

# Polarization in diffractive electroproduction of light vector mesons<sup>1</sup>

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**Abstract:** We study in perturbative QCD the helicity amplitudes of the process  $\gamma^*p \rightarrow \rho p$  at large virtualities  $Q$  of the photon  $\gamma^*$ . We estimate all spin flip amplitudes taking into account an important effect of the scale behaviour of the gluon density. The transition of a transverse virtual photon to a longitudinal vector meson is not small at typical HERA conditions. This helicity non-conserving amplitude leads by interference to a measurable effect in the distribution of the angle between the electron scattering and the meson production planes.

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<sup>1</sup>Supported by German Bundesministerium für Bildung, Wissenschaft, Forschung und Technologie, grant No. 05 7LP91 P0

# 1 Introduction

The process of diffractive  $\rho$  meson electroproduction

$$\gamma^* p \rightarrow \rho p \tag{1.1}$$

is now under intensive experimental study at the  $ep$  collider HERA in a new energy range as compared to the previous fixed target experiments. The word diffractive means that the energy of  $\gamma^* p$  collision  $W$  is much larger than the virtuality of the photon  $Q$ . Therefore this process is mediated by the pomeron exchange. On the other hand the photon virtuality is large in comparison with the typical hadron scale and we deal here with hard diffraction.

The diagram of this process and our notations are given at Fig.1

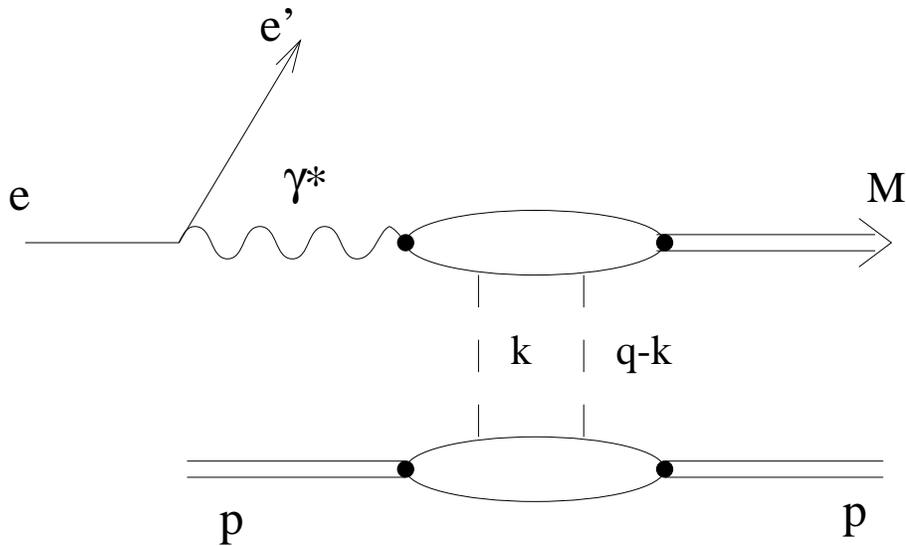


Figure 1: Diffractive electroproduction of vector meson on a proton target.

Theoretically this process was considered both in nonperturbative and perturbative approaches in many papers. The references can be found in the review [10]. It turns out that the perturbative models where pomeron is represented by the hard two-gluon exchange are able to reproduce the main features of the

HERA data for hard diffraction. In these models the amplitude is proportional to the gluon distribution in the proton

$$M \propto xG(x, \tilde{Q}^2) , \quad (1.2)$$

where  $x = \frac{Q^2}{W^2}$ ,  $\tilde{Q}$  is the typical hard scale of the process.

The aim of this paper is to consider in the framework of perturbative QCD the polarization effects in diffractive electroproduction of vector meson at large  $Q^2$ . Experimentally, information about the polarization state of produced meson is extracted from the angular distributions of the meson decay products ( $\pi^+\pi^-$  for  $\rho$ ). The present day results of such investigations at HERA are consistent with the  $s$ - channel helicity conservation (SCHC), which means that the produced meson retains the helicity of incoming virtual photon. The expected increase of the HERA luminosity gives us a hope that we will have in the near future much more precise experimental information about helicity properties of the  $\gamma^*M$  transition. Such an information will give us valuable insight into the underlying dynamics.

We shall discuss experiments with unpolarized protons, therefore the proton can be formally considered as a spinless target and we shall indicate further only the polarization states of the virtual photon and the produced meson.

Under the assumption of SCHC there are only two independent helicity amplitudes  $M_{\gamma_L^* \rightarrow \rho_L}$  and  $M_{\gamma_T^* \rightarrow \rho_T}$ , where  $L(T)$  denotes longitudinal(transverse) polarized state.

The longitudinal amplitude dominates at large  $Q^2$ . It has been considered in the papers [1, 2, 3, 4, 6]. A QCD factorization theorem for this amplitude was proven in [5]. The situation for the transverse amplitude is more complicated. Formal power counting gives  $\sim \frac{m_\rho}{Q}$  suppression in comparison with the longitudinal amplitude. However, this suppression factor is too small for typical values of  $Q^2 \sim 10 \text{ GeV}^2$  for HERA electroproduction experiments and does not agree with the measured ratio of  $\frac{\sigma_T}{\sigma_L}$ . For the transverse amplitude the integral over the longitudinal fraction  $z$  of the quark momentum is logarithmically divergent in the end points of the integration region. This means that essential transverse distances which scale like  $\frac{1}{Q\sqrt{z(1-z)}}$  can be large even for large  $Q^2$  and that the transverse amplitude can receive large or even dominant contribution from the nonperturbative region. As was noticed in [6], if the nonperturbative contribution would be dominant we would expect for  $\sigma_T$  features similar to the ones observed in photoproduction experiments as: a "softer" as compared to  $\sigma_L$  energy behaviour  $\sim W^{0.2}$  and a larger slope  $b \sim 9 - 10 \text{ GeV}^{-2}$ . However these expectations are not supported by the data.

In [6] it was assumed that the scale behaviour of gluon distribution  $G(x, Q^2) \sim \left(\frac{Q^2}{Q_0^2}\right)^\gamma$ , where  $\gamma$  is the anomalous dimension of gluon density, plays a very important role in the physics underlying the transverse amplitude. Taking into account this dependence it can be seen that the typical transverse distances are  $\sim \frac{1}{Q\sqrt{\gamma}}$

and can be smaller than  $\frac{1}{\Lambda_{QCD}}$  at large  $Q$  even for small  $\gamma$ . Therefore perturbative QCD can be applicable to the transverse amplitude. The estimate derived in [6] under the assumption of constant anomalous dimension

$$\frac{\sigma_L}{\sigma_T} = \frac{Q^2}{M_\rho^2} \left( \frac{\gamma}{1+\gamma} \right)^2 \quad (1.3)$$

shows the role of the scaling violation of the gluon density and agrees qualitatively with the data.

Under the assumption of natural parity exchange in the  $t$ - channel there are five independent helicity amplitudes. These are the two helicity conserving amplitudes

$$M_{(0,0)} = M_{\gamma_0^* \rightarrow \rho_0}, M_{(+1,+1)} = M_{\gamma_{+1}^* \rightarrow \rho_{+1}} \quad (M_{(-1,-1)} = M_{(+1,+1)}) ; \quad (1.4)$$

and the amplitudes violating SCHC: two single spin-flip amplitudes

$$M_{(+1,0)} = M_{\gamma_{+1}^* \rightarrow \rho_0}, \quad (M_{(-1,0)} = -M_{(+1,0)}) , \quad (1.5)$$

$$M_{(0,+1)} = M_{\gamma_0^* \rightarrow \rho_{+1}}, \quad (M_{(0,-1)} = -M_{(0,+1)}) ; \quad (1.6)$$

and one helicity double-flip amplitude

$$M_{(+1,-1)} = M_{\gamma_{+1}^* \rightarrow \rho_{-1}}, \quad (M_{(-1,+1)} = M_{(+1,-1)}) . \quad (1.7)$$

As it will be shown below, similar to the non-flip amplitudes (1.4), the single spin-flip amplitudes can be expressed through the gluon density in the proton. In the leading order of  $1/Q$  expansion the double spin-flip amplitude does not receive a logarithmic contribution from the integration over the  $t$ - channel gluon momenta, see Fig. 1. Therefore in the first term of the  $1/Q$  expansion of the double spin-flip amplitude the large factor  $xG(x, \tilde{Q}^2)$  is absent.

