{\tau}$ )), so that the observed value of  $\delta m^2$  in fact represents the (mass)<sup>2</sup> of  $\nu_{\text{L}}^{\tau}$ . Such a hierarchical pattern, as opposed to near degeneracy of two or three neutrino flavors, is of course naturally expected within the see-saw formula. Following a very recent work by Babu, Wilczek and myself[[4\]](#page-15-0), I furthermore note that by combining contributions to the oscillation angle from the neutrino and the charged lepton-sectors, and by following familiar ideas on the understanding of Cabibbo-like mixing angles in the quark-sector, one can quite plausibly obtain a large  $\nu_{\rm L}^{\mu} - \nu_{\rm L}^{\tau}$ -oscillation angle, as observed, in spite of hierarchical masses of the light neutrinos: e.g. with  $m(\nu_L^{\mu})$  $L_{\text{L}}^{\mu}$ /m( $\nu_{\text{L}}^{\tau}$ )  $\approx 1/10 - 1/20$ . In this case,  $\nu_{\text{L}}^e - \nu_{\text{L}}^{\mu}$  oscillation can be relevant to the small angle MSW explanation of the solar neutrino puzzle. The results on  $\delta m^2$  and mixing obtained in the context of  $G(224)$  can of course be obtained within any extension of  $G(224)$ , such as  $SO(10)$ [\[5\]](#page-16-0), together with supersymmetry. At the end, implications of the neutrino mass-scale observed at SuperKamiokande on proton decay are noted. Comments are made on how the SK result supplements that of LEP in selecting out the route to higher unification.

2. The Need for New Physics: First, as we know, the standard model (SM), based on the gauge symetry  $SU(2)_L \times U(1)_Y \times SU(3)^C$ , contains 15 two-component objects in each family – e.g. for the electron-family they are:  $[Q = (u_L, d_L), L = (\nu_L^e, e_L^-)$  $_L^-$ ,  $u_R$ ,  $d_R$  and  $e_R$ ] - and the Higgs doublet  $H = (H^+, H^o)$ . Notice that in the standard model, the left-handed neutrino  $\nu_L$ is an odd ball in that it is the only member in each family which does not have a right-handed counterpart $\nu_R$ . This feature in fact carries over to its grand unifying extension SU(5) as well [[6\]](#page-16-0). In other words, the standard model (as also SU(5)) provides a clear distinction between left and right, in the spectrum as well as in the gauge interactions, and thus explicitly violates parity and charge conjugation.

Can the neutrinos acquire masses in the standard model? Without a right-handed counterpart, a left-handed neutrino  $\nu$ <sub>L</sub> cannot acquire a Dirac mass. But it may still acquire a Majorana mass (like  $m_L \nu_L^T C^{-1} \nu_L$ ), by utilizing the effects of quantum gravity, which of course exists even for the SM, and which may induce a lepton-number violating non-renormalizable operator (written schematically) in the form[\[7](#page-16-0)]

$$
\lambda_{\rm L} L L H H / M_{\rm Pl} + \text{hc.} \tag{1}
$$

Here,  $M_{Pl}$  denotes the reduced Planck mass =  $2 \times 10^{18}$  GeV and  $\lambda_L$  is the effective dimensionless coupling. Apriori, we would expect  $\lambda_L$  to be of order one, unless there are symmetries that are respected by quantum gravity, like local (B-L), which may suppress  $\lambda_L$ ; in this case, it would be less than one. In the SM, however, there is no such symmetry. Using the VEV of  $\lt H \gt \approx$ 250GeV, such an operator would then give:

$$
m(\nu_{L}) \approx \lambda_{L} \frac{(250 \text{GeV})^{2}}{2 \times 10^{18} \text{GeV}} \approx (\lambda_{L})(3 \times 10^{-5} \text{eV})
$$
\n(2)

Such a mass would lead to values of  $\delta m^2$  (for any two light neutrino-species)  $\leq \lambda_L^2 (10^{-9} eV^2)$ . This is far too small (even for ridiculously large  $\lambda_L \sim 10^2$ , say) compared to the observed value of  $\delta m^2 \approx 10^{-2} - 10^{-3} eV^2$ .<sup>1</sup> It thus follows rather conclusively that the specific range of values

<sup>&</sup>lt;sup>1</sup>One might have asked whether the mass-scale in the denominator of eq. (1) could plausibly be the GUT scale  $(\approx 2 \times 10^{16} \text{GeV})$ , instead of the reduced Planck mass. That would have given  $m(\nu) \approx \lambda_L(3 \times 10^{-3} \text{eV})$ , which is closer but still a bit low compared to the SuperKamiokande value of  $(10^{-1}$  to  $3 \times 10^{-2}$ eV), unless  $\lambda_L \approx 30$  to 10.

of δm<sup>2</sup> reported by SuperKamiokande cannot reasonably be accommodated within the standard model, even with the inclusion of quantum gravity, and thus there <u>must</u> exist new physics beyond the standard model.

