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## ***D*-wave heavy quarkonium production in fixed target experiments**

Feng Yuan and Cong-Feng Qiao

*Department of Physics, Peking University, Beijing 100871, People's Republic of China*

Kuang-Ta Chao

*China Center of Advanced Science and Technology (World Laboratory), Beijing 100080, People's Republic of China*

*and Department of Physics, Peking University, Beijing 100871, People's Republic of China*

### Abstract

We calculate the *D*-wave heavy quarkonium production at fixed target experiments under the NRQCD factorization formalism. We find that the color octet contributions are two orders of magnitude larger than color-singlet contributions if color-octet matrix elements are taken according to the NRQCD velocity scaling rules. Within the theoretical uncertainties, the prediction for the production rate of  $2^{--}$  *D*-wave charmonium state agrees with the preliminary result of E705 and other experiments. Searching for the  $1^{--}$  *D*-wave state  $\psi(3770)$  is further suggested.

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Studies of heavy quarkonium production in high energy collisions provide important information on both perturbative and nonperturbative QCD. Recent progress in this area was stimulated by the experiment results of **CDF** at the Fermilab Tevatron. In the 1992-1993 run, the **CDF** data [1] for the prompt production rates of  $\psi$  and  $\psi'$  at large transverse momentum were observed to be orders of magnitude larger than the lowest order perturbative calculation based on the color-singlet model. At the same time, a new framework in treating quarkonium production and decays has been advocated by Bodwin, Braaten and Lepage in the context of nonrelativistic quantum chromodynamics (NRQCD) [2]. In this approach, the production process is factorized into short and long distance parts, while the latter is associated with the nonperturbative matrix elements of four-fermion operators. This factorization formalism gives rise to a new production mechanism called the color-octet mechanism, in which the heavy-quark and antiquark pair is produced at short distance in a color-octet configuration and subsequently evolves nonperturbatively into physical quarkonium state. The color-octet terms in the gluon fragmentation to  $J/\psi(\psi')$  have been considered to explain the  $J/\psi(\psi')$  surplus problems discovered by CDF [3,4]. Taking the nonperturbative  $\langle \mathcal{O}_8^{J/\psi}(^3S_1) \rangle$  and  $\langle \mathcal{O}_8^{\psi'}(^3S_1) \rangle$  as input parameters, the CDF surplus problem for  $J/\psi$  and  $\psi'$  can be explained as the contributions of color-octet terms due to gluon fragmentation. In the past few years, applications of the NRQCD factorization formalism to  $J/\psi(\psi')$  production at various experimental facilities have been studied [5].

Even though the color-octet mechanism has gained some successes in describing the production and decays of heavy quark bound systems, it still needs more effort to go before finally setting its position and role in heavy quarkonium physics. (For instance, the photoproduction data from HERA [6] puts a question about the universality of the color-octet matrix elements [7], in which the fitted values of the matrix elements  $\langle \mathcal{O}_8^{J/\psi}(^1S_0) \rangle$  and  $\langle \mathcal{O}_8^{J/\psi}(^3P_J) \rangle$  are one order of magnitude smaller than those determined from the Tevatron data [4])<sup>1</sup> So, we must find other processes to test the color-octet mechanism in the heavy quarkonium production.

In our previous studies [9–12], we propose the  $D$ -wave heavy quarkonia production as a crucial test for the color-octet mechanism. We have calculated the  $D$ -wave quarkonium production via gluon fragmentation at the hadron colliders [10], in the  $Z^0$  decays [11], and the  $D$ -wave charmonia production in  $B$  decays [12]. All these results show that the color-octet mechanism is crucially important to  $D$ -wave charmonium production, and the color-octet

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<sup>1</sup>Possible solutions for this problem have been suggested recently in [8].

contributions are found to be over two orders of magnitude larger than the color-singlet contributions.

In this paper, we analyze the  $D$ -wave heavy quarkonium production in fixed target experiments, at energies in which the color-octet gluon fragmentation is not dominant. However, the analysis of  $S$ -wave charmonium and bottomonium production in fixed target experiments show that the color-octet contribution is important in removing large discrepancies between experiments and predictions from the color-singlet model [13]. We can further expect that the color-octet contribution may be dominant in the production of  $D$ -wave quarkonium production in fixed target, because within the color-singlet model the  $D$ -wave heavy quarkonia production is always suppressed by the second derivative of the wave function at the origin. But this suppression can be avoided in the color-octet model. Therefore, the  $D$ -wave heavy quarkonium production may provide another test for the color-octet mechanism. In addition, on the experimental side, there are some clues for the  $D$ -wave  $2^{--}$  charmonium state in  $E705$  300 GeV  $\pi^\pm$ - and proton-Li interaction experiment [14]. In this experiment there is an abnormal phenomenon that in the  $J/\psi\pi^+\pi^-$  mass spectrum, two peaks at  $\psi(3686)$  and at 3.836 GeV (given to be the  $2^{--}$  state) are observed and they have almost the same height. Obviously, this situation is difficult to explain based upon the color-singlet model. However, it might be explained within the scope of NRQCD.

In NRQCD the Fock state expansion for  ${}^3D_J$  states is

$$|{}^3D_J\rangle = O(1)|Q\bar{Q}({}^3D_J, \underline{1})\rangle + O(v)|Q\bar{Q}({}^3P_J, \underline{8})g\rangle + O(v^2)|Q\bar{Q}({}^3S_1, \underline{8} \text{ or } \underline{1})gg\rangle + \dots \quad (1)$$

In the above expansion, the contributions to the NRQCD matrix elements for the production of the  $D$ -wave charmonium from the three terms are the same order of  $v$  according to the NRQCD velocity scaling rules. So, in the factorization formula of  $D$ -wave quarkonium production, all of these three terms should be considered. Accordingly, the production cross section for the physical  $D$ -wave quarkonium state  $\delta_J$  in hadron process <sup>2</sup>

$$A + B \rightarrow \delta_J + X \quad (2)$$

can be written as

$$\sigma(A + B \rightarrow \delta_J + X) = \sum_{i,j} \int_0^1 dx_1 dx_2 f_{i/A}(x_1) f_{j/B}(x_2) \hat{\sigma}(ij \rightarrow \delta_J), \quad (3)$$

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<sup>2</sup>Here, the symbol  $\delta_J$  denotes the physical spin-triplet  $D$ -wave heavy quarkonium state. The notation  ${}^{2S+1}D_J$  represents the  $c\bar{c}$  or  $b\bar{b}$  pair configurations with angular momentum  $L = 2$ .

$$\hat{\sigma}(ij \rightarrow \delta_J) = \sum_n F(ij \rightarrow n) \langle \mathcal{O}_n^{\delta_J} \rangle. \quad (4)$$