We will show that the largest amplitude violating SCHC is  $M_{(+1,0)}$ . The other amplitudes violating SCHC can be neglected in a first approximation. Our result derived in the approximation of constant gluon anomalous dimension  $\gamma$  reads

$$\beta = \frac{M_{(+1,0)}}{M_{(0,0)}} = \frac{\sqrt{|t|}}{\sqrt{2}Q\gamma} . \quad (1.8)$$

The observed  $t$ - dependence for  $\rho$  meson electroproduction is  $\frac{d\sigma}{dt} \sim e^{-bt}$ , with the slope  $b = 5 \dots 6 \text{ GeV}^{-2}$ . At typical values of  $t \sim 1/b$  this amplitude is not too small and, as will be shown below, leads to a sizable interference effect.

In the perturbative QCD approach the effect of the scale behaviour of the gluon density manifests itself qualitatively in the similar way in both transverse amplitudes: the helicity non-flip  $M_{(+1,+1)}$  and the helicity single-flip amplitude

$M_{(+1,0)}$ . Therefore the measurement of  $M_{(+1,0)}$  at HERA would give us an important check of whether perturbative QCD describes correctly the physics underlying the amplitudes of vector meson electroproduction initiated by a transverse photon.

The present work is based on the experience gained in a previous study of the diffractive vector meson production [7].

The paper is organized as follows. In section 2 we present the main steps of our calculation method. The results for helicity amplitudes and the discussion of the underlying physical effects are given in section 3. The influence of the helicity-flip amplitudes on the vector meson product distributions is discussed in the section 4. Our conclusions are summarized in section 5.

## 2 Impact parameter representation and the meson wave functions

The amplitude of the diffractive process  $\gamma^*p \rightarrow \rho p$  can be represented as the integral over the transverse momenta of gluons in the  $t$ - channel (impact representation)

$$M_{\gamma^*p \rightarrow \rho p} = i s_{\gamma^*p} \int \frac{d^2k}{\mathbf{k}^2(\mathbf{q} - \mathbf{k})^2} J_{\gamma^*\rho} J_p . \quad (2.1)$$

Here  $\mathbf{q}$  is the momentum transfer which is transverse with high accuracy,  $q^2 = t = -\mathbf{q}^2$ . Throughout the paper all vectors, if it is not mentioned separately, are two-dimensional vectors in the transverse space. The accuracy of the representation (2.1) is expected to be  $\sim Q^2/s_{\gamma^*p}$ .

The space-time picture of the process in diffractive (high energy) region is the following. The virtual photon fluctuates into the  $q\bar{q}$  pair long time before and the  $q\bar{q}$  pair converts into the vector meson long time after the interaction with the proton. Therefore it is possible to represent the photon impact factor as the convolution of the impact factor for the  $q\bar{q}$  dipole scattering with the light cone wave functions of the incoming virtual photon and the outgoing vector meson (Fig.2)

$$J_{\gamma^* \rightarrow \rho}(\mathbf{k}, \mathbf{q}) = \int \frac{d^2l_1 dz_1}{16\pi^3} \frac{d^2l_2 dz_2}{16\pi^3} \Psi_{\gamma^*}(\mathbf{l}_1, z_1) \Phi^{dipole}(\mathbf{l}_1, \mathbf{l}_2, z_1, z_2, \mathbf{k}, \mathbf{q}) \Psi_{\rho}^*(\mathbf{l}_2, z_2) , \quad (2.2)$$

where

$$\begin{aligned} \Phi^{dipole}(\mathbf{l}, \mathbf{l}_1, z, z_1, \mathbf{k}, \mathbf{q}) = & 16\pi^3 \frac{\alpha_S \delta^{ab}}{N} \delta(z - z_1) [\delta(\mathbf{l} - \mathbf{l}_1 - \mathbf{q}z) + \\ & \delta(\mathbf{l} - \mathbf{l}_1 + \mathbf{q}\bar{z}) - \delta(\mathbf{l} - \mathbf{l}_1 + \mathbf{k} - \mathbf{q}z) - \delta(\mathbf{l} - \mathbf{l}_1 - \mathbf{k} + \mathbf{q}\bar{z})] . \end{aligned} \quad (2.3)$$

$N = 3$  is the number of colors,  $(\delta^{ab})^2 = N^2 - 1$ .  $\alpha_S$  is the strong coupling constant.

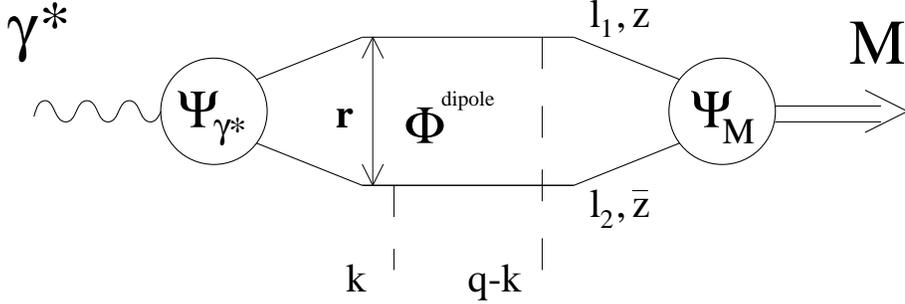


Figure 2: The  $\gamma^* \rightarrow M$  impact factor. Only one contribution of the interaction of the  $q\bar{q}$  dipole with the two exchanged gluons (dashed lines) is shown.

The light cone wave function of the photon

$$\Psi_{\gamma^*}(\mathbf{l}, z) = -e_q \sqrt{z\bar{z}} \frac{\bar{u}\hat{\mathbf{e}}v}{\mathbf{l}^2 + Q^2 z\bar{z}} \quad (2.4)$$

describes the probability amplitude for the splitting of the photon into the  $q\bar{q}$  pair with electric charge  $e_q$ , ( $e_q = \frac{2}{3}e$  for the  $u$  quark). The quark carries the transverse momentum  $\mathbf{l}$  relative to the photon momentum and the fraction  $z$  of the photon longitudinal momentum (the fraction for antiquark is  $\bar{z} = 1 - z$ ). It should be noted that the vector  $\mathbf{l}_2$  in eq. (2.2) is transverse relative to the momentum of the outgoing vector meson which received non-zero momentum transfer  $\mathbf{q}$ .

The polarization state of the photon is described by the vector  $\mathbf{e}$ . We choose the following convention for the polarization four vectors describing a transversely polarized vector meson and a transverse photon

$$\mathbf{e}_{\pm} = \frac{\mp 1}{\sqrt{2}}(0, 1, \pm i, 0) , \quad (2.5)$$

where the  $x$  axis is chosen in the direction of the momentum transfer

$$\mathbf{q} = q \cdot (0, 1, 0, 0) . \quad (2.6)$$

It can be seen that the difference in the polarization vectors of the vector meson from (2.5) related to the non-zero value of momentum transfer can be neglected. The polarization vector of the longitudinally polarized virtual photon is

$$\mathbf{e}_0 = \frac{1}{Q}(p_1, 0, 0, \sqrt{p_1^2 - Q^2}) , \quad (2.7)$$

where  $p_1$  is the value of the longitudinal momentum of the photon. And the similar convention is adopted for the polarization vector of the longitudinally polarized vector meson.

The perturbative expression (2.4) for the splitting of a photon into the light  $\bar{q}q$  pair has the important property that the total helicity of the produced massless quark–antiquark pair is zero irrespective of the polarization state of the photon:  
a) for longitudinally polarization

$$\sqrt{z\bar{z}}\bar{u}_\lambda\hat{e}_0v_{\lambda'} = 2Qz\bar{z}\delta_{\lambda,-\lambda'} , \quad (2.8)$$

b) for transverse polaritation

$$\sqrt{z\bar{z}}\bar{u}_\lambda\hat{e}_\pm v_{\lambda'} = \delta_{\lambda,-\lambda'}\{(1-2z)\mp\lambda\}(\mathbf{e}_\pm\mathbf{1}) , \quad (2.9)$$

The quark helicities  $\pm\frac{1}{2}$  are represented in the above equations by  $\lambda = \pm 1$ .

The splitting of the transverse photon into the quark pair with the total helicity  $\pm 1$  is proportional to the current quark mass. Since light quarks have very small current masses we will neglect here this splitting. Of course it becomes important in the case of a heavy flavour production.

The physics of the spin flip transitions which is the main subject of this paper looks more transparent in the space of impact parameters of the  $q\bar{q}$  pair. The helicities of quarks coincide in this case with the projection of the quark spins onto the  $z$  axes. Since the total helicity of the quark pair is zero, the projection of the orbital momentum of the incoming pair onto the  $z$  axis should coincide with the helicity of the incoming virtual photon. Another important property of perturbative QCD is that the interaction of  $t$ -channel gluons with the pair does not change with high accuracy the helicity states of the quarks. This interaction does not change also the impact parameters of the pair even if the momentum transfer is not zero. Therefore the helicity state of the produced meson should coincide with the projection of the orbital momentum of the outgoing quark pair onto the  $z$  axis. We can conclude that in the frame of perturbative QCD the only possibility to have the change of the helicity state during the diffractive  $\gamma^*\rho$  transition is to change the  $z$  projection of the angular momentum of the  $q\bar{q}$  pair in the interaction with the proton.

