GLUON RESUMMATION IN VECTOR BOSON PRODUCTION AND DECAY^a

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After a brief review on the technique of resumming the large logarithmic terms $\alpha_s^n \ln^m (Q^2/Q_T^2)$ due to the effects of multiple gluon emission predicted by the QCD theory, I discuss its application in the production of W^{\pm} , Z and $\gamma\gamma$ in hadron collision.

1 Introduction

It is the prediction of the QCD theory that at hadron colliders the production of Drell-Yan pairs or weak gauge bosons (W^{\pm} and Z) are often accompanied by gluon radiation. Therefore, to test the QCD theory or to probe the electroweak properties in the vector boson productions, it is necessary to include the effects of multiple gluon emission. At the Tevatron (a $p\bar{p}$ collider), we expect about $2 \times 10^6 W^{\pm}$ and $6 \times 10^5 Z$ bosons produced at $\sqrt{S} = 1.8 \text{ TeV}$, per 100 pb^{-1} of luminosity. This large sample of data is useful for (i) QCD studies (single and multiple scale cases), (ii) precision measurement of the W boson mass and width, and (iii) probing for new physics (e.g., Z'), etc. To achieve the above physics goals requires detailed information on the distributions of the rapidity and the transverse momentum of W^{\pm}/Z bosons and of their decay products.

Consider the production process $h_1h_2 \to VX$. Denote Q_T and Q to be the transverse momentum and the invariant mass of the vector boson V, respectively. When $Q_T \sim Q$, there is only one hard scale in this problem, and the fixed-order perturbation calculation is reliable. When $Q_T \ll Q$, this becomes a two scale problem, and the convergence of the conventional perturbative expansion becomes impaired. Hence, it is necessary to apply the technique of QCD resummation to resum the *singular* terms:

$$\begin{split} \frac{d\sigma}{dQ_T^2} &\sim \frac{1}{Q_T^2} \left\{ \begin{array}{ll} \alpha_S(L+1) &+ \alpha_S^2(L^3+L^2) &+ \alpha_S^3(L^5+L^4) + \alpha_S^4(L^7+L^6) + \dots \\ &+ \alpha_S^2(L+-1) &+ \alpha_S^3(L^3+L^2) + \alpha_S^4(L^5+L^4) + \dots \\ &+ \alpha_S^3(L+-1) + \alpha_S^4(L^3+L^2) + \dots \end{array} \right\}, \end{split}$$

where L denotes $\ln(Q^2/Q_T^2)$ and the explicit coefficients multiplying the logs are suppressed.

Resummation of large logarithms yields a Sudakov form factor and cures divergence at $Q_T \to 0$. It was pioneered by Dokshitzer, D'yakonov and Troyan (DDT) who performed the analysis in Q_T -space which lead to the leading-log resummation formalism¹. Later, Parisi-Petronzio showed² that for large Q the $Q_T \to 0$ region can be calculated perturbatively by imposing the condition of the transverse momentum conservation,

$$\delta^{(2)}\left(\sum_{i=1}^{n} \vec{k}_{T_{i}} - \vec{q}_{T}\right) = \int \frac{d^{2}b}{4\pi^{2}} e^{i\vec{Q}_{T}\cdot\vec{b}} \prod_{i=1}^{n} e^{i\vec{k}_{T_{i}}\cdot\vec{b}},$$

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in the *b*-space (the impact parameter, the Fourier conjugate to Q_T). This improved formalism sums also some subleading-logs. As $Q \to \infty$, events at $Q_T \sim 0$ may be obtained asymptotically by the emission of at least two gluons whose transverse momenta are not small and add to zero. The intercept at $Q_T = 0$ is predicted to be²

$$\left. \frac{d\sigma}{dQ_T^2} \right|_{Q_T \to 0} \sim \sigma_0 \left(\frac{\Lambda^2}{Q^2} \right)^{\eta_0},$$

where $\eta_0 = A \ln \left[1 + \frac{1}{A}\right]$ with $A = 12C_F/(33 - 2n_f)$, and $\eta_0 \simeq 0.6$ for $n_f = 4$ and $C_F = 4/3$. Collins-Soper-Sterman (CSS) extended ³ the work by Parisi-Petronzio in *b*-space and applied the properties of the renormalization group invariance to set up a formalism that resums all the large log terms to all orders in α_s . This is the formalism I will concentrate on in this talk.

