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**GENERALIZED NEUTRINO MIXING  
FROM THE ATMOSPHERIC ANOMALY**

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**Abstract**

We determine the neutrino mixing and mass parameters that are allowed by the Super-Kamiokande atmospheric neutrino data in a three-neutrino model with one mass-squared difference contributing to the oscillations. We find that although  $\nu_\mu \rightarrow \nu_\tau$  oscillations are favored,  $\nu_\mu \rightarrow \nu_e$  oscillations with amplitude as large as 0.18 are allowed even after accounting for the limit from the CHOOZ reactor experiment. The range of allowed parameters permit observable  $\nu_\mu \leftrightarrow \nu_e$  and  $\nu_e \rightarrow \nu_\tau$  oscillations in future long-baseline experiments.

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*Introduction.* It was suggested long ago [1] that the atmospheric neutrino anomaly [2] could be explained by the oscillation of muon neutrinos and antineutrinos into another neutrino species. This interpretation has been confirmed by the zenith angle dependence measured by the Super–Kamiokande (SuperK) experiment [3]. Neutrino oscillations can also be invoked to separately explain the solar neutrino deficit [4, 5] and the results of the LSND experiment [6]. Because confirmation of the LSND results awaits future experiments and recent measurements in the KARMEN detector exclude part of the LSND allowed region [7], a conservative approach is to assume that oscillations need only account for the solar and atmospheric data. Then the two mass–squared difference scales in a three–neutrino model are sufficient to describe the data. Interest in the implications for models of the atmospheric neutrino anomaly has recently intensified [8, 9, 10]. An attractive possibility is that both the atmospheric  $\nu_\mu$  and solar  $\nu_e$  oscillate maximally or near–maximally at the  $\delta m_{atm}^2$  and  $\delta m_{sun}^2$  scales, respectively [9, 10].

In this letter we use the recent Super–Kamiokande atmospheric neutrino data [3] to determine the allowed values for the general three–neutrino mixing matrix under the assumption that one mass–squared difference,  $\delta m_{atm}^2$ , explains the atmospheric neutrino oscillations. In our scenario the other mass–squared difference,  $\delta m_{sun}^2 \ll \delta m_{atm}^2$ , can explain the solar neutrino oscillations via either an MSW [11] or vacuum long–wavelength scenario [12]. We find that although pure  $\nu_\mu \rightarrow \nu_\tau$  oscillations of atmospheric neutrinos are favored, there exist three–neutrino solutions with non–negligible  $\nu_\mu \leftrightarrow \nu_e$  oscillations, even after applying the constraints from the CHOOZ reactor experiment [13]. Consequently  $\nu_\mu \leftrightarrow \nu_e$  and  $\nu_e \rightarrow \nu_\tau$  oscillations may be observable in future long–baseline experiments.

*Oscillation probabilities.* We begin our analysis with the survival probability for a given neutrino flavor in a vacuum [14]

$$P(\nu_\alpha \rightarrow \nu_\alpha) = 1 - 4 \sum_{k < j} P_{\alpha j} P_{\alpha k} \sin^2 \Delta_{jk}, \quad (1)$$

where

$$P_{\alpha j} \equiv |U_{\alpha j}|^2, \quad (2)$$

$U$  is the neutrino mixing matrix (in the basis where the charged–lepton mass matrix is diagonal),  $\Delta_{jk} \equiv \delta m_{jk}^2 L/4E = 1.27(\delta m_{jk}^2/\text{eV}^2)(L/\text{km})/(E/\text{GeV})$ ,  $\delta m_{jk}^2 \equiv m_j^2 - m_k^2$ , and the sum is over all  $j$  and  $k$ , subject to  $k < j$ . The matrix elements  $U_{\alpha j}$  are the mixings between the flavor ( $\alpha = e, \mu, \tau$ ) and the mass ( $j = 1, 2, 3$ ) eigenstates, and we assume without loss of generality that  $m_1 < m_2 < m_3$ . The solar oscillations are driven by  $|\Delta_{21}| \equiv \Delta_{sun}$  and the atmospheric oscillations are driven by  $|\Delta_{31}| \simeq |\Delta_{32}| \equiv \Delta_{atm} \gg \Delta_{sun}$ .