To make this discussion more quantitative let us transform eqs. (2.2,2.3, 2.4) into the space of impact parameters. By Fourier transformation we obtain the photon (meson) wave function in the representation of impact parameters

$$\Psi(\mathbf{r}, z) = \int \frac{d^2\mathbf{l}}{2\pi} \Psi(\mathbf{l}, z) \cdot e^{-i\mathbf{l}\mathbf{r}} , \quad (2.10)$$

where  $\mathbf{r}$  is the difference between the quark and antiquark impact parameters.

$$\Psi_{\gamma^*}^T(\mathbf{r}, z) = \mp ieQ\sqrt{z\bar{z}} \left[ \delta_{\lambda,-\lambda'}\{(1-2z)\mp\lambda\}(\mathbf{e}_\pm\frac{\mathbf{r}}{r})K_1(rQ\sqrt{z\bar{z}}) \right] \quad (2.11)$$

$$\Psi_{\gamma^*}^L(\mathbf{r}, z) = -2eQz\bar{z}\delta_{\lambda,-\lambda'}K_0(rQ\sqrt{z\bar{z}}) \quad (2.12)$$

In these eqs. (2.11,2.12)  $K_{0,1}(mr)$  are the Bessel (MacDonald) functions.

We can represent the impact factor describing the  $\gamma^* \rightarrow \rho$  transition (2.10) in the form (Fig.2)

$$J_{\gamma^* \rightarrow \rho}(\mathbf{k}, \mathbf{q}) = \int \frac{d^2 \mathbf{r}_1 dz_1}{16\pi^3} \Psi_{\gamma^*}(\mathbf{r}_1, z) \Phi^{dipole}(\mathbf{r}_1, \mathbf{r}_2, \mathbf{k}, \mathbf{q}, z_1, z_2) \Psi_{\rho}^*(\mathbf{r}_2, z) \frac{d^2 \mathbf{r}_2 dz_2}{16\pi^3} \quad (2.13)$$

where  $\Phi^{dipole}(\mathbf{r}_1, \mathbf{r}_2, \mathbf{k}, \mathbf{q}, z_1, z_2)$  is the Fourier transform of (2.3)

$$\begin{aligned} \Phi^{dipole}(\mathbf{r}_1, \mathbf{r}, \mathbf{k}, \mathbf{q}, z_1, z_2) &= 16\pi^3 \frac{\alpha_S \delta^{ab}}{N} \delta(\mathbf{r} - \mathbf{r}_1) \delta(z_1 - z_2) f(\mathbf{k}, \mathbf{q}, \mathbf{r}, z_1) , \\ f(\mathbf{r}, \mathbf{k}, \mathbf{q}, z) &= e^{i\mathbf{q}\mathbf{r}z} (1 - e^{-i\mathbf{k}\mathbf{r}}) (1 - e^{-i(\mathbf{q}-\mathbf{k})\mathbf{r}}) . \end{aligned} \quad (2.14)$$

The expression for the dipole impact factor is proportional to  $\delta(\mathbf{r} - \mathbf{r}_1)$  which reflects the mentioned above property that the interaction does not change the impact parameters of the pair. The main part of the dipole impact factor is the factor  $f(\mathbf{r}, \mathbf{k}, \mathbf{q}, z)$ . This factor tends to zero if the momentum of one of the  $t$ -channel gluon ( $\mathbf{k}$  or  $\mathbf{q} - \mathbf{k}$ ) or if the transverse separation between the quarks  $\mathbf{r}$  vanishes.

Let us discuss now the wave functions of a vector meson. They can be constructed using the analogy with the photon wave functions. The wave function of the longitudinally polarized photon can be rewritten in the following way

$$\begin{aligned} \Psi_{\gamma^*}^L(\mathbf{l}, z) &\sim z\bar{z} \frac{\Phi(M)}{z\bar{z}} , \\ \Phi(M) &= \frac{1}{Q^2 + M^2} , M^2 = \frac{l^2}{z\bar{z}} . \end{aligned} \quad (2.15)$$

Where  $M$  is the invariant mass of the  $q\bar{q}$  pair. We shall adopt the natural assumption that the meson wave functions depend on the invariant mass of  $q\bar{q}$  pair. But in the  $\rho$  meson case the corresponding  $\Phi(M)$  should fall off faster at large  $M$  as compared to the photon wave function. Therefore

$$\Psi_{\rho}^L(\mathbf{l}, z) = -\frac{3}{2} \delta_{\lambda, -\lambda'} f_{\rho} z\bar{z} \frac{\Phi(l^2/(z\bar{z}))}{z\bar{z}} . \quad (2.16)$$

The dimensionful coupling constant  $f_{\rho} \sim 200$  GeV is related with the  $e^+e^-$  decay width of the  $\rho$  meson,

$$\Gamma = \frac{2\pi\alpha^2 f_{\rho}^2}{3m_{\rho}} , \alpha = e^2/4\pi = 1/137 . \quad (2.17)$$

The normalization for function  $\Phi(M)$  is

$$\int \frac{d^2 l}{(16\pi^3) z\bar{z}} \Phi\left(\frac{l^2}{z\bar{z}}\right) = 1 . \quad (2.18)$$

The wave function of the transversely polarized  $\rho$  meson

$$\Psi_\rho^T(\mathbf{l}, z) = \pm \frac{3}{4} \delta_{\lambda, -\lambda'} f_\rho \sqrt{z\bar{z}} \{ (1 - 2z) \mp \lambda \} (\mathbf{e}_\pm \frac{\mathbf{l}}{l}) \frac{\Phi(l^2/(z\bar{z}))}{z\bar{z}} \quad (2.19)$$

is constructed in analogy with the wave function of the transverse photon. In the meson rest frame the meson splitting looks like as follows: the quark and the antiquark fly out back to back, the polar angles of the quark momentum relative to the direction of the meson momentum in the boosted frame are  $(\theta, \phi)$ . The amplitude of the transition of a spin 1  $\rho$ -meson in a state with the helicity  $\lambda$  into the  $q\bar{q}$  pair with the total helicity  $\lambda'$  is proportional to the rotation matrix  $d_{\lambda, \lambda'}^1(\theta, \phi)$ . We are interested in such meson splittings when the quark and antiquark have the opposite helicities (zero total helicity) in the boosted frame, that corresponds to the total helicity of the pair  $\pm 1$  in the rest frame. Since the angle  $\theta$  in the rest frame and the variable  $z$  in the boosted frame (the fraction of meson momentum carrying by the quark) are related as

$$z = \frac{1 + \cos(\theta)}{2} ,$$

it can be seen that the ratio of  $\Psi_\rho^L(\mathbf{l}, z)$  and  $\Psi_\rho^T(\mathbf{l}, z)$  is equal to the ratio of the corresponding rotation matrices. This observation justifies eq. (2.19) for the wave functions of transverse  $\rho$  as well as the relative sign between the transverse and longitudinal wave functions.

The meson wave functions in the representation of the impact parameters are given by the Fourier transform of eqs. (2.16, 2.19). Since the  $q\bar{q}$  fluctuation of the transverse polarized meson has the projection  $\pm 1$  of the angular momentum onto the  $z$  axis its wave function has a factor  $(\mathbf{e}_\pm \mathbf{r})$ .

Now let us return to eq. (2.13) and discuss the virtual photon to meson impact factor

$$J_{\gamma^* \rightarrow \rho}(\mathbf{k}, \mathbf{q}) = \frac{\alpha_S \delta^{ab}}{N} \int \frac{d^2 \mathbf{r} dz}{16\pi^3} \Psi_{\gamma^*}(\mathbf{r}, z) f(\mathbf{r}, \mathbf{k}, \mathbf{q}, z) \Psi_\rho^*(\mathbf{r}, z) \quad (2.20)$$

for the various helicity transitions.