Recently, there is another theory model in Q_T -space (extension of DDT) proposed by Ellis and Veseli⁴, which does not have either the exact transverse momentum conservation or the renormalization group invariance conditions. The reader can find a detailed discussion of this formalism in the talk by K. Ellis at this conference. Despite of the imperfectness of the formalism, it can still be useful for the W^{\pm} and Z physics program at the Tevatron. I shall come back to this point in the conclusion section.

2 Collins-Soper-Sterman (CSS) resummation formalism

In the resummation formalism by Collins, Soper and Sterman³, the cross section is written in the form

$$\frac{d\sigma(h_1h_2 \to VX)}{dQ^2 \, dQ_T^2 dy} = \frac{1}{(2\pi)^2} \int d^2 b \, e^{i\vec{Q}_T \cdot \vec{b}} \widetilde{W}(b,Q,x_1,x_2) + \ Y(Q_T,Q,x_1,x_2),$$

where Y is the *regular* piece which can be obtained by subtracting the *singular* terms from the exact fixed-order result. \widetilde{W} satisfies a renormalization group equation. Its solution is of the form

$$\widetilde{W}(Q, b, x_1, x_2) = e^{-\mathcal{S}(Q, b, C_1, C_2)} \widetilde{W}\left(\frac{C_1}{C_2 b}, b, x_1, x_2\right),$$

where the Sudakov exponent is defined as

$$\mathcal{S}(Q,b,C_1,C_2) = \int_{C_2^2/b^2}^{C_2^2Q^2} \frac{d\overline{\mu}^2}{\overline{\mu}^2} \left[A\left(\alpha_s(\overline{\mu}),C_1\right) \ln\left(\frac{C_2^2Q^2}{\overline{\mu}^2}\right) + B\left(\alpha_s(\overline{\mu}),C_1,C_2\right) \right],$$

and the x_1 and x_2 dependence of \widetilde{W} factorizes as

$$\widetilde{W}\left(\frac{C_1}{C_2b}, b, x_1, x_2\right) = \sum_j e_j^2 \, \mathcal{C}_{jh_1}\left(\frac{C_1}{C_2b}, b, x_1\right) \, \mathcal{C}_{jh_2}\left(\frac{C_1}{C_2b}, b, x_2\right),$$

where C_{jh} is a convolution of the parton distribution with a calculable Wilson coefficient, called C_{ja} function:

$$\mathcal{C}_{jh}(Q,b,x) = \sum_{a} \int_{x}^{1} \frac{d\xi}{\xi} C_{ja}\left(\frac{x}{\xi}, b, \mu = \frac{C_3}{b}, Q\right) f_{a/h}\left(\xi, \mu = \frac{C_3}{b}\right),$$

where a sums over incoming partons, and j denotes the quark flavors with (electroweak) charge e_j . A few comments about this formalism is listed below:

- The A, B and C functions can be calculated order-by-order in α_s .
- A special choice of the renormalization constants C_i can be made so that the singular terms obtained from the expansion of the CSS resummed calculation agrees with that from the fixed-order calculation. This is the canonical choice. It has $C_1 = C_3 = 2e^{-\gamma_E} \equiv b_0$ and $C_2 = C_1/b_0 = 1$, where $\gamma_E = 0.577...$ is Euler's constant.
- b is integrated from 0 to ∞ . For $b \gg 1/\Lambda_{QCD}$, the perturbative calculation is no longer reliable. Hence, a non-perturbative function is needed in this formalism to compare theory prediction with experimental data.

We refer the readers to Ref.⁵ for a more detailed discussion on how to apply the CSS resummation formalism to phenomenological physics at hadron colliders.