The off-diagonal vacuum oscillation probabilities of this three–neutrino model are

$$P(\nu_e \rightarrow \nu_\mu) = 4 P_{e3} P_{\mu3} \sin^2 \Delta_{atm} - 4 \text{Re}\{U_{e1} U_{e2}^* U_{\mu1}^* U_{\mu2}\} \sin^2 \Delta_{sun} - 2 J \sin 2\Delta_{sun}, \quad (3)$$

$$P(\nu_e \rightarrow \nu_\tau) = 4 P_{e3} P_{\tau3} \sin^2 \Delta_{atm} - 4 \text{Re}\{U_{e1} U_{e2}^* U_{\tau1}^* U_{\tau2}\} \sin^2 \Delta_{sun} + 2 J \sin 2\Delta_{sun}, \quad (4)$$

$$P(\nu_\mu \rightarrow \nu_\tau) = 4 P_{\mu3} P_{\tau3} \sin^2 \Delta_{atm} - 4 \text{Re}\{U_{\mu1} U_{\mu2}^* U_{\tau1}^* U_{\tau2}\} \sin^2 \Delta_{sun} - 2 J \sin 2\Delta_{sun}, \quad (5)$$

where the  $CP$ –violating “Jarlskog invariant” [15] is  $J = \sum_{k,\gamma} \epsilon_{ijk} \epsilon_{\alpha\beta\gamma} \text{Im}\{U_{\alpha i} U_{\alpha j}^* U_{\beta i}^* U_{\beta j}\}$  for any  $\alpha, \beta, i$ , and  $j$  (e.g.,  $J = \text{Im}\{U_{e2} U_{e3}^* U_{\mu2}^* U_{\mu3}\}$  for  $\alpha = e, \beta = \mu, i = 2$ , and  $j = 3$ ). The  $CP$ –odd term changes sign under reversal of the oscillating flavors. We note that the  $CP$ –violating

probability at the atmospheric scale is suppressed to order  $\delta m_{sun}^2/\delta m_{atm}^2$ , the leading term having cancelled in the sum over the two light-mass states; thus,  $P(\nu_\alpha \rightarrow \nu_\beta) = P(\nu_\beta \rightarrow \nu_\alpha)$  at the atmospheric scale.

*Fit to atmospheric neutrino data.* For the  $L/E$  values of the atmospheric and long-baseline experiments,  $\Delta_{sun}$  can be neglected, and the vacuum oscillation probabilities become simply

$$P(\nu_\mu \rightarrow \nu_\mu) = 1 - 4 P_{\mu 3}(1 - P_{\mu 3}) \sin^2 \Delta_{atm}, \quad (6)$$

$$P(\nu_e \rightarrow \nu_e) = 1 - 4 P_{e 3}(1 - P_{e 3}) \sin^2 \Delta_{atm}, \quad (7)$$

$$P(\nu_e \leftrightarrow \nu_\mu) = 4 P_{e 3} P_{\mu 3} \sin^2 \Delta_{atm}, \quad (8)$$

$$P(\nu_e \leftrightarrow \nu_\tau) = 4 P_{e 3}(1 - P_{e 3} - P_{\mu 3}) \sin^2 \Delta_{atm}, \quad (9)$$

$$P(\nu_\mu \leftrightarrow \nu_\tau) = 4 P_{\mu 3}(1 - P_{e 3} - P_{\mu 3}) \sin^2 \Delta_{atm}. \quad (10)$$

In Eqs. (9) and (10) we have used the unitarity condition  $P_{\tau 3} = 1 - P_{e 3} - P_{\mu 3}$ . Thus for oscillations at the atmospheric scale there are only two independent mixing matrix parameters, e.g.,  $P_{e 3}$  and  $P_{\mu 3}$ , that are relevant. All predictions for atmospheric and long-baseline experiments are completely determined by the three parameters  $\delta m_{atm}^2$ ,  $P_{e 3}$ , and  $P_{\mu 3}$ . We define the oscillation amplitudes  $A_{atm}^{\mu\mu}$ ,  $A_{atm}^{e\ell}$ ,  $A_{atm}^{\mu e}$ ,  $A_{atm}^{e\tau}$ , and  $A_{atm}^{\mu\tau}$ , as the coefficients of the  $\sin^2 \Delta_{atm}$  terms in Eqs. (6)–(10), respectively. The parameters  $P_{e 3}$  and  $P_{\mu 3}$  can then be determined from the atmospheric neutrino data by the relations