Looking at the eqs. (2.16, 2.19, 2.11, 2.12) for the meson and photon wave functions it is easy to see that:

a) the transitions obeying SCHC are proportional to the dipole factor  $f(\mathbf{r}, \mathbf{k}, \mathbf{q}, z)$  averaged over the polar angle of the vector  $\mathbf{r}$

$$\langle f(\mathbf{r}, \mathbf{k}, \mathbf{q}, z) \rangle = [J_0(rqz) - J_0(r|\mathbf{k} - \mathbf{q}z|)] + [z \leftrightarrow \bar{z}] , \quad (2.21)$$

b) the single spin-flip transitions are proportional to the projection of this dipole factor onto  $\frac{\mathbf{r}}{r}$

$$\langle \frac{\mathbf{r}}{r} f(\mathbf{r}, \mathbf{k}, \mathbf{q}, z) \rangle = -i \left[ \frac{\mathbf{q}}{q} J_1(rqz) + \frac{\mathbf{k} - \mathbf{q}z}{|\mathbf{k} - \mathbf{q}z|} J_1(r|\mathbf{k} - \mathbf{q}z|) - (z \leftrightarrow \bar{z}) \right] , \quad (2.22)$$

c) the double spin-flip transitions are related to the projections of this dipole factor onto  $\frac{\mathbf{r}_\mu \mathbf{r}_\nu}{r^2}$ ,  $\langle \frac{\mathbf{r}_\mu \mathbf{r}_\nu}{r^2} f(\mathbf{r}, \mathbf{k}, \mathbf{q}, z) \rangle$ . They are proportional to

$$\left[ 2 \frac{(k - \mathbf{q}z, \mathbf{q})^2}{(\mathbf{k} - \mathbf{q}z)^2} J_2(r|\mathbf{k} - \mathbf{q}z|) - q^2 (J_2(r|\mathbf{k} - \mathbf{q}z|) + J_2(rqz)) \right] + [z \leftrightarrow \bar{z}] . \quad (2.23)$$

Using the general formulae derived in this section we shall calculate in the next section the helicity amplitudes in the high  $Q^2$  limit using the approximations to be described in the following.

### 3 Calculating the helicity amplitudes

Assuming some functional form for the meson wave function allows to calculate according to the eq. (2.20) the  $\gamma^* \rightarrow \rho$  impact factor without any approximation. Unfortunately the meson wave functions are purely known. But, as it will be seen further, if we proceed to calculate the amplitude in the leading order of  $\frac{1}{Q}$  expansion only a limited information about these wave functions is needed.

The eqs. (2.11,2.12) show that the typical size of the virtual photon fluctuation is  $r \sim \frac{1}{Q\sqrt{z\bar{z}}}$ . Outside of the end point regions of  $z$  this size is much smaller than the meson transverse size. Therefore to calculate the leading in  $\frac{1}{Q}$  behaviour of the impact factor we have to calculate the first term of the Taylor expansion for the meson wave function and the expansion of the dipole factor  $f(\mathbf{r}, \mathbf{k}, \mathbf{q}, z)$  in the region of small  $\mathbf{r}$ .

Making the Fourier transform of eqs. (2.16,2.19) and then expanding them at small  $\mathbf{r}$  we find

$$\Psi_\rho^L(\mathbf{r}, z) \approx -\frac{16\pi^3}{2\pi} \frac{3}{2} \delta_{\lambda, -\lambda'} f_\rho z \bar{z} , \quad (3.1)$$

$$\Psi_\rho^T(\mathbf{r}, z) \approx \mp i \frac{16\pi^3}{2\pi} \frac{3}{8} \delta_{\lambda, -\lambda'} f_\rho z \bar{z} \{ (1 - 2z) \mp \lambda \} (\mathbf{e}_\pm \mathbf{r}) \langle M \rangle , \quad (3.2)$$

where

$$\langle M \rangle = \int \frac{d^2 l}{(16\pi^3) z \bar{z}} \frac{l}{\sqrt{z \bar{z}}} \Phi\left(\frac{l^2}{z \bar{z}}\right) \quad (3.3)$$

is the mean invariant mass of the  $q\bar{q}$  fluctuation. This mean invariant mass is expected to be of order of the  $\rho$  meson mass. We would like to note that the Taylor expansion for the transversely polarized meson wave function starts from the term  $\sim r$ , whereas the longitudinal polarization wave function is constant at small  $r$ . The appearance of this suppression factor  $r$  is related with the nonzero projection of the orbital momentum ( $\pm 1$ ) of the quark pair in the transverse case.

The azimuthal projections of the dipole factor  $f(\mathbf{r}, \mathbf{k}, \mathbf{q}, z)$  for the various helicity transitions, see eqs. (2.21–2.23), can be simplified in the region of small  $\mathbf{r}$  by expanding the Bessel functions.

a) for the SCHC transitions:

$$(J_0(x) \approx 1 - \frac{x^2}{4})$$

$$\langle f(\mathbf{r}, \mathbf{k}, \mathbf{q}, z) \rangle = \frac{r^2}{2}(k^2 - (\mathbf{kq})) + O(r^4) ; \quad (3.4)$$

b) for the single spin–flip transitions:

$$(J_1(x) \approx \frac{x}{2} - \frac{x^3}{24})$$

$$\langle \frac{\mathbf{r}}{r} f(\mathbf{r}, \mathbf{k}, \mathbf{q}, z) \rangle = i \frac{r^3(z - \bar{z})}{24} [\mathbf{k}(q^2 - 2(\mathbf{kq})) - \mathbf{q}(k^2 - 2(\mathbf{kq}))] + O(r^5) ; \quad (3.5)$$

c) for the double spin–flip transitions:

$$(J_2(x) \approx \frac{x^2}{8} - \frac{x^4}{8 \cdot 12})$$

$$\begin{aligned} & \frac{r^2}{4q^2} [2(\mathbf{kq})^2 - (\mathbf{kq})q^2 - k^2q^2] \\ & - \frac{r^4}{4 \cdot 12q^2} [2k^2(\mathbf{kq})^2 - k^4q^2 - 2(\mathbf{kq})^3 + \\ & 3(\mathbf{kq})^2q^2(z^2 + \bar{z}^2) - 2(\mathbf{kq})q^4(z^3 + \bar{z}^3)] + O(r^6) . \end{aligned} \quad (3.6)$$

We quote in the last case the expansion up to the next-to-leading term, the importance of which will be discussed in what follows.

Let us discuss now the proton impact factor and the integration over the  $t$ -channel gluon momenta in the eq. (2.1).

The proton is a colorless state. Therefore its impact factor vanishes if the transverse momentum of any of the  $t$ -channel gluon tends to zero. On the other hand if the transverse momenta of gluons are large, much larger than the inverse transverse size of the proton (the value of the momentum transfer  $q$  is expected to be small, therefore  $\mathbf{k} \approx \mathbf{k} - \mathbf{q}$  in this region), both of the gluons couple to the same parton inside the proton and as a function of  $\mathbf{k}$  the impact factor is approximately a constant in this region. According to eqs. (3.4,3.5) the impact factors of the helicity conserving and single helicity flip  $\gamma^* \rightarrow \rho$  transitions are proportional to the square of the  $t$ -channel gluon momentum  $k^2$  at large  $\mathbf{k}$ :

a) for the SCHC transitions

$$\langle f(\mathbf{r}, \mathbf{k}, \mathbf{q}, z) \rangle \approx \frac{r^2 k^2}{2} ; \quad (3.7)$$

b) for the single spin–flip transitions

$$\langle \frac{\mathbf{r}}{r} f(\mathbf{r}, \mathbf{k}, \mathbf{q}, z) \rangle \approx -i \mathbf{q} \frac{r^3 k^2 (z - \bar{z})}{2^3} . \quad (3.8)$$

Therefore the main, logarithmic, contribution to the helicity non–flip and helicity single–flip amplitudes originates from the broad region of large  $\mathbf{k}$ ,  $k \leq \tilde{Q} = \frac{1}{Q\sqrt{z\bar{z}}}$ .

In the leading log  $\tilde{Q}^2$  approximation and at  $t = 0$  these amplitude are proportional to the gluon distribution

$$x \cdot G(x, \tilde{Q}^2) = \frac{\delta^{ab}}{2\pi} \int^{\tilde{Q}^2} J_p^{ab} \frac{dk^2}{k^2} . \quad (3.9)$$

Let us calculate these amplitudes in the leading log approximation.

### 3.1 SCHC amplitudes

Let us start with the dominant at large  $Q^2$  longitudinal amplitude. Using (3.9) and inserting eqs. (2.12,3.1,3.7) into eq. (2.20) and performing the sum over the quark helicities we obtain the following expression

$$M_{(0,0)} = is_{\gamma^*p} \int d^2r dz r^2 \frac{3\pi e\alpha_S f_\rho}{\sqrt{2}N} Q(z\bar{z})^2 K_0(rQ\sqrt{z\bar{z}}) xG(x, Q^2 z\bar{z}) . \quad (3.10)$$

We took into account that the mean electric charge of the quarks inside the  $\rho$  meson ( $|\rho\rangle = \frac{1}{\sqrt{2}N}(|u\bar{u}\rangle - |d\bar{d}\rangle)$ ) is  $e/\sqrt{2}$ . Performing the integral over  $\mathbf{r}$ , using  $\int_0^\infty K_0(r)r^3 dr = 4$ , we obtain the result

$$M_{(0,0)} = is_{\gamma^*p} \int dz \frac{8 \cdot 3\pi^2 e\alpha_S f_\rho}{\sqrt{2}NQ^3} xG(x, Q^2 z\bar{z}) . \quad (3.11)$$

If we neglect the  $z$  dependence of the argument of the gluon density and take the integral over  $z$  in eq. (3.11) we will reproduce the known result for the longitudinal amplitude [1, 2], ( for the asymptotical form of the meson distribution amplitude).