Here,  $n$  denotes  $c\bar{c}$  pair configuration in the three Fock states of Eq. (1) (including angular momentum  $^{2S+1}L_J$  and color index 1 or 8).  $F(ij \rightarrow n)$  is the short distance coefficient for the subprocess  $ij \rightarrow n$ .  $\langle \mathcal{O}_n^{\delta_J} \rangle$  is the long distance non-perturbative matrix element which represents the property of the  $\bar{c}c$  pair in  $n$  configuration evolving into the physical state  $\delta_J$ . The short distance coefficient  $F$  can be calculated by using perturbative QCD in expansion of powers of  $\alpha_s$ . The long distance matrix elements are still not available from the first principle at present, which can be obtained by fitting the theoretical prediction with the experimental result in literatures. By the naive NRQCD velocity scaling rules, these three matrix elements are of the same order in powers of  $v$ . Their scaling properties are,

$$\langle \mathcal{O}_1^{\delta_J}({}^3D_J) \rangle \sim M_c^7 v^7, \quad \langle \mathcal{O}_8^{\delta_J}({}^3S_1) \rangle \sim M_c^3 v^7, \quad \langle \mathcal{O}_8^{\delta_J}({}^3P'_J) \rangle \sim M_c^5 v^7. \quad (5)$$

That is to say that the color-octet and color-singlet processes are of the same order in powers of  $v$ . However, the color-octet processes are enhanced by an order of  $\alpha_s$  over the color-singlet processes. So, in the hadroproduction of  $D$ -wave heavy quarkonium at fixed target experiments the color-octet contributions should be included.

First, we will calculate the color-singlet contribution to  ${}^3D_J$  quarkonium hadroproduction. The leading order color-singlet contribution comes from the gluon-gluon fusion process  $gg \rightarrow {}^3D_J g$ . The production rate of this process can be calculated by making use of the covariant formalism. The general form of the wave function of the spin-triplet heavy quarkonium state (with angular momentum  ${}^3L_J$ ) can be written as

$$\Phi(P, \vec{q}) = \frac{1}{M} \sum_{sm} \langle JM | 1sLm \rangle \Lambda_+^1(\vec{p}_1) \gamma_0 \not{\epsilon}^{(s)}(M + \not{P}) \gamma_0 \Lambda_-^2(\vec{p}_2) \psi_{Lm}(\vec{q}), \quad (6)$$

where  $\epsilon^{(s)}$  is the polarization vector associated with the spin-triplet states.  $\Lambda_+^1(\vec{p}_1)$  and  $\Lambda_-^2(\vec{p}_2)$  are positive energy projection operators of quark and antiquark .

$$\Lambda_+^1(\vec{p}_1) = \frac{E_1 + \gamma_0 \vec{\gamma} \cdot \vec{p}_1 + M_c \gamma_0}{2E_1}, \quad \Lambda_-^2(\vec{p}_2) = \frac{E_2 - \gamma_0 \vec{\gamma} \cdot \vec{p}_2 - M_c \gamma_0}{2E_2}. \quad (7)$$

For the  $D$ -wave function,  $\Phi(P, \vec{q})$  must be expanded to the second order in the relative momentum  $\vec{q}$ . The first and second derivatives of the wave function are

$$\Phi_\alpha(\vec{q}) = \frac{-1}{2M_c^2 M} \sum_{sm} \langle JM | 1sLm \rangle [M_c \gamma_\alpha \not{\epsilon}^{(s)}(M + \not{P}) + M_c \not{\epsilon}^{(s)}(M + \not{P}) \gamma_\alpha] \psi_{Lm}(\vec{q}); \quad (8)$$

$$\Phi_{\alpha\beta}(\vec{q}) = \frac{-1}{2M_c^2 M} \sum_{sm} \langle JM | 1sLm \rangle \gamma_\alpha \not{\epsilon}^{(s)}(M + \not{P}) \gamma_\beta \psi_{Lm}(\vec{q}). \quad (9)$$

After integrating over the relative momentum  $\vec{q}$ , the orbit angular momentum part of the wave function will depend on the radial wave function or its derivatives at the origin  $R_S(0)$ ,  $R'_P(0)$ , and  $R''_D(0)$ , respectively, for  $S$ -wave ( $L = 0$ ),  $P$ -wave ( $L = 1$ ), and  $D$ -wave ( $L = 2$ ) states,

$$\int \frac{d^3q}{(2\pi)^3} \psi_{00}(\mathbf{q}) = \frac{1}{\sqrt{4\pi}} R_S(0), \quad (10)$$

$$\int \frac{d^3q}{(2\pi)^3} q^\alpha \psi_{1m}(\mathbf{q}) = i\epsilon^\alpha \sqrt{\frac{3}{4\pi}} R'_P(0), \quad (11)$$

$$\int \frac{d^3q}{(2\pi)^3} q^\alpha q^\beta \psi_{2m}(\mathbf{q}) = e_m^{\alpha\beta} \sqrt{\frac{15}{8\pi}} R''_D(0), \quad (12)$$

where the polarization tensor's label  $m$  is magnetic quantum number. For the spin-triplet case where  $J = 1, 2, 3$ , using explicit Clebsch-Gordan coefficients, there are following relations for the three cases [15].

$$\begin{aligned} \sum_{sm} \langle 1J_z | 1s2m \rangle \epsilon_{\alpha\beta}^{(m)} \epsilon_\rho^{(s)} &= -\left[\frac{3}{20}\right]^{1/2} \left[ (g_{\alpha\rho} - \frac{p_\alpha p_\rho}{4M_c^2}) \epsilon_\beta^{(J_z)} + (g_{\beta\rho} - \frac{p_\beta p_\rho}{4M_c^2}) \epsilon_\alpha^{(J_z)} \right. \\ &\quad \left. - \frac{2}{3} (g_{\alpha\beta} - \frac{p_\alpha p_\beta}{4M_c^2}) \epsilon_\rho^{(J_z)} \right], \end{aligned} \quad (13)$$

$$\sum_{sm} \langle 2J_z | 1s2m \rangle \epsilon_{\alpha\beta}^{(m)} \epsilon_\rho^{(s)} = \frac{i}{2\sqrt{6}M_c} (\epsilon_{\alpha\sigma}^{(J_z)} \epsilon_{\tau\beta\rho\sigma'} p^\tau g^{\sigma\sigma'} + \epsilon_{\beta\sigma}^{(J_z)} \epsilon_{\tau\alpha\rho\sigma'} p^\tau g^{\sigma\sigma'}), \quad (14)$$

$$\sum_{sm} \langle 3J_z | 1s2m \rangle \epsilon_{\alpha\beta}^{(m)} \epsilon_\rho^{(s)} = \epsilon_{\alpha\beta\rho}^{(J_z)}. \quad (15)$$

