The $K^+ \to \pi^+ \nu\bar{\nu}$ Rare Decay in Two Higgs Doublet Model

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Abstract

The rare $K^+ \to \pi^+ \nu\bar{\nu}$ decay is investigated in the context of type II two-Higgs-doublet model (2HDM). By using the existing experimental data of the branching ratio, restrictions on the free parameters of the model m_H, and $tan\beta$ are obtained: $0.7 \leq tan\beta \leq 0.8$, and $500GeV \leq m_H \leq 700GeV$.

I. INTRODUCTION

The determination of the elements of the Cabibbo-Kobayashi -Maskawa matrix (CKM) is still an important issue in the flavor physics. The precise determination of the CKM parameters will be one of the most important progresses to understand the nature, physics of violated symmetry.

In this sense the rare $K^+ \to \pi^+ \nu\bar{\nu}$ decay has attached a special interest due its sensitivity for the determination of CKM parameters, in particular the element V_{td}, and considered one of the cleanest decays from a theoretical standpoint. Moreover this decay occupies a special place, since for this decay the short distance effects dominated over the long distance effects. Over the years important refinements have been added in the theoretical treatment of $K^+ \to \pi^+ \nu\bar{\nu}$, long-distance contributions to the branching ratio were estimated quantitatively and could be shown to be essentially negligible as expected, two to three orders of magnitude smaller than the short distance contribution at the level of the branching ratio [1]. On the other hand, the calculation [2] of next-to-leading QCD correction reduced considerably the theoretical uncertainty due to the choice of the renormalization scales present in the leading order expression. Since the relevant hadronic matrix element of the operator $\bar{s}\gamma_\mu(1-\gamma_5)d\bar{\nu}\gamma_\mu(1-\gamma_5)\nu$ can be extracted in the leading decay $K^+ \to \pi^0 e^+ \nu$. Conventionally, the Br($K^+ \to \pi^+ \nu\bar{\nu}$) is related to the experimental well-known quantity Br($K^+ \to \pi^+ e^+ \nu$)=0.0482, measured to 1% accuracy. The resulting theoretical expression for Br($K^+ \to \pi^+ \nu\bar{\nu}$) is only a function of the CKM parameters, the QCD scale $\Lambda_{\bar{M}s}$ and the quark masses m_t and m_c.

Experiments in the K meson system have entered new period. That the branching ratio of the flavor-changing neutral current (FCNC) process $K^+ \to \pi^+ \nu\bar{\nu}$ has been recently measured, and it has turned out to be $Br(K^+ \to \pi^+ \nu\bar{\nu}) = (4.2^{+9.7}_{-3.5}).10^{-10}$ [3]. The central value seems to be 4-6 times larger than the predictions of the Standard Model (SM) Br(0.6-1.5)x10^{-10} [4]. Hence the rare $K^+ \to \pi^+ \nu\bar{\nu}$ decay is very sensitive to a new physics beyond the SM. Therefore, careful investigation of this decay can provide useful information about new physics [5]. For this reason different new physics scenarios for this decay will become very actual.

In this work the decay $K^+ \to \pi^+ \nu\bar{\nu}$ is investigated in the framework of the two Higgs doublet model (2HDM). We estimate the constraints of the 2HDM parameters namely, $tan\beta$ and m_H, using the result coming from the measurement of [3]. Subsequently, this paper is organized as follows: In Section 2, the relevant effective Hamiltonian for the decay $K^+ \to \pi^+ \nu\bar{\nu}$ in 2HDM is presented. Section 3, being devoted to the numerical analysis of our results; and finally a brief discussion of the results is given.

