Supersymmetric Neutrino Masses and Mixing with R-parity Violation

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Abstract

In the context of the minimal supersymmetric standard model, nonzero neutrino masses and mixing can be generated through renormalizable lepton number (and thus R-parity) violating operators. It is examined whether neutrino mass matrices from tree and one-loop contributions can account for two mass-squared differences and mixing angles that explain current experimental data. By accommodating, in particular, the solar and atmospheric neutrino data, we find interesting restrictions not only on the free parameters of the theory, such as lepton number violating couplings and soft-parameters, but also on the oscillation parameters of atmospheric neutrinos.

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I. INTRODUCTION

There exists some evidence for nonzero neutrino masses and mixing. The observations of the solar neutrino deficit have been indicating neutrino oscillation [1,2]. The resonant conversion of ν_e inside the Sun [3] would provide the most favorable explanation for the solar neutrino data. Several neutrino experiments have also observed deficit in the atmospheric neutrino flux [4]. The evidence for atmospheric neutrino oscillations was recently presented by the Super-Kamiokande group [5], which favors ν_μ–ν_τ (or ν_s) oscillation. The ν_μ–ν_e oscillation interpretation can be ruled out by CHOOZ experiment [6]. The last one is laboratory evidence for the neutrino oscillation $\bar{\nu}_\mu \to \bar{\nu}_e$ coming from the LSND experiment [7]. This evidence is recently being challenged by KARMEN experiment [8], and is to be checked in the near future. The above neutrino data are known to require three distinct mass-squared differences and mixing angles;

$$\left.\begin{array}{rcl} \Delta m^2_{\rm sol} & \simeq & (4-10)\times 10^{-6}\ {\rm eV}^2 \\ \sin^2 2\theta_{\rm sol} & \simeq & (0.12-1.2)\times 10^{-2} \end{array}\right\} [9] \tag{1}$$

$$\left.\begin{array}{rcl} \Delta m^2_{\rm atm} & \simeq & (0.5-6)\times 10^{-3}\ {\rm eV}^2 \\ \sin^2 2\theta_{\rm atm} & \simeq & (0.82-1) \end{array}\right\} [5,10] \tag{2}$$

$$\left.\begin{array}{rcl} \Delta m^2_{\rm LSND} & \simeq & (0.3-2.2)\ {\rm eV}^2 \\ \sin^2 2\theta_{\rm LSND} & \simeq & (0.1-4)\times 10^{-2} \end{array}\right\} [7] \tag{3}$$

With ordinary three neutrinos, any two of the mass-squared differences in the above equations can be obtained: that is, those corresponding to (i) solar and atmospheric (S+A), (ii) solar and LSND (S+L), or (iii) atmospheric and LSND (A+L) neutrino data. For (S+A), the LSND result has to be disregarded. In the case of (S+L) or (A+L), the presence of a sterile neutrino is necessary for the explanation of the atmospheric or solar neutrino experiment, respectively.

One of the desirable features of the supersymmetric extension of the standard model would be the generation of small neutrino masses within its context, as the supersymmetric standard model with the minimal particle content (MSSM) allows for the lepton (L) and baryon number (B) violating operators. In order to ensure the longevity of a proton, one usually assumes the conservation of R-parity, forbidding both (renormalizable) B and L violating operators. As a consequence, the lightest supersymmetric particle (LSP) is stable and thus cold dark matter of the universe may consist of neutral LSP's. However, there is no obvious theoretical reason why R-parity needs to be conserved, or why both B and L conservation have to be imposed. L-violation would be present in the MSSM and it may be the origin of nonzero neutrino masses and mixing that explain current experimental data, while proton stability is ensured by B conservation alone.

The L-violating operators in the MSSM are

$$\mu_i L_i H_2\,, \quad \lambda'_{ijk} L_i Q_j D^c_k\,, \quad \text{and} \quad \lambda_{ijk} L_i L_j E^c_k\,. \tag{4}$$

As is well-known [11], ordinary neutrinos can obtain nonzero masses in tree-level via nonzero vacuum expectation values (VEVs) of sneutrinos, as well as at the one-loop level through squark or slepton exchanges. Recently, there have been many works studying neutrino phenomenology in the context of R-parity breaking supersymmetric models [12]. Typically, the tree mass is much larger than the loop mass. The key observation we wish to emphasize is that it is, however, possible to find the soft supersymmetry breaking parameter space for which the tree mass is rather close to the loop mass, and the solutions to the actual neutrino problems can be provided. This was first recognized by Hempfling in Ref. [12] where a scatter plot study of the supersymmetric grand unification model allowing only bilinear operators shows that the solar and atmospheric neutrino data can be accounted for.

In this paper, assuming the presence of trilinear L-violating terms, we will examine how the soft parameter space viable for neutrino physics is constrained depending on the choice of the L-violating couplings and $\tan\beta$. Furthermore, we will find that there are certain correlated patterns among soft parameters and predicted masses and mixing for given L-violating couplings and $\tan\beta$.

II. PATTERNS OF NEUTRINO MASS MATRICES

Before investigating neutrino masses arising in the L-violating MSSM, let us first discuss the patterns of mass matrices required for each case (i), (ii) or (iii).

(i) (S+A)

(a) hierarchical neutrino structure: The R-parity violation may generate at least two nonzero mass eigenvalues satisfying $0 < m_{\nu_2} < m_{\nu_3}$, and thus $\Delta m^2_{\rm sol} \simeq m^2_{\nu_2}$ and $\Delta m^2_{\rm atm} \simeq m^2_{\nu_3}$ required for the explanation of the solar and atmospheric neutrino data. Then, we need a mass matrix in the ν_μ, ν_τ directions which yields a large mixing, $\sin^2 2\theta_{\mu\tau} \sim 1$, and a *rather small* ratio between two eigenvalues,

$$\chi \equiv m_{\nu_3}/m_{\nu_2} = (7-40) \tag{5}$$

as can be obtained from Eqs.(1) and (2). In this case, the components along the ν_e direction should be able to explain the mixing of ν_e with $\nu_{\mu,\tau}$ reproducing $\sin^2 2\theta_{sol}$ given in Eq. (1).
(b) degenerate neutrino structure: Another way to accommodate the solar and atmospheric neutrino data is to have almost degenerate three neutrinos. This could be achieved in our scheme if the amount of L-violation along the e, μ and τ directions are the same. Then, the solar neutrino data could be explained by a large mixing resonant conversion or vacuum oscillation effect [9] which will be disregarded in this paper.