3. The Nature of New Physics: We now go further and turn to the second point about the nature of the new physics, suggested by the SK result. The only reasonable way to understand a mass for the neutrino or  $\delta m^2$ , as observed, it seems to me, is to introduce a right-handed (RH) neutrino  $(\nu_R)$  and utilize the see-saw mechanism (described below).<sup>2</sup> This in turn has far-reaching implications. The existence of a RH neutrino becomes compelling by extending the SM symmetry toinclude either SU(4)-color or the left-right symmetric gauge-structure  $SU(2)_L \times SU(2)_R$ , [[2\]](#page-15-0). Thus the SK result motivates, on observational ground, the route to higher unification via the gauge-structure:

$$
G(224) = SU(2)L \times SU(2)R \times SU(4)C.
$$
\n(3)

This is the minimal extension of the SM that specifies all quantum numbers (given a representation), quantizes electric charge and introduces  $\nu_R$ . With respect to G(224), quarks and leptons of a given family fall into the neat pattern[[2\]](#page-15-0):

$$
\mathbf{F}_{\mathrm{L,R}}^{\mathrm{e}} = \begin{bmatrix} u_r & u_y & u_b & \nu_e \\ d_r & d_y & d_b & e^- \end{bmatrix}_{\mathrm{L,R}} \tag{4}
$$

with the transformation properties  $F_L^e = (2, 1, 4)$ , and  $F_R^e = (1, 2, 4)$ ; likewise for the  $\mu$  and the  $\tau$ families. We see that the RH neutrino  $(\nu_R)$  arises as the fourth color partner of the RH up-quarks and, also, as the left-right conjugate partner of the LH neutrino  $(\nu_L)$ . It is worth noting that the symmetry G(224), subject to L-R discrete symmetry [2,8], possesses some additional advantages, But, more to the point, in the context of the standard model, supplemented by just gravity, while Planck mass seems to have every reason to appear in eq. (1), there does not seem to be any simple reason for the relevance of the GUT scale. Putting it another way, if the GUT scale is needed in eq. (1) for numerical agreement, that by itself calls for new physics beyond the Standard model. I thank S. Weinberg, who had considered operators like eq. (1) long ago [\[7](#page-16-0)] for raising this point and for discussions.

<sup>2</sup>The alternative of giving a Majorana mass to  $\nu$ <sub>L</sub> through renormalizable interaction by introducing a SU(2)<sub>L</sub> Higgs-triplet  $\xi$  and choosing the corresponding (Yukawa coupling)  $\times$  (VEV of  $\xi$ ) to be nearly (1/10 - 1/30)eV seems to be rather arbitrary.

even without being embedded into a simple group like  $SO(10)$  [\[5](#page-16-0)] or  $E_6$  [\[9](#page-16-0)]. These include: (i) inclusion of all members of a family into one multiplet, (ii) quark-lepton unification through SU(4)-color, (iii) quantization of electric charge, mentioned above, (iv) spontaneous violations of parity $[2, 8]$  $[2, 8]$  $[2, 8]$  and of CP  $[10]$  $[10]$ ,  $(v)$   $(B-L)$ , as a local symmetry whose spontaneous violation may be needed to implement baryogenesis [\[11](#page-16-0)], (vi) a promising solution to the strong CP problem inthe context of supersymmetry [[12\]](#page-16-0), and (vii) a possible resolution of the  $\mu$ -problem in the same context [\[13](#page-16-0)]. Embedding  $G(224)$  into  $SO(10)$ , for which  $(F_L^e + \overline{F_R^e})$  yield the 16 of  $SO(10)$ , would of course retain most of these advantages, except possibly (vii). Last, but not least, the symmetry G(224) can emerge from strings with three chiral families (see e.g. Refs. 14 and 15). In this case, the gauge coupling unification[[16\]](#page-16-0) at string scale would still hold[[17](#page-17-0)] even without having the covering  $SO(10)$ , below the string scale.<sup>3</sup> It is worth noting that in the string context there is a distinct advantage if the preferred string solution would contain  $G(224)$  rather than SO(10), because it appears rather difficult to implement doublet-triplet splitting for string-derived  $SO(10)$  so as to avoid rapid proton decay.[\[20](#page-17-0)]For string-derived  $G(224)$  [[14\]](#page-16-0), on the other hand, the dangerous color triplets are either projected out or naturally become superheavy.

**4. The Mass of**  $\nu_{\text{L}}^{\tau}$ : I now turn to an estimate of the masses of the light neutrinos, that are observed in the laboratory, especially the  $\nu_L^{\tau}$ , allowing for the existence of the RH neutrinos  $(\nu_R's)$ . For this purpose, I will work with either  $G(224)$  or its natural extension  $SO(10)$ . With a string or a GUT-origin, one can motivate the symmetry-breaking scale for either  $G(224)$  or  $SO(10)$ , to be around  $M_{string}/10$ , which is nearly the (empirical) GUT-scale  $\approx 2 \times 10^{16}$ GeV.

The amusing thing is that, in contrast to the case of the SM (eq.(1)), now the mass of  $\nu_{\text{L}}^{\tau}$ comes out to be just in the right range, so as to be relevant to the SK result.

The simplest reason for the known neutrinos to be so light  $( $30 \text{eV}$  (say)) is provided by$ the so-called see-saw mechanism [\[3](#page-15-0)]. It utilizes the fact that neutrinos being electrically neutral

<sup>&</sup>lt;sup>3</sup>Possible resolutions of a mismatch between MSSM and string-unification scales by about a factor of 20 have been proposed, including one that suggests two vector-like families  $(16 + \overline{16})$  at the TeV-scale, that leads to semiperturbative unification and raises  $M_X$  to a few  $\times 10^{17}$  GeV[\[18](#page-17-0)]; and also one that makes use of string duality[\[19](#page-17-0)] and allows for a re-evaluation of  $M_{string}$  compared to that of Ref. [17]. In general, both ideas may play a role.

