HADRONIC CHARMLESS B DECAYS AND NONFACTORIZABLE EFFECTS

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Hadronic charmless B decays and their nonfactorizable effects are reviewed.

1 Introduction

A remarkable progress in the study of exclusive charmless B decays has been made recently. Experimentally, CLEO [1] has discovered many new two-body decay modes

$$B \to \eta' K^\pm,\ \eta' K_S^0,\ \pi^\pm K_S^0,\ \pi^\pm K^\mp,\ \omega K^\pm, \tag{1}$$

and a possible evidence for $B \to \phi K^*$. Moreover, CLEO has provided new improved upper limits for many other decay modes. Some of the CLEO data are surprising from the theoretical point of view: The measured branching ratios for $B^\pm \to \eta' K^\pm$ and $B^\pm \to \omega K^\pm$ are about several times larger than the naive theoretical estimate.

2 Difficulties with naive factorization

The relevant effective weak Hamiltonian for hadronic weak B decay is of the form

$$\mathcal{H}_{\rm eff} = \frac{G_F}{\sqrt{2}}\left[\lambda_u(c_1O_1^u + c_2O_2^u) + \lambda_c(c_1O_1^c + c_2O_2^c) - \lambda_t\sum_{i=3}^{10}c_iO_i\right], \tag{2}$$

where $\lambda_i = V_{ib}V_{iq}^*$, O_{3-6} are the QCD penguin operators and O_{7-10} the electroweak penguin operators. The Wilson coefficients $c_i(\mu)$ in Eq. (2) have been evaluated to the next-to-leading order (NLO) and they depend on the choice of the renormalization scheme. The mesonic matrix elements are customarily evaluated using the factorization hypothesis. Under this assumption, the 3-body hadronic matrix element $\langle M_1M_2|O|B\rangle$ is approximated as the product of two matrix elements $\langle M_1|J_{1\mu}|0\rangle$ and $\langle M_2|J_2^\mu|B\rangle$. Although this approach for matrix elements is very simple, it encounters two major difficulties. First, the hadronic matrix element under factorization is renormalization scale μ

independent as the vector or axial-vector current is partially conserved. Consequently, the amplitude $c_i(\mu)\langle O\rangle_{\rm fact}$ is not truly physical as the scale dependence of Wilson coefficients does not get compensated from the matrix elements. Second, in the naive factorization approach, the relevant Wilson coefficient functions for color-allowed external W-emission (or so-called "class-I") and color-suppressed (class-II) internal W-emission amplitudes are given by $a_1 = c_1 + c_2/N_c$, $a_2 = c_2 + c_1/N_c$, respectively, with N_c the number of colors. However, naive factorization fails to describe class-II decay modes. For example, the ratio $R = \Gamma(D^0 \to \overline{K}^0\pi^0)/\Gamma(D^0 \to K^-\pi^+)$ is predicted to be $\sim \frac{1}{50}$, while experimentally [2] $R = 0.51 \pm 0.07$. This implies that it is necessary to take into account nonfactorizable contributions to the decay amplitude in order to render the color suppression of internal W-emission ineffective.

2.1 Scale and scheme independence of physical amplitudes

Under the factorization hypothesis, we would like to know if it is possible to obtain physical amplitudes independent of the choice of the renormalization scale and scheme. The answer is yes. The scale and scheme dependence of the hadronic matrix elements can be calculated in perturbation theory at the one-loop level [3,4,5]. Schematically,

$$\langle O(\mu)\rangle = g(\mu)\langle O\rangle_{\rm tree}, \qquad \langle \mathcal{H}_{\rm eff}\rangle = c^{\rm eff}\langle O\rangle_{\rm tree}, \tag{3}$$

with $g(\mu)$ being the perturbative corrections to the four-quark operators renormalized at the scale μ. Formally, one can show that $c^{\rm eff} = g(\mu)c(\mu)$ is μ and renormalization scheme independent. It is at this stage that the factorization approximation is applied to the hadronic matrix elements of the operator O at tree level. The physical amplitude obtained in this manner is guaranteed to be renormalization scaheme and scale independent. [a]

The penguin-type corrections to $g(\mu)$ depend on k^2, the gluon's momentum squared, so are the effective Wilson coefficient functions. To NLO, we obtain [7]

$$\begin{aligned}
&c_1^{\rm eff} = 1.149, && c_2^{\rm eff} = -0.325, \\
&c_3^{\rm eff} = 0.0211 + i0.0045, && c_4^{\rm eff} = -0.0450 - i0.0136, \\
&c_5^{\rm eff} = 0.0134 + i0.0045, && c_6^{\rm eff} = -0.0560 - i0.0136, \\
&c_7^{\rm eff} = -(0.0276 + i0.0369)\alpha, && c_8^{\rm eff} = 0.054\,\alpha, \\
&c_9^{\rm eff} = -(1.318 + i0.0369)\alpha, && c_{10}^{\rm eff} = 0.263\,\alpha,
\end{aligned} \tag{4}$$

at $k^2 = m_b^2/2$.