(ii) (S+L)

For the explanation of the atmospheric neutrino oscillation in addition to the solar neutrino and LSND data, a sterile neutrino almost maximally mixed with ν_μ has to be invoked [14]. Let us denote the 4×4 neutrino mass matrix by m_{ij} in $(\nu_e, \nu_\mu, \nu_\tau, \nu_s)$ basis,

$$\begin{pmatrix} m_{ee} & m_{e\mu} & m_{e\tau} & m_{es} \\ m_{e\mu} & m_{\mu\mu} & m_{\mu\tau} & m_{\mu s} \\ m_{e\tau} & m_{\mu\tau} & m_{\tau\tau} & m_{\tau s} \\ m_{es} & m_{\mu s} & m_{\tau s} & m_{ss} \end{pmatrix} \tag{6}$$

where the components m_{is} come from a certain origin beyond the MSSM. A natural way to obtain a large mixing required by the atmospheric neutrino experiment is to have the almost Dirac structure with $m_{\mu s} \gg m_{es}, m_{\tau s}$. There exist two possibilities to realize this (S+L).
(a) $m_{\nu_e}, m_{\nu_\tau} \gg m_{\nu_\mu}, m_{\nu_s}$: This is the case where the largest mass scale for $\Delta m^2_{\rm LSND}$ is determined by the (e,τ) block of m_{ij} which is larger than the (μ, s) block. As the ν_e–ν_μ oscillation explains the LSND experiment, the solar neutrino data is explained by the ν_e–ν_τ oscillation. Therefore, it is required that ν_e and ν_τ are almost degenerate: $m_{\nu_e} \simeq m_{\nu_\tau} \sim 1$ eV, and $m_{\nu_\tau} - m_{\nu_e} \sim 10^{-6}(10^{-10})$ for the MSW (vacuum) solution [9]. This extreme degeneracy is, however, hard to achieve in our scheme as will become clear in Section IV.
(b) $m_{\nu_\mu}, m_{\nu_s} \gg m_{\nu_e}, m_{\nu_\tau}$: This possibility is to have $\nu_{e,\tau}$ lighter than $\nu_{\mu,s}$, for which the solar neutrino data can be explained by small mixing in Eq. (1). Then, the mass-squared differences for the atmospheric and LSND data are determined by

$$\Delta m^2_{\rm LSND} \simeq m^2_{\mu s}\,, \quad \Delta m^2_{\rm atm} \simeq 2 m_{\mu s}(m_{\mu\mu} + m_{ss})\,. \tag{7}$$

The mass-squared difference $\Delta m^2_{\rm atm}$ for the ν_μ–ν_s oscillation can be different from that for the ν_μ–ν_τ oscillation due to matter effects in the Earth. Recent analysis [13] based on the Super-Kamiokande data shows that $\Delta m^2_{\rm atm} = (0.2-1)\times 10^{-2}$ eV^2 for the ν_μ–ν_s oscillation which is a bit shifted up compared with the value in Eq. (2). For this value of $\Delta m^2_{\rm atm}$ and $\Delta m^2_{\rm LSND}$ in Eq. (3), we get

$$\begin{aligned} m_{\mu s} &\simeq (0.55-1.5)\ \text{eV} \\ m_{\mu\mu} + m_{ss} &\simeq (1.3-18)\times 10^{-3}\ \text{eV}\,. \end{aligned} \tag{8}$$

Since the mixing angle required for the explanation of the LSND result is given by $\theta_{\rm LSND} \simeq m_{es}/m_{\mu s}$, one needs $m_{es} \simeq (2.4-5.5)\times 10^{-2}$ eV.

Let us now estimate the sizes of m_{ij} for active neutrinos coming from R-parity violation. If the lepton number violation in the μ direction is suppressed so that $m_{\mu\mu}, m_{\mu e}, m_{\mu\tau} \ll m_{ss}$, one needs to generate only the ν_e–ν_τ oscillation for the solar neutrino for suitable values of $m_{e\tau}$ and $m_{\tau\tau}$ which can be rather trivially obtained in terms of the lepton number violations in the direction of e and τ. This case requires an explanation for the origin of the mass scale $m_{ss} \sim 10^{-2}$ eV. It would be more natural to have $m_{ss} \approx 0$ as in Ref. [14] and thus $m_{\mu\mu} \gg m_{ss}$. In this case, one needs $m_{\mu\mu} \simeq (1.3-18)\times 10^{-3}$ eV. Under the assumption that $m_{\tau s}/m_{\mu s} \ll 1$, the solar neutrino data can be explained if $m_{\tau\tau} = \sqrt{\Delta m^2_{\rm sol}}$ and $m_{e\tau}/m_{\tau\tau} - m_{\mu\tau} m_{es}/m_{\tau\tau} m_{\mu s} = \theta_{\rm sol}$. From this, the component $m_{\mu\tau}$ has to be restricted as $m_{\mu\tau} \lesssim \sqrt{\Delta m^2_{\rm sol}}\theta_{\rm sol}/\theta_{\rm LSND}$. Therefore, as in the previous case (i), the L-violation in the direction of μ and τ has to generate *not so large* ratios:

$$m_{\mu\mu}/m_{\tau\tau} \simeq (0.41-9)\,, \quad m_{\mu\tau}/m_{\tau\tau} \lesssim 3.5\,. \tag{9}$$

For this calculation, we have used the value of $m_{\mu s}$ varying from 0.55 eV to 1.5 eV and the largest value of $m_{\mu\tau}$.