The integral over  $z$  is convergent in eq. (3.11). Therefore the end point regions of  $z$  do not bring the essential contributions, and typical transverse distances of the process are small,  $\sim \langle \frac{1}{Q\sqrt{z\bar{z}}} \rangle$ . This justifies the application of the perturbative QCD in this case.

The situation is more complicated for the case of the transverse amplitude. Inserting eqs. (2.11,3.2,3.7) into (2.20) we obtain

$$M_{(+1,+1)} = is_{\gamma^*p} \int d^2r dz r^3 \frac{3\pi e\alpha_S f_\rho}{8N\sqrt{2}} \langle M \rangle Q(z\bar{z})^{3/2} (z^2 + \bar{z}^2) K_1(rQ\sqrt{z\bar{z}}) xG(x, Q^2 z\bar{z}) . \quad (3.12)$$

The integration over  $\mathbf{r}$  gives, using  $\int_0^\infty K_1(r)r^4 dr = 16$ ,

$$M_{(+1,+1)} = is_{\gamma^*p} \int dz \frac{4 \cdot 3\pi^2 e\alpha_S f_\rho \langle M \rangle}{\sqrt{2}NQ^4(z\bar{z})} (z^2 + \bar{z}^2) xG(x, Q^2 z\bar{z}) . \quad (3.13)$$

If we would neglect the scale dependence of the gluon density  $G(x, \bar{Q}^2 = Q^2 z \bar{z})$  the integration over  $z$  would be logarithmically divergent at  $z \rightarrow 0, 1$  in the above integral. On the other hand in the small  $x$  region the gluon density increases rapidly with  $\bar{Q}^2$ . This increase can be simulated in a first approximation as

$$G(x, Q^2 z \bar{z}) = G(x, Q_0^2) \left[ (Q^2 z \bar{z}) / Q_0^2 \right]^\gamma, \quad (3.14)$$

with a constant anomalous dimension  $\gamma$  of the gluon density. Then the integral over  $z$  in the eq. (3.13) is convergent and we confirm the result of [6]

$$\alpha = \frac{M_{(+1,+1)}}{M_{(0,0)}} = \frac{\langle M \rangle}{Q} \frac{1 + \gamma}{\gamma}. \quad (3.15)$$

### 3.2 The helicity single-flip amplitudes

The calculations of the two independent helicity single-flip amplitudes are quite similar to the ones for the SHCH amplitudes.

Using eqs. (2.11,3.1,3.8) we obtain the following expression for the single spin-flip transition of the transversely polarized initial photon

$$M_{(+1,0)} = i s_{\gamma^* p} \int d^2 r dz r^3 \frac{3\pi e \alpha_S f_\rho}{16N} \sqrt{|t|} Q(z\bar{z})^{3/2} (z - \bar{z})^2 K_1(rQ\sqrt{z\bar{z}}) x G(x, Q^2 z \bar{z}). \quad (3.16)$$

The integration over  $\mathbf{r}$  gives

$$M_{(+1,0)} = i s_{\gamma^* p} \int dz \frac{2 \cdot 3\pi^2 e \alpha_S f_\rho \sqrt{|t|}}{N Q^4(z\bar{z})} (z - \bar{z})^2 x G(x, Q^2 z \bar{z}). \quad (3.17)$$

According to eqs. (2.12,3.2,3.8) the single spin-flip transition of the longitudinally polarized initial photon is

$$M_{(0,+1)} = -i s_{\gamma^* p} \int d^2 r dz r^4 \frac{3\pi e \alpha_S f_\rho}{32N} \sqrt{|t|} \langle M \rangle Q(z\bar{z})^2 (z - \bar{z})^2 K_0(rQ\sqrt{z\bar{z}}) x G(x, Q^2 z \bar{z}). \quad (3.18)$$

After the integration over  $\mathbf{r}$ , using  $\int_0^\infty K_0(r) r^5 dr = 64$ , we have

$$M_{(0,+1)} = -i s_{\gamma^* p} \int dz \frac{4 \cdot 3\pi^2 e \alpha_S f_\rho \sqrt{|t|} \langle M \rangle}{N Q^5(z\bar{z})} (z - \bar{z})^2 x G(x, Q^2 z \bar{z}). \quad (3.19)$$

We calculate the integrals over  $z$  in (3.11,3.17,3.19) using the assumption of the constant gluon anomalous dimension (3.14). The following results for the

ratios of the single-flip amplitudes to the longitudinal non-flip amplitude can be derived

$$\beta = \frac{M_{(+1,0)}}{M_{(0,0)}} = \frac{\sqrt{|t|}}{Q} \frac{1}{\sqrt{2}\gamma}, \quad (3.20)$$

$$\delta = \frac{M_{(0,+1)}}{M_{(0,0)}} = -\frac{\langle M \rangle \sqrt{|t|} \sqrt{2}}{Q^2 \gamma}. \quad (3.21)$$

### 3.3 The helicity double-flip amplitude

The helicity double spin-flip amplitude  $M_{(+1,-1)}$  can be written as the sum of the two parts

$$M_{(+1,-1)} = M_{(+1,-1)}^0 + M_{(+1,-1)}^1, \quad (3.22)$$

where  $M_{(+1,-1)}^0$  corresponds to the first ( $\sim r^2$ ) term of eq. (3.6), and  $M_{(+1,-1)}^1$  corresponds to the second ( $\sim r^4$ ) term of eq. (3.6). Keep in mind that the expansion in  $r$  is equivalent to the expansion of the amplitude in  $\frac{1}{Q\sqrt{z\bar{z}}}$ .

The integrations over the transverse momenta of the  $t$ - channel gluons, see eq. (2.1), and over the  $q\bar{q}$  longitudinal momentum fraction  $z$  are different for  $M_{(+1,-1)}^0$  and  $M_{(+1,-1)}^1$ .

Let us discuss  $M_{(+1,-1)}^0$  first. Using (2.11,3.2,3.6) we obtain

$$M_{(+1,-1)}^0 = -is_{\gamma^*p} \int \frac{d^2r dz r^3}{2\pi} \frac{3ef_\rho}{8N\sqrt{2}} \langle M \rangle Q(z\bar{z})^{5/2} K_1(rQ\sqrt{z\bar{z}}) I_0, \quad (3.23)$$

where the factor  $I_0$  represents the integration over the transverse momenta of the  $t$ - channel gluons

$$I_0 = \frac{\alpha_S \delta^{ab}}{q^2} \int \frac{d^2k}{\mathbf{k}^2(\mathbf{k}-\mathbf{q})^2} \left[ 2(\mathbf{k}\mathbf{q})^2 - k^2 q^2 - (\mathbf{k}\mathbf{q})q^2 \right] J_p^{ab}(\mathbf{k}, \mathbf{q}). \quad (3.24)$$

Performing the integration over  $\mathbf{r}$  in (3.23) we have

$$M_{(+1,-1)}^0 = is_{\gamma^*p} \int dz \frac{3 \cdot 2ef_\rho \langle M \rangle}{\sqrt{2}NQ^4} I_0. \quad (3.25)$$

The above equation should be compared with the corresponding eq. (3.13) for the helicity non-flip transverse amplitude  $M_{(+1,+1)}$ . In contrast to (3.13), the integration over  $z$  in (3.25) is not singular in the end point regions. This difference is related with the helicity flip. Looking at the expressions (2.11) and (3.2) for the photon and vector meson wave functions it is seen that in the flip case the sum over the helicities of the intermediate  $q\bar{q}$  pair is proportional to the additional factor  $z\bar{z}$ . For the non-flip case the sum over  $q\bar{q}$  helicities gives the factor  $\sim (z^2 + \bar{z}^2)$ , which does not vanish at  $z = 0, 1$ .

Now let us discuss the integration over the  $t$ - channel gluon momenta (3.24). This integral is convergent on the upper limit. Therefore  $M_{(+1,-1)}^0$  can not be expressed through the gluon density using eq. (3.9). Moreover, since the main part of the integral (3.24) originates from the region of small transverse momenta of the  $t$ - channel gluons,  $k \sim q$ , we are dealing here with soft physics.