$$N_\mu/N_\mu^o = \alpha \left[ (1 - \langle S \rangle A_{atm}^{\mu\mu}) + r \langle S \rangle A_{atm}^{\mu e} \right], \quad (11)$$

and

$$N_e/N_e^o = \alpha \left[ (1 - \langle S \rangle A_{atm}^{e\ell}) + r^{-1} \langle S \rangle A_{atm}^{\mu e} \right], \quad (12)$$

where  $N_e^o$  and  $N_\mu^o$  are the expected numbers of atmospheric  $e$  and  $\mu$  events, respectively,  $r \equiv N_e/N_\mu^o$ ,  $\langle S \rangle$  is  $\sin^2 \Delta_{atm}$  appropriately averaged, and  $\alpha$  is the overall neutrino flux normalization, which we allow to vary following the SuperK analysis [3]. SuperK reports  $N_\mu/N_\mu^o$  and  $N_e/N_e^o$  from a 535 day exposure for eight different  $L/E$  bins [3]. The data were obtained by inferring an  $L/E$  value for each event from the zenith angle  $\theta_\ell$  and energy of the observed charged lepton  $E_\ell$  and comparing it to expectations from a monte carlo simulation based on the atmospheric neutrino spectrum [16] folded with the differential cross section.

Due to the fact that the charged lepton energy and direction in general differ from the corresponding values for the incident neutrino (or antineutrino), the  $L/E$  distribution involves substantial smearing. We estimate this smearing by a monte carlo integration over the neutrino angle and energy spectrum [17] weighted by the differential cross section. We generate events with  $E_\nu$  and  $\theta_\nu$ , and determine the corresponding  $E_\ell$  and  $\theta_\ell$  for the charged lepton. We bin the events in  $L/E_\nu$ , using  $\theta_\ell$  to determine  $L$  and an estimated neutrino energy inferred from the average ratio of lepton momentum to neutrino energy,  $E_\nu^{\text{est}} = E_\ell \langle E_\nu/E_\ell \rangle$ , analogous to the SuperK analysis [3]. We then calculate a value for  $\langle \sin^2 \Delta_{atm} \rangle$  for each  $L/E$  bin for a given value of  $\delta m_{atm}^2$ . Finally we make a fit to Eqs. (11) and (12) to determine  $P_{e 3}$ ,  $P_{\mu 3}$ ,  $\delta m_{atm}^2$ , and  $\alpha$  [18]. Without loss of generality we take  $\delta m_{atm}^2$  to be positive.

Our best fit values for the four parameters are

$$\delta m_{atm}^2 = 2.8 \times 10^{-3} \text{ eV}^2, \quad (13)$$

$$P_{e3} = 0.00, \quad (14)$$

$$P_{\mu3} = 0.50, \quad (15)$$

$$\alpha = 1.16, \quad (16)$$

with  $\chi_{min}^2 = 7.1$  for 12 degrees of freedom. This best fit is close to the result of the SuperK simulation that assumed only two-neutrino oscillations. In Fig. 1a we show the allowed region for  $P_{\mu3}$  versus  $P_{e3}$  for  $\alpha = 1.16$  and  $\delta m_{atm}^2 = 2.8 \times 10^{-3} \text{ eV}^2$ . Although  $P_{e3} = 0$  is favored, small nonzero values are allowed, which permit some  $\nu_\mu \leftrightarrow \nu_e$  and  $\nu_e \rightarrow \nu_\tau$  oscillations of atmospheric neutrinos. In Fig. 1b we show the allowed region for the overall flux normalization  $\alpha$  versus  $\delta m_{atm}^2$  for  $P_{e3} = 0$  and  $P_{\mu3} = 0.50$ .

Another limit on  $P_{e3}$  comes from the CHOOZ reactor experiment [13] that measures  $\bar{\nu}_e$  disappearance

$$A_{atm}^{e\ell} = 4P_{e3}(1 - P_{e3}) \lesssim 0.2, \quad (17)$$

which applies for  $\delta m_{atm}^2 \gtrsim 2 \times 10^{-3} \text{ eV}^2$ . The exact limit on  $A_{atm}^{e\ell}$  varies with  $\delta m_{atm}^2$ , and for  $\delta m_{atm}^2 < 10^{-3} \text{ eV}^2$  there is no limit at all. For  $\delta m_{atm}^2 = 2.8 \times 10^{-3} \text{ eV}^2$  and  $\alpha = 1.16$ ,  $P_{e3}$  is constrained to be less than 0.04. The result of this additional constraint is shown in Fig. 1a. In Fig. 1c we show the effect of the CHOOZ constraint on the allowed region of  $P_{e3}$  versus  $\delta m_{atm}^2$  for  $\alpha = 1.16$  and  $P_{\mu3} = 0.50$ .