(iii) (A+L)

This is the most popular case with four neutrinos [15]. The solar neutrino problem is solved by ν_e–ν_s mixing, and the atmospheric neutrino problem by maximally mixed ν_μ–μ_τ oscillation. Typically, one would expect that $\nu_{e,s}$ are lighter than $\nu_{\mu,\tau}$. Combined with the LSND result, it is then required that the two mass eigenvalues satisfy $m_{\nu_2} \simeq m_{\nu_3} \simeq (0.55-1.5)$ eV (therefore $\chi \simeq 1$) and $m^2_{\nu_3} - m^2_{\nu_2} = \Delta m^2_{\rm atm}$. The LSND mixing angle requires also $m_{e\mu}/m_{\mu\mu}$ or $m_{e\tau}/m_{\mu\tau} \simeq (2.4-5.5) \times 10^{-2}$. In the scheme of generating the neutrino masses from the R-parity violating couplings, it is again difficult to produce a small number: $(m_{\nu_3} - m_{\nu_2})/(m_{\nu_3} + m_{\nu_2}) = \Delta m^2_{\rm atm}/4m^2_{\nu_3} \sim 10^{-3}$. Recall that it is also possible to have almost degenerate $\nu_{e,s}$ with mass around 1 eV, and lighter $\nu_{\mu,\tau}$. In this case, one would need to fine-tune some parameters in the ν_e–ν_s mass matrix to achieve small mixing and a very small mass difference, $\sim 10^{-6}$ or 10^{-10} eV.

III. MSSM WITH R-PARITY AND L VIOLATION

Our framework is the conventional MSSM [16] in which soft supersymmetry breaking parameters arise from the gravitational coupling to the hidden sector and are assumed to be universal at the grand unification (GUT) scale where the three gauge couplings meet. In this framework, there are five independent parameters; the scalar mass m_0, the trilinear soft parameter A_0, the gaugino mass $m_{1/2}$, as well as $\tan\beta$ and the sign of μ. Here μ represents the mass parameter of the bilinear operator $H_1 H_2$.

Without loss of generality, one can take only dimensionless L-violating couplings λ', λ at the GUT scale. Recall that, in models where all the L-violating terms appear generically, the universality condition allows us to redefine the superfields L_i and H_1 in such a way that $\mu_i = D_i = 0$ at the GUT scale. Here D_i is a dimension-two soft parameter corresponding to the bilinear operator $L_i H_2$. Even if μ_i and D_i are zero at the GUT scale, upon renormalizing the soft SUSY breaking parameters and the L-violating couplings down to the weak scale, the universality condition breaks down and nonzero μ_i, D_i and $m^2_{L_i H_1}$ are generated. For the calculation of the renormalization group equations (RGEs), it is convenient to use the basis [17] where the μ_i terms are continuously rotated away in the following approximate manner:

$$\begin{aligned} H_1 + \frac{\mu_i}{\mu} L_i &\longrightarrow H_1 \\ L_i - \frac{\mu_i}{\mu} H_1 &\longrightarrow L_i \,. \end{aligned} \tag{10}$$

This definition of new basis is valid up to the leading order of λ' and λ, which can be justified if they are small enough. The merit of this basis is that the RGEs of λ' and λ do not mix each other as shown in Appendix. Nonvanishing D_i and $m^2_{L_i H_1}$ at the weak scale induce nonzero VEVs of sneutrinos [11]. Keeping only the leading terms of the L-violating soft-parameters, one finds the sneutrino VEVs [17]:

$$\langle \tilde{\nu}_i \rangle \simeq -\frac{D_i v_2 + m^2_{L_i H_1} v_1}{m^2_{L_i} + M_Z^2 \cos 2\beta/2} \,. \tag{11}$$

The tree-level neutrino mass matrix is then given by

$$\left(m_\nu^{\rm tree}\right)_{ij} = \frac{\langle\tilde{\nu}_i\rangle\langle\tilde{\nu}_j\rangle}{m_{1/2}/4\pi\alpha_{GUT} + v^2\sin\beta/\mu} \,. \tag{12}$$

Given the L-violating couplings λ' and λ at the weak scale, the neutrinos acquire radiative masses whose matrix elements are given by

$$\begin{aligned}\left(m_\nu^{\rm loop}\right)_{ij} &\simeq \frac{3}{16\pi^2}\frac{\Delta_{d_k}m_{d_l}}{\bar{m}^2_{d_k}}\left[\lambda'_{ikl}\lambda'_{jlk} + (i\leftrightarrow j)\right] \\ &+ \frac{1}{16\pi^2}\frac{\Delta_{e_k}m_{e_l}}{\bar{m}^2_{e_k}}\left[\lambda_{ikl}\lambda_{jlk} + (i\leftrightarrow j)\right] \,,\end{aligned} \tag{13}$$

where $m_{d,e}$ denote the down-type quark and charged lepton masses, respectively, $\Delta_{d,e}$ denote the mixing masses of the corresponding squarks or sleptons, and $\bar{m}^2$ are the functions of the squark or slepton mass eigenvalues $\tilde{m}_{1,2}$ given by $\bar{m}^2 = (\tilde{m}_1^2 - \tilde{m}_2^2)/\ln(\tilde{m}_1^2/\tilde{m}_2^2)$ for each generation. Here, i, j, k, l are generation indices. The full neutrino mass matrix is now given by $(m_\nu) = (m_\nu^{\rm tree}) + (m_\nu^{\rm loop})$, which will be computed with specifying the input values: the L-violating couplings λ', λ, the soft parameters $m_0, A_0, m_{1/2}$ (fixed at the GUT scale), as well as $\tan\beta$ and the sign of μ. For the computation, we took the ranges of the soft parameters as follows: m_0 from 100 GeV to 1 TeV, the absolute value of A_0 from 100 GeV to $3m_0$, and the absolute value of $m_{1/2}$ from 100 GeV to 1 TeV, fixing the sign of μ to be positive. Note that changing the signs of μ, A_0 and $m_{1/2}$ simultaneously yields the same results. We also fixed the top quark mass to be 175 GeV, and the strong coupling constant $\alpha_s(M_Z)$ to be 0.118.