Non-perturbative function 2.1

As noted in the previous section, it is necessary to include a non-perturbative function in the CSS resummation formalism to incorporate some long distance physics that is not accounted by the perturbative derivation. Collins-Soper postulated³

$$\widetilde{W}_{j\bar{k}}(b) = \widetilde{W}_{j\bar{k}}(b_*)\widetilde{W}_{j\bar{k}}^{NP}(b)$$

with

$$b_* = \frac{b}{\sqrt{1 + (b/b_{max})^2}} \,,$$

so that b never exceeds b_{max} and $\widetilde{W}_{i\bar{k}}(b_*)$ can be reliably calculated perturbatively. (In a numerical calculation, b_{max} is set to be, say, $1/(2 \,\text{GeV})$.) Based upon a renormalization group analysis, they found that the non-perturbative function can be generally written as

$$\widetilde{W}_{j\bar{k}}^{NP}(b,Q,Q_0,x_1,x_2) = \exp\left[-F_1(b)\ln\left(\frac{Q^2}{Q_0^2}\right) - F_{j/h_1}(x_1,b) - F_{\bar{k}/h_2}(x_2,b)\right],$$

where F_1 , F_{j/h_1} and $F_{\bar{k}/h_2}$ must be extracted from data with the constraint

$$\widetilde{W}_{j\bar{k}}^{NP}(b=0) = 1.$$

Furthermore, F_1 only depends on Q, while F_{j/h_1} and $F_{\bar{k}/h_2}$ in general depend on x_1 or x_2 . Later, in Ref. ⁶, it was shown that the $F_1(b) \ln \left(\frac{Q^2}{Q_0^2}\right)$ dependence is also suggested by analyzing the infrared renormalon contribution in Q_T distribution.

Testing the universality of \widetilde{W}_{ik}^{NP} 2.2

The CSS resummation formalism suggests the non-perturbative function to be universal. Its role is similar to the parton distribution function (PDF) in any fixed order perturbative calculation, and its value needs to be determined by data. The first attempt to determine such a universal non-perturbative function was done by Davies, Webber and Stirling (DWS)⁷ in 1985 using Duke and Owens parton distribution function. In 1994, Ladinsky and Yuan (LY)⁸ observed that the prediction of the DWS set of $\widetilde{W}_{i\bar{k}}^{NP}$ largely deviates from the R209 data $(p + p \rightarrow \mu^+ \mu^- + X)$ at $\sqrt{S} = 62 \,\text{GeV}$) using the CTEQ2M PDF. To incorporate possible $\ln(\tau)$ dependence, LY postulated

$$\widetilde{W}_{j\bar{k}}^{NP}(b,Q,Q_0,x_1,x_2) = \exp\left[-g_1b^2 - g_2b^2\ln\left(\frac{Q}{2Q_0}\right) - g_1g_3b\ln\left(100x_1x_2\right)\right],$$



Figure 1: Comparison of 4 pb^{-1} CDF-Z data with two different theory model predictions.

where $x_1x_2 = \tau$. A "two-stage fit" of the R209, CDF-Z (4 pb⁻¹ data) and E288 (p + Cu) data gave $g_1 = 0.11_{-0.03}^{+0.04} \text{ GeV}^2$, $g_2 = 0.58_{-0.2}^{+0.1} \text{ GeV}^2$, and $g_3 = -1.5_{-0.1}^{+0.1} \text{ GeV}^{-1}$, for $Q_0 = 1.6 \text{ GeV}$ and $b_{max} = 0.5 \text{ GeV}^{-1}$. Unfortunately, a fortran code error in calculating the parton densities inside the neutron lead to a wrong g_3 value. (An erratum with corrected values will be submitted after the completion of our revised analysis.) Currently, Brock, Ladinsky, Landry, and Yuan⁹ are revisiting this problem using the R209 (p + p), CDF-Z ($p + \bar{p}$ with 4 pb⁻¹ data), E288 (p + Cu), and E605 (p + Cu) data, with CTEQ3M PDF. The preliminary results show that, for $Q_0 = 1.6 \text{ GeV}$ and $b_{max} = 1/(2 \text{ GeV})$, two forms for $\widetilde{W}_{j\bar{k}}^{NP}(b, Q, Q_0, x_1, x_2)$ give good fits ($\chi^2/dof \simeq 1.4$): (i) $g_1 = 0.24$, $g_2 = 0.34$ and $g_3 = 0.0$ (DWS form, pure Gaussian form in *b*-space, without *x* dependence), and (ii) $g_1 = 0.15$, $g_2 = 0.48$ and $g_3 = -0.58$ (LY form, with a linear *b* term and *x* dependence). We are in the process of determining the uncertainties of these fitted parameters q_i .