can have two sources of mass: (i) first, with both  $\nu_{\rm L}^{\rm i}$  and  $\nu_{\rm R}^{\rm i}$ , neutrino of the ith family would naturally acquire a Dirac mass  $m(\nu^i_D)$  which would be related to the up-flavor quark-mass  $(m_u, m_c)$ or  $m_t$ ), depending upon the Higgs representation (see below), by SU(4)-color. (ii) Second, since RH neutrinos are standard model singlets they can acquire superheavy Majorana masses  $(M_{iR})$ , preserving the SM symmetry; by utilizing the VEV of a suitable Higgs multiplet (call it  $\Sigma$ ), which would be involved in breaking  $SO(10)$  or  $G(224)$  to the SM symmetry  $G(213)$ . Before discussing the choice of  $\Sigma$  and its coupling, let us recall that a mass-matrix involving Dirac and superheavy Majorana masses, as mentioned above, would diagonalize to yield three superheavy RH neutrinos with masses  $M_{iR}$  and three light LH neutrinos with masses [\[3](#page-15-0)]:

$$
m(\nu_L^i) \approx m(\nu^i)_D^2 / M_{iR}
$$
 (5)

In writing this, we have neglected (for simplicity) possible off diagonal mixings between different flavors. Since we will be interested in this note primarily in the mass of the heaviest one among the light neutrinos (i.e.  $\nu_{\text{L}}^{\tau}$ ), such mixings will not be so important. (For a more general analysis, see e.g. Ref. 4 and 21). Since the Dirac masses enter quadratically into (5), and are highly hierarchical (e.g.  $m_u : m_c : m_t \approx 1 : 300 : 10^5$ ), we expect, even allowing for a rather large hierarchy (by successive factors of order 100, say) in  $M_{iR}$ , that the masses of the left-handed neutrinos will be light but hierarchical (m( $\nu_{\rm L}^{\rm e}$ ) << m( $\nu_{\rm L}^{\mu}$  $_{\mathrm{L}}^{\mu}$ ) << m( $\nu_{\mathrm{L}}^{\tau}$ )).

The Higgs multiplet  $\Sigma$ , mentioned above, and its conjugate  $\overline{\Sigma}$  (needed for supersymmetry), can either be in a symmetric tensorial representation  $[3]$  $[3]$  $[3]$  - i.e.  $(126_H, 126_H)$  of SO(10) or equivalently  $[(1,3,10), (1,3,\overline{10})]$  of G(224) - or in the spinorial representation - i.e.  $(16_H, \overline{16}_H)$  [\[22\]](#page-17-0) of SO(10) - or equivalently in  $[(1, 2, 4)]_H$ ,  $(1, 2, 4)$  [\[2](#page-15-0)] of G(224), like the quarks and the leptons. For a string-derived G(224), the L-R conjugate multiplets (like  $(3, 1, 10)$ <sub>H</sub> or  $(2, 1, 4)$ <sub>H</sub> etc.) should also exist if L-R discrete symmetry (i.e. parity) is preserved in the Higgs-sector, following stringcompactification. (In general, even if  $G(224)$  emerges as a gauge symmetry, after compactification, and the spectrum of 3 chiral families respect L-R discrete symmetry, the full spectrum need not. See e.g. Ref. 14, where the multiplets  $(1, 2, 4)$ <sub>H</sub> and  $(1, 2, 4)$ <sub>H</sub> do emerge, but not their (L-R) conjugates  $(2, 1, 4)$ <sub>H</sub> and  $(2, 1, 4)$ <sub>H</sub>.)

We first remark that, in string theory, the tensorial representations  $126<sub>H</sub>$  and  $126<sub>H</sub>$ , and likewise  $(1, 3, 10)$ <sub>H</sub> and  $(1, 3, \overline{10})$ <sub>H</sub>, which can have renormalizable Yukawa interactions with quarks and leptons, are hard, perhaps impossible, to realize[[23](#page-17-0)], and have not been realized in any solution yet. By contrast, the spinorial  $16_H$  and  $\overline{16}_H$ , as also  $(1, 2, 4)_H$  and  $(1, 2, \overline{4})_H$ , do emerge quite simply in string-solutions (see e.g. Ref. 14 for  $G(224)$  and Ref. 20 for  $SO(10)$ ). Taking this as a good guide, and believing in the string-origin of the effective theory just above the GUT-scale, we will work only with the spinorial  $16_H$  and  $\overline{16}_H$ , or equivalently with  $(1, 2, 4)_H$  and  $(1, 2, \overline{4})_H$ .

The effective non-renormalizable interaction, involving these multiplets, which we expect might be induced by Planck-scale physics, and would give Majorana masses to the RH neutrinos, are then<sup>4</sup>

$$
\mathcal{L}_{M}(SO(10)) = \lambda_{R}^{ij} 16_{i} \cdot 16_{j} \overline{16}_{H} \cdot \overline{16}_{H} / M_{P\ell} + hc
$$
\n(6)

$$
\mathcal{L}_{M}(G(224)) = \lambda_{R}^{ij}(1, 2, 4)_{i}(1, 2, 4)_{j}(1, 2, \overline{4})_{H}(1, 2, \overline{4}_{H})/M_{p\ell} + hc
$$
\n(7)

Here, i, j = 1, 2, 3, correspond respectively to e,  $\mu$  and  $\tau$ -families. Note that in each case, we have set the scale of the interaction to be given by the reduced Planck mass, as in eq. (1). Such effective non-renormalizable interactions may well arise – in part or dominantly – by renormalizable interactions through tree-level exchange of superheavy states, such as those in the string-tower (see remarks later).