We shall assume that this soft  $t$ - channel exchange can be described in the frame of the two-gluon exchange model, see eq. (2.1), with some simple functional form for the nonperturbative impact factor of the proton  $J_p$ . The model of such type was used successfully in [8] to describe the total  $pp$  cross sections. Following [8], the proton impact factor is

$$J_p(\mathbf{k}, \mathbf{q}) = \bar{\alpha}_S \delta^{ab} \left[ \frac{A^2}{A^2 + q^2/4} - \frac{A^2}{A^2 + (\mathbf{k} - \mathbf{q}/2)^2} \right]. \quad (3.26)$$

The first term in (3.26) describes the contribution of the diagrams where both  $t$ -channel gluons are coupled to the same quark inside the proton. This contribution is similar to the electromagnetic form factor, therefore it is natural to adopt that

$$A = \frac{m_\rho}{2}. \quad (3.27)$$

The strength of the nonperturbative coupling  $\bar{\alpha}_S = \frac{\bar{g}^2}{4\pi}$  is a free parameter. To choose its value, let us calculate the value of the total  $pp$  ( $pp$ ) cross section which is related through the optical theorem to the forward amplitude

$$\sigma_{tot}^{pp} = \frac{Im(M^{pp}(t=0))}{s_{pp}}. \quad (3.28)$$

Using eqs. (2.1) and (3.24) it can be shown that

$$Im(M^{pp}(t=0)) = \frac{8\pi\bar{\alpha}_S^2 s_{pp}}{A^2}. \quad (3.29)$$

Describing the  $pp$  total cross section with this simple model leads to an parameter  $\bar{\alpha}_S$  increasing with energy. We understand that one can do better by replacing the two gluon exchange by a pomeron exchange. For our aim of a simple estimate we shall adopt the relations (3.28,3.29).

Performing the integral (3.24) (using the expression (3.26) for the proton impact factor) we find

$$I_0 = 8\pi\bar{\alpha}_S^2 \frac{A^2}{A^2 + q^2/4} \left[ \left(1 + \frac{4A^2}{q^2}\right) \log\left(1 + \frac{q^2}{4A^2}\right) - 1 \right]. \quad (3.30)$$

At small  $q^2$ ,  $q^2 \ll A^2$ ,

$$I_0 = \pi \frac{q^2}{A^2} = \pi \frac{4|t|}{m_\rho^2}. \quad (3.31)$$

Our final result for  $M_{(+1,-1)}^0$  is

$$M_{(+1,-1)}^0 = -is_{\gamma^*p} \frac{3 \cdot 8\pi e \bar{\alpha}_S^2 f_\rho |t| \langle M \rangle}{\sqrt{2} N Q^4 m_\rho^2} . \quad (3.32)$$

Performing the integral over  $z$  in (3.11) we find

$$\eta^0 = \frac{M_{(+1,-1)}^0}{M_{(0,0)}} = -\frac{\bar{\alpha}_S^2 |t| \langle M \rangle}{\pi \alpha_S Q m_{\rho^0}^2} \frac{1}{\left[ 4^\gamma \frac{\Gamma^2(\gamma+1)}{\Gamma(2\gamma+2)} x G(x, Q^2/4) \right]} . \quad (3.33)$$

Now let us discuss  $M_{(+1,-1)}^1$ . Using (2.11,3.2) and the second term of (3.6) we can write

$$M_{(+1,-1)}^1 = is_{\gamma^*p} \int \frac{d^2 r d z r^5}{2\pi} \frac{3e f_\rho}{8 \cdot 12N\sqrt{2}} \langle M \rangle Q(z\bar{z})^{5/2} K_1(rQ\sqrt{z\bar{z}}) I_1 . \quad (3.34)$$

In this case the integration over the transverse momenta of the  $t$ - channel gluons is different as compared to  $M_{(+1,-1)}^0$  case

$$I_1 = \frac{\alpha_S \delta^{ab}}{q^2} \int \frac{d^2 k}{\mathbf{k}^2 (\mathbf{k}-\mathbf{q})^2} [2k^2 (\mathbf{k}\mathbf{q})^2 - k^4 q^2 - 2(\mathbf{k}\mathbf{q})^3 + 3(\mathbf{k}\mathbf{q})^2 q^2 (z^2 + \bar{z}^2) - 2(\mathbf{k}\mathbf{q}) q^4 (z^3 + \bar{z}^3)] J_p^{ab}(\mathbf{k}, \mathbf{q}) . \quad (3.35)$$

Extracting the main, logarithmic, part from the above integral we find

$$I_1 = \frac{\alpha_S \delta^{ab}}{q^2} \frac{q^4 [3(z^2 + \bar{z}^2) - 1]}{2} \int \frac{d^2 k}{k^2} J_p^{ab}(\mathbf{k}, \mathbf{q}) . \quad (3.36)$$

Therefore we can relate  $M_{(+1,-1)}^1$ , using the relation (3.9), to the gluon density.

Performing the integral over  $\mathbf{r}$ , using  $\int_0^\infty K_1(r) r^6 dr = 384$ , we have

$$M_{(+1,-1)}^1 = is_{\gamma^*p} \int dz \frac{4 \cdot 3\pi^2 e \alpha_S f_\rho |t| \langle M \rangle [3(z^2 + \bar{z}^2) - 1]}{\sqrt{2} N Q^6 (z\bar{z})} x G(x, Q^2 z\bar{z}) . \quad (3.37)$$

Note that the  $z$  integration is different for  $M_{(+1,-1)}^1$  and  $M_{(+1,-1)}^0$ . The additional factor  $(z\bar{z})Q^2$  appears in the denominator of (3.37) by virtue of two additional powers of  $r$  in the numerator of eq. (3.34). As a result the  $z$  integration for  $M_{(+1,-1)}^1$  becomes similar to ones for  $M_{(+1,+1)}^1$ ,  $M_{(0,+1)}^1$  and  $M_{(+1,0)}^1$ . Calculating the integrals over  $z$  ( using (3.14) in eqs. (3.11) and (3.37)) we find

$$\eta^1 = \frac{M_{(+1,-1)}^1}{M_{(0,0)}} = \frac{|t| \langle M \rangle}{Q^3} \frac{2(\gamma + 2)}{\gamma} . \quad (3.38)$$

$$\eta = \frac{M_{(+1,-1)}}{M_{(0,0)}} = \eta^0 + \eta^1 . \quad (3.39)$$

It should be noted that although  $M_{(+1,-1)}^1$  is suppressed as compared to  $M_{(+1,-1)}^0$  by the factor  $\sim \frac{m_p^2}{Q^2}$ , the perturbative part  $M_{(+1,-1)}^1$  can be dominant at high energies since it has a steeper energy dependence ( $\sim xG(x, \bar{Q}^2)$ ) as compared to the soft part  $M_{(+1,-1)}^0$ .  $M_{(+1,-1)}^1$  contains also an enhancement factor  $1/\gamma$  which originates from the more singular  $z$  integration. We will show below that  $|M_{(+1,-1)}^1| > M_{(+1,-1)}^0$  at typical for HERA kinematical conditions.

### 3.4 Additional remarks on the amplitudes

We have calculated above the helicity amplitudes for the diffractive vector meson electroproduction at  $s_{\gamma^*} \gg Q^2 \gg \Lambda_{QCD}^2$  (large  $Q^2$ , small  $x$ ). Let us discuss now the assumptions used and the physical issues related with the helicity amplitudes.

We assume that perturbative QCD can be applied to describe the process (1.1) at large  $Q^2$ . Although the QCD factorization theorem has been proven only for the leading scalar amplitude  $M_{(0,0)}$ , we extend here the perturbative approach, following [6] (where transverse helicity non-flip amplitude  $M_{(+1,+1)}$  was considered), to describe helicity-flip amplitudes.

The main features of the perturbative QCD approach are the following. The virtual photon splits into the massless  $q\bar{q}$  pair having the total helicity 0. As a result, the helicity of the photon coincides with the  $z$  projection of the quark angular momentum. The helicity states of the quarks do not change during the interaction with the proton. Therefore the helicity of the meson is equal to the projection of the angular momentum of the outgoing  $q\bar{q}$  pair onto the direction of the meson momentum. The helicity flip comes from the change of the projection of the  $q\bar{q}$  angular momentum during the interaction. This change originates in our approach from the non-forward kinematics ( $t \neq 0$ ). Therefore there is no suppression of the helicity-flip amplitudes with energy, they are driven by the leading gluon (pomeron) exchange.  $M_{(+1,0)}$ ,  $M_{(0,+1)}$  and  $M_{(+1,-1)}^1$  have the energy dependences which are similar to the energy dependences of the helicity non-flip amplitudes. They are proportional (at small  $t$ ) to  $xG(x, \bar{Q}^2)$ .