Here,  $\epsilon_\alpha$ ,  $\epsilon_{\alpha\beta}$ ,  $\epsilon_{\alpha\beta\rho}$  are the spin-one, spin-two and spin-three polarization tensors which obey the projection relations [15]

$$\sum_m \epsilon_\alpha^{(m)} \epsilon_\beta^{(m)} = (-g_{\alpha\beta} + \frac{p_\alpha p_\beta}{4M_c^2}) \equiv \mathcal{P}_{\alpha\beta}, \quad (16)$$

$$\sum_m \epsilon_{\alpha\beta}^{(m)} \epsilon_{\alpha'\beta'}^{(m)} = \frac{1}{2} [\mathcal{P}_{\alpha\alpha'} \mathcal{P}_{\beta\beta'} + \mathcal{P}_{\alpha\beta'} \mathcal{P}_{\beta\alpha'}] - \frac{1}{3} \mathcal{P}_{\alpha\beta} \mathcal{P}_{\alpha'\beta'}, \quad (17)$$

$$\begin{aligned} \sum_m \epsilon_{\alpha\beta\rho}^{(m)} \epsilon_{\alpha'\beta'\rho'}^{(m)} &= \frac{1}{6} (\mathcal{P}_{\alpha\alpha'} \mathcal{P}_{\beta\beta'} \mathcal{P}_{\rho\rho'} + \mathcal{P}_{\alpha\alpha'} \mathcal{P}_{\beta\rho'} \mathcal{P}_{\beta\rho'} + \mathcal{P}_{\alpha\beta'} \mathcal{P}_{\beta\alpha'} \mathcal{P}_{\rho\rho'} \\ &\quad + \mathcal{P}_{\alpha\beta'} \mathcal{P}_{\beta\rho'} \mathcal{P}_{\rho\alpha'} + \mathcal{P}_{\alpha\rho'} \mathcal{P}_{\beta\beta'} \mathcal{P}_{\rho\alpha'} + \mathcal{P}_{\alpha\rho'} \mathcal{P}_{\beta\alpha'} \mathcal{P}_{\rho\beta'}) \\ &\quad - \frac{1}{15} (\mathcal{P}_{\alpha\beta} \mathcal{P}_{\rho\alpha'} \mathcal{P}_{\beta'\rho'} + \mathcal{P}_{\alpha\beta} \mathcal{P}_{\rho\beta'} \mathcal{P}_{\alpha'\rho'} + \mathcal{P}_{\alpha\beta} \mathcal{P}_{\rho\rho'} \mathcal{P}_{\alpha'\beta'} \\ &\quad + \mathcal{P}_{\alpha\rho} \mathcal{P}_{\beta\alpha'} \mathcal{P}_{\beta'\rho'} + \mathcal{P}_{\alpha\rho} \mathcal{P}_{\beta\beta'} \mathcal{P}_{\alpha'\rho'} + \mathcal{P}_{\alpha\rho} \mathcal{P}_{\beta\rho'} \mathcal{P}_{\alpha'\beta'} \\ &\quad + \mathcal{P}_{\beta\rho} \mathcal{P}_{\alpha\alpha'} \mathcal{P}_{\beta'\rho'} + \mathcal{P}_{\beta\rho} \mathcal{P}_{\alpha\beta'} \mathcal{P}_{\alpha'\rho'} + \mathcal{P}_{\beta\rho} \mathcal{P}_{\alpha\rho'} \mathcal{P}_{\alpha'\beta'}). \end{aligned} \quad (18)$$

So, the differential cross section for the color-singlet process  $gg \rightarrow \delta_J X$  can be calculated, as an expansion of the form in Eq.(3),

$$\frac{d\sigma(gg \rightarrow \delta_J g)_1}{dt} = F(gg \rightarrow {}^3D_J g) \times \langle \mathcal{O}_1^{\delta_J}({}^3D_J) \rangle, \quad (19)$$

where the color-singlet matrix element  $\langle \mathcal{O}_1^{\delta_J}({}^3D_J) \rangle$  can be related to the second derivative of the nonrelativistic radial wave function at the origin  $|R_D''(0)|^2$  for  $D$ -wave by

$$\langle \mathcal{O}_1^{\delta_J}({}^3D_J) \rangle = \frac{15(2J+1)N_c}{4\pi} |R_D''(0)|^2. \quad (20)$$

The calculation of the short distance coefficient  $F(gg \rightarrow {}^3D_J)g$  is straightforward. Because the results are lengthy, we give the expressions in the Appendix. As in the case of gluon fragmentation to color-singlet  ${}^3D_J$  [10],  $gg \rightarrow {}^3D_J g$  processes also have the infrared divergences involved, which are associated with the soft gluon in the final state. In our calculations of the cross section for these processes, we follow the way in Ref. [10] by imposing a lower cutoff  $\Lambda$  on the energy of the outgoing gluon in the quarkonium rest frame. The cutoff  $\Lambda$  can be set to be  $m_Q$  to avoid large logarithms in the divergence terms.

For the color-octet contributions in the  $gg, gq, q\bar{q}$  subprocesses, we readily have [4,13]

$$\hat{\sigma}(gg \rightarrow \delta_J)_8 = \frac{5\pi^3\alpha_s^2}{12(2m_Q)^3 s} \delta(x_1 x_2 - 4m_Q^2/s) \left[ \frac{3}{m_Q^2} \langle \mathcal{O}_8^{\delta_J}({}^3P_0) \rangle + \frac{4}{5m_Q^2} \langle \mathcal{O}_8^{\delta_J}({}^3P_2) \rangle \right], \quad (21)$$

$$\hat{\sigma}(gq \rightarrow \delta_J)_8 = 0, \quad (22)$$

$$\hat{\sigma}(q\bar{q} \rightarrow \delta_J)_8 = \frac{16\pi^3\alpha_s^2}{27(2m_Q)^3 s} \delta(x_1 x_2 - 4m_Q^2/s) \langle \mathcal{O}_8^{\delta_J}({}^3S_1) \rangle. \quad (23)$$

Here  $\sqrt{s}$  is the center-of-mass energy, and  $\alpha_s$  is normalized at the scale  $2m_Q$ .

Our numerical results are displayed in Fig.1 to Fig.4. We use the Glück-Reya-Vogt (GRV) LO parton distributions for the proton and the pion [16,17]. We set the renormalization scale to be  $2m_Q$ .