[a] This formulation is different from the one advocated in [6] in which the μ dependence of the Wilson coefficients $c_i(\mu)$ are canceled out by that of the nonfactorization parameters $\varepsilon_8(\mu)$ and $\varepsilon_1(\mu)$ so that the effective parameters $a_i^{\rm eff}$ are μ independent.

2.2 Generalized factorization

Because there is only one single form factor (or Lorentz scalar) involved in the class-I or class II decay amplitude of $B \to PP,\ PV$ decays, the effects of nonfactorization can be lumped into the effective parameters a_1 and a_2 [8]:

$$a_1^{\rm eff} = c_1^{\rm eff} + c_2^{\rm eff}\left(\frac{1}{N_c} + \chi_1\right), \qquad a_2^{\rm eff} = c_2^{\rm eff} + c_1^{\rm eff}\left(\frac{1}{N_c} + \chi_2\right), \tag{5}$$

where χ_i are nonfactorizable terms and receive main contributions from the color-octet current operators. Since $|c_1/c_2| \gg 1$, it is evident from Eq. (5) that even a small amount of nonfactorizable contributions will have a significant effect on the color-suppressed class-II amplitude. If $\chi_{1,2}$ are universal (i.e. process independent) in charm or bottom decays, then we still have a generalized factorization scheme in which the decay amplitude is expressed in terms of factorizable contributions multiplied by the universal effective parameters $a_{1,2}^{\rm eff}$. For $B \to VV$ decays, this new factorization implies that nonfactorizable terms contribute in equal weight to all partial wave amplitudes so that $a_{1,2}^{\rm eff}$ *can* be defined. It should be stressed that, contrary to the naive one, the improved factorization does incorporate nonfactorizable effects in a process independent form. Phenomenological analyses of two-body decay data of D and B mesons indicate that while the generalized factorization hypothesis in general works reasonably well, the effective parameters $a_{1,2}^{\rm eff}$ do show some variation from channel to channel, especially for the weak decays of charmed mesons [8,9,10]. An eminent feature emerged from the data analysis is that $a_2^{\rm eff}$ is negative in charm decay, whereas it becomes positive in the two-body decays of the B meson [8,12,6]:

$$a_2^{\rm eff}(D \to \overline{K}\pi) \sim -0.50\,, \qquad a_2^{\rm eff}(B \to D\pi) \sim 0.20 - 0.28\,. \tag{6}$$

It follows that

$$\chi_2(D \to \overline{K}\pi) \sim -0.36\,, \qquad \chi_2(B \to D\pi) \sim 0.12 - 0.19\,. \tag{7}$$

The observation $|\chi_2(B)| \ll |\chi_2(D)|$ is consistent with the intuitive picture that nonperturbative soft gluon effects become stronger when the final-state particles move slower, allowing more time for significant final-state interactions after hadronization [8]. Phenomenologically, it is often to treat the number of colors N_c as a free parameter to model the nonfactorizable contribution to hadronic matrix elements and its value can be extracted from the data of two-body nonleptonic decays. Theoretically, this amounts to defining an effective number of colors $N_c^{\rm eff}$, called $1/\xi$ in [11], by $1/N_c^{\rm eff} \equiv (1/N_c) + \chi$. It is clear from

(7) that

$$N_c^{\rm eff}(D\to \overline{K}\pi)\gg 3, \qquad N_c^{\rm eff}(B\to D\pi)=1.8-2.2\approx 2\,. \tag{8}$$

3 Nonfactorizable effects in charmless B decays

We next study the nonfactorizabel effects in charmless rare B decays. We note that the effective Wilson coefficients appear in the factorizable decay amplitudes in the combinations $a_{2i}=c_{2i}^{\rm eff}+\frac{1}{N_c}c_{2i-1}^{\rm eff}$ and $a_{2i-1}=c_{2i-1}^{\rm eff}+\frac{1}{N_c}c_{2i}^{\rm eff}$ ($i=1,\cdots,5$). As discussed in Sec. 2.2, nonfactorizable effects in the decay amplitudes of $B\to PP,\ VP$ can be absorbed into the parameters $a_i^{\rm eff}$. This amounts to replacing N_c in $a_i^{\rm eff}$ by $(N_c^{\rm eff})_i$. Explicitly,

$$a_{2i}^{\rm eff}=c_{2i}^{\rm eff}+\frac{1}{(N_c^{\rm eff})_{2i}}c_{2i-1}^{\rm eff}, \qquad a_{2i-1}^{\rm eff}=c_{2i-1}^{\rm eff}+\frac{1}{(N_c^{\rm eff})_{2i-1}}c_{2i}^{\rm eff}. \tag{9}$$