According to our calculations the helicity single-flip amplitudes are proportional to  $\sqrt{|t|}$ , the double spin-flip one is  $\sim |t|$ . These factors come from the expansion of the  $\gamma^* \rightarrow \rho$  transition impact factors describing these amplitudes. Since the typical transverse separation between the quarks is small,  $\sim \frac{1}{Q\sqrt{\gamma}}$ , this expansion is expected to be valid up to the rather large values of  $|t|$ ,  $|t| \leq Q^2\gamma$ . On the other hand, equation (3.9) relating the proton impact factor with the gluon density and its implications eqs. (3.11,3.13,3.17,3.19,3.37) are valid only at very small (vanishing) momentum transfer. The coupling of the two gluon system with the proton decreases with the growth of the momentum transfer. We are not able to describe the  $t$  behaviour of this coupling from the first principles. But we see that at small  $t$  the structure of the integration over the transverse momenta of the  $t$ -channel gluons is similar for both the helicity non-flip and the helicity-

flip amplitudes (with the exception of the nonperturbative part of the double-flip amplitude  $M_{(+1,-1)}^0$ ). Therefore it is natural to expect an universal  $t$  behaviour for all helicity amplitudes coming from the coupling of the  $t$ - channel gluons with the proton. And we believe that this universal  $t$  dependence is canceled in the ratios (3.15,3.20,3.21,3.38) and, therefore, these results are valid in the broad  $t$  region extending up to  $|t| \leq Q^2\gamma$ . This assumption is based on the observation that in all cases we deal with the scattering of a  $q\bar{q}$  pair the transverse size of which is much smaller than the size of the proton.

We will parametrize this universal  $t$  behaviour as  $\sim e^{-b|t|/2}$  for the amplitudes ( $\sim e^{-b|t|}$  for the cross sections) with the slope  $b$  which is of the order of the square of the proton size. Due to the large value of this slope,  $b = 5 \dots 6 \text{ GeV}^{-2}$  according to the HERA data [10], the helicity flip amplitudes are peaked as well as the helicity non-flip ones at small  $t$  ( $|t| \leq t_0 \sim 1/b$ ) in spite of the fact that they contain the factors  $\sqrt{|t|}$  for single-flip and  $|t|$  for double-flip.

According to eqs. (3.15,3.20,3.21,3.39) for the typical  $|t|$  values,  $|t| \approx 1/b$ , assuming that  $\langle M \rangle \sim m_\rho$ , we have

$$1 > \alpha > \beta > |\delta| > |\eta|. \quad (3.40)$$

Therefore the largest among the amplitudes violating SCHC, the helicity single-flip amplitude  $M_{(+1,0)}$ , is smaller than the transverse non-flip amplitude  $M_{(+1,+1)}$ .

Let us estimate the ratios in eq. (3.40) for the kinematical conditions relevant for the HERA experiments. We choose  $Q^2 = 10 \text{ GeV}^2$ ,  $x = 10^{-3}$  which corresponds to  $W = \sqrt{s_{\gamma^*p}} = 100 \text{ GeV}$ . We will give the estimates for the simplest situation when the momentum transfer is not restricted during the helicity analysis, i.e. the experimental sample is not divided into the  $t$  bins. In this case we can substitute the factors  $\sqrt{|t|}$  and  $|t|$  by their mean values

$$|t| \rightarrow \langle |t| \rangle = \frac{\int dt |t| e^{-b|t|}}{\int dt e^{-b|t|}} = \frac{1}{b},$$

$$\sqrt{|t|} \rightarrow \langle \sqrt{|t|} \rangle = \frac{\int dt \sqrt{|t|} e^{-b|t|}}{\int dt e^{-b|t|}} = \frac{\sqrt{\pi}}{2\sqrt{b}}.$$

We shall use  $b = 6 \text{ GeV}^{-2}$ ,  $\langle M \rangle = m_\rho$  in our estimates. For the effective gluon anomalous dimension at these values of  $Q^2$  and  $x$  we use two values  $\gamma = 0.7$  and  $\gamma = 0.5$ . These values of  $\gamma$  are close to the ones presented in Fig. 3 of [6].

The results of our estimates for  $\gamma = 0.7$  ( $\gamma = 0.5$ ) are the following:

$$\alpha = 0.59(0.73), \quad (3.41)$$

$$\beta = 0.12(0.16), \quad (3.42)$$

$$\delta = 0.056(0.079) , \quad (3.43)$$

$$\eta^1 = 0.031(0.041) , \quad (3.44)$$

$$\eta^0 = -0.016(-0.015) . \quad (3.45)$$

Estimating  $\eta^0$  in eq. (3.33) we use  $\alpha_S = 0.3$ ,  $\bar{\alpha}_S = 0.87$ . This value of  $\bar{\alpha}_S = 0.87$  used in eq. (3.29) reproduces the correct value of the total  $pp$  cross section at  $\sqrt{s_{pp}} = 100$  GeV. Combining eqs. (3.44,3.45) we obtain a very small number for the ratio of  $M_{(+1,-1)}$  to  $M_{(0,0)}$

$$\eta = 0.015(0.026) . \quad (3.46)$$

$M_{(+1,0)}$  is approximately 7...8 times smaller than  $M_{(0,0)}$ . The other helicity-flip amplitudes are considerably more suppressed.

We have calculated above the dominant at high energy imaginary parts of the helicity amplitudes. We will give here only an argument why the real parts can be not too important in the polarization phenomena. The real parts of the amplitudes are related to the imaginary ones through the dispersion relations. Since the imaginary parts of all helicity amplitudes have according to our consideration similar energy behaviour we expect that the helicity amplitudes have phases which are close to each other. Therefore observable effects related to the differences of these phases will be additionally suppressed.

## 4 Vector meson decay angular distribution at HERA kinematics

The polarization of the  $\rho$  meson is experimentally accessible through the measurement of the angular distributions of the decay products. For the relations between the helicity amplitudes and the angular distributions we shall use the results and standard conventions of [9], see also [10]. The definition of the three independent angles involves three planes: 1) the electron scattering plane, 2) the vector meson production plane (which contains the photon and meson momentum vectors), 3) the meson decay plane. The orientation of the meson decay plane is described by the polar and the azimuthal angles ( $\theta$ ) and ( $\phi$ ). The third angle ( $\Phi$ ) is the angle between the electron scattering and the meson production planes.

The polarization parameter of the virtual photon density matrix,

$$\epsilon = \frac{1 - y}{1 - y + \frac{y^2}{2}} ,$$

is close to 1 at HERA kinematics. For  $W = 100$  GeV ( $y = \frac{s_{\gamma^*p}}{s_{ep}} = 1/9$ ) its value is  $\epsilon = 0.993$ .

The decay distribution  $W(\cos\theta, \phi, \Phi)$  contains the parameters, the matrix elements  $r_{ik}^\alpha$ , which are known bilinear combinations of the helicity amplitudes [9]. These matrix elements can be determined experimentally by the analysing moments of the observed decay angular distribution.

According to our estimate the helicity–flip amplitudes are substantially smaller than the helicity non–flip ones. Nevertheless, as we shall show, the largest among them  $M_{(+1,0)}$  leads to a sizable effect.

The ratio of the longitudinal to the transverse cross section is expressed through the ratios of the helicity amplitudes as follows

$$R = \sigma_L/\sigma_T = N_L/N_T , \quad (4.1)$$

where

$$N_T = \alpha^2 + \beta^2 + \eta^2 ; N_L = 1 + 2\delta^2 . \quad (4.2)$$

The matrix elements entering the decay angular distribution have the following expressions in terms of the ratios of the helicity amplitudes ( assuming that the helicity amplitudes are purely imaginary):

$$\begin{aligned} r_{00}^{04} &= B(\epsilon + \beta^2) , \quad Re(r_{10}^{04}) = B(2\epsilon\delta + \beta\alpha - \beta\eta)/2 , \quad r_{1-1}^{04} = B(\alpha\eta - \epsilon\delta^2) ; \\ r_{11}^1 &= B\alpha\eta , \quad Re(r_{10}^1) = B\beta(\eta - \alpha)/2 , \quad r_{00}^1 = -B\beta^2 , \quad r_{1-1}^1 = B(\alpha^2 + \eta^2)/2 ; \\ Im(r_{10}^2) &= B\beta(\alpha + \eta)/2 , \quad Im(r_{1-1}^2) = B(\eta^2 - \alpha^2)/2 ; \\ r_{11}^5 &= \frac{B}{\sqrt{2}}\delta(\alpha - \eta) , \quad Re(r_{10}^5) = \frac{B}{\sqrt{2}}(2\beta\delta + \alpha - \eta)/2 , \\ r_{00}^5 &= \frac{B}{\sqrt{2}}2\beta , \quad r_{1-1}^5 = \frac{B}{\sqrt{2}}\delta(\eta - \alpha) ; \\ Im(r_{10}^6) &= -\frac{B}{\sqrt{2}}(\alpha + \eta)/2 , \quad Im(r_{1-1}^6) = \frac{B}{\sqrt{2}}\delta(\alpha + \eta)/2 . \end{aligned} \quad (4.3)$$