In Fig.1 and Fig.2, we plot the total cross section of  $D$ -wave heavy quarkonium for proton-nucleon collisions for  $x_F > 0$ , where we choose the color-octet matrix elements to be related to the color-singlet matrix elements according to the naive NRQCD velocity scaling rules,

$$\frac{\langle \mathcal{O}_8^{\delta_c}({}^3S_1) \rangle}{M_c^3} \approx \frac{\langle \mathcal{O}_8^{\delta_c}({}^3P_1) \rangle}{M_c^5} \approx \frac{\langle \mathcal{O}_1^{\delta_c}({}^3D_1) \rangle}{M_c^7}, \quad (24)$$

$$\frac{\langle \mathcal{O}_8^{\delta_b}({}^3S_1) \rangle}{M_b^3} \approx \frac{\langle \mathcal{O}_8^{\delta_b}({}^3P_1) \rangle}{M_b^5} \approx \frac{\langle \mathcal{O}_1^{\delta_b}({}^3D_1) \rangle}{M_b^7}. \quad (25)$$

And the heavy quark spin symmetry relations for the color-octet matrix elements have been used,

$$\langle \mathcal{O}_8^{\delta_J}({}^3S_1) \rangle \approx \frac{2J+1}{3} \langle \mathcal{O}_8^{\delta_1}({}^3S_1) \rangle, \quad (26)$$

$$\langle \mathcal{O}_8^{\delta_J}({}^3P_1) \rangle \approx \frac{2J+1}{3} \langle \mathcal{O}_8^{\delta_1}({}^3P_1) \rangle. \quad (27)$$

The values of color-singlet matrix elements are obtained from the potential model calculation,  $|R_D''(0)|_c^2 = 0.015 GeV^7$ ,  $|R_D''(0)|_b^2 = 0.637 GeV^7$  [18]. Fig.1 is the cross section for the  $D$ -wave charmonium and Fig.2 for the  $D$ -wave bottomonium respectively. From these figures, we can see that the color-octet contributions are two orders of magnitude larger than the color-singlet contributions.

It must be noted that all of the cross sections are expressed in terms of heavy quark mass as  $m_c$  and  $m_b$ , for which we take values as

$$m_c = 1.5 GeV, \quad M_b = 4.9 GeV.$$

If we choose the charm quark mass as one half of the charmonium mass (e.g.,  $\delta^c \approx 3.8 GeV$ ), the theoretical prediction of the cross section will be reduced by an order of magnitude for the color-singlet contributions and by a factor of five for the color-octet contributions at a typical energy scale  $\sqrt{s} = 24 GeV$ . Compared with this large uncertainty, the variation due to the choice of parton distribution functions and  $\alpha_s(\mu)$  is negligible.

In Fig.3 and Fig.4, we plot the cross sections of  $D$ -wave charmonium and bottomonium states in pion-nucleon collisions for  $x_F > 0$ . Similar to the case of proton-nucleon collisions, in pion-nucleon collisions we can see from these figures that the color-octet contributions are two order of magnitude larger than the color-singlet contributions.

Among the three triplet states of  $D$ -wave charmonium,  $\delta_2^c$  is the most promising candidate to discover firstly. Its mass falls in the range of  $3.810 \sim 3.840$  GeV in the potential model calculation, that is below the  $D\bar{D}^*$  threshold but above the  $D\bar{D}$  threshold. However, the parity conservation forbids it decaying into  $D\bar{D}$ . It, therefore, is a narrow resonance. Its main decay modes are expected to be,

$$\delta_2^c \rightarrow J/\psi \pi \pi, \quad \delta_2^c \rightarrow {}^3P_J \gamma (J = 1, 2), \quad \delta_2^c \rightarrow 3g. \quad (28)$$

The hadronic transition rate of  $\delta_2^c \rightarrow J/\psi \pi^+ \pi^-$  is estimated to be [11]

$$\Gamma(\delta_2^c \rightarrow J/\psi \pi^+ \pi^-) = \Gamma({}^3D_1 \rightarrow J/\psi \pi^+ \pi^-) \approx 46 keV. \quad (29)$$

For the E1 transition  $\delta_c^2 \rightarrow {}^3P_J \gamma (J = 1, 2)$ , using the potential model with relativistic effects being considered [19], we find

$$\Gamma(\delta_2^c \rightarrow \chi_{c1}\gamma) = 250 \text{ keV}, \quad \Gamma(\delta_2^c \rightarrow \chi_{c2}\gamma) = 60 \text{ keV}, \quad (30)$$

where the mass of  $\delta_2^c$  is set to be  $3.84\text{GeV}$ . As for the  $\delta_2^c \rightarrow 3g$  annihilation decay, an estimate gives [15]

$$\Gamma(\delta_2^c \rightarrow 3g) = 12 \text{ keV}. \quad (31)$$

From (29), (30), and (31), we find

$$\begin{aligned} \Gamma_{tot}(\delta_2^c) &\approx \Gamma(\delta_2^c \rightarrow J/\psi\pi\pi) + \Gamma(\delta_2^c \rightarrow \chi_{c1}\gamma) + \Gamma(\delta_2^c \rightarrow \chi_{c2}\gamma) + \Gamma(\delta_2^c \rightarrow 3g) \\ &\approx 390 \text{ keV}, \end{aligned} \quad (32)$$

and

$$B(\delta_2^c \rightarrow J/\psi\pi^+\pi^-) \approx 0.12. \quad (33)$$

Comparing (33) with  $B(\psi' \rightarrow J/\psi\pi^+\pi^-) = 0.324 \pm 0.026$ , the branching ratio of  $\delta_2^c \rightarrow J/\psi\pi^+\pi^-$  is only smaller by a factor of 3, and therefore the decay mode of  $\delta_2^c \rightarrow J/\psi\pi^+\pi^-$  could be observable in various experiments, such as at hadron colliders and at fixed target experiments.

Multiplied by the assumed branching ratio of  $\delta_2^c \rightarrow J/\psi\pi^+\pi^-$  above, we can estimate the production rate of  $\delta_2^c$  in pion-nucleon collisions. From Fig.3, at  $\sqrt{s} \approx 24\text{GeV}$ , the cross section is predicted to be  $\sigma(\pi^- N \rightarrow \delta_2^c + X)B(\delta_2^c \rightarrow J/\psi\pi^+\pi^-) = 4\text{nb}$  per nucleon. This value is in agreement with the preliminary result of E705, where the cross section is estimated to be  $5.3 \pm 1.9 \pm 1.3\text{nb}$  [14]. From Fig.3, we can see that the experimental result of E705 collaboration can not be explained within the color-singlet model, because the prediction of the color-singlet model is two orders of magnitude smaller than the color-octet model. This indicates that though the agreement with the E705 data can not be taken too seriously (due to large theoretical uncertainties, e.g., the choice of charm quark mass; the naive estimate for the color-octet matrix elements), an appreciable  $D$ -wave signal can only be explained by the color-octet mechanism. Experimentally, the strong signal of  $J/\psi\pi^+\pi^-$  at  $3.836\text{GeV}$  observed by E705 is now questioned by other experiments. (E705 observed  $(77 \pm 21)$  and  $(58 \pm 21)$  background-subtracted events at  $\psi'(2S)$  and  $3.836 \text{ GeV}$  respectively [14]; while E672-E706 reported  $(224 \pm 44)$  and  $(52 \pm 30)$  background-subtracted events at  $\psi'(2S)$  and  $3.836 \text{ GeV}$  respectively [20]). Nevertheless, if the E705 result is confirmed (even with a smaller rate, say, by a factor of 3, for the signal at  $3.836\text{GeV}$ , as might be implied by other experiments [20]), the color-octet production mechanism may provide a quite unique explanation for the  $D$ -wave charmonium production.