Let us now specify more on the required L-violation. Since there are too many L-violating couplings, it would be impossible to draw sensible conclusions allowing such arbitrariness in explaining the neutrino oscillation parameters in Eqs. (1)–(3). Our basic assumption in this regard is to take the couplings with a "natural" hierarchy: the L-violating Yukawa couplings have a hierarchical structure similar to the corresponding quark or lepton mass matrix. In other words, e.g., $\lambda'_{ijk} \propto h^d_{jk}$ where h^d_{jk} is the Yukawa matrix for the down-type quarks before diagonalization. This will be a consequence of models that explain the fermion hierarchies in terms of flavor symmetry [18]. In such a scheme, one expects that λ'_{i33} and λ_{i33} are the largest components and thus give the leading contribution to the neutrino masses. The subleading contribution comes from the couplings, e.g., λ'_{i32} or λ'_{i23}. Their contribution to the component $(m_\nu)_{ij} = m_{ij}$ would be suppressed by the factor of $(\lambda'_{i32}\lambda'_{j23}/\lambda'_{i33}\lambda'_{j33})(m_s/m_b)$ compared with the leading contribution. It is to be understood that this expression is taken after diagonalization of the quark mass matrices. It was realized recently by Drees et.al. [19] that the ratio m_s/m_b could account for the ratio χ^{-1} if $\lambda'_{i32}\lambda'_{j23} \sim \lambda'_{i33}\lambda'_{j33}$. But, for the couplings with the natural hierarchy, it is expected that $\lambda'_{i32}\lambda'_{j23}/\lambda'_{i33}\lambda'_{j33} \lesssim m_s/m_b$, and thus the contribution of the smaller couplings is suppressed by a factor $(m_s/m_b)^2$. This is too small to account for the size of various components of the neutrino mass matrices leading to the case (i), (ii) or (iii). Therefore, *the couplings other than the (33)-components,* λ'_{i33} *(or* λ_{i33}*), can be neglected* in our discussion. Now, the question one may ask first is whether the phenomenologically desirable neutrino mass matrices can be obtained by taking a minimal set of the L-violating couplings, that is, the trilinear couplings other than $\lambda'_i \equiv \lambda'_{i33}$ are negligibly small. After answering this, we will examine the effect of the couplings $\lambda_{i33} \equiv \lambda_i (i \neq 3)$. We concentrate on the couplings of $\lambda'_{2,3}$ or λ_2 since the coupling λ'_1 or λ_1 (smaller than others) can be adjusted to reproduce the desirable mixing of ν_e to other neutrinos as already discussed.

IV. NEUTRINO MASSES FROM R-PARITY VIOLATION

Our task is now to examine whether the neutrino masses coming from the R-parity violation in the MSSM can reproduce the mass matrix patterns studied in Section II. Before presenting our main results, it is useful to look into some qualitative features of the neutrino masses coming from R-parity violation. First thing to note is that typically the tree mass is much larger than the loop mass. This can be seen by taking a crude approximation to integrate the RGEs in the Appendix. That is, taking the constant coefficients in the RGEs for D_i and $m^2_{L_i H_1}$, one can easily obtain their values at the weak scale which yield the sneutrino VEVs

$$\langle \tilde{\nu}_i \rangle \approx v_1 \frac{\ln \frac{M_{GUT}}{M_Z}}{8\pi^2} a_i (\lambda'_i h_b + b_i \lambda_i h_\tau) \tag{14}$$

where a_i, b_i are the parameters encoding the RGE effects for the soft masses and couplings. Therefore, the tree and loop mass can be written as

$$\begin{aligned} \left(m_\nu^{\rm tree}\right)_{ij} &\approx \kappa_0 a_i a_j (\lambda'_i h_b + b_i \lambda_i h_\tau)(\lambda'_j h_b + b_j \lambda_j h_\tau) \\ \left(m_\nu^{\rm loop}\right)_{ij} &\approx \kappa_1 (\lambda'_i \lambda'_j h_b^2 + b_{ij} \lambda_i \lambda_j h_\tau^2) \end{aligned} \tag{15}$$

where $\kappa_0 \sim (3\ln(M_{GUT}/M_Z)/8\pi^2)^2 (M_Z^2 \cos^2\beta/m_{1/2})$ and $\kappa_1 \sim 3v_1^2/8\pi^2\tilde{m}$. Generically, one would find $m^{\rm tree}/m^{\rm loop} \sim \kappa_0/\kappa_1 \gtrsim 10^2$ for $\tilde{m}/m_{1/2} \gtrsim 0.1$. The second observation is that, for $\lambda'_i \gg \lambda_i$, the tree mass can be aligned with the loop mass; that is, $a_i \approx a_j$ for $i \neq j$. The alignment can be weakened as $\tan\beta$ and λ_i become larger. For a large $\tan\beta$, the misalignment of, e.g., $A'_3/\lambda'_3 - A'_2/\lambda'_2$ can be amplified by the RGE effect (and thus $a_3 - a_2$ also) since

$$8\pi^2 \frac{d}{dt} \left(\frac{A'_3}{\lambda'_3} - \frac{A'_2}{\lambda'_2} \right) \approx h_\tau A_\tau \, . \tag{16}$$

For large λ_i, it is obvious to have a large misalignment since $\lambda_3 = 0$ due to the antisymmetry of λ_{ijk}.

The above properties play important roles in the case (S+A), for which one needs *two distinctive* nonzero mass eigenvalues. Since the tree mass is much larger than the loop mass, the loop mass can fit better the solar neutrino mass scale with vacuum oscillation (requiring $\Delta m^2_{\rm sol} \simeq 10^{-10}{\rm eV}^2$) while the tree mass gives rise to the atmospheric neutrino mass scale. This is basically why one finds more soft parameter space for the vacuum oscillation solution to the solar neutrino problem than for the MSW solution in the scatter plot study by Hempfling [12]. The vacuum oscillation of solar neutrinos requires a large mixing between ν_e and ν_τ in our scheme since the large mixing for vacuum oscillation implies $\lambda'_1 \approx \lambda'_{2,3}$. This might be in conflict with the Superkamiokande data as well as the CHOOZ result

[5,6]. Therefore, we prefer the small mixing MSW solution to the solar neutrino problem as mentioned before. It is now clear that, in order to get the neutrino mass hierarchy viable for the atmospheric neutrino oscillation and the MSW solar neutrino conversion, the tree mass has to be suppressed requiring $a_i \ll 1$. This can occur for certain soft parameters which admit a cancellation in $\langle\tilde{\nu}_i\rangle$, that is, $m^2_{L_iH_1}/D_i \simeq -\tan\beta$. Then, it could be that the ratio of the tree and loop mass is responsible for the two distinctive neutrino masses differing by the factor $\chi = (7-40)$.