II. EFFECTIVE HAMILTONIAN

In the Standard Model (SM) the process $K^+ \to \pi^+ \nu\bar{\nu}$ is described at quark level by the $s \to d\nu\bar{\nu}$ transitions and received contributions from Z^0-penguin and box diagrams. The effective Hamiltonian relevant to $s \to d\nu\bar{\nu}$ transition is described by only one Wilson coefficient, and its explicit form is:

$$H_{eff} = \frac{G}{\sqrt{2}} \frac{\alpha}{2\pi sin^2\theta_w} V_{ts}^* V_{td} C_{11}^{SM} \bar{s}\gamma_\mu(1-\gamma_5) d\bar{\nu}\gamma_\mu(1-\gamma_5)\nu, \quad (1)$$

where G is the Fermi coupling constant, α is the fine structure constant, $V_{tb}V_{ts}^*$ are products of Cabibbo-Kabayashi-Maskawa matrix elements and $x_t = \frac{m_t^2}{m_w^2}$. The resulting expression of Wilson coefficient C_{11}, which was derived in the context of the SM including $O(\alpha_s)$ corrections is [6,7]

$$C_{11}^{SM} = \left[X_0(x) + \frac{\alpha_s}{4\pi} X_1(x) \right], \tag{2}$$

with

$$X_0(x) = \eta \frac{x}{8} \left[\frac{x+2}{x-1} + \frac{3x-6}{(x-1)^2} lnx \right], \tag{3}$$

$$X_1(x) = \frac{4x^3 - 5x^2 - 23x}{3(x-1)^2} - \frac{x^4 + x^3 - 11x^2 + x}{(x-1)^3} lnx + \frac{x^4 - x^3 - 4x^2 - 8x}{2(x-1)^3} ln^2 x$$
$$+ \frac{x^3 - 4x}{(x-1)^2} Li_2(1-x) + 8x \frac{\partial X_0(x)}{\partial x} lnx_\mu. \tag{4}$$

Here $Li_2(1-x) = \int_1^x \frac{lnt}{1-t} dt$ and $x_\mu = \frac{\mu^2}{m_w^2}$ with $\mu = O(m_t)$.

At $\mu = m_t$, the QCD correction for $X_1(x)$ term is very small (around $\sim$ 3%), and $\eta = 0.985$ is the next-to-leading order (NLO) QCD correction to the t- exchange calculated in [2].

From the theoretical point of view, the transition $s \to d\nu\bar{\nu}$ is a very clean process as pointed out, since it is practically free from the scale dependence, and free from any long distance effects. In addition, the presence of a single operator governing the inclusive $s \to d\nu\bar{\nu}$ transition is an appealing property. The theoretical uncertainty within the SM is only related to the value of the Wilson coefficient C_{11} due to the uncertainty in the top quark mass. In this work, we have considered possible new physics in $s \to d\nu\bar{\nu}$ only through the value of that Wilson coefficient.

In this spirit, the process $s \to d\nu\bar{\nu}$ in the context of the 2HDM has additional contributions from Z^0-penguin and box diagrams through H boson exchanges. The relevant Feynman diagrams correspond to the transition $s \to d\nu\bar{\nu}$ has been given in [8,9]. The first three diagrams describe the effective Hamiltonian in the SM, while the last three diagrams represent the 2HDM contributions to the $s \to d\nu\bar{\nu}$ transition, due to the charged Higgs boson exchanges. The interaction lagrangian between the charged Higgs bosons fields and fermions are then given by:

$$L = (2\sqrt{2}G_F)^{1/2}\left[tan\beta\bar{U_L}V_{CKM}M_D D_R + ctg\beta\bar{U_R}M_U V_{CKM}D_L + tan\beta\bar{N_L}M_E E_R\right]H^+ + h.c. \quad (5)$$

Here, H^+ represents the charged physical Higgs field. U_L and D_R represent left-handed up and right-handed down quark fields. N_L and E_R are left-handed neutral and right-handed charged leptons. M_D, M_U, and M_E are the mass matrices for the down quarks, up quarks, and charged leptons respectively. V_{CKM} is the Cabibbo-Kobayashi- Maskawa matrix. $tan\beta$- is the ratio of the vacuum expectation values of the two Higgs doublets in 2HDM, and it is a free parameter of the model.