Judging from the string-side, one naturally expects the VEVs of fields which break GUTlike symmetries – i.e.  $SO(10)$  or  $G(224)$  – to the standard model symmetry to be of order  $M_{string}/(5 \text{ to } 20) \approx 2-8 \times 10^{16} \text{GeV}$  [see, e.g. Ref. 24 and 14], where  $M_{string} \approx 4 \times 10^{17} \text{ GeV}.$  [[17\]](#page-17-0). Interestingly enough, this is also nearly the GUT-scale ( $M_{GUT} \approx 2 \times 10^{16} GeV$ ), as judged from the MSSM extrapolation of the three gauge-couplings,<sup>3</sup> which should therefore represent the VEVs of fields like  $\langle \overline{16}_H \rangle$  or  $\langle (1, 2, 4)_H \rangle$ , which break SO(10) or G(224) to the SM. (For SO(10), the VEV of  $\langle \bar{16}_{\rm H} \rangle$  may possibly be somewhat larger than  $M_{\rm GUT}$ , because  $\langle \bar{16}_{\rm H} \rangle$  breaks SO(10) to SU(5) rather than the Standard model.) Thus, both from the viewpoint of connection

<sup>&</sup>lt;sup>4</sup>We are not exhibiting the interactions of  $(2, 1, 4)$ <sub>H</sub> because, either it is absent (as in Ref. 14) or has zero VEV. 3

with string theory, as well as comparison with the MSSM unification-scale, we expect the VEV's of the respective fields to be given by:

For SO(10): 
$$
{}_{16}S_{\text{H}} > {}_{16}S_{\text{H}} > {}_{16}S_{\text{H}} > {}_{16}S_{\text{H}} > 3 \times 10^{16} \text{ GeV}.\eta
$$
 (8)

For G(224): 
$$
((1, 2, 4)_H > = \langle (1, 2, 4)_H \rangle \simeq 3 \times 10^{16} \text{ GeV.} \eta
$$
\n
$$
(9)
$$

with  $\eta \approx 1/2$  to 2, being the most plausible range. Thus, using  $(6) - (7)$  and  $(8) - (9)$ , for either  $SO(10)$  or  $G(224)$ , the Majorana masses of the RH neutrinos are given by:

$$
M_{iR} \approx \lambda_{ii} \frac{(3 \times 10^{16} \text{GeV})^2}{2 \times 10^{18} \text{GeV}} \eta^2 \approx \lambda_{ii} (4.5 \times 10^{14} \text{GeV}) \eta^2 \tag{10}
$$

In writing (10), we have ignored the effects of off-diagonal mixing. This is justified, especially for the third family, if we assume, as we do, that the Majorana couplings are family-hierarchical,  $\lambda_{33}$ being the leading one, somewhat analogous to those that give the Dirac masses.

Now using SU(4)-color and the Higgs multiplet  $(2, 2, 1)$ <sub>H</sub> for G(224) or equivalently 10<sub>H</sub> for SO(10), one obtains the relation  $m_\tau(M_X) = m_b(M_X)$ , which is known to be successful. Thus, there is a good reason to believe that the third family gets its masses primarily from the  $10<sub>H</sub>$ or equivalently  $(2, 2, 1)$ <sub>H</sub>, which automatically gives the same Dirac mass to the quark and the lepton of a given flavor. (In the context of SUSY, one would need two 10's or two (2,2,1)'s, or effective non-renormalizable operators, to induce CKM mixings). In turn this implies:

$$
m(\nu_D^{\tau}) \approx m_{\text{top}}(M_X) \approx (100 - 120) \text{GeV}
$$
\n(11)

combining  $(10)$  and  $(11)$  via the see-saw relation  $(5)$ , we obtain:

$$
m(\nu_{L}^{\tau}) \approx \frac{(100 \text{GeV})^2 (1 \text{ to } 1.44)}{\lambda_{33} (4.5 \times 10^{14} \text{GeV}) \eta^2} \approx (1/45) \text{eV} (1 \text{ to } 1.44) / \lambda_{33} \eta^2 \tag{12}
$$

Now, considering that we expect  $m(\nu_L^{\mu})$  $L_{\text{L}}^{\mu}$   $<<$  m( $\nu_{\text{L}}^{\tau}$ ) (by using eq. (5)), and assuming that SuperKamiokande observation represents  $\nu_L^{\mu} \to \nu_L^{\tau}$ -oscillation, so that the observed  $\delta m^2 \approx 10^{-2}$ to $10^{-3}$ eV<sup>2</sup> corresponds to  $m(\nu_L^{\tau})_{obs} \approx 1/10$  to  $1/30$ eV, it seems *truly remarkable* that the expected magnitude of m( $\nu_{\text{L}}^{\tau}$ ), given by eq. (12), is just about what is observed, if  $\lambda_{33}\eta^2 \approx 1$  to 1/4. Such a range for  $\lambda_{33}\eta^2$  seems most plausible and natural (see remarks below). This observation regarding

the agreement between the expected and the observed value of  $\delta m^2$  (in this case  $m(\nu_L^{\tau})$ ), in the context of the ideas mentioned above, is the main point of this note.