Moreover, the  $1^{--}$   $D$ -wave charmonium state  $\psi(3770)$  may also be observable at the fixed target experiments, with a rate smaller by a factor of  $\frac{5}{3}$  than the  $2^{--}$  state, provided the decay channel  $\psi(3770) \rightarrow D\bar{D}$  (with about 100% decay branching ratio to the charmed meson pairs) can be detected.

In conclusion, in this paper, we have calculated the  $D$ -wave heavy quarkonium production in fixed target experiments. We find that the color-octet mechanism plays an important role in the production. The color-octet contributions are two order of magnitude higher than the color-singlet contributions both in proton-nucleon and pion-nucleon collisions. Despite of some theoretical uncertainties, the prediction of the color-octet model is found to be in agreement with the preliminary result of E705 collaboration. This may provide another positive support to the color-octet production mechanism of heavy quarkonium.

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**APPENDIX:**

The short distance coefficients for the color-singlet processes  $gg \rightarrow {}^3D_J$  in Eq.(19) are,

$$\begin{aligned}
 F(gg \rightarrow {}^3D_{1g}) = & \frac{16\alpha_s^3\pi^2}{81M^3s^2(M^2-s)^5(M^2-t)^5(s+t)^5} \{ 102M^{20}s^3 + 302M^{20}s^2t + 302M^{20}st^2 \\
 & + 102M^{20}t^3 - 286M^{18}s^4 - 1732M^{18}s^3t - 2844M^{18}s^2t^2 - 1732M^{18}st^3 \\
 & - 286M^{18}t^4 + 275M^{16}s^5 + 3840M^{16}s^4t + 10289M^{16}s^3t^2 + 10289M^{16}s^2t^3 \\
 & + 3840M^{16}st^4 + 275M^{16}t^5 - 227M^{14}s^6 - 5004M^{14}s^5t - 19569M^{14}s^4t^2 \\
 & - 29536M^{14}s^3t^3 - 19569M^{14}s^2t^4 - 5004M^{14}st^5 - 227M^{14}t^6 + 410M^{12}s^7 \\
 & + 5137M^{12}s^6t + 23585M^{12}s^5t^2 + 47908M^{12}s^4t^3 + 47908M^{12}s^3t^4 \\
 & + 23585M^{12}s^2t^5 + 5137M^{12}st^6 + 410M^{12}t^7 - 470M^{10}s^8 - 4220M^{10}s^7t \\
 & - 19534M^{10}s^6t^2 - 47528M^{10}s^5t^3 - 63536M^{10}s^4t^4 - 47528M^{10}s^3t^5 \\
 & - 19534M^{10}s^2t^6 - 4220M^{10}st^7 - 470M^{10}t^8 + 245M^8s^9 + 2190M^8s^8t \\
 & + 10358M^8s^7t^2 + 28602M^8s^6t^3 + 47093M^8s^5t^4 + 47093M^8s^4t^5 \\
 & + 28602M^8s^3t^6 + 10358M^8s^2t^7 + 2190M^8st^8 + 245M^8t^9 - 49M^6s^{10} \\
 & - 580M^6s^9t - 2822M^6s^8t^2 - 8984M^6s^7t^3 - 17653M^6s^6t^4 - 21968M^6s^5t^5 \\
 & - 17653M^6s^4t^6 - 8984M^6s^3t^7 - 2822M^6s^2t^8 - 580M^6st^9 - 49M^6t^{10} \\
 & + 67M^4s^{10}t + 210M^4s^9t^2 + 774M^4s^8t^3 + 2006M^4s^7t^4 + 3147M^4s^6t^5 \\
 & + 3147M^4s^5t^6 + 2006M^4s^4t^7 + 774M^4s^3t^8 + 210M^4s^2t^9 + 67M^4st^{10} \\
 & + 25M^2s^{10}t^2 + 100M^2s^9t^3 + 220M^2s^8t^4 + 340M^2s^7t^5 + 390M^2s^6t^6 \\
 & + 340M^2s^5t^7 + 220M^2s^4t^8 + 100M^2s^3t^9 + 25M^2s^2t^{10} + 5s^{10}t^3 \\
 & + 25s^9t^4 + 60s^8t^5 + 90s^7t^6 + 90s^6t^7 + 60s^5t^8 + 25s^4t^9 + 5s^3t^{10} \},
 \end{aligned}$$

$$\begin{aligned}
 F(gg \rightarrow {}^3D_{2g}) = & \frac{32\alpha_s^3\pi^2}{27M^3s^2(M^2-s)^5(M^2-t)^5(s+t)^5} \{ M^{20}s^3 + 9M^{20}s^2t + 9M^{20}st^2 + M^{20}t^3 \\
 & - 3M^{18}s^4 - 64M^{18}s^3t - 130M^{18}s^2t^2 - 64M^{18}st^3 - 3M^{18}t^4 + 5M^{16}s^5 \\
 & + 180M^{16}s^4t + 611M^{16}s^3t^2 + 611M^{16}s^2t^3 + 180M^{16}st^4 - 11M^{14}s^6 \\
 & - 280M^{14}s^5t - 1369M^{14}s^4t^2 - 2208M^{14}s^3t^3 - 1369M^{14}s^2t^4 - 280M^{14}st^5 \\
 & - 11M^{14}t^6 + 20M^{12}s^7 + 269M^{12}s^6t + 1716M^{12}s^5t^2 + 3927M^{12}s^4t^3 \\
 & + 3927M^{12}s^3t^4 + 1716M^{12}s^2t^5 + 269M^{12}st^6 + 20M^{12}t^7 - 20M^{10}s^8 \\
 & - 1283M^{10}s^6t^2 - 3888M^{10}s^5t^3 - 5450M^{10}s^4t^4 - 3888M^{10}s^3t^5 \\
 & - 1283M^{10}s^2t^6 - 144M^{10}st^7 - 20M^{10}t^8 + 10M^8s^9 + 16M^8s^8t + 5M^{16}t^5
 \end{aligned}$$