It is customary to assume in the literature that $(N_c^{\rm eff})_1\approx(N_c^{\rm eff})_2\cdots\approx(N_c^{\rm eff})_{10}$; that is, the nonfactorizable term is usually assumed to behavor in the same way in penguin and non-penguin decay amplitudes. A closer investigation shows that this is not the case. We have argued in [7] that nonfactorizable effects in the matrix elements of $(V-A)(V+A)$ operators are *a priori* different from that of $(V-A)(V-A)$ operators. One reason is that the Fierz transformation of the $(V-A)(V+A)$ operators $O_{5,6,7,8}$ is quite different from that of $(V-A)(V-A)$ operators $O_{1,2,3,4}$ and $O_{9,10}$. As a result, contrary to the common assumption, $N_c^{\rm eff}(LR)$ induced by the $(V-A)(V+A)$ operators are theoretically different from $N_c^{\rm eff}(LL)$ generated by the $(V-A)(V-A)$ operators [7]. Hence, it is plausible to assume that

$$\begin{aligned} N_c^{\rm eff}(LL)&\equiv\left(N_c^{\rm eff}\right)_1\approx\left(N_c^{\rm eff}\right)_2\approx\left(N_c^{\rm eff}\right)_3\approx\left(N_c^{\rm eff}\right)_4\approx\left(N_c^{\rm eff}\right)_9\approx\left(N_c^{\rm eff}\right)_{10}, \\ N_c^{\rm eff}(LR)&\equiv\left(N_c^{\rm eff}\right)_5\approx\left(N_c^{\rm eff}\right)_6\approx\left(N_c^{\rm eff}\right)_7\approx\left(N_c^{\rm eff}\right)_8, \end{aligned} \tag{10}$$

and that $N_c^{\rm eff}(LR)\neq N_c^{\rm eff}(LL)$. In principle, $N_c^{\rm eff}$ can vary from channel to channel, as in the case of charm decay. However, in the energetic two-body B decays, $N_c^{\rm eff}$ is expected to be process insensitive as supported by data [6].

3.1 Classification of charmless B decays

By studying the $N_c^{\rm eff}$-dependence of the effective parameters a_i's (for simplicity, we will drop the superscript "eff" henceforth), we learn that (i) the dominant coefficients are a_1, a_2 for current-current amplitudes, a_4 and a_6 for QCD penguin-induced amplitudes, and a_9 for electroweak penguin-induced amplitudes, and (ii) a_1, a_4, a_6 and a_9 are $N_c^{\rm eff}$-stable, while the other coefficients depend strongly on $N_c^{\rm eff}$. Therefore, for those charmless B decays whose decay amplitudes depend dominantly on $N_c^{\rm eff}$-stable coefficients, their decay rates can be reliably predicted within the factorization approach even in the absence of information of nonfactorizable effects. By contrast, the decay modes involving the coefficients a_2, a_3 and a_5 are sensitive to $N_c^{\rm eff}$ and hence the nonfactorizable effects.

In order to study hadronic charmless B decays, it is useful to classify the decay modes into several different categories. Besides the widely used three classes I, II, III for tree-dominated decay modes, the penguin-dominated charmless rare B decays also can be classified into three classes:

- Class-IV for those decays whose amplitudes are governed by the parameters a_4 and a_6 in the combination $a_4 + Ra_6$, where the coefficient R arises from the $(S-P)(S+P)$ part of the operator O_6. In general, $R = 2m_{P_b}^2/[(m_1+m_2)(m_b-m_3)]$ for $B \to P_a P_b$ with the meson P_b being factored out in the factorizable approximation, $R = -2m_{P_b}^2/[(m_1+m_2)(m_b+m_3)]$ for $B \to V_a P_b$, and $R = 0$ for $B \to P_a V_b$ and $B \to V_a V_b$ with V_b being factorizable. Note that a_4 is always accompanied by a_{10}, and a_6 by a_8. Examples are $\overline{B}_d \to K^-\pi^+$, $\overline{K}^0\pi^0$, $B^- \to K^-\pi^0$, $\overline{B}_s \to K^+K^-$, $K^0\overline{K}^0, \cdots$.
- Class-V modes for those decays whose amplitudes are governed by the effective coefficients a_3, a_5, a_7 and a_9 in the combinations $a_3 \pm a_5$ and/or $a_7 \pm a_9$. Examples are $\overline{B}_d \to \phi\pi^0$, $B^- \to \phi\pi^-$, $\overline{B}_s \to \phi\pi^0$.
- Class-VI involving the interference of class-IV and class-V decays, e.g. $B \to K\eta'$, $K\omega$, $K\phi$ $(B = B_u, B_d, B_s)$.

