## Comment on the paper: "The perturbative proton form factor reexamined" by Kundu et al.

J. Bolz<sup>1</sup>, R. Jakob<sup>2</sup>, P. Kroll<sup>1</sup> and N. G. Stefanis<sup>3</sup>

 $1$ Fachbereich Physik, Universität Wuppertal, D-42097 Wuppertal, Germany

 $2$ Dipartimento di Fisica Nucleare e Teorica, Università di Pavia, I-27100 Pavia, Italy

 $3$ Institut für Theoretische Physik II, Ruhr-Universität Bochum, D-44780 Bochum, Germany

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## Abstract

We point out some serious problems in the investigation of Kundu et al. which claims agreement with the existing data of the proton form factor, calculated without taking into account the intrinsic  $k_{\perp}$ -dependence of the proton wave function.

In 1993 Li[[1](#page-2-0)] applied the modified factorization scheme of perturbative QCD to exclusive processes, developed in [\[2\]](#page-2-0), to the proton form factor at large momentum transfer. One of the key ingredients of his approach was the use of strongly end-point concentrated distribution amplitudes of the Chernyak-Zhitnitsky type [\[3\]](#page-2-0). With such distribution amplitudes good agreement of the calculated perturbative contribution to  $G_M^p$  with the experimental data wasclaimed. However, as it was subsequently pointed out in [[4\]](#page-2-0), Li's analytical expression for the form factor embodied uncontrolled logarithmic singularities in the soft end-point (kinematic) regions. As a result, the seemingly good agreement of his numerical results with experiment was merely the product of an incorrect numerical integration.

Indeed,in [[4\]](#page-2-0) we showed in detail where the singularities come from, and proposed another infrared cut-off prescription which suffices to render the integrands finite. Our analysis not only comprised gluonic radiative corrections, encoded in Sudakov form factors as Li's work, it also took into account the intrinsic  $k_{\perp}$ -dependence of the proton wave function. Now this effect is well-known, see, e.g., [\[5](#page-2-0)–[10\]](#page-2-0): it has to do with the finite size of the bound state and has to be taken into account for consistency. Both the Sudakov form factor and the intrinsic transverse momentum dependence strongly suppress the end-point regions, yielding numerical results for the proton form factor much lower than the data for a whole class of proton distribution amplitudes, determined in [\[11\]](#page-2-0). On the other hand, self-consistency is improved in our approach. Furthermore, we recall that our results for the proton form factor should be considered as a kind of upper bound because they are obtained under the proviso of normalizing the wave functions to 1. A more realistic valence Fock state normalization (of the order of 0.1 to 0.2) leads to substantially smaller perturbative contributions.

Now to the present investigation. Kundu, Li, Samuelsson, and Jain [\[12](#page-2-0)] have taken up the calculation of the proton form factor again, claiming now agreement with experiment for one of the end-point concentrated distribution amplitudes. The crucial difference between ourinvestigation [[4\]](#page-2-0) and the new one [[12](#page-2-0)] is the neglect of the intrinsic  $k_1$ -dependence. As we explained above, neglecting this effect has the consequence of artificially enhancing the perturbative result at the expense of strong contributions from the end-point regions, where perturbative QCD is not applicable. There are two more technical points in which the papers [\[4](#page-2-0)] and[[12\]](#page-2-0) differ. One concerns the use of an improved version of the Sudakov form factor in [\[12\]](#page-2-0), which though formally rectifies the expression given in [\[2,1](#page-2-0)], is numerically irrelevant (see, for instance, the discussion in[[13\]](#page-2-0)). The other point is the use of a larger infrared cut-off in [\[12](#page-2-0)], namely the maximum transverse distance, proposed in our approach [\[4](#page-2-0)], multiplied by a parameter  $c > 1$ . This cut-off slightly increases the numerical values of the form factor.

Moreover, it is to be stressed that in the analyses of  $[1,4,12]$  the transverse momentum dependence of the quark propagators is neglected. As discussed in[[4\]](#page-2-0) (where we were mainly interested in estimating the maximum size of the perturbative contribution) this overestimates the results for the form factor strongly.

Last but not least we want to mention that there are also soft contributions of the overlap type (Feynman mechanism). Contrary to a statement made in [\[12\]](#page-2-0), these soft contributions do not constitute an alternative approach to the form factor but are an essential ingredient of the QCD expansion of the form factors[[14–17](#page-2-0)]. Such contributions are rather to be added to the perturbative contribution. For the end-point concentrated distribution amplitudes combined with a plausible assumption on the intrinsic transverse momentum dependence, these soft contributions are extremely large, overshooting the data by an order of magnitude [\[18,19\]](#page-2-0). Such distribution amplitudes also lead to inconsistencies with the valence quark distribution functions [\[20,19](#page-2-0)].

To summarize: We still believe that the perturbative contribution to the proton form factor is much smaller than the experimental data, despite the claims and the results of [\[12\]](#page-2-0), for the reasons explained above. At accessible momentum-transfer values, the proton form factor seems to be controlled by soft physics.

## REFERENCES

- <span id="page-2-0"></span>[1] H.-n. Li, Phys. Rev. D 48 (1993) 4243.
- [2] J. Botts and G. Sterman, Nucl. Phys. B325 (1989) 62; H.-n. Li and G. Sterman, Nucl. Phys. B381 (1992) 129.
- [3] V.L. Chernyak and I.R. Zhitnitsky, Phys. Rep. 112 (1984) 173; V.L. Chernyak, A.A. Ogloblin, and I.R. Zhitnitsky, Z. Phys. C 42 (1989) 569.
- [4] J. Bolz, R. Jakob, P. Kroll, M. Bergmann, and N.G. Stefanis, Z. Phys. C 66 (1995) 267.
- [5] J.C. Collins and D.E. Soper, Nucl. Phys. B193 (1981) 381; ibid. B194 (1982) 445.
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- [6] R. Jakob and P. Kroll, Phys. Lett. B 315 (1993) 463; ibid. B 319 (1993) 545(E).
- [7] M.G. Sotiropoulos and G. Sterman, Nucl. Phys. B425 (1994) 489.
- [8] B. Chibisov and A.R. Zhitnitsky, Phys. Rev. D 52 (1995) 5273.
- [9] I.V. Musatov and A.V. Radyushkin, Phys. Rev. D 56 (1997) 2713.
- [10] R. Akhoury, A. Sinkovics and M.G. Sotiropoulos, Phys. Rev. D 58 (1998) 013011.
- [11] M. Bergmann and N.G. Stefanis, Phys. Rev. D 48 (1993) R2990.
- [12] B. Kundu, H.-N. Li, J. Samuelsson, and P. Jain, [hep-ph/9806419.](http://arxiv.org/abs/hep-ph/9806419)
- [13] M. Dahm, R. Jakob, and P. Kroll, Z. Phys. C 68 (1995) 595.
- [14] G.P. Lepage and S.J. Brodsky, Phys. Rev. D 22 (1980) 2157.
- [15] A.V. Radyushkin, Nucl. Phys. A532 (1991) 141c.
- [16] A.V. Radyushkin, [hep-ph/9803316](http://arxiv.org/abs/hep-ph/9803316).
- [17] S.J. Brodsky, [hep-ph/9807212.](http://arxiv.org/abs/hep-ph/9807212)
- [18] N. Isgur and C.H. Llewellyn Smith, Nucl. Phys. B317 (1989) 526.
- [19] J. Bolz and P. Kroll, Z. Phys. A 356 (1996) 327.
- [20] A. Schäfer, L. Mankiewicz, and Z. Dziembowski, Phys. Lett. B233 (1989) 217.