We introduce for short the notation

$$B = 1/(N_T + \epsilon N_L) .$$

Substituting our estimates for the ratios of the helicity amplitudes derived in the previous section we have

$$\begin{aligned} r_{00}^{04} &= 0.74(0.65) , \quad Re(r_{10}^{04}) = -0.015(-0.014) , \\ r_{1-1}^{04} &= 0.0042(0.0082) ; \\ r_{11}^1 &= 0.0065(0.012) , \quad Re(r_{10}^1) = -0.025(-0.036) , \\ r_{00}^1 &= -0.011(-0.016) , \quad r_{1-1}^1 = 0.13(0.17) ; \\ Im(r_{10}^2) &= 0.027(0.039) , \quad Im(r_{1-1}^2) = -0.13(-0.17) ; \\ r_{11}^5 &= -0.017(-0.025) , \quad Re(r_{10}^5) = 0.15(0.15) , \\ r_{00}^5 &= 0.12(0.14) , \quad r_{1-1}^5 = 0.017(0.025) ; \\ Im(r_{10}^6) &= -0.16(-0.17) , \quad Im(r_{1-1}^6) = -0.0088(-0.013) . \end{aligned} \quad (4.4)$$

The matrix elements which have nonzero values in the case of SCHC ( if  $\beta, \delta, \eta = 0$ ) are  $r_{00}^{04}, r_{1-1}^1, Im(r_{1-1}^2), Re(r_{10}^5), Im(r_{10}^6)$ . The relations which would hold between them in the the case of SHCH are only slightly violated since  $\delta, \eta$  are very small:

$$\frac{1}{2}(1 - r_{00}^{04}) - r_{1-1}^1 = B\epsilon\delta^2 \approx 0.002(0.004) , \quad (4.5)$$

$$r_{1-1}^1 + Im(r_{1-1}^2) = B\eta^2 \approx 2(4) \cdot 10^{-4} , \quad (4.6)$$

$$Re(r_{10}^5) + Im(r_{10}^6) = \frac{B}{\sqrt{2}}(\beta\delta - \eta) \approx -0.011(-0.017) . \quad (4.7)$$

Therefore these matrix elements should be measured with high precision to see the violation of the SCHC. Note also that the value of  $R$  calculated using the SCHC relation  $R = \frac{1}{\epsilon} \frac{r_{00}^{04}}{1 - r_{00}^{04}}$  exceeds the one calculated using eq. (4.1) by 3...4%.

It is natural that the effects of the violation of SCHC manifest themselves more transparent in other matrix elements which would be zero in the case of SCHC. The largest among them is  $r_{00}^5 \approx 0.12(0.14)$ . This matrix element has a very clear meaning. It is related to the interference between the two amplitudes describing the two possibilities to produce the longitudinally polarized vector meson. These are the dominant at large  $Q^2$  amplitude  $M_{(0,0)}$  and the helicity single-flip amplitude  $M_{(+1,0)}$ .

Since this interference exists on the level of the production of the vector meson and does not depend on the kinematical variables describing the meson decay  $(\theta, \phi)$ , the correspondig effect survives without the loose of the analysing power after the integration of the angular distribution over the angles  $(\theta, \phi)$ . The resulting distribution over the relative angle between the electron scattering and the meson production planes is

$$W(\Phi) = \left[ 1 + \sqrt{2\epsilon(1 + \epsilon)} \cos \Phi (r_{00}^5 + 2r_{11}^5) \right] . \quad (4.8)$$

We skipped in the above equation the term  $\sim \cos 2\Phi$  which is proportional to the small matrix elements  $r_{00}^1, r_{11}^1$ . Substituting our estimates for  $r_{00}^5, r_{11}^5$  we obtain a substantial deviation of the  $\Phi$  distribution from the flat one

$$W(\Phi) = [1 + 0.18(0.19) \cos \Phi] . \quad (4.9)$$

The other matrix element that could be potentially large is  $Re(r_{10}^{04})$ . Since it contains the term  $\sim 2\epsilon\delta$  which is linear in  $\delta$ , this matrix element has a large sensitivity to the second single-flip amplitude  $M_{(0,+1)}$ . But its value turns out to be small due to a large cancelation between the terms  $2\epsilon\delta$  and  $\beta\alpha$ . This cancelation is related with the opposite signs of  $M_{(+1,0)}$  and  $M_{(0,+1)}$ . Note that in

the case of positive sign of  $\delta$  this matrix element would be estimated as  $Re(r_{10}^{04}) = 0.066(0.086)$ .

The matrix elements (4.3) parametrize the angular distribution for the unpolarized initial positron. A few additional matrix elements can be measured if the initial positron is longitudinally polarized. But these matrix elements do not exhibit an advantage in sensitivity to the violation of SCHC as compared to the unpolarized ones.

## 5 Conclusions

We have considered in perturbative QCD the polarization effects in diffractive  $\rho$  meson electroproduction. We assume that perturbative QCD with the account of the important effect of the gluon scale behaviour is applicable to all helicity amplitudes. We estimate the helicity amplitudes in the approximation of the constant effective anomalous dimension of the gluon. Our results are summarized in the eqs. (3.15,3.20,3.21, 3.38,3.39).

The equations derived here can be applied also to other light vector mesons  $\omega, \phi$  with corresponding changes in the coupling constants and parameters  $\langle M \rangle$  describing the meson wave functions.

The perturbative QCD leads to a very definite qualitative picture for the violation of SCHC at high  $Q^2$ . The largest among the helicity-flip amplitudes is  $M_{(+1,0)}$ . At typical values of  $t$  this amplitude is smaller than the transverse helicity

non-flip amplitude  $M_{(+1,+1)}$ ,  $M_{(+1,0)} \sim \frac{\sqrt{|t|}}{m_\rho} M_{(+1,+1)}$ . The other independent single-flip amplitude  $M_{(0,+1)}$  is suppressed compared to  $M_{(+1,0)}$  by the factor  $\frac{2m_\rho}{Q}$ .

The double-flip amplitude  $M_{(+1,-1)}$  consists of the two parts. The perturbative contribution  $M_{(+1,-1)}^1$  is larger at small  $x$  than the nonperturbative one. But it

is suppressed compared to  $M_{(0,+1)}$  by the additional factor  $\sim \frac{\sqrt{|t|}}{Q}$ . Therefore at very high  $Q^2$  we have  $M_{(+1,0)} \gg |M_{(0,+1)}| \gg M_{(+1,-1)}$ . For the kinematical region typical for the HERA experiments we find that  $|M_{(0,+1)}|$  is about two times smaller than  $M_{(+1,0)}$ , and  $M_{(+1,-1)}$  is about 10 times smaller than  $M_{(+1,0)}$ .

This hierarchy between the helicity amplitudes leads to the peculiar predictions for the parameters of the meson decay angular distribution. We predict that the only one parameter among that vanishing in the case of SCHC,  $r_{00}^5$ , deviates substantially from zero. The relations between the parameters, which are nonzero in the case of SCHC, are only slightly violated. It would be very interesting to confront these predictions with the data.

Let us note that the parameter  $r_{00}^5 \sim M_{(+1,0)}$  is sensitive to the meson wave function. Since single spin-flip transitions are proportional to the factor vanishing if  $z = \bar{z}$ , see (2.22,3.8), the helicity amplitude  $M_{(+1,0)}$  would be zero if the wave

function of the meson is a non-relativistic one  $\sim \delta(z - 1/2)$ . It is a consequence of perturbative QCD that the fluctuation of the light meson into the pair of current quarks is described by the broad wave function. On the other hand in the nonperturbative models a meson is often considered as a weakly bounded system of constituent quarks having a mass  $m_q \sim m_M/2$  and, therefore, described by a function close to  $\delta(z - 1/2)$ . Therefore the large value of  $r_{00}^5$  is a characteristic prediction of perturbative QCD.

Note that our work is only a first step in the investigation of the helicity–flip amplitudes. More detailed numerical calculations can be done using the equations derived in this paper. It is possible to do this without the approximation of the constant gluon anomalous dimension and to investigate in more details the dependence of the amplitudes on the meson wave functions. Also the real parts of the helicity amplitudes have to be considered.

**Acknowledgements.** In the final stage of this work we had encouraging discussions with P. Marage, K. Piotrkowski and J. Crittenden. D.Yu. I. would like to acknowledge the warm hospitality extended to him at University of Leipzig.

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