Sometimes the tree and penguin contributions are comparable. For example, decays $\overline{B}_s \to K^0\omega, K^{*0}\omega$ fall into the classes of II and VI.

3.2 Some general features for penguin-dominated proceses

For penguin-dominated decay modes, some observations can be made:

- For class-IV modes, the decay rates obey the pattern:

$$\Gamma(B \to P_a P_b) > \Gamma(B \to P_a V_b) \sim \Gamma(B \to V_a V_b) > \Gamma(B \to V_a P_b), \quad (11)$$

  where $M = P_b$ or V_b is factorizable under the factorization assumption. For example, the branching ratios for $\overline{B}^0 \to K^-\pi^+, K^{*-}\pi^+, K^{*-}\rho^+, K^-\rho^+$ are predicted to be $\sim 1.5\times10^{-5}$, 0.7×10^{-5}, 0.6×10^{-5} and 0.5×10^{-6}

respectively. This hierarchy follows from various interference between the penguin terms characterized by the effective coefficients a_4 and a_6. On the contrary, in general $\Gamma(B \to P_a V_b) > \Gamma(B \to P_a P_b)$ for tree-dominated decays because the vector meson has three different polarization states.

- Among the two-body charmless B decays, the class-III decay modes $B^- \to \eta' K^-$, $\overline{B}_d \to \eta' K^0$ and $\overline{B}_s \to \eta\eta', \eta'\eta'$ have the largest branching ratios. Theoretically, $\mathcal{B}(B^- \to \eta' K^-) \approx \mathcal{B}(\overline{B}_d \to \eta' K^0) \sim 4 \times 10^{-5}$ and $\mathcal{B}(\overline{B}_s \to \eta\eta', \eta'\eta') \sim 2 \times 10^{-5}$. These decay modes receive two different sets of penguin terms proportional to $a_4 + Ra_6$ with $R > 0$. By contrast, VP, VV modes in charm decays or bottm decays involving charmed mesons usually have larger branching ratios than the PP mode.

- The decay amplitudes of $B \to M\pi^0, M\rho^0$ with $\pi^0(\rho^0)$ being factored out contain the electroweak penguin contributions proportional to $\frac{3}{2}(-a_7 + a_9)X_u^{(BM,\pi^0)}$ and $\frac{3}{2}(a_7 + a_9)X_u^{(BM,\rho^0)}$, respectively, [13] with

$$X_u^{(BM,\pi^0)} = \langle \pi^0 | (\bar{u}u)_{_{V-A}} | 0 \rangle \langle M | (\bar{q}b)_{_{V-A}} | \overline{B} \rangle. \tag{12}$$

  For $\overline{B}_s \to \eta'\pi, \eta'\rho, \phi\pi, \phi\rho$, QCD penguin contributions are canceled out so that these decays are dominated by electroweak penguins. Hence, a measurement of them can be used to determine the effective coefficient a_9. [b]

3.3 Nonfactorizable effects in spectator amplitudes

We focus on class-III decay modes dominated by the spectator diagrams induced by the current-current operators O_1 and O_2 and are sensitive to the interference between external and internal W-emission amplitudes. Good examples are the class-III modes: $B^\pm \to \omega\pi^\pm,\ \pi^0\pi^\pm,\ \eta\pi^\pm,\ \pi^0\rho^\pm, \cdots$, etc. Considering $B^\pm \to \omega\pi^\pm$, we find that the branching ratio is sensitive to $1/N_c^{\rm eff}$ and has the lowest value of order 2×10^{-6} at $N_c^{\rm eff} = \infty$ and then increases with $1/N_c^{\rm eff}$. The 1997 CLEO measurement yields [15]

$$\mathcal{B}(B^\pm \to \omega\pi^\pm) = \left(1.1^{+0.6}_{-0.5} \pm 0.2\right) \times 10^{-5}. \tag{13}$$

[b] It has been suggetsed in [14] that $\overline{B}_d \to \overline{K}^0\rho^0$ can be utilized to extract a_9. However, this method relies on the cancellation between the QCD penguin terms characterized by a_4 and a_6. In general, this cancellation is not complete and this makes this decay mode less clean than $\overline{B}_s \to (\eta', \phi)(\pi, \rho)$ for determining a_9.