From eq.(5), it follows that the box diagrams contribution to the process $s \rightarrow d\nu\bar{\nu}$ in 2HDM are proportional to the charged lepton masses; and therefore, they are giving a negligible contribution. So in this model, the transition $s \rightarrow d\nu\bar{\nu}$ in eq.(1) can only include extra contribution due to the charged Higgs interactions. Hence, the charged Higgs contribution modify only the value of the Wilson coefficient C_{11} (see eq.(1)), and it does not induce any new operators (see also [8,9]):

$$C_{11}^{2HDM} = -\frac{1}{8}xyctg^2\beta\left\{\frac{1}{y-1} - \frac{lny}{(y-1)^2}\right\}, \quad (6)$$

where $x = \frac{m_t^2}{m_W^2}$ and $y = \frac{m_t^2}{m_H^2}$.

As we noted earlier the QCD corrections practically do not change the value of C_{11}. If so, eq. (2) and eq.(6), are plugged in eq.(1), to obtain a modified effective Hamiltonian, which represents $s \rightarrow d\nu\bar{\nu}$ decay in 2HDM:

$$H_{eff} = \frac{G}{\sqrt{2}}\frac{\alpha}{2\pi sin^2\theta_w}V_{td}V_{ts}^*[X_{tot}]\bar{s}\gamma_\mu(1-\gamma_5)d\bar{\nu}\gamma_\mu(1-\gamma_5)\nu, \quad (7)$$

where $X_{tot} = C_{11}^{SM} + C_{11}^{2HDM}$.

However, in spite of such theoretical advantages, it would be a very difficult task to detect the inclusive $s \to d\nu\bar{\nu}$ decay experimentally, because the final state contains two missing neutrinos and many hadrons. Therefore, only the exclusive channels are expected, namely $K^+ \to \pi^+\nu\bar{\nu}$, are well suited to search for and constrain for possible "new physics" effects.

In order to compute $K^+ \to \pi^+\nu\bar{\nu}$ decay, we need the matrix elements of the effective Hamiltonian eq.(7) between the final and initial meson states. This problem is related to the non-perturbative sector of QCD and can be solved only by using non-perturbative methods. The matrix element $< \pi^+ \mid H_{eff} \mid K^+ >$ has been investigated in a framework of different approaches, such as chiral perturbation theory [10], three point QCD sum rules [11], relativistic quark model by the light front formalism [12], effective heavy quark theory [13], and light cone QCD sum rules [14,15]. As a result, the hadronic matrix element for the $K^+ \to \pi^+\nu\bar{\nu}$ can be parameterized in terms of form factors:

$$< \pi \mid \bar{s}\gamma_\mu(1-\gamma_5)d \mid K > \; = f_+^{\pi^+}(q^2)(p_K + p_\pi)_\mu + f_- q_\mu, \tag{8}$$

where $q_\mu = p_K - p_\pi$, is the momentum transfer. In our calculations the form factor f_- part do not give any contributions since its contribution $\sim m_\nu$=0. After performing the mathematics and taking into account the number of light neutrinos $N_\nu = 3$ the differential decay width is expressed as:

$$\frac{d\Gamma(K^+ \to \pi^+\nu\bar{\nu})}{dq} = \frac{G^2\alpha^2\eta^2}{2^8\pi^5 sin^4\theta_w} m_{K^+}^3 \mid V_{tb}V_{ts}^* \mid^2 \lambda^{3/2}(1, r_+, s) \mid X_{tot} \mid^2 \mid f_+^{\pi^+}(q^2) \mid^2, \tag{9}$$

where $r_+ = \frac{m_{\pi^+}^2}{m_{K^+}^2}$ and $s = \frac{q^2}{m_{K^+}^2}$.