With the minimal number of L-violating couplings (or, $\lambda_i \ll \lambda'_i$), the mere suppression of the tree mass is not enough because the tree and loop masses are almost aligned as discussed above (that is, $\langle\tilde{\nu}_i\rangle \propto \lambda'_i$), which renders the second eigenvalue m_{ν_2} close to zero. However, it turns out that a partial cancellation between the tree and loop mass can result in the breakdown of the alignment and the production of the desirable value of χ. In other words, it is required that $|m_\nu^{\rm tree}| \simeq |m_\nu^{\rm loop}|$ and they have opposite sign. In order to quantify these statements, let us give some examples and calculate the amount of the misalignment measured by $\xi \equiv |\lambda'_2\langle\tilde{\nu}_3\rangle - \lambda'_3\langle\tilde{\nu}_2\rangle|/|\lambda'||\langle\tilde{\nu}\rangle|$ where $|\lambda'| = \sqrt{{\lambda'_2}^2 + {\lambda'_3}^2}$, etc. For $\tan\beta = 5$, $m_0 = 201$ GeV, $m_{1/2} = -109$ GeV, and $A_0 = -550$ GeV, the muon and tau neutrino mass matrix is found to be

$$m_\nu/\text{eV} \approx -\begin{pmatrix} 0.67 & 0.68 \\ 0.68 & 0.69 \end{pmatrix} + \begin{pmatrix} 0.65 & 0.65 \\ 0.65 & 0.65 \end{pmatrix} \tag{17}$$

where the first (second) one is tree (loop) contribution. As is obvious from Eq. (17), the tree and loop masses are aligned very closely ($\xi = 7\times10^{-3}$) and the cancellation of the largest digit results in $\chi \approx 38$ and $\sin^2 2\theta \approx 0.9$. For $\tan\beta = 40$, taking $m_0 = 20$ GeV, $m_{1/2} = -110$ GeV, $A_0 = -250$ GeV, one finds the mass matrix with $\chi \approx 16$ and $\sin^2 2\theta \approx 0.9$;

$$m_\nu/\text{eV} \approx -\begin{pmatrix} 0.69 & 1.22 \\ 1.22 & 2.14 \end{pmatrix} + \begin{pmatrix} 4.01 & 3.87 \\ 3.87 & 3.73 \end{pmatrix} \tag{18}$$

which shows that the alignment is badly broken ($\xi = 0.1$). Still, one needs a cancellation to get the right value for the mass ratio, in particular. It turns out that negative values of $m_{1/2}$ and A_0 are necessary to yield opposite signs for the tree and loop masses. Now that the alignment becomes severer for smaller $\tan\beta$, the soft parameters have to be tuned more precisely for smaller $\tan\beta$. In the numerical computation, we calculated the ratio χ of two mass eigenvalues and the atmospheric neutrino mixing $\sin^2 2\theta$ for some sample set of $\tan\beta$ and the ratios between the couplings $\lambda'_{2,3}$, scanning the soft parameters up to the smallest digit above point. The results of the computations are presented in TABLE I and II. Notice that the neutrino mass matrix is proportional to the overall scale of $\lambda'^2_{2,3}$, and thus the ratio χ and the mixing $\sin^2 2\theta$ depend only on the ratio of two couplings λ'_3/λ'_2. As expected, the number of the desirable soft parameters gets smaller for smaller $\tan\beta$, and we find *no parameter space for* $\tan\beta \lesssim 4$. Furthermore, the values of χ and $\sin^2 2\theta$ depend very sensitively on the soft parameters, and the acceptable soft parameters are scattered for a small $\tan\beta$. For a large $\tan\beta$, χ becomes a slowly varying function of the soft parameters, and one can isolate some finite region for given values of χ and $\sin^2 2\theta$.

Unexpectedly, some patterns have emerged for values of the ratio χ and the mixing angle that are realized in our scheme depending on $\tan\beta$ and the L-violating couplings, as presented in TABLE I. For illustration, we took some variations of λ'_3 larger than λ'_2. Similar results can be obtained also in the cases with λ'_2 larger than λ'_3. For given $\tan\beta$ and $\lambda'_{2,3}$, χ and $\sin^2 2\theta$ are found to lie inside some restricted ranges, and to be correlated in the way that *larger mass ratio corresponds to larger mixing for* $\lambda'_2 = \lambda'_3$, *or to smaller mixing for* $\lambda'_2 < \lambda'_3$. The correlation becomes weaker for larger $\tan\beta$. From TABLE I, one sees that the maximal mixing is easily realized with $\lambda'_3/\lambda'_2 \simeq 2$. We found practically *no desirable parameter space for* $\lambda'_3/\lambda'_2 \gtrsim 5$ which appears inconsistent with large $\sin^2 2\theta$. The eigenvalue m_{ν_2} depends on m_0 in the way that larger m_0 produces smaller neutrino mass as implied by the formulae (11)–(13). The right values of the L-violating couplings can be obtained by the rescaling: $\lambda'_{2,3} = 10^{-4}(\Delta m^2_{\rm sol}/m^2_{\nu_2})$ where m_{ν_2} is a value given in TABLE I. For instances, the actual values one needs are $\lambda'_2 = \lambda'_3 = (10^{-3} - 10^{-4})$ for $\tan\beta = 5$, $\lambda'_2 = \lambda'_3 = (10^{-4} - 10^{-5})$ for $\tan\beta = 20$ and $\lambda'_2 = \lambda'_3 = (3\times 10^{-5} - 3\times 10^{-6})$ for $\tan\beta = 40$. As a consequence of partial cancellation between the tree and loop mass, the size of the L-violating couplings yielding the phenomenologically viable neutrino masses becomes slightly larger than the values for which the loop mass yields the atmospheric neutrino mass scale, say 5×10^{-2} eV.

It is also amusing to find that there are preferable ranges of the acceptable soft parameters: *larger* $|A_0|$ *and smaller* $|m_{1/2}|$ are preferred. To be specific, we show in TABLE II the ranges of soft parameters within which viable neutrino masses and mixing can be realized with $\lambda'_2 = \lambda'_3 = 10^{-4}$. For a given m_0, the values of A_0 and $m_{1/2}$ should reside in between two end values shown in the table. There is a correlation between A_0 and $m_{1/2}$: larger $|A_0|$ corresponds to larger $|m_{1/2}|$. A similar pattern follows even with a slight variation of two couplings $\lambda'_{2,3}$ such as given in TABLE I.