$$\begin{aligned}
& + 568M^8s^7t^2 + 2365M^8s^6t^3 + 4181M^8s^5t^4 + 4181M^8s^4t^5 + 2365M^8s^3t^6 \\
& + 568M^8s^2t^7 + 16M^8st^8 + 10M^8t^92M^6s^{10} + 20M^6s^9t - 144M^{10}s^7t \\
& - 156M^6s^8t^2 - 1072M^6s^7t^3 - 2230M^6s^6t^4 - 2664M^6s^5t^5 - 2230M^6s^4t^6 \\
& - 1072M^6s^3t^7 - 156M^6s^2t^8 + 20M^6st^9 - 2M^6t^{10} - 6M^4s^{10}t \\
& + 50M^4s^9t^2 + 472M^4s^8t^3 + 1172M^4s^7t^4 + 1570M^4s^6t^5 + 1570M^4s^5t^6 \\
& + 1172M^4s^4t^7 + 472M^4s^3t^8 + 50M^4s^2t^9 - 6M^4st^{10} - 16M^2s^{10}t^2 \\
& - 160M^2s^9t^3 - 496M^2s^8t^4 - 832M^2s^7t^5 - 960M^2s^6t^6 - 832M^2s^5t^7 \\
& - 496M^2s^4t^8 - 160M^2s^3t^9 - 16M^2s^2t^{10} + 16s^{10}t^3 + 80s^9t^4 \\
& + 192s^8t^5 + 288s^7t^6 + 288s^6t^7 + 192s^5t^880s^4t^9 + 16s^3t^{10}\},
\end{aligned}$$

$$\begin{aligned}
F(gg \rightarrow^3 D_{3g}) &= \frac{256\alpha_s^3\pi^2}{189M^3s^2(M^2-s)^5(M^2-t)^5(s+t)^5} \{8M^{20}s^3 + 18M^{20}s^2t + 18M^{20}st^2 \\
& + 8M^{20}t^3 - 24M^{18}s^4 - 128M^{18}s^3t - 206M^{18}s^2t^2 - 128M^{18}st^3 \\
& - 24M^{18}t^4 + 25M^{16}s^5 + 300M^{16}s^4t + 826M^{16}s^3t^2 + 826M^{16}s^2t^3 \\
& + 300M^{16}st^4 + 25M^{16}t^5 - 13M^{14}s^6 - 356M^{14}s^5t - 1556M^{14}s^4t^2 \\
& - 2424M^{14}s^3t^3 - 1556M^{14}s^2t^4 - 356M^{14}st^5 - 13M^{14}t^6 + 10M^{12}s^7 \\
& + 1680M^{12}s^5t^2 + 3717M^{12}s^4t^3 + 3717M^{12}s^3t^4 + 1680M^{12}s^2t^5 \\
& + 283M^{12}st^6 + 10M^{12}t^7 - 10M^{10}s^8 - 180M^{10}s^7t - 1201M^{10}s^6t^2 \\
& - 3342M^{10}s^5t^3 - 4624M^{10}s^4t^4 - 3342M^{10}s^3t^5 - 1201M^{10}s^2t^6 \\
& - 180M^{10}st^7 - 10M^{10}t^8 + 5M^8s^9 + 80M^8s^8t + 602M^8s^7t^2 \\
& + 1943M^8s^6t^3 + 3307M^8s^5t^4 + 3307M^8s^4t^5 + 1943M^8s^3t^6 \\
& + 602M^8s^2t^7 + 80M^8st^8 + 5M^8t^9 - M^6s^{10} - 20M^6s^9t - 198M^6s^8t^2 \\
& - 776M^6s^7t^3 - 1502M^6s^6t^4 - 1812M^6s^5t^5 - 1502M^6s^4t^6 - 776M^6s^3t^7 \\
& - 198M^6s^2t^8 - 20M^6st^9 - M^6t^{10} + 3M^4s^{10}t + 40M^4s^9t^2 + 221M^4s^8t^3 \\
& + 514M^4s^7t^4 + 698M^4s^6t^5 + 698M^4s^5t^6 + 514M^4s^4t^7 + 221M^4s^3t^8 \\
& + 3M^4st^{10} - 5M^2s^{10}t^2 - 50M^2s^9t^3 - 155M^2s^8t^4 - 260M^2s^7t^5 \\
& - 300M^2s^6t^6 - 260M^2s^5t^7 - 155M^2s^4t^8 - 50M^2s^3t^9 \\
& - 5M^2s^2t^{10} + 5s^{10}t^3 + 25s^9t^4 + 60s^8t^5 + 90s^7t^6 + 90s^6t^7 \\
& + 60s^5t^8 + 25s^4t^9 + 5s^3t^{10} + 283M^{12}s^6t\},
\end{aligned}$$

where  $M = 2m_Q$  and  $s, t, u$  are Mandelstern invariants for the processes  $gg \rightarrow^3 D_{Jg}$ .

## Figure Captions

Fig.1.  $D$ -wave charmonium production in proton-nucleon collisions for  $x_F > 0$ . The dashed lines are the color-singlet contributions and the solid lines are color-octet contributions. For the solid lines, from up to down, they are for  $J = 3, 2, 1$   $D$ -wave states respectively. For the dashed lines, from up to down, they are for  $J = 3, 1, 2$  states respectively.

Fig.2  $D$ -wave bottomonium production in proton-nucleon collisions for  $x_F > 0$ . The lines are the same as those in Fig.1.

Fig.3  $D$ -wave charmonium production in pion-nucleon collisions for  $x_F > 0$ . The lines are the same as those in Fig.1.

Fig.4  $D$ -wave bottomonium production in pion-nucleon collisions for  $x_F > 0$ . The lines are the same as those in Fig.1.