Consequently, $1/N_c^{\rm eff} > 0.35$ is preferred by the data [7]. Because this decay is dominated by tree amplitudes, this in turn implies that $N_c^{\rm eff}(V-A) < 2.9$. If the value of $N_c^{\rm eff}(V-A)$ is fixed to be 2, the branching ratio for positive ρ, which is preferred by the current analysis [16], will be of order $(0.9-1.0)\times 10^{-5}$, which is very close to the central value of the measured one. Unfortunately, the significance of $B^\pm \to \omega\pi^\pm$ is reduced in the recent CLEO analysis and only an upper limit is quoted [17]: $\mathcal{B}(B^\pm \to \pi^\pm\omega) < 2.3\times 10^{-5}$. Since $\mathcal{B}(B^\pm \to K^\pm\omega) = (1.5^{+0.7}_{-0.6} \pm 0.2)\times 10^{-5}$ and $\mathcal{B}(B^\pm \to h^\pm\omega) = (2.5^{+0.8}_{-0.7} \pm 0.3)\times 10^{-5}$ with $h=\pi,\ K$, the central value of $\mathcal{B}(B^\pm \to \pi^\pm\omega)$ remains about the same as (11). The fact that $N_c^{\rm eff}(LL)\sim 2$ is preferred in charmless two-body decays of the B meson is consistent with the nonfactorizable term extracted from $B\to (D,D^*)\pi,\ D\rho$ decays, namely $N_c^{\rm eff}(B\to D\pi)\approx 2$. Since the energy release in the energetic two-body decays $B\to\omega\pi$, $B\to D\pi$ is of the same order of magnitude, it is thus expected that $N_c^{\rm eff}(LL)|_{B\to\omega\pi}\approx 2$.

Just like the decay $B^-\to\pi^-\omega$, the branching ratio of $B^-\to\pi^-\pi^0$ also increases with $1/N_c^{\rm eff}$. The CLEO measurement is [18]

$$\mathcal{B}(B^\pm \to \pi^\pm\pi^0) = \left(0.9^{+0.6}_{-0.5}\right)\times 10^{-5} \quad < 2.0\times 10^{-5}. \tag{14}$$

However, the errors are so large that it is meaningless to put a sensible constraint on $N_c^{\rm eff}(LL)$. Nevertheless, we see that in the range [5] $0\le 1/N_c^{\rm eff}\le 0.5$, $N_c^{\rm eff}(LL)\approx 2$ is most favored.

In analogue to the decays $B\to D^{(*)}\pi(\rho)$, the interference effect of spectator amplitudes in class-III charmless B decay can be tested by measuring the ratios:

$$R_1 \equiv 2\,\frac{\mathcal{B}(B^-\to\pi^-\pi^0)}{\mathcal{B}(\bar B^0\to\pi^-\pi^+)},\quad R_2 \equiv 2\,\frac{\mathcal{B}(B^-\to\rho^-\pi^0)}{\mathcal{B}(\bar B^0\to\rho^-\pi^+)},\quad R_3 \equiv 2\,\frac{\mathcal{B}(B^-\to\pi^-\rho^0)}{\mathcal{B}(\bar B^0\to\pi^-\rho^+)}. \tag{15}$$

Evidently, the ratios R_i are greater (less) than unity when the interference is constructive (destructive). Numerically we find

$$R_1 = \begin{cases} 1.74, \\ 0.58, \end{cases} \qquad R_2 = \begin{cases} 1.40, \\ 0.80, \end{cases} \qquad R_3 = \begin{cases} 2.50 & \text{for } N_c^{\rm eff}=2, \\ 0.26 & \text{for } N_c^{\rm eff}=\infty. \end{cases} \tag{16}$$

Hence, a measurement of R_i (in particular R_3), which has the advantage of being independent of the Wolfenstein parameters ρ and η, will constitute a very useful test on the effective number of colors $N_c^{\rm eff}(LL)$. The present experimental information on $\overline{B}^0\to\pi^+\pi^-$ is [18]

$$\mathcal{B}(\overline{B}^0\to\pi^\pm\pi^\mp) = (0.7\pm 0.4)\times 10^{-5} \quad < 1.5\times 10^{-5}. \tag{17}$$

As far as the experimental central value of R_1 is concerned, it appears that $1/N_c^{\rm eff}\sim 0.5$ is more favored than any other small values of $1/N_c^{\rm eff}$.