Similar calculations for $K^+ \to \pi^0 e^0 \bar{\nu}$ lead to the following result:

$$\frac{d\Gamma(K^+ \to \pi^0 e^+ \bar{\nu})}{dq} = \frac{G^2}{192\pi^3} \mid V_{us} \mid^2 \lambda^{3/2}(1, r_-, s) m_{K^+}^3 \mid f_+^{\pi^0}(q^2) \mid^2, \tag{10}$$

where $r_- = \frac{m^2_{\pi^0}}{m^2_{K^+}}$, and $\lambda(1, r_\pm, s) = 1 + r^2_\pm + s^2 - 2r_\pm s - 2r_\pm - 2s$ is the usual triangle function. In derivation of eq.(10) we neglect the electron mass, and we use the form factors $f_+^{\pi^+} = \sqrt{2} f_+^{\pi^0}$ which follows from isotopic symmetry. Using eq.(9) and eq.(10) one can relate the branching ratio of $K^+ \to \pi^+ \nu\bar{\nu}$ to the well known measured decay $K^+ \to \pi^0 e^0 \bar{\nu}$ branching ratio:

$$B(B^+ \to K^+ \nu\bar{\nu}) = k \left[\left(\frac{Im\lambda_t}{\lambda^5} X_{tot} \right)^2 + \left(\frac{Re\lambda_c}{\lambda} P_0(K^+) + \frac{Re\lambda_t}{\lambda^5} X_{tot} \right)^2 \right], \qquad (11)$$

where $k = r_{K^+} \frac{3\alpha^2 B(K^+ \to \pi^0 e^+ \nu)}{2\pi^2 sin^4\theta_w} \lambda^8 = 4.11.\ 10^{-11}$.

Here $r_{K^+} = 0.901$ summaries the isospin-breaking corrections which come from phase space factors due to the difference of masses of π^+ and π^0.

In derivation eq.(11) we have used the wolfenstein parametrization of the CKM matrix, in which each element is expanded as a power series in the small parameter
$\lambda =\mid V_{us} \mid = 0.22$, $\lambda_i = V^*_{is} V_{id}$ and $P(K^+)$ represent the sum of charm contributions to the two diagrams including the (NLO) QCD corrections [2]. At $m_c = 1.3$ GeV, $\Lambda_{\bar{M}s} = 0.325$ GeV and at renormalization scale $\mu_c = m_c$ in [16] it is found that $P_0(K^+) = 0.4 \pm 0.06$.

III. NUMERICAL ANALYSIS

In the numerical analysis, the following values have been used as input parameters:
$G_F = 1.17\ .10^{-5}\ GeV^{-2}$, $\alpha = 1/137$, and $\lambda = 0.22$ As we noted early we used the wolfeustein parametrization of CKM matrix elements. In this parametrization
$Im\lambda_t = A^2\lambda^5\eta, \qquad Re\lambda_c = -\lambda(1 - \lambda^2/2)$, and $Re\lambda_t = -A^2\lambda^5(1-\rho)$.

The parameter A determines from $b \to c$ transition and its $A = 0.80 \pm 0.075$ [17]. The other two CKM parameters ρ and η are constrained by the measurements of $\mid V_{ub}/V_{cb} \mid$, $x_d(B_d^0 - \bar{B}_d^0)$ mixing, and $\mid \epsilon \mid$ (the CP violation parameter in the kaon system). For typical values of the necessary input parameters of ρ and η we have adopt the following two sets :

$$\text{set I} : \begin{cases} \rho = 0.06 \\ \eta = 0.35 \end{cases} \qquad \text{set II} : \begin{cases} \rho = -0.25 \\ \eta = 0.3 \end{cases} . \tag{12}$$