## FIGURES

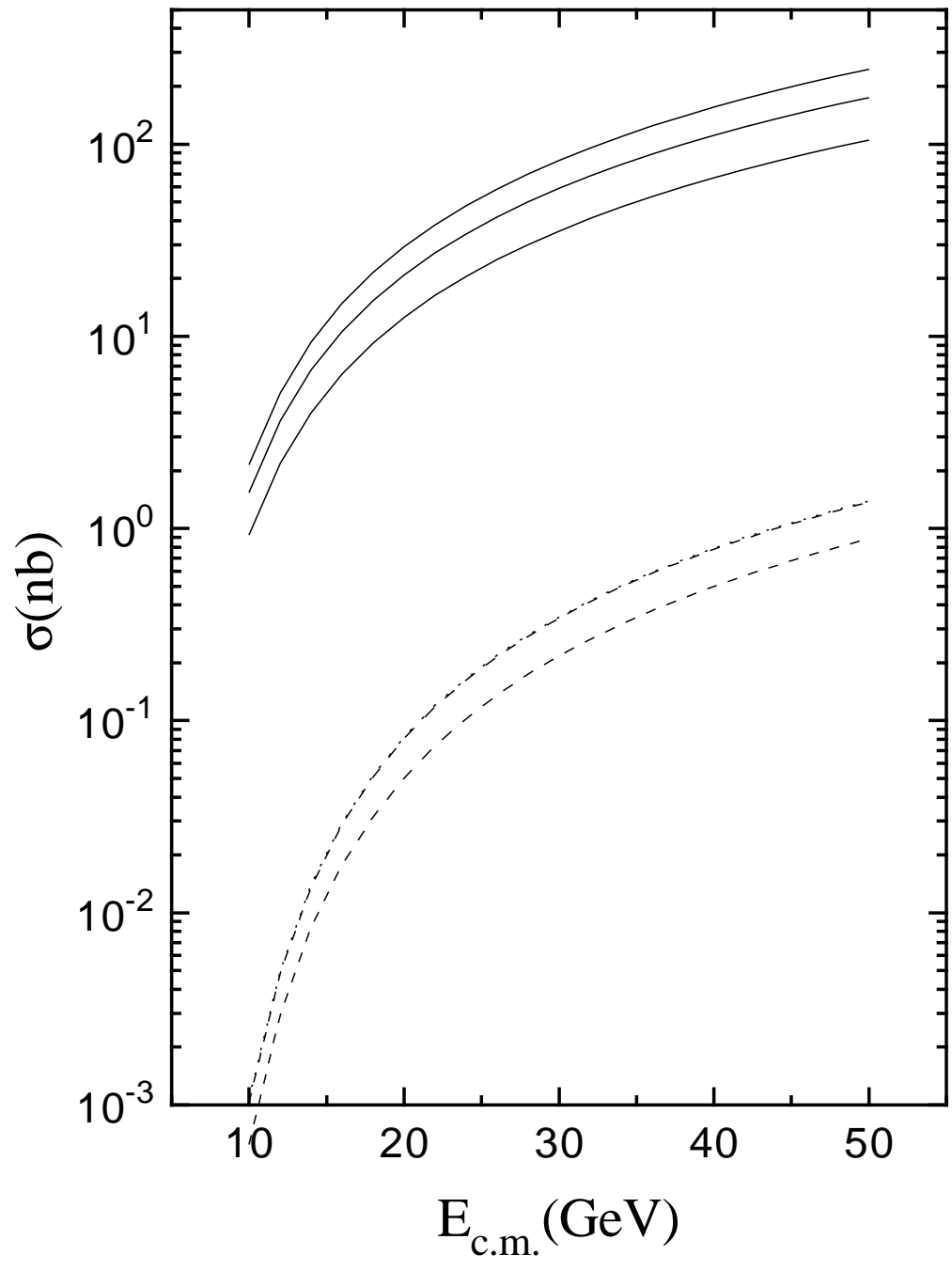


Fig.1

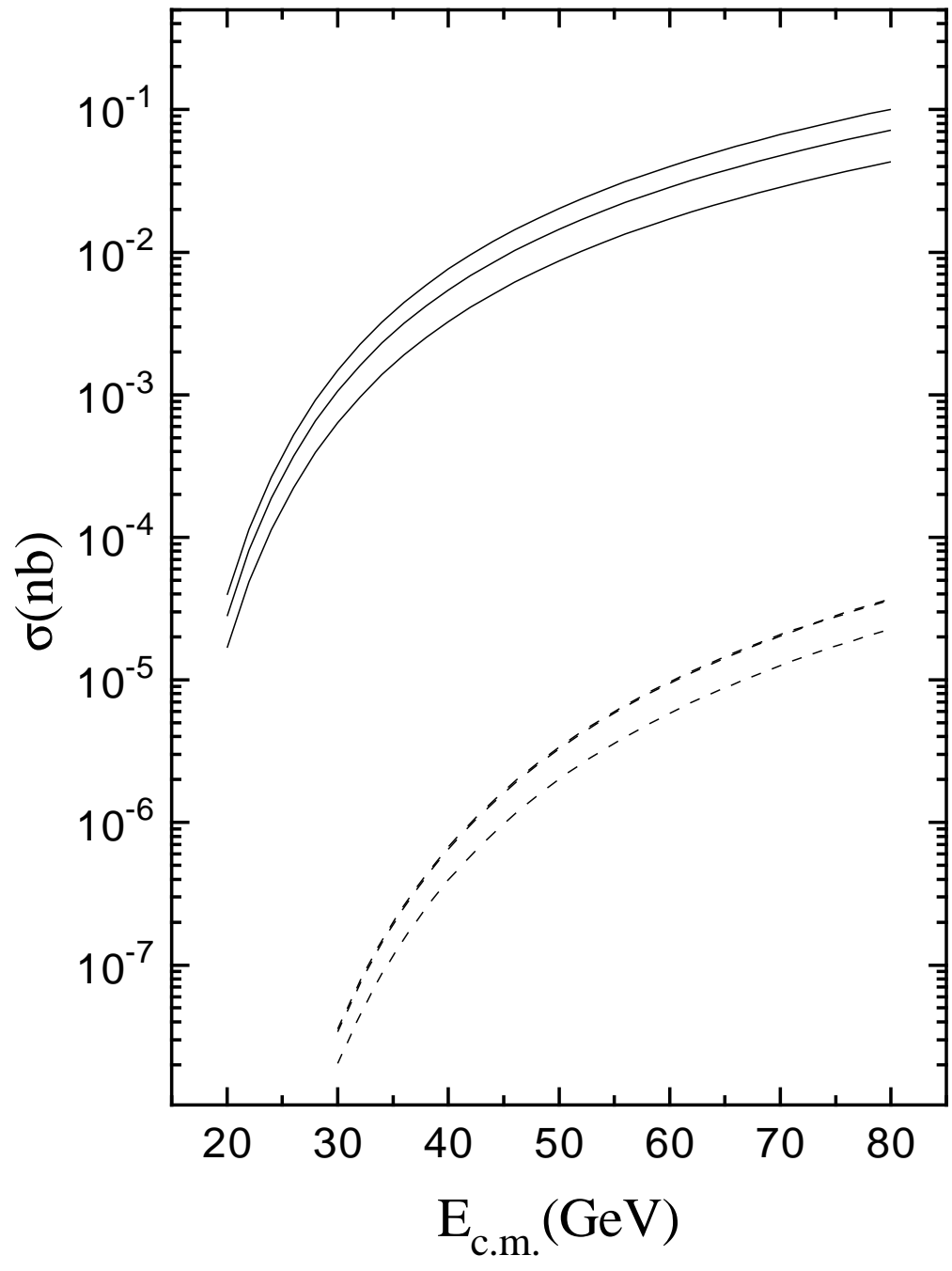