Varying over the entire parameter space, and imposing the CHOOZ constraint, the ranges of allowed values at 68% (95%) C.L. are

$$0.8 (0.5) \leq \delta m_{atm}^2 / (10^{-3} \text{ eV}^2) \leq 7.9 (10.0), \quad (18)$$

$$0.00 (0.00) \leq P_{e3} \leq 0.05 (0.08), \quad (19)$$

$$0.29 (0.25) \leq P_{\mu3} \leq 0.71 (0.75), \quad (20)$$

$$1.07 (1.04) \leq \alpha \leq 1.24 (1.28). \quad (21)$$

The allowed ranges of some related oscillation parameters are

$$0.29 (0.25) \leq P_{\tau3} \leq 0.71 (0.75) \quad (22)$$

$$0.83 (0.75) \leq A_{atm}^{\mu\ell} \leq 1.00 (1.00), \quad (23)$$

$$0.00 (0.00) \leq A_{atm}^{e\ell} \leq 0.18 (0.29), \quad (24)$$

$$0.00 (0.00) \leq A_{atm}^{\mu e} \leq 0.11 (0.18), \quad (25)$$

$$0.00 (0.00) \leq A_{atm}^{e\tau} \leq 0.10 (0.16), \quad (26)$$

$$0.82 (0.74) \leq A_{atm}^{\mu\tau} \leq 1.00 (1.00). \quad (27)$$

Although  $\nu_\mu \rightarrow \mu_\tau$  oscillations are strongly favored,  $\nu_\mu \leftrightarrow \nu_e$  are allowed with amplitude as large as 0.18.

We do not consider matter effects in the above analysis. For the  $\delta m_{atm}^2$  favored by our fit, matter effects are small for the sub-GeV neutrinos that constitute most of the data. Also, as evidenced by our fit, the dominant oscillation is  $\nu_\mu \rightarrow \nu_\tau$ , which is not affected by matter. However, matter effects could be important for neutrinos with smaller  $\delta m_{atm}^2/E$ , i.e., especially for multi-GeV data in solutions with  $\delta m_{atm}^2 \lesssim 10^{-3} \text{ eV}^2$ , which could be observable as more data becomes available at higher energies.

New physics predictions. The new physics predicted when  $P_{e3} \neq 0$  is  $\nu_e \rightarrow \nu_\tau$  oscillations with leading probability (i.e., the  $\Delta_{atm}$  terms) given by Eq. (9). From Eq. (26) the maximal amplitude  $A_{atm}^{e\tau}$  for these oscillations is 0.16 at 95% C.L. Figure 2 shows the allowed values of  $A_{atm}^{e\tau}$  versus  $A_{atm}^{\mu e}$  for  $\delta m^2 = 2.8 \times 10^{-3} \text{ eV}^2$  and  $\alpha = 1.16$ ; the effect of the CHOOZ constraint is also shown. The  $\nu_e \rightarrow \nu_\tau$  oscillations could be observed by long-baseline neutrino experiments with proposed high intensity muon sources [19, 20, 21], which can also make precise measurements of  $\nu_\mu \leftrightarrow \nu_e$  and  $\nu_\mu \rightarrow \nu_\tau$  oscillations. Sensitivity to  $A_{atm}^{e\tau} (\delta m_{atm}^2 / \text{eV}^2)^2 > 2.5 \times 10^{-9}$  is expected [19] for the parameter ranges of interest here; for  $\delta m^2 = 2.8 \times 10^{-3} \text{ eV}^2$ ,  $A_{atm}^{e\tau}$  could be measured down to  $3 \times 10^{-4}$ . The measurement of  $\nu_e \rightarrow \nu_\tau$  and  $\nu_\mu \leftrightarrow \nu_e$  oscillations in such a long-baseline experiment would test the three-neutrino model.