We remark that this agreement has come about without making any parameter unnaturally small or large. In particular, the effective Majorana coupling of the third family  $(\lambda_{33})$  is needed to be nearly one or order one for this agreement to hold. One is tempted to compare with the top-Yukawa coupling  $(h_{top})$  which is also nearly one. This common feature regarding maximality of the dimensionless couplings associated with the third family (i.e.  $\lambda_{33} \sim h_{top} \sim 1$ ) may well find its explanation in the context of string solutions for which such couplings may be given just bythe gauge coupling [e.g.  $h_{top} = \sqrt{2}g \approx 1$ , [see e.g. Ref. [[24](#page-17-0)]] and are thus of order one<sup>5</sup>, while those associated with the second and the first families are progressively smaller, because, subject to string symmetries and selection rules, they are induced only at the level of higher dimensional operators utilizing VEV's of fields which are small (by nearly factor of 10) compared to the stringscale. In addition to  $\lambda_{33}$ , the value of m( $\nu_{\rm L}^{\tau}$ ) depends on two other parameters - i.e. on the Dirac mass m( $\nu_{\rm D}^{\tau}$ ) (see eq. (5)) and on the VEV of  $\langle \overline{16}_{\rm H} \rangle$  or  $\langle (1,2,4)_{\rm H} \rangle$ , and thus on  $\eta^2$  (See eqs.  $(8)/(9)$ ,  $(10)$  and  $(5)$ .) As regards the Dirac mass, the use of SU(4)-color plays a crucial role in that it enables one to determine  $m(\nu_D^{\tau})$  fairly reliably from  $m_{top}$ , extrapolated to the GUT-scale (see eq. (11)). As regards determining the VEVs of fields mentioned above, the use of string as well as GUT-related ideas yield nearly the same value for the VEV of  $< 16<sub>H</sub> >$  or  $< (1, 2, \overline{4})<sub>H</sub> >$ , within a factor of 2 to 4, which is reflected in the uncertainty in  $\eta(\approx 1/2 \text{ to } 2)$  (see eqs.  $(8)/(9)$ ). It is for these reasons that the value of  $m(\nu_L^{\tau})$  obtained in eq. (12), with  $\lambda_{33}\eta^2 \approx 1$  to 1/4, seems most plausible.  $^6\,$ 

<sup>&</sup>lt;sup>5</sup>Although  $\lambda_{ij}$  are associated with effective non-renormalizable couplings, as mentioned before, they may well arise, in part or dominantly, through the exchange of superheavy states  $\{\phi_{\alpha}\}\$  (such as those in the string-tower or just below string-scale), if these possess Yukawa couplings of the form  $h_{i\phi}16_i\overline{16}_H\phi$ , together with invariant mass-term ( $M_{\phi}\phi\phi + hc$ ). If  $h_{(i\phi)}$  are family-hierarchical with  $h_{3\phi}$  being maximal (i.e. O(1) like  $h_{top}$ ) and leading,  $\lambda'_{ij}$ s would also be hierarchical, with  $\lambda_{33} (= h_{3\phi}^2(M_{Pl}/M_{\phi}))$  being maximal  $(\mathcal{O}(1))$  and leading.

<sup>&</sup>lt;sup>6</sup>Note that m( $\nu_{\rm L}^{\tau}$ ) depends in fact only on the *product*  $\lambda_{33}\eta^2$  (see eqs. (10) and (12)). A more precise understanding of  $(\lambda_{33}\eta^2)$  and thereby of m( $\nu_{\rm L}^{\tau}$ ) would of course still need a sharpening of an understaning of  $\eta$ , as well as of  $\lambda_{33}$ , e.g. in the context of string-solutions [see remarks in Footnote 5].

Together with the result  $\delta m^2 \simeq 10^{-2} - 10^{-3} eV^2$ , the SuperKamiokande group reports another puzzling feature that  $\nu_{\mu} \to \nu_{\tau}$  (or  $\nu_{\text{X}}$ ) -oscillation angle is nearly maximal - i.e.  $\sin^2 2\theta > 0.8$ . Ordinarily, such large oscillation angle is attributed to nearly degenerate masses of the  $(\nu_{\mu} - \nu_{\tau})$ or  $(\nu_{\mu} - \nu_{\rm X})$  systems, as many authors in fact have. In this case, the large oscillation angle is attributed almost entirely to a large or maximal mixing in the mass eigenstates of the respective neutrinos. However, considering that nearly degenerate masses for the light neutrinos seem to be rather unnatural in the context of the see-saw formula, Babu, Wilczek and I have very recently ob-served [\[4](#page-15-0)] that such degeneracy is not even needed to obtain large oscillation angle. By *combining* the contributions from the mixing angle of the neutrinos (i.e.  $\nu_L^{\mu} - \nu_L^{\tau}$ ) with that from the charged leptons  $(\mu-\tau)$ , and by following familiar ideas on the understanding of Cabibbo-like quark-mixing angles, one can in fact obtain, quite simply and naturally, large  $(\nu_L^{\mu} - \nu_L^{\tau})$  oscillation angle, as observed, in spite of a highly non-degenerate  $\nu^{\mu} - \nu^{\tau}$  system<sup>7</sup> e.g. with m( $\nu_{\rm L}^{\mu}$  $\binom{\mu}{L} / m(\nu_L^{\tau}) \approx 1/10 - 1/20.$ Briefly, a simple and plausible origin of the large mixing angle is as follows. If one assumes that the lighter eigenvalue for a hierarchical  $2 \times 2$ -system arises entirely or primarily by the off-diagonal mixing of the (would-be) light with the heavier state (as in a symmetrical see-saw type mass matrix), one obtains the familiar square root-formula[\[25\]](#page-17-0) for the mixing angle, like  $\theta_{d,u} \approx (\sqrt{m_d/m_s}, \sqrt{m_u/m_c})$ , and the Cabibbo angle is obtained by combining  $\theta_d$  with  $\theta_u$ , allowing for a relative phase between them. Regardless of the phase, such an expression for the Cabibbo angle is known to be fairly successful (to better than 30 %). Assuming analogous mass-matrices for the  $\nu_{\mu} - \nu_{\tau}$  system (Dirac and Majorana) as well as for the charged leptons  $(\mu - \tau)$ , one obtains, ignoring CP violation (and assuming the exact see-saw form for each of the three matrices):  $\theta_{\rm osc}(\nu_\mu-\nu_\tau) = \theta(\nu_{\rm L}^\mu-\nu_{\rm L}^\tau) \pm \theta(\mu-\tau) \approx [\rm m(\nu_{\rm L}^\mu$  $\frac{\mu}{\mu}$ /m( $\nu_L^{\tau}$ )]<sup>1/2</sup>  $\pm$ [m<sub> $\mu$ </sub>/m<sub> $\tau$ </sub>]<sup>1/2</sup>  $\approx 0.31 \pm 0.25 \approx 0.56$  or 0.06, where we have put  $m(\nu_L^{\mu})$  $L(L) / m(\nu_L^{\tau}) \approx 1/10$ . This yields, choosing a positive relative sign between the two mixing angles,  $\sin^2 2\theta_{osc} \approx 0.8$ . In short, a large oscillation angle can arise quite plausibly, without near degeneracy and without large mixing in the mass eigenstates of the neutral and the charged leptons. Various sources of departures from the simple square root formula for the mixing angle (corresponding to departures from exactly symmetrical see-saw mass matrices), which can