Fig.2



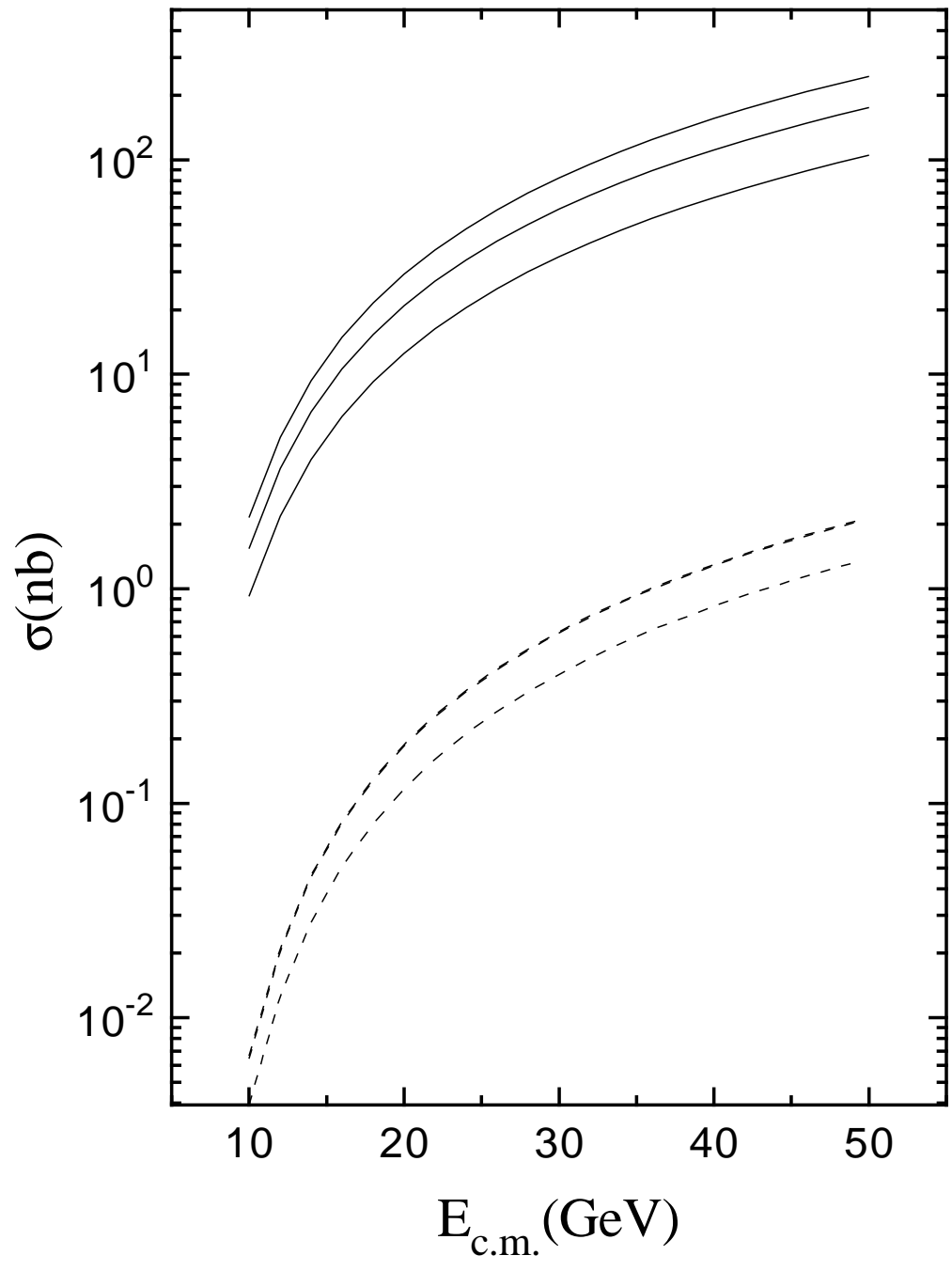


Fig.3

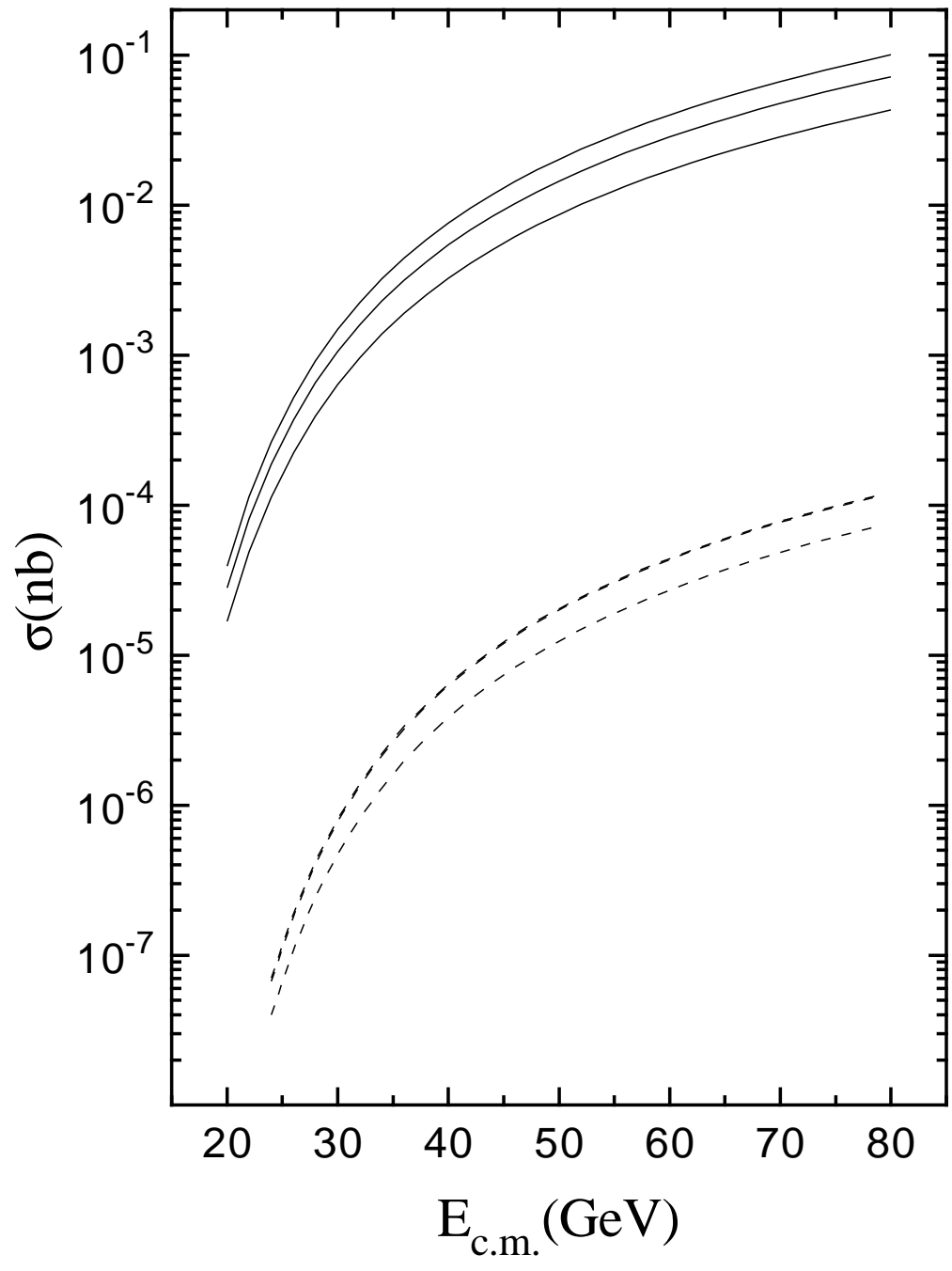


Fig.4