Discussion. Further measurements of atmospheric neutrinos will more precisely determine the parameters  $P_{e3}$ ,  $P_{\mu 3}$ , and  $\delta m_{atm}^2$ . The MINOS [22], K2K [23], and ICARUS [24] experiments are expected to test for  $\nu_\mu \rightarrow \nu_e$  and  $\nu_\mu \rightarrow \nu_\tau$  oscillations for  $\delta m_{atm}^2 > 10^{-3} \text{ eV}^2$ . Together these measurements could put strong limits on  $P_{e3}$ , which governs the  $\nu_e \rightarrow \nu_\tau$  oscillations that could be seen in future long-baseline experiments such as those utilizing a muon storage ring at Fermilab [19]. Full three-neutrino fits including the solar neutrino data [25] can then determine one of the remaining two independent parameters in the mixing matrix, e.g.,  $P_{e1}$ , using the  $\nu_e$  survival probability for  $\Delta_{atm} \gg 1$

$$P(\nu_e \rightarrow \nu_e) = 1 - 2P_{e3}(1 - P_{e3}) - 4P_{e1}(1 - P_{e1} - P_{e3}) \sin^2 \Delta_{sun}. \quad (28)$$

The considerations in this paper can also be extended to a four-neutrino model [21].

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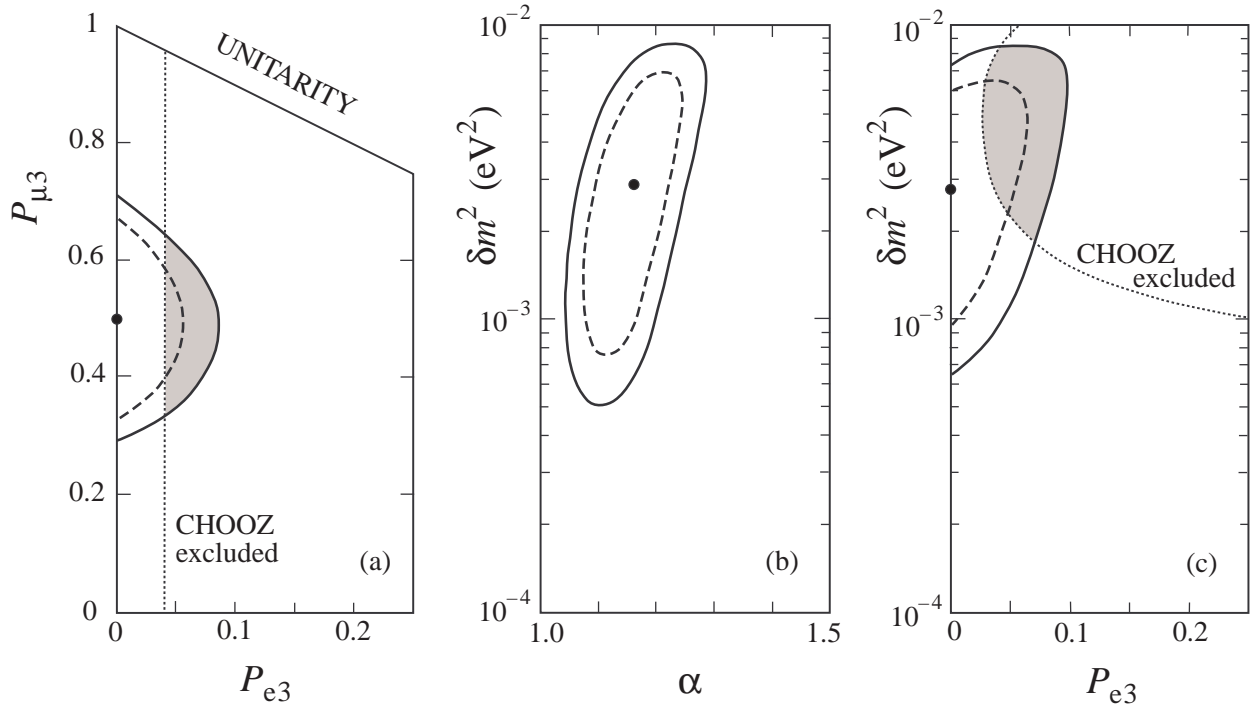


Figure 1: Allowed regions of the four-parameter space  $(\delta m_{atm}^2, P_{e3}, P_{\mu3}, \alpha)$  at 68% (dashed line) and 95% C.L. (solid line) for (a)  $P_{\mu3}$  versus  $P_{e3}$  with  $\delta m_{atm}^2 = 2.8 \times 10^{-3}$  eV<sup>2</sup> and  $\alpha = 1.16$ , (b)  $\delta m_{atm}^2$  versus  $\alpha$  with  $P_{e3} = 0$  and  $P_{\mu3} = 0.50$ , and (c)  $\delta m_{atm}^2$  versus  $P_{e3}$  with  $P_{\mu3} = 0.50$  and  $\alpha = 1.16$ . The best fit values are indicated by the filled circles. The CHOOZ constraint [13], shown by a dotted line, excludes the shaded region.