<sup>&</sup>lt;sup>7</sup>Preliminary aspects of this joint work [\[4](#page-15-0)] were presented by J.C. Pati at the  $\nu$  − 98 Conference held at Takayama, June 6, 1998.

lead to even larger oscillation angles (for  $m(\nu_L^{\mu})$  $\binom{\mu}{L}/m(\nu_L^{\tau}) \approx 1/10 - 1/20$ , are discussed in Ref[[4\]](#page-15-0): In this case,  $\nu_e - \nu_\mu$  - oscillation can become relevant to the small angle MSW explanation[\[26\]](#page-17-0) of the solar neutrino-puzzle. I refer the reader to Ref. 4 for a full discussion of this explanation of the large oscillation angle for the  $\nu_{\mu} - \nu_{\tau}$  system, with hierarchical masses for the neutrinos. The purpose of the present note has primarily been to emphasize the implications of the observed magnitude of  $\delta m^2$ - or equivalently, in our case of  $m(\nu_L^{\tau})$ , on the nature of new physics.

## 5. Link Between Neutrino Masses and Proton Decay.

Proton decay is one of the hallmarks of grand unification  $[2], [6]$  $[2], [6]$  $[2], [6]$ . As discussed here, light neutrino masses ( $<< m_{e,\mu,\tau}$ ) are also an important characteristic of symmetries such as  $G(224)$ and SO(10), assuming that they are supplemented by the see-saw mechanism. Ordinarily, except for the scale of new physics, involved in the two cases, however, proton decay, especially its decay modes, are considered to be essentially unrelated to the pattern of neutrino masses. In a recent paper, Babu, Wilczek and I noted that, contrary to this common impression, in a class of supersymmetric unified theories such as SUSY  $SO(10)$  or SUSY  $G(224)$ , there is likely to be an *intimate link* between the neutrino masses and proton decay [\[21](#page-17-0)]  $\text{8}$ . This is because, in the process of generating light neutrino masses via the see-saw mechanism, one inevitably introduces a new set of color-triplets (unrelated to electroweak doublets), with effective couplings to quarks and leptons, which are related to the superheavy Majorana masses of the RH neutrinos (see eqs.  $(6)$  and  $(7)$ ). Exchange of these new color-triplets give rise to a new set of  $d=5$  proton decay operators, which are thus directly related to the neutrino-masses. Assuming that  $\nu_e-\nu_\mu$  oscillation is relevant to the MSW explanation of the solar neutrino puzzle, so that  $m(\nu_L^{\mu})$  $_{\rm L}^{\mu}$ )  $\approx 3 \times 10^{-3} \rm{eV}$ , which corresponds to  $M(\nu_R^{\mu}) \approx 2 \times 10^{12}$  GeV, the strength of the new d=5 operators turns out to be just about right  $(\tau_P \approx 10^{32.5\pm 2})$  yrs, for proton decay to be observable at SuperKamiokande[[21](#page-17-0)].

The flavor-structure of the new d=5 operators are, however, expected to be distinct from

<sup>&</sup>lt;sup>8</sup> The link is most compelling for the case of  $\overline{126_H}$  giving Majorana masses to the RH neutrinos. It becomes compelling also for the case of  $(\overline{16_H}, 16_H)$ , serving the same purpose, when one attempts to understand not only the masses but also the CKM mixings of quarks[[21\]](#page-17-0).

those of the standard d=5 operators, which are related to the highly hierarchical Dirac masses of quarks and leptons. In contrast to the standard  $d=5$  operators, the new ones can lead to prominent (or even dominant) charged lepton decay modes, such as  $\ell^+\pi^o, \ell^+K^o$  and  $\ell^+\eta$ , where  $\ell = e$  or  $\mu$ , even for low or moderate values of  $tan\beta \leq 10$ . The intriguing feature thus is that owing to the underlying  $SO(10)$  or just  $SU(4)$ -color symmetry, proton decay operator knows about neutrino masses and vice versa.