3.4 Nonfactorizable effects in penguin amplitudes

The penguin amplitude of the class-VI mode $B \to \phi K$ is proportional to $(a_3 + a_4 + a_5)$ and hence sensitive to the variation of $N_c^{\rm eff}$. Neglecting W-annihilation and space-like penguin diagrams, we find [7] that $N_c^{\rm eff}(LR) = 2$ is evidently excluded from the present CLEO upper limit [17]

$$\mathcal{B}(B^\pm \to \phi K^\pm) < 0.5 \times 10^{-5}, \tag{18}$$

and that $1/N_c^{\rm eff}(LR) < 0.23$ or $N_c^{\rm eff}(LR) > 4.3$. A similar observation was also made in [19]. The branching ratio of $B \to \phi K^*$, the average of ϕK^{*-} and ϕK^{*0} modes, is also measured recently by CLEO with the result [17]

$$\mathcal{B}(B \to \phi K^*) = \left(1.1^{+0.6}_{-0.5} \pm 0.2\right) \times 10^{-5}. \tag{19}$$

We find that the allowed region for $N_c^{\rm eff}(LR)$ is $4 \gtrsim N_c^{\rm eff}(LR) \gtrsim 1.4$. This is in contradiction to the constraint $N_c^{\rm eff}(LR) > 4.3$ derived from $B^\pm \to \phi K^\pm$. In fact, the factorization approach predicts that $\Gamma(B \to \phi K^*) \approx \Gamma(B \to \phi K)$ when the W-annihilation type of contributions is neglected. The current CLEO measurements (18) and (19) are obviously not consistent with the prediction based on factorization. One possibility is that generalized factorization is not applicable to $B \to VV$. Therefore, the discrepancy between $\mathcal{B}(B \to \phi K)$ and $\mathcal{B}(B \to \phi K^*)$ will measure the degree of deviation from the generalized factorization that has been applied to $B \to \phi K^*$. It is also possible that the absence of $B \to \phi K$ events is a downward fluctuation of the experimental signal. At any rate, in order to clarify this issue and to pin down the effective number of colors $N_c^{\rm eff}(LR)$, we urgently need measurements of $B \to \phi K$ and $B \to \phi K^*$, especially the neutral modes, with sufficient accuracy.

The decay mode $B \to \eta' K$ also provides another useful information on $N_c^{\rm eff}(LR)$. The discrepancy between the experimental measurements

$$\begin{aligned} \mathcal{B}(B^\pm \to \eta' K^\pm) &= \left(6.5^{+1.5}_{-1.4} \pm 0.9\right) \times 10^{-5}, \\ \mathcal{B}(B^0 \to \eta' K^0) &= \left(4.7^{+2.7}_{-2.0} \pm 0.9\right) \times 10^{-5} \end{aligned} \tag{20}$$

and the theoretical estimates [20,4,21] of order 1×10^{-5} seems to call for some new mechanisms unique to the η' production or even some new physics beyond the Standard Model.

In the conventional way of treating $N_c^{\rm eff}(LR)$ and $N_c^{\rm eff}(LL)$ in the same manner, the branching ratio of $B^\pm \to \eta' K^\pm$ can be enhanced to the order of $(2-3) \times 10^{-5}$ due to the small running strange quark mass at $\mu = m_b$ and $SU(3)$ breaking in the decay constants f_8 and f_0 (corresponding to the dashed

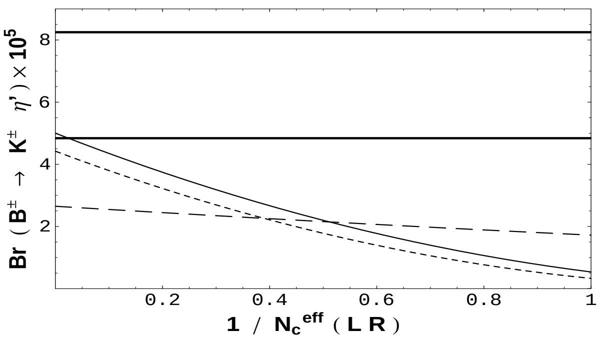

Figure 1: The branching ratio of $B^\pm \to \eta' K^\pm$ as a function of $1/N_c^{\rm eff}(LR)$ with $N_c^{\rm eff}(LL)$ being fixed at the value of 2 and $\eta = 0.34$ and $\rho = 0.16$. The charm content of the η' with $f_{\eta'}^c = -6\,{\rm MeV}$ contributes to the solid curve, but not to the dotted curve. The anomaly contribution to $\langle\eta'|\bar s\gamma_5 s|0\rangle$ is included. For comparison, the prediction for the case that $N_c^{\rm eff}(LL) = N_c^{\rm eff}(LR)$ as depicted by the dashed curve is also shown. The solid thick lines are the CLEO measurements with one sigma errors.