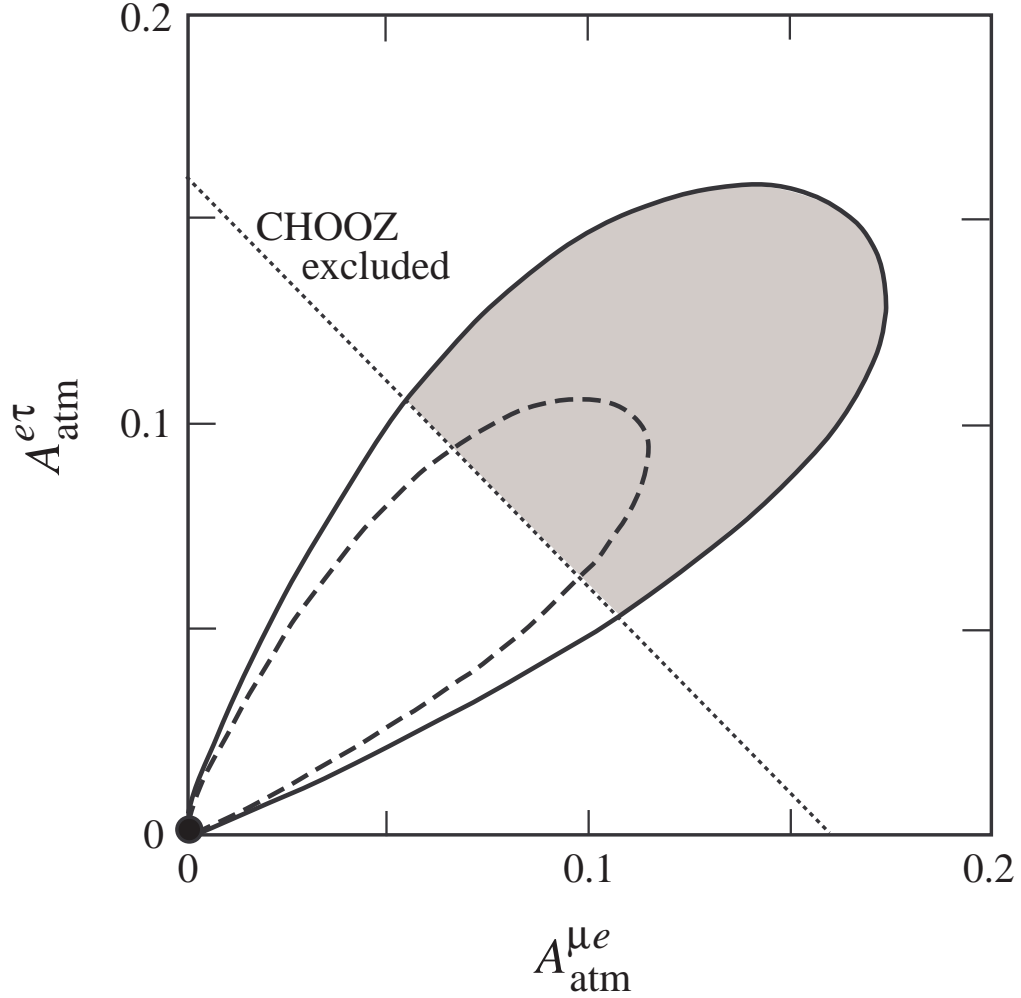


Figure 2: Allowed regions at 68% (dashed line) and 95% C.L. (solid line) for the oscillation amplitudes  $A_{e\tau}$  versus  $A_{\mu e}$  with  $\delta m^2 = 2.8 \times 10^{-3} \text{ eV}^2$  and  $\alpha = 1.16$ . The best fit values are indicated by the filled circle. The CHOOZ constraint [13], shown by a dotted line, excludes the shaded region.