With a *maximal* effective Majorana-coupling for the third family (i.e.  $\lambda_{33} \sim \mathcal{O}(1)$ ), as suggested here, that corresponds to  $M_{3R} \approx$  (few  $\times 10^{14}$ GeV) (see eq. (10)) and thereby to  $m(\nu_L^{\tau})$ agreeing with the SuperKamiokande value (eq. (12)), one might however worry that proton may decay too fast, because of an enhancement in the new d=5 operators, relative to that considered in Ref. 21. It turns out, however, that because  $\tau^+$  is heavier than the proton and because  $\bar{\nu}_{\tau}K^+$ mode receives a strong suppression-factor from the small mixing angle associated with the third family ( $V_{ub} \approx 0.002 - 0.005$ ), a maximal Majorana-coupling of the third family ( $\lambda_{33} \sim \mathcal{O}(1)$ ), and thus  $m(\nu_L^{\tau}) \approx (1/10 - 1/30)$ eV, is perfectly compatible with present limit on proton lifetime[[4](#page-15-0)]. With a family-hierarchical Majorana coupling - i.e.  $\lambda_{33} \sim \mathcal{O}(10) \lambda_{23} \approx \mathcal{O}(10^2) \lambda_{22}$  etc. -  $\nu_{\rm L}^{\tau}$  and  $\nu_{\rm L}^{\mu}$ L - masses can be relevant respectively to the atmospheric and the solar-neutrino-problems, yet the new neutrino-mass related d=5 operator does not conflict with proton lifetime. They would still give observable rates for proton-decay, with prominent charged lepton decay modes, involving at least the second family (i.e.  $(\mu^+\pi^0, \mu^+K^o$  etc.), together with  $\bar{\nu}K^+$  modes [\[4](#page-15-0)]. Observation of proton decay, together with prominence of charged lepton modes, would thus be a double confirmation of both susy-unification through  $G(224)/SO(10)$ , as well as of the ideas on neutrino masses in this context.

6. Concluding Remarks and a Summary: As noted in the introduction and the subsequent sections, the impressive result of SuperKamiokande clearly has far-reaching implications on the nature of new physics. These are summarized below and some remarks are added:

(1) The Right-Handed Neutrino: A New Form of Matter: As noted in the introduction, the most reasonable explanation for the neutrino mass-scale observed at SuperKamiokande needs a RH neutrino  $(\nu_R)$ . Many in the past, motivated by the possible masslessness of neutrinos, have preferred to view the neutrino as an "odd ball," believing that it is the messenger that nature is intrinsically left-right asymmetric (parity-violating). This is reflected by the two-component neutrino hypothesis of Lee, Yang, Landau and Salam, as well as by the hypothesis of the grand unification-symmetry SU(5). The SuperKamiokande result (especially its value for  $\delta$ m<sup>2</sup>) clearly suggests, however, that that is in fact not the case. Neutrino is "elusive" but not an odd ball after all. It has its RH counterpart (one for each flavor) just like all the other fermions.

Nevertheless, the neutrino has a *unique character*. It is the only fundamental fermion, among the members of a quark-lepton family, that is electrically neutral (not counting possible SUSY gauge matter such as photino or gluino). Therefore, it is the only fermion that can acquire both a Dirac mass ( $\Delta F = \Delta L = 0$ ), combining  $\nu_R$  and  $\nu_L$ , and a Majorana mass for either  $\nu_R$  or  $ν$ <sub>L</sub> ( $ΔF = ΔL = Δ(B – L) = 2$ ), conserving electric charge. The Majorana masses of the RH neutrinos can be superheavy, because they do not break the Standard model symmetry. As mentioned before, this unique character of possessing both a Dirac and a superheavy Majorana mass for the RH  $\nu_R$  allows the LH neutrinos to be naturally light via the see-saw mechanism. The lightness of  $\nu$ <sub>L</sub> is in fact a reflection of the heaviness of  $\nu$ <sub>R</sub>. By the same token, the light neutrinos know about both mass-scales – the Dirac and the Superheavy Majorana – and thereby simultaneously of the physics at the electroweak and the string/GUT-scales. In short, neutrino masses carry a *gold mine of information* about the nature of new physics.

(2) Minimal Extension Needed of the Standard Model: In suggesting the need for the RH neutrino, the SuperKamiokande result in turn suggests, following discussions presented here, that the standard model symmetry must be extended minimally to the symmetry-structure  $G(224) = SU(2)_L \times SU(2)_R \times SU(4)^c$ . The need for  $SU(4)^c$  has been noted above and is summarized below. Strictly speaking, for an understanding of  $(\delta m)^2$ , as presented here, the extension of the SM symmetry to just  $G(214) = SU(2)_L \times I_{3R} \times SU(4)^c$  would suffice.<sup>9</sup> The further extension of  $G(214)$  to  $G(224)$  (that also quantizes electric charge by replacing  $I_{3R}$  by  $SU(2)_R$ ) may however

<sup>&</sup>lt;sup>9</sup>For a string-origin of  $G(214)$ , see Ref. [\[27](#page-17-0)].

be needed by some of the other considerations, listed in Sec. 3, as well as those of fermion masses and mixings.

(3) The Three Necessary Ingredients: Understanding the neutrino mass-scale observed at SuperKamiokande, as discussed here, utilizes three concepts in an essential manner. They are: (a) SU(4)-color that not only enforces  $\nu_R$ , but more importantly gives the Dirac mass of  $\nu^{\tau}$ , fairly reliably, by relating it to the mass of the top quark (eq.  $(11)$ );<sup>10</sup> (b) String/GUT-scale physics that determines the Majorana mass of the RH tau-neutrino (subject to maximality of the effective coupling) (eqs.  $(8)-(10)$ ); and  $(c)$  the see-saw relation (eq.  $(5)$ ). Given the sensitivity of the final result to both the Dirac mass and the VEV that determines the Majorana mass, $^{11}$  the agreement of the expected value with the observed one (for most plausible values of  $\lambda_{33}\eta^2 \approx \mathcal{O}(1)$ ) seems to suggest the correctness of each of the three ideas.