If the coupling λ_2 is comparable to $\lambda'_{2,3}$, then the above mentioned properties are significantly modified as anticipated in the beginning of this section. We find that χ and $\sin^2 2\theta$ become very slowly varying functions of the soft parameters even for a small $\tan\beta$, as a consequence of a large misalignment between λ'_i and $\langle\tilde\nu_i\rangle$. But one still needs $m_\nu^{\rm tree} \sim m_\nu^{\rm loop}$ requiring a cancellation in the sneutrino VEVs, which essentially restricts the soft parameter space. To show this explicitly, let us take two examples for $\tan\beta = 5$. Taking $m_0 = 500$ GeV, $m_{1/2} = -100$ GeV and $A_0 = -780$ GeV, one finds the mass matrix in which the tree mass is larger than the loop mass as in the above minimal cases:

$$m_\nu/\mathrm{eV} \approx -\begin{pmatrix} 0.581 & 0.527 \\ 0.527 & 0.479 \end{pmatrix} + \begin{pmatrix} 0.394 & 0.390 \\ 0.390 & 0.390 \end{pmatrix} \tag{19}$$

giving $\chi \approx 38$ and $\sin^2 2\theta \approx 0.89$. There is a small contribution (0.003 eV) to the $m^{\rm loop}_{\nu_\mu\nu_\mu}$ from the λ_2 coupling. The other one is of different type with dominant loop contribution: for $m_0 = 500$ GeV, $m_{1/2} = 400$ GeV and $A_0 = 100$ GeV, one gets

$$m_\nu/\mathrm{eV} \approx -\begin{pmatrix} 0.071 & 0.023 \\ 0.023 & 0.0077 \end{pmatrix} + \begin{pmatrix} 0.136 & 0.127 \\ 0.127 & 0.127 \end{pmatrix} \tag{20}$$

which gives $\chi \approx 20$ and $\sin^2 2\theta \approx 0.95$. In this case, the λ_2 contribution (0.009 eV) is more significant than before. In both examples, any of the tree and loop contributions cannot be neglected in order to produce the right values of χ and $\sin^2 2\theta$. In TABLE III, we illustrate some set of soft parameter ranges yielding the right values of the ratio χ and the mixing angle, taking $\lambda'_2 = \lambda'_3 = \lambda_2$. The ranges of A_0 and $m_{1/2}$ for given m_0 are shown. In TABLE

IV, we take some variations of $\lambda_2/\lambda'_{2,3}$. Compared to the previous case, one finds more parameter space open up for larger λ_2. To have the right value of m_{ν_2}, we need a bit smaller values for the couplings λ' than before: that is, $\lambda'_{2,3} = \lambda_2 \simeq (10^{-4} - 3 \times 10^{-5})$ for $\tan\beta = 5$ and $\lambda'_{2,3} = \lambda_2 \simeq (10^{-5} - 2 \times 10^{-6})$ for $\tan\beta = 40$. From TABLE III and IV, one can also see that larger $\tan\beta$ are still needed to destroy the alignment in a sufficient amount even with sizable λ_2. It can be found that a large soft parameter space opens up for $\tan\beta \lesssim 2$ only when $\lambda_2 \gtrsim 4\lambda'_2$. Contrary to the case with negligible λ_2, almost all ranges of χ, $\sin^2 2\theta$ can be realized as shown in TABLE III.

Now let us turn to the other cases. For the case (S+L), it is important to realize that one *needs not to generate two nonzero eigenvalues* in the neutrino mass matrix along the ν_μ, ν_τ directions. Furthermore, the ratios $m_{\mu\tau}/m_{\tau\tau}$ and $m_{\mu\mu}/m_{\tau\tau}$ (9) are determined roughly by the input values λ'_2/λ'_3 and $(\lambda'_2/\lambda'_3)^2$, respectively. Therefore, it is required from the previous discussion, $\lambda'_2/\lambda'_3 \simeq (0.64 - 3)$, and the value of λ'_3 to be determined by the condition $m^\nu_{\tau\tau} = \sqrt{\Delta m^2_{\rm sol}} \sim 10^{-3}$eV. Generically, the tree mass is much larger than the loop mass. Taking $\lambda'_{i33} = 10^{-4}$, the tree mass can be as large as 100 eV for a small $\tan\beta$ [12]. Therefore, for a small $\tan\beta$, one needs $\lambda'_3 \gtrsim 10^{-7}$. This value can be as large as 10^{-4} when the tree mass is suppressed as we discuss above. Since λ_i cannot give rise to the nonzero component $m^\nu_{\tau\tau}$, taking the largest components being $\lambda'_2 \sim \lambda'_3$ as above is the best way to explain $m^\nu_{\mu\mu} \sim m^\nu_{\mu\tau} \sim m^\nu_{\tau\tau}$.

Let us finally comment on the case (A+L). In order to achieve $(m_{\nu_3} - m_{\nu_2})/(m_{\nu_3} + m_{\nu_2}) \sim 10^{-3}$, much finer tuning of the soft parameters is required and thus it is very difficult to be realized in our scheme.

V. CONCLUSIONS

In conclusion, we have examined the possibility of obtaining the realistic neutrino masses and mixing in the context of the R-parity violating minimal supersymmetric standard model. We have assumed an ultraviolet theory which has generic L-violating Yukawa couplings in the basis where the L-violating bilinear terms are rotated away. The L-violating Yukawa couplings induce the L-violating bilinear soft terms through the renormalization group evolution which takes a simple form in the basis (valid for small L-violating couplings) as shown in Appendix. Analyzing the neutrino masses arising both from the sneutrino vacuum expectation values generated by the L-violating soft terms and from the loop corrections through squark or slepton exchange, we found restrictions on the soft parameters, the L-violating couplings and $\tan\beta$, under which realistic neutrino mass matrices can be obtained.

With three ordinary neutrinos, one can account for any two of the three distinct mass-squared differences required by the solar, atmospheric and LSND neutrino data. First, we have discussed the phenomenological mass matrices along the ν_μ, ν_τ directions which are required by the data. If the solar neutrino and LSND data are due to the active neutrinos, and a sterile neutrino is introduced to explain the atmospheric data, then it is not necessary to have two distinct mass eigenvalues for the mass matrix in the ν_μ, ν_τ directions but its components should not differ by a factor of more than a few. This can be easily achieved if λ'_{i33} are the largest L-violating couplings and $\lambda'_{233} \sim \lambda'_{333}$. In the case of solving the atmospheric neutrino and LSND data, one needs to generate two almost degenerate mass eigenvalues for ν_μ, ν_τ with a very small mass difference. This case can hardly be realizable in our scheme.