2.3 Run-1B W/Z data at the Tevatron

The Run-1B W/Z data at the Tevatron can be useful to test the universality and the x dependence of the non-perturbative function $\widetilde{W}_{j\bar{k}}^{NP}(b, Q, Q_0, x_1, x_2)$. In Fig. 1, we show the prediction of the two different global fits (2-parameter and 3-parameter fits) obtained in the previous section using the CTEQ3M PDF. (The CTEQ4M PDF gives the similar results.)

We note that for $Q_T > 10 \text{ GeV}$, the non-perturbative function has little effect on the Q_T distribution although in principle it affects the whole Q_T range (up to Q). This is also clearly shown in the figures of Ref. ⁵. (Because of the limited space of this article, I will not reproduce here the figures that can be found in Ref. ⁵.) To study the resolution power of the Tevatron Run-1B Z data on determining the non-perturbative function, we have performed a "toy global fit" as follows. First, we generate a set of "fake Run-1B Z data" (assuming 5,000 reconstructed Z bosons) using the original LY fit ($g_1 = 0.11, g_2 = 0.58$ and $g_3 = -1.5$). Then, we combine this set of "fake" data with the low energy Drell-Yan data as listed above to perform a global fit. Using the 3-parameter form (the LY form), we get back the g_1 and g_2 values from the fit but the

 g_3 value is smaller by a factor of 2. The best fit gives $g_1 = 0.11$, $g_2 = 0.57$ and $g_3 = -0.88$. (This amounts to shift the prediction on the mass and the width of the W boson by about 5 MeV and 10 MeV, respectively¹⁰.) With this large sample of Z data, it can be clearly illustrated that a single parameter without Q dependence (i.e. the parameter g_1 alone) cannot yield a good global fit.

3 W/Z Production and Decay at the 1.8 TeV Tevatron

In Ref. ⁵, we have presented the results of a detailed study on the distributions of the decay leptons from the W/Z boson produced at the Tevatron. Here, we shall only present a few of the most interesting results.

3.1 "Matching" from low to high Q_T

First of all, let us examine the distribution of the transverse momentum of W boson produced at the 1.8 TeV Tevatron. It is obvious that the CSS formalism gives automatic matching in the Q_T distribution if A, B, C and Y functions were known to all orders in α_s . However, in practice we only calculate those functions to some fixed order in α_s , therefore, some "switching" procedure (to switch from the total resummed result to the fixed-order perturbative result) should be applied to have a continuous distribution that would match the fixed-order result in the large transverse momentum Q_T (of the order Q) where $\ln(Q^2/Q_T^2)$ is small and the resummed result is less accurate. Our matching prescription is to switch from the resummed prediction to the fixed-order perturbative calculation as they cross around $Q_T \sim Q$. This switching procedure is done for any given Q (invariant mass) and y (rapidity) of the W/Z boson. We note that this procedure is different from that proposed in Ref. ¹¹. As shown in the figures of Ref. ⁵, including a higher order Y term, as expected, improves the "matching" between the resummed and the fixed-order results. At $O(\alpha_s)$, that crossing occurs at about 50 GeV, while at $O(\alpha_s^2)$, it occurs at about 70 GeV for W boson production.

3.2 Total cross section

It was shown by an analytical calculation in Ref.⁵ that if "matching" (switching) is chosen to be at $Q_T = Q$, then the CSS resummed rate is the same as the NLO rate. However, in that case, the Q_T distribution is not smoothly continuous for Q_T close to Q. Since the region of $Q_T > 50 \text{ GeV}$ only contributes to the total rate by less than 2%, and the CSS resummed formalism includes also some even higher order contributions beyond NLO, we regard the difference between the resummed rate and the NLO rate as a gauge of the uncalculated higher order contribution which is shown to be small at the Tevatron.