The free parameters of the 2HDM model which we have used namely $tan\beta$ and m_H are not arbitrary, but there are some constraints on them by using the existing experimental data. These constraints are usually obtained from $B^0 - \bar{B}^0$, $K^0 - \bar{K}^0$ mixings, $b \to s\gamma$ decay width, $R_b = \frac{\Gamma(z \to b\bar{b})}{\Gamma(z \to hadrons)}$, and semileptonic $b \to c\bar{\nu_\tau}\tau$ decay which are given by [18] as

$$0.7 \leq tan\beta \leq 0.6(\frac{m_H^+}{1GeV}), \tag{13}$$

where as a lower bound for the charged Higgs mass $m_H \geq 300$ GeV at $\mu = 5$ scale has been estimated in 2HDM [19]. If these constraints are respected, an upper and lower bound for $ctg\beta$ is extracted:

$$0.004 \leq ctg\beta \approx 2. \tag{14}$$

In Figures 1 and 2, we represent the branching ratio of $K^+ \to \pi^+\nu\bar{\nu}$ as a function of $ctg\beta$ for various values of m_H, and as a function of m_H for various values of $ctg\beta$. For illustrative purposes we consider three values of $ctg\beta$, namely $ctg\beta$ =1, 1.5 and 2 and we allow m_H to range between 300 GeV and 1000 GeV, and then we consider three values of m_H, namely m_H=300, 500, 1000 GeV and we allow $ctg\beta$ to range between 0 to 2. It can be seen that for $ctg\beta = 1$, the branching ratio (BR) for $K^+ \to \pi^+\nu\bar{\nu}$ decay increases slowly with the increasing of m_H; whereas, for larger values of $ctg\beta$, the BR decreases at all values of m_H.

Furthermore, when the $ctg\beta$ is increased the BR rapidly grows up. Therefore, it can be concluded that the main contribution to the decay width comes from the charged Higgs exchange diagrams (see [8,9]).

The question now is; what kind of restrictions on $tan\beta$ and m_H can be obtained if the recent experimental result of $Br(K^+ \to \pi^+ \nu\bar{\nu}) = (4.2^{+9.7}_{-3.5}).\ 10^{-10}$ [3] is respected that is:

$$(0.7 \leq BR^{exp.} \leq 13.9).\ 10^{-10}, \tag{15}$$

and whether or not it coincide with the restrictions given in [18]. For this aim, in Figure 3 we present the dependence of $tan\beta$ on m_H using both sets of values of ρ and η. We see that when $300GeV \leq m_H \leq 1TeV$ it gives:

$$0.18 \leq tan\beta \leq 0.5 \pm 0.2 \qquad (setI), \tag{16}$$

$$0.18 \leq tan\beta \leq 0.8 \pm 0.3 \qquad (setII). \tag{17}$$

If we use the lowest bound for $tan\beta = 0.7$ (see eq.(13)) we see that the set I predictions is ruled out and for set II we have small room for $tan\beta$, namely from eq.(16) and from eq.(17) we have:

$$0.7 \leq tan\beta \leq 0.8. \tag{18}$$

If we increase a little bit the upper bound and if we put a lower value for m_H=500 GeV we can see that in this case

$$0.7 \leq tan\beta \leq 0.9. \tag{19}$$

Using these results we can conclude that the mass of the charged Higgs boson must be lie in the interval

$$500GeV \leq m_H \leq 700GeV. \quad (20)$$

In conclusion, using the experimental result of the branching ratio for $K^+ \to \pi^+ \nu \bar{\nu}$ and the CLEO measurements on $b \to s\gamma$ [20] we find new restrictions on the free parameters $tan\beta$ and m_H of the 2HDM model. In summay it is found that the contribution of type II two-Higgs-doublet model to the branching ratio is exceed at most by $\sim 20\%$ from the standard model ones.

Figure Captions

Figure 1 : The dependence of the Br($K^+ \to \pi^+ \nu \bar{\nu}$) on $ctg\beta$ at fixed values of m_H.

Figure 2 : The dependence of the Br($K^+ \to \pi^+ \nu \bar{\nu}$) on m_H at fixed values of $ctg\beta$.

Figure 3 : The dependence of $tan\beta$ on the charged Higgs boson mass m_H. Curves (A, B), and (C, D) describes upper and lower bound of the experimental values of the Br($K^+ \to \pi^+ \nu \bar{\nu}$) for Set I and Set II values of ρ and η respectively.

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