curve in Fig. 1). It should be emphasized that this prediction has taken into account the anomaly effect in the matrix element $\langle\eta'|\bar s\gamma_5 s|0\rangle$. Specifically,

$$\langle\eta'|\bar s\gamma_5 s|0\rangle = -i\frac{m_{\eta'}^2}{2m_s}\left(f_{\eta'}^s - f_{\eta'}^u\right), \tag{21}$$

where the QCD anomaly effect is manifested by the decay constant $f_{\eta'}^u$. Since $f_{\eta'}^u \sim \frac{1}{2}f_{\eta'}^s$ and the decay amplitude is dominated by $(S-P)(S+P)$ matrix elements, it is obvious that the decay rate of $B \to \eta' K$ would be (wrongly) enhanced considerably in the absence of the anomaly term in $\langle\eta'|\bar s\gamma_5 s|0\rangle$.

It has been advocated that the new internal W-emission contribution coming from the Cabibbo-allowed process $b \to c\bar c s$ followed by a conversion of the $c\bar c$ pair into the η' via two gluon exchanges may play an important role since its mixing angle $V_{cb}V_{cs}^*$ is as large as that of the penguin amplitude and yet its Wilson coefficient a_2 is larger than that of penguin operators. The decay constant $f_{\eta'}^c$, defined by $\langle 0|\bar c\gamma_\mu\gamma_5 c|\eta'\rangle = if_{\eta'}^c q_\mu$, has been estimated to be $f_{\eta'}^c = (50-180)$ MeV, based on the OPE, large-N_c approach and QCD low energy theorems [22]. Recent refined estimates [23,24] give $f_{\eta'}^c = -(2 \sim 15)$ MeV, which is in strong contradiction in magnitude and sign to the estimate of [22]. It turns out that if $N_c^{\rm eff}(LL)$ is treated to be the same as $N_c^{\rm eff}(LR)$, this new mechanism is not welcome for explaining $\mathcal{B}(B \to \eta' K)$ at small $1/N_c^{\rm eff}$ due to the fact that its contribution is proportional to a_2, which is negative at small $1/N_c^{\rm eff}$.

We have shown in [7] that if $N_c^{\rm eff}(LL)\sim 2$ and $N_c^{\rm eff}(LR)>N_c^{\rm eff}(LL)$, $\mathcal{B}(B^\pm\to\eta' K^\pm)$ at $1/N_c^{\rm eff}(LR)\leq 0.2$ will be enhanced considerably from $(2.5-3)\times 10^{-5}$ to $(3.7-5)\times 10^{-5}$ (see Fig. 1). First, the η' charm content contribution now contributes in the right direction to the decay rate irrespective of the value of $N_c^{\rm eff}(LR)$ as a_2 now is always positive. Second, the interference between the spectator amplitudes of $B^\pm\to\eta' K^\pm$ is constructive. Third, the term proportional to $2(a_3-a_5)X_u^{(BK,\eta')}+(a_3+a_4-a_5)X_s^{(BK,\eta')}$ is enhanced when $(N_c^{\rm eff})_3=(N_c^{\rm eff})_4=2$. The agreement with experiment for $B^\pm\to\eta' K^\pm$ thus provides another strong support for $N_c^{\rm eff}(LL)\sim 2$ and for the relation $N_c^{\rm eff}(LR)>N_c^{\rm eff}(LL)$.

4 Final-state interactions and $B\to\omega K$

The CLEO observation [17] of a large branching ratio for $B^\pm\to\omega K^\pm$

$$\mathcal{B}(B^\pm\to\omega K^\pm)=\left(1.5^{+0.7}_{-0.6}\pm 0.2\right)\times 10^{-5}, \tag{22}$$

is difficult to explain at first sight. Its factorizable amplitude is of the form

$$A(B^-\to\omega K^-)\propto(a_4+Ra_6)X^{(B\omega,K)}+(2a_3+2a_5+\frac{1}{2}a_9)X^{(BK,\omega)}+\cdots, \tag{23}$$

with $R=-2m_K^2/(m_b m_s)$, where ellipses represent for contributions from W-annihilation and space-like penguin diagrams. It is instructive to compare this decay mode closely with $B^-\to\rho K^-$

$$A(B^-\to\rho^0K^-)\propto(a_4+Ra_6)X^{(B\rho,K)}+\frac{3}{2}a_9X^{(BK,\rho)}+\cdots. \tag{24}$$