Most nontrivial and realistic case is to accommodate the solar and atmospheric data within the ordinary context of three active neutrinos (disregarding the LSND data). In this case, one needs to generate two distinct mass eigenvalues for ν_μ, ν_τ whose ratio, χ, should reside roughly between 7 and 40, and the mixing angle $\sin^2 2\theta_{\rm atm} \gtrsim 0.82$.

Under the assumption of the natural Yukawa hierarchy in the L-violating couplings, we have argued that the relevant contribution to the phenomenological neutrino mass matrices comes from the components λ'_{i33} and λ_{i33}. With the minimal choice of the L-violating couplings (namely, other than λ'_{i33} are negligible), we needed not only the suppression of tree mass (that is, $\tan\beta \simeq -m^2_{L_iH_1}/D_i$), but also some partial cancellation between the tree and loop mass. This basically strongly constrains the soft parameter space. Our study have shown that the realistic neutrino masses and mixing prefer a large trilinear soft parameter A_0 and a small gaugino mass $m_{1/2}$. The desirable soft parameter space becomes more restricted and finer tuned for smaller $\tan\beta$, and thus a reasonable parameter space can be found only for fairly large $\tan\beta$, say, $\tan\beta \gtrsim 40$. We have found indeed no parameter space for $\tan\beta \lesssim 4$. The large mixing explaining the atmospheric neutrino data requires that λ'_{233} and λ'_{333} should not differ by more than a factor of 5. Interestingly, the mass ratio $\chi = m_{\nu_3}/m_{\nu_2}$ and the atmospheric neutrino mixing $\sin^2 2\theta$ are found to be restricted in a certain range and correlated in the way that smaller ratio χ has smaller or larger mixing depending on the values of $\lambda'_{2,3}$. This tendency becomes also weaker for larger $\tan\beta$ and larger λ_{233}. It appears more reasonable to have a sizable λ_{233}, for which the experimental quantities of neutrino oscillations become very stable under the variation of the soft parameters, and thus there exist fairly extended regions of parameters fitting into the experimental data. Still, there exist significant constraints on the soft parameters coming from the fact that the tree mass has to be suppressed as above. More soft parameter space can be found for larger $\tan\beta$ and λ_{233}.

APPENDIX

Renormalization group equations for the lepton number violating parameters in the basis where L_iH_1 terms are rotated away in the superpotential.

$$16\pi^2 \frac{d\lambda'_i}{dt} = \lambda'_i(\delta_{i3}h_\tau^2 + h_t^2 + 3h_b^2 - \frac{7}{9}g_1^2 - 3g_2^2 - \frac{16}{3}g_3^2) \tag{21}$$

$$16\pi^2 \frac{d\lambda_i}{dt} = \lambda_i(3h_\tau^2 - 3g_1^2 - 3g_2^2) \tag{22}$$

$$\begin{aligned} 16\pi^2 \frac{dA'_i}{dt} &= A'_i(\delta_{i3}h_\tau^2 + h_t^2 + 9h_b^2 - \frac{7}{9}g_1^2 - 3g_2^2 - \frac{16}{3}g_3^2) + A_i(2h_bh_\tau) \\ &+ 2\lambda'_i(\delta_{i3}h_\tau A_\tau + h_tA_t + 2h_bA_b + \frac{7}{9}g_1^2M_1 + 3g_2^2M_2 + \frac{16}{3}g_3^2M_3) \end{aligned} \tag{23}$$

$$16\pi^2 \frac{dA_i}{dt} = A_i(5h_\tau^2 - 3g_1^2 - 3g_2^2) + A'_i(6h_bh_\tau) + \lambda_i(6h_\tau A_\tau + 6g_1^2M_1 + 6g_2^2M_2) \tag{24}$$

$$
\begin{aligned}
16\pi^2 \frac{dm^2_{L_iH_1}}{dt} &= m^2_{L_iH_1}(3\delta_{i3}h^2_\tau + h^2_\tau + 3h^2_b) - 6A'_iA_b - 2A_iA_\tau \\
&\quad - 6\lambda'_i h_b(m^2_{L_i} + m^2_{Q_3} + m^2_{D_3}) - 2\lambda_i h_\tau(m^2_{L_i} + m^2_{L_3} + m^2_{E_3})
\end{aligned} \tag{25}
$$

$$
16\pi^2 \frac{dD_i}{dt} = D_i(3h^2_t - g^2_1 - 3g^2_2) - \mu(6A'_ih_b + 2A_ih_\tau) \tag{26}
$$

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TABLES

TABLE I. Ranges of the atmospheric neutrino mixing angle, the ratio of two mass eigenvalues corresponding to $\sqrt{\Delta m^2_{\rm atm}/\Delta m^2_{\rm sol}}$, and the smallest mass eigenvalue (for solar neutrino) which can be realized in the soft parameter space for given $\tan\beta$ and L-violating couplings $\lambda'_{2,3}$. $\lambda'_2 = 10^{-4}$ is taken.

	$\tan\beta$	5	20	40
	$\sin^2 2\theta$	$0.82 \sim 0.91$	$0.82 \sim 0.95$	$0.82 \sim 1.0$
$\lambda'_3/\lambda'_2 = 1$	m_{ν_3}/m_{ν_2}	$19 \sim 40$	$14 \sim 40$	$7 \sim 40$
	m_{ν_2} (eV)	$3 \times 10^{-5} - 3 \times 10^{-3}$	$10^{-3} \sim 0.1$	$0.03 \sim 2$
	$\sin^2 2\theta$	$0.89 \sim 1.0$	$0.86 \sim 1.0$	$0.82 \sim 1.0$
$\lambda'_3/\lambda'_2 = 2$	m_{ν_3}/m_{ν_2}	$40 \sim 7$	$40 \sim 7$	$7 \sim 40$
	m_{ν_2} (eV)	$3 \times 10^{-3} \sim 10^{-2}$	$0.01 \sim 0.4$	$0.05 \sim 4$
	$\sin^2 2\theta$	$0.82 \sim 0.96$	$0.82 \sim 0.97$	$0.82 \sim 0.99$
$\lambda'_3/\lambda'_2 = 3$	m_{ν_3}/m_{ν_2}	$16 \sim 7$	$16 \sim 7$	$28 \sim 7$
	m_{ν_2} (eV)	$2 \times 10^{-4} \sim 2 \times 10^{-2}$	$5 \times 10^{-3} \sim 0.8$	$0.1 \sim 5$
	$\sin^2 2\theta$	$0.82 \sim 0.88$	$0.82 \sim 0.89$	$0.82 \sim 0.94$
$\lambda'_3/\lambda'_2 = 4$	m_{ν_3}/m_{ν_2}	$9 \sim 7$	$9 \sim 7$	$9 \sim 7$
	m_{ν_2} (eV)	$4 \times 10^{-4} \sim 2 \times 10^{-2}$	$8 \times 10^{-3} \sim 1$	$0.4 \sim 2$