Total	cross sections o	f $p\bar{p} \to (W$	V^+ or $Z)X$	at the	Tevatron,	calculated i	n different	prescriptions
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in units of nb.													
V	E_{cm}	Fixed Order		Res. $(1,1,1)$	Res. $(2,1,1)$	Res. $(2,1,2)$	Experiment						
	(TeV)	$\mathcal{O}(lpha_S^0)$	$\mathcal{O}(\alpha_S)$	\oplus Pert. $\mathcal{O}(\alpha_S)$	\oplus Pert. $\mathcal{O}(\alpha_S)$	\oplus Pert. $\mathcal{O}(\alpha_S^2)$							
W^+	1.8	8.81	11.1	11.3	11.3	11.4	11.5 ± 0.7						
W^+	2.0	9.71	12.5	12.6	12.6	12.7							
\mathbf{Z}^{0}	1.8	5.23	6.69	6.79	6.79	6.82	6.86 ± 0.36						
\mathbf{Z}^{0}	2.0	6.11	7.47	7.52	7.52	7.57							

In the above Table, the values of the strong coupling constants used with the CTEQ4L and CTEQ4M PDF's are $\alpha_S^{(1)}(M_Z) = 0.132$ and $\alpha_S^{(2)}(M_Z) = 0.116$, respectively. (Res. (2,1,2) denotes the result with A and B calculated to α_S^2 order, C to α_S , and Y to α_S^2 order, etc.)



Figure 2: NLO (dashed) and resummed $\mathcal{O}(\alpha_S)$ (solid) calculations Using the LY fit with cuts ($Q_T < 30 \text{ GeV}, p_T^{e^+, \nu} > 25 \text{ GeV}$).

We note that the total rate is not sensitive to either non-perturbative function or matching prescription.

3.3 Lepton transverse momentum

At the Run-2 of Tevatron, the number of interactions per crossing increases, and the transverse mass measurement becomes less accurate. It is useful to also measure the inclusive lepton $P_T^{e^+}$ spectrum. In Fig. 2, we show that the difference between the resummed and the NLO results is much larger than the dependence on the non-perturbative function in the resummed calculation (although its dependence is not negligible).

3.4 Lepton charge asymmetry

The distribution of the rapidity of the charged lepton from W^{\pm} boson decay provides useful information to determine the ratio of the up and down quark parton distribution functions. It was shown in Ref. ⁵ that without imposing the kinematic cuts, the prediction of the resummed calculation is the same as that of the NLO. This is obvious because an integration over the entire Q_T space in the CSS formalism reproduces the NLO rate as discussed above. However, with kinematic cuts, the rapidity distribution of the charged lepton or the W boson is different between the resummed and the NLO calculations. Furthermore, for this distribution, the dependence on the non-perturbative function is negligible.

The resummed calculation also predicts different correlations between the two decay leptons from Z boson. A couple of examples are the distribution showing the balance in transverse momentum $\Delta p_T = |\vec{p}_T^{\ell_1}| - |\vec{p}_T^{\ell_2}|$, and the angular correlation $z = -\vec{p}_T^{\ell_1} \cdot \vec{p}_T^{\ell_2} / [\max(p_T^{\ell_1}, p_T^{\ell_2})]^2$. We refer the reader to Ref. ⁵ for more details.



Figure 3: The predicted distribution for the invariant mass of the photon pair $M_{\gamma\gamma}$ from the resummed calculation compared to the CDF data, with the CDF cuts imposed in the calculation.

4 Di-photon rates and distributions

It is straightforward to extend the above CSS formalism to calculate the distribution of photons from $h_1h_2 \rightarrow \gamma\gamma X$. In Ref. ¹², we have included the full content of NLO contributions from $q\bar{q}$ and qg subprocesses. The accuracy of the resummed result is similar to that for the Drell-Yan and W/Z productions. In addition, we also included part of the higher order contribution from the gg process whose lowest order contribution comes from one-loop box diagrams. This was done by including $A^{(2)}$ term in the Sudakov factor to resum higher order large logs due to initial state radiation, and the NLO contribution, of $\mathcal{O}(\alpha_{em}^2 \alpha_s^3)$, to the hard scattering subprocess is added in an approximate fashion.