Due to the destructive interference between a_4 and a_6 penguin terms, the branching ratio of $B^\pm\to\rho^0K^\pm$ is estimated to be of order 5×10^{-7}. The question is then why is the observed rate of the ωK mode much larger than the ρK mode ? By comparing (23) with (24), it is natural to contemplate that the penguin contribution proportional to $(2a_3+2a_5+\frac{1}{2}a_9)$ accounts for the large enhancement of $B^\pm\to\omega K^\pm$. However, this is not the case: The coefficients a_3 and a_5, whose magnitudes are smaller than a_4 and a_6, are not large enough to accommodate the data unless $N_c^{\rm eff}(LR)<1.1$ or $N_c^{\rm eff}(LR)>20$ (see Fig. 9 of [7]).

So far we have neglected three effects in the consideration of $B^\pm\to\omega K^\pm$: W-annihilation, space-like penguin diagrams and final-state interactions (FSI). It turns out that FSI may play the dominant role for $B^\pm\to\omega K^\pm$. The weak decays $B^- \to K^{*-}\pi^0$ via the penguin process $b \to su\bar{u}$ and $B^- \to K^{*0}\pi^-$ via $b \to sd\bar{d}$ followed by the quark rescattering reactions $\{K^{*-}\pi^0,\ K^{*0}\pi^-\} \to \omega K^-$ contribute constructively to $B^- \to \omega K^-$, but destructively to $B^- \to \rho K^-$. Since the branching ratios for $B^- \to K^{*-}\pi^0$ and $K^{*0}\pi^-$ are large, of order $(0.5-0.8)\times 10^{-5}$, it is conceivable that a large branching ratio for $B^\pm \to \omega K^\pm$ can be achieved from FSI via inelastic scattering. Moreover, if FSI dominate, it is expected that $\mathcal{B}(B^\pm \to \omega K^\pm) \approx (1+\sqrt{2})^2\mathcal{B}(B^0 \to \omega K^0)$.

5 Conclusions

To summarize, the CLEO data of $B^\pm \to \omega\pi^\pm$ available last year clearly indicate that $N_c^{\rm eff}(LL)$ is favored to be small, $N_c^{\rm eff}(LL) < 2.9$. This is consistent with the observation that $N_c^{\rm eff}(LL) \approx 2$ in $B \to D\pi$ decays. Unfortunately, the significance of $B^\pm \to \omega\pi^\pm$ is reduced in the recent CLEO analysis and only an upper limit is quoted. Therefore, a measurement of its branching ratio is urgently needed. In analogue to the class-III $B \to D\pi$ decays, the interference effect of spectator amplitudes in charged B decays $B^- \to \pi^-\pi^0,\ \rho^-\pi^0,\ \pi^-\rho^0$ is sensitive to $N_c^{\rm eff}(LL)$; measurements of them [see (15)] will be very useful to pin down the value of $N_c^{\rm eff}(LL)$.

As for $N_c^{\rm eff}(LR)$, we found that the constraints on $N_c^{\rm eff}(LR)$ derived from $B^\pm \to \phi K^\pm$ and $B \to \phi K^*$ are no consistent. Under the factorization hypothesis, the decays $B \to \phi K$ and $B \to \phi K^*$ should have almost the same branching ratios, a prediction not borne out by current data. Therefore, it is crucial to measure the charged and neutral decay modes of $B \to \phi(K, K^*)$ in order to see if the generalized factorization approach is applicbale to $B \to \phi K^*$ decay. Nevertheless, our analysis of $B \to \eta' K$ indicates that $N_c^{\rm eff}(LL) \approx 2$ is favored and $N_c^{\rm eff}(LR)$ is preferred to be larger. Since the energy release in the energetic two-body charmless B decays is not less than that in $B \to D\pi$ decays, it is thus expected that

$$|\chi(2-\text{body rare B decay})| \lesssim |\chi(B \to D\pi)|. \tag{25}$$

It follows from Eq. (7) that $N_c^{\rm eff}(LL) \approx N_c^{\rm eff}(B \to D\pi) \sim 2$ and $N_c^{\rm eff}(LR) \sim 2-5$, depending on the sign of χ. Therefore, we conjecture that $N_c^{\rm eff}(LR) \sim 5 > N_c^{\rm eff}(LL) \sim 2$.

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