(4) Selecting the Route to Higher Unification: Unlike proton decay, which can probe directly into the full grand unification symmetry (including gauge transformations of  $q \to \overline{q}$  and  $q \to \overline{\ell}$ , neutrino physics probes directly into  $SU(2)_L \times SU(2)_R \times SU(4)^c$ , but not necessarily beyond. For example, the results discussed here, such as determinations of the Dirac and the Majorana mass of the  $\tau$  neutrino utilize only G(224), but not the full SO(10).<sup>12</sup> They have also utilized supersymmetry, at least indirectly, because without it, there would be no rationale for the use of string-GUT-related scale for the VEV of  $\overline{16}_H$  or  $(1, 2, 4)_H$ .

<sup>10</sup>It is, of course, possible that a string-derived solution containing, for example, only  $G(2213) = SU(2)_L \times$  $SU(2)_R \times (B - L) \times SU(3)^C$  or  $G(2113) = SU(2)_L \times I_{3R} \times (B - L) \times SU(3)^C$  [[24\]](#page-17-0), or flipped  $SU(5) \times U(1)$ [[28\]](#page-17-0), all of which yield RH neutrinos, may still relate  $m(\nu_D^{\tau})$  to  $m_{top}$  at string-scale. This comes about because such a solution still remembers its origin through  $SU(4)$ -color or  $SO(10)$ . Here, I am only discussing the minimal underlying symmetry needed to remove arbitrariness in the choice of  $m(\nu_D^{\tau})$ , which appears to be SU(4)-color.

<sup>&</sup>lt;sup>11</sup>Because of the quadratic dependence of  $m(\nu_L^{\tau})$  on both the Dirac mass  $m(\nu_D^{\tau})$  and the VEV of  $(1,2,4)_H$  or  $\overline{16}_H$ , that determines the Majorana mass, with error in either one by a factor of 10 (say), one could have been off by orders of magnitude in the final answer.

<sup>&</sup>lt;sup>12</sup>The scale of the VEV determining the Majorana mass assumes the relevance of string/GUT scale physics. But that can hold in a string theory, even if it gives, after compactificatin, only  $G(224)$  and not the full  $SO(10)$ (See also remarks in footnote 3).

At the same time, by providing clear support for G(224), the SK result selects out SO(10) or  $E_6$  as the underlying grand unification symmetry, rather than SU(5). Either SO(10) or  $E_6$ or both of these symmetries ought to be relevant at some scale, and in the string context, that may, of course, well be in higher dimensions, above the compactification-scale, below which there need be no more than just the  $G(224)$ -symmetry. If, on the other hand,  $SU(5)$  were regarded as a fundamental symmetry, first, there would be no compelling reason, based on symmetry alone, to introduce a  $\nu_R$ , because it is a singlet of SU(5). Second, even if one did introduce  $\nu_R^i$  by hand, the Dirac masses, arising from the coupling  $h^{i\overline{5}}<sub>i</sub> < 5<sub>H</sub> > \nu_{R}^{i}$ , would be unrelated to the up-flavor masses and thus rather arbitrary (contrast with eq. (11)). So also would be the Majorana masses of the  $\nu_R^i$ 's, which are SU(5)-invariant and thus can even be of order Planck scale (contrast with Eq. (10)). This would give  $m(\nu_L^{\tau})$  in gross conflict with the observed value. We thus see that the SK result clearly disfavors SU(5) as a fundamental symmetry, with or without supersymmetry.

It is worth noting that the precision LEP-data, exhibiting coupling unification[\[29](#page-17-0)], as also proton-decay searches [\[30](#page-17-0)], are known to disfavor non-supersymmetric grand unification, but are compatible with either SUSY SU(5) or SUSY SO(10). It is thus interesting that the neutrino data[[1\]](#page-15-0) revises this conclusion in a major way, by disfavoring SUSY SU(5), and selecting out either string-derived SUSY  $G(224)$ , or SUSY SO $(10)$ .

In summary, it seems that the single discovery of atmospheric neutrino-oscillation has brought to light the existence of the right-handed neutrino and has reinforced the ideas of SU(4)-color, left-right symmetry and see-saw. The agreement between the simplest estimate of the mass of the tau-neutrino, presented here, and the"observed value" suggests the correctness of these three ideas. Simultaneously, it suggests the relevance of the string/GUT-scale-symmetry-breaking, as opposed to intermediate or TeV-scale breaking of  $(B-L)$ . Any symmetry containing  $G(224)$  =  $SU(2)_{L} \times SU(2)_{R} \times SU(4)^{c}$ , such as  $SO(10)$  or  $E_6$ , would of course possess the same desirable features as regards neutrino physics, as G(224). Given the wealth of insight already provided by the SuperKamiokande result, one looks forward eagerly to further revelations of deeper physics in the coming years from the neutrino-system through the many existing and the forthcoming facilities, involving atmospheric, solar and accelerator neutrinos. In particular, one would like

<span id="page-15-0"></span>a clarification of whether the SK result is observing  $\nu_{\rm L}^{\mu} - \nu_{\rm L}^{\tau}$  (as assumed here) as opposed to  $\nu_L^{\mu} - \nu_X$  oscillation, and whether the resolution to the solar neutrino-problem would favor the MSW solution (supported here) as opposed to  $\nu_e - \nu_X$  or vacuum oscillation. One of course also looks forward to learning much about further aspects of unification from searches for proton decay,which, as we saw  $\lfloor 21 \rfloor$  4, is intimately related to neutrino masses, because of SU(4)-color and supersymmetry.

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