We found that the $gg \to \gamma\gamma X$ rate is not small at the Tevatron. Including part of the NLO $gg \to \gamma\gamma$ approximately doubles the LO gg box contribution to the cross section. This is clearly seen in the distribution of the invariant mass of the photon pair, as shown in Fig. 3. The other distributions, such as the distribution of the transverse momentum of the photon pair Q_T , compared with CDF and DØ data, can be found in Ref. ¹². We note that to accurately predict the production rates of the di-photon pairs at the CERN Large Hadron Collider (LHC) requires a better calculation of the gg fusion rate. Namely, a full NLO result is needed to predict the distribution of the di-photon pair with large Q_T or small $\Delta\phi$.

In Ref. ¹², we also presented our results for the di-photon production rate and distributions at the fixed-target experiment (E706 at Fermilab): $p + Be \rightarrow \gamma\gamma + X$ at $\sqrt{S}=31.5$ GeV. In this case, we found that the gg rate is small, and the distribution of the transverse momentum of the photon pair Q_T is dominated by the non-perturbative contribution. Therefore, this data can be useful for determining the non-perturbative function associated with the gg initiated processes in the CSS formalism. We refer the readers to Ref. ¹² for more details.

5 Conclusion

In conclusion, the effects of QCD gluon resummation are important in many precision measurements. A Monte Carlo package <u>ResBos</u> ((<u>Res</u>ummed <u>Bos</u>on Production and Decay)) is available¹³ for studying the effects of gluon resummation (with NLO hard part corrections) in hadron collisions.

Before closing, I would like to comment on the Q_T space resummation formalism proposed in Ref. 4 , and compare that with the *b* space resummation formalism (CSS formalism) discussed in this article. Despite of its imperfectness in the theory structure, in practice, the Q_T space resummation formalism may still prove to be useful because it does not require a "switching" in the large Q_T region and its calculation takes much less CPU time due to the fact that there is no need to perform a Fourier transformation from the b space to the Q_T space. This is obvious for Q_T above 10 GeV where the non-perturbative contribution is not important, as discussed above. However, in the small Q_T region, it is not obvious that both formalisms will give the same prediction of the Q_T distribution of the W^{\pm} boson after the non-perturbative part is fit by the Q_T of the Z boson using the Tevatron Run-1B and Run-2 data. Nevertheless, if one is not interested in testing the universality of the non-perturbative function, as suggested by the CSS formalism, then it seems likely that it is possible to choose a proper form of the non-perturbative function in the Q_T space resummation formalism to reproduce (within the experimental uncertainties) the prediction of the b space formalism when considering the Tevatron W/Z data alone. If one is interested in the universality property of the formalism⁴ so that one can use the same values of the non-perturbative function to predict future data at different hadron colliders (such as the LHC), then it becomes unclear whether these two formalisms will always give the same physics prediction.

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References

- 1. Y.I. Dokshitser, D.I. D'Yakonov, S.I. Troyan, Phys. Lett. B79 (1978) 269.
- 2. G. Parisi, R. Petronzio, Nucl. Phys. B154 (1979) 427.
- J. Collins, D. Soper, Nucl. Phys. B193 (1981) 381; Erratum B213 (1983) 545; B197 (1982) 446;
 - J. Collins, D. Soper, G. Sterman, Nucl. Phys. **B250** (1985) 199.
- 4. R.K. Ellis and S. Veseli, Nucl. Phys. B511 (1998) 649.
- 5. C. Balázs and C.-P. Yuan, Phys. Rev. D56 (1997) 5558, and the references therein.
- 6. G.P. Korchemsky and G. Sterman, Nucl. Phys. **B437** (1995) 415.
- 7. C. Davies, B. Webber, W. Stirling, Nucl. Phys. **B256** (1985) 413.
- 8. G.A. Ladinsky, C.–P. Yuan, Phys. Rev. D50 (1994) 4239.
- 9. R. Brock, G. Ladinsky, F. Landry, and C.-P. Yuan, in preparation.
- E. Flattum, private communication;
 B. Ashmanskas, private communication.
- 11. P.B. Arnold, R.P. Kauffman, Nucl. Phys. B349 (1991) 381.
- 12. C. Balázs, E.L. Berger, S. Mrenna, and C.–P. Yuan, Phys. Rev. **D57** (1998) 6934, and the references therein.
- 13. It can be obtained from http://www.pa.msu.edu/~balazs/ResBos/.

 $^{^{}b}$ Needless to say, this postulation has to be further tested by data.