TABLE II. Illustrated ranges of soft parameters in units of GeV within which the solar and atmospheric neutrino data can be accommodated by suitable choices of soft parameters in the case of $\lambda'_2 = \lambda'_3 = 10^{-4}$. The sign of μ is fixed to be positive.

$\tan\beta$	m_0	A_0	$m_{1/2}$
5	200	$-590 \sim -500$	$-150 \sim -100$
	400	$-1000 \sim -730$	$-270 \sim -100$
	600	$-950 \sim -900$	$-150 \sim -100$
20	200	$-600 \sim -230$	$-270 \sim -100$
	500	$-1000 \sim -300$	$-430 \sim -150$
	800	$-1000 \sim -370$	$-430 \sim -100$
40	200	$-600 \sim -200$	$-300 \sim -100$
	500	$-1000 \sim -500$	$-500 \sim -100$
	800	$-1000 \sim -240$	$-480 \sim -100$

TABLE III. Illustrated set of soft parameter ranges yielding realistic values of the mixing and the mass ratio in the case of $\lambda'_2 = \lambda'_3 = \lambda_2 = 10^{-4}$. The sign of μ is again taken to be positive. Soft parameters are in units of GeV and neutrino masses are in units of eV.

$\tan\beta$	m_0	A_0	$m_{1/2}$	m_{ν_2}	$\sin^2 2\theta$	m_{ν_3}/m_{ν_2}
	100	$-100 \sim -170$	$430 \sim 570$	$(2 \sim 7) \times 10^{-2}$	$0.82 \sim 0.96$	$7 \sim 40$
3	300	$-310 \sim -830$	$490 \sim 210$	$(1 \sim 3) \times 10^{-2}$	$0.82 \sim 0.94$	$7 \sim 40$
	500	$-830 \sim -970$	$510 \sim 470$	$(0.9 \sim 1.1) \times 10^{-2}$	$0.82 \sim 0.88$	$25 \sim 40$
	100	$100 \sim 310$	$160 \sim 490$	$(5 \sim 16) \times 10^{-2}$	$0.82 \sim 0.91$	$7 \sim 40$
	500	$-770 \sim -990$	$-100 \sim -240$	$(0.6 \sim 2) \times 10^{-2}$	$0.82 \sim 1.0$	$7 \sim 40$
5		$100 \sim 530$	$370 \sim 1000$	$(1 \sim 3) \times 10^{-2}$	$0.82 \sim 1.0$	$7 \sim 40$

	900	$-100 \sim -170$	$690 \sim 550$	$(5 \sim 6) \times 10^{-3}$	$0.99 \sim 1.0$	$37 \sim 40$
		$100 \sim 530$	$690 \sim 1000$	$(5 \sim 10) \times 10^{-3}$	$0.88 \sim 1.0$	$14 \sim 40$
	300	$-630 \sim -910$	$-290 \sim -430$	$2.8 \sim 3.5$	$0.82 \sim 1.0$	$7 \sim 12$
		$530 \sim 910$	$290 \sim 510$	$2.5 \sim 3.5$	$0.82 \sim 1.0$	$7 \sim 14$
40	500	$-250 \sim -330$	$-100 \sim -130$	$0.4 \sim 0.9$	$0.82 \sim 1.0$	$7 \sim 19$
		$930 \sim 1000$	$510 \sim 570$	$1 \sim 2$	$0.82 \sim 1.0$	$7 \sim 9$
	900	$-300 \sim -590$	$-100 \sim -230$	$0.1 \sim 0.5$	$0.82 \sim 1.0$	$7 \sim 40$
		$100 \sim 280$	$110 \sim 210$	$0.2 \sim 0.6$	$0.82 \sim 1.0$	$7 \sim 40$

TABLE IV. Same as in TABLE III with different set of λ_2. Here two approximate end values of soft parameters A_0 and $m_{1/2}$ with the corresponding neutrino parameters are shown.

		$\lambda_2 = 3 \times 10^{-5}$		$\lambda_2 = 3 \times 10^{-4}$	
$\tan\beta$	m_0	A_0	$m_{1/2}$	A_0	$m_{1/2}$
3	100	$-300 \sim -120$	$120 \sim 240$	$100 \sim 160$	$380 \sim 1000$
	300	$-900 \sim -640$	$320 \sim 420$	$-100 \sim -900$	$150 \sim 1000$
	900	none	none	$-1000 \sim -620$	$880 \sim 1000$
5	100	$100 \sim 300$	$140 \sim 380$	$100 \sim 300$	$220 \sim 1000$
	500	$-1000 \sim -820$	$-220 \sim -120$	$-1000 \sim -100$	$-200 \sim 460$
		none	none	$100 \sim 340$	$580 \sim 1000$
	900	none	none	$-1000 \sim -100$	$120 \sim 1000$
40	300	$-300 \sim -240$	$-120 \sim -100$	$-600 \sim -320$	$-320 \sim -160$
		$180 \sim 260$	$120 \sim 140$	$240 \sim 560$	$160 \sim 360$
	500	$-560 \sim -300$	$-240 \sim -120$	$-1000 \sim -540$	$-540 \sim -260$
		$140 \sim 460$	$120 \sim 240$	$400 \sim 980$	$280 \sim 680$
	900	$-1000 \sim -420$	$-420 \sim -100$	$-320 \sim -300$	$-100 \sim -120$
		$100 \sim 800$	$120 \sim 480$	$740 \sim 1000$	$520 \sim 680$