

Masses of Orbitally Excited Baryons in Large N_c QCD

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Abstract

We present the first phenomenological study of the masses of orbitally excited baryons in large N_c QCD. Restricting here to the nonstrange sector of the $\ell = 1$ baryons, the $1/N_c$ expansion is used to order and select a basis of effective operators that spans the nine observables (seven masses and two mixing angles). Fits are performed using subsets of the complete set of nine operators, including corrections up to $O(1/N_c)$ where leading order is N_c^1 . This study shows that the $1/N_c$ expansion provides an excellent framework for analyzing the mass spectrum, and uncovers a new hierarchy of operator contributions.

I. INTRODUCTION

It appears that QCD admits a useful and elegant expansion in powers of $1/N_c$, where N_c is the number of colors [1]. There are explicit rules that determine the order in N_c of any given Feynman diagram or matrix elements of any given operator. One can thus isolate first the leading contributions to the observable under consideration and then systematically include contributions that are proportional to higher powers of $1/N_c$.

One may question whether $1/N_c = 1/3$ is small enough to provide a valid phenomenological expansion parameter. Experience suggests that it is. For the ground state baryons, the large N_c approach has been used successfully to study SU(6) spin-flavor symmetry [2–6], masses [4,7–9], magnetic moments [4,8,10–12], and axial current matrix elements [2,4,8,12].

The next natural step in this progression is studies of excited baryons, in particular of the mixed symmetry, negative parity **70**-plet of SU(6). There have been studies of the strong [13,14] and radiative [15] decays of these states and of the structure of axial operator matrix elements [16]. Progress [17] along the lines of the present work has already been made with the first non-leading mass operators.

In this paper, we study the relative order $1/N_c$ and $1/N_c^2$ corrections to the masses of the nonstrange members of the **70**-plet. Qualitatively, we find that the large N_c limit is accurate for the **70**-plet, in the sense that all the mass operators—if one wishes, all the mass terms in an effective Hamiltonian—give contributions at or below the level estimated from large N_c considerations. None are larger. Quantitative detail is added to this statement in Section III.

Before proceeding, we make some comments on the nature of the **70**-plet and the nomenclature we use. We describe the state as a symmetrized “core” of $(N_c - 1)$ quarks in the ground state plus one excited quark in a relative P state. The wave function is antisymmetric in color and symmetric in SU(6) \times O(3), where SU(6) is the spin-flavor symmetry and O(3) is the rotation group. We use SU(6) to classify the large N_c baryon states and the transformation properties of the operators; however, we need not assume that SU(6) is

an exact symmetry. In fact, while the leading order contribution to the masses is of $O(N_c)$, $SU(6)$ is broken at $O(N_c^0)$ for our states.

One can analyze the masses in the **70**-plet by expressing the effective Hamiltonian as a sum of operators, one of which is proportional to the identity and the rest are products of $SU(6) \times O(3)$ generators times numerical coefficients. Each operator contributes to the mass at a definite order in N_c , which is determined by rules delineated in Sec. II.

The operator analysis for the excited baryons involves more distinguishable generators than the ground state baryons, because of the orbital angular momentum and because one quark is singled out. This leads to a much larger collection of mass operators. A list is given in the next section, which shows that there is one operator of $O(N_c)$, two operators of $O(1)$, and many of $O(1/N_c)$ or smaller.

Regarding the states, nonstrange mixed symmetry $SU(6)$ states with one quark singled out have total quark spin and isospin related by $S = I$ or $I \pm 1$, with each of S and I in the range $1/2$ to $N_c/2$. (There is one exception: There are no doubly maximal mixed symmetry $S = I = N_c/2$ states.) Thinking of the states as a core of $N_c - 1$ ground state quarks with spin S_c and isospin I_c combined with an excited quark with angular momentum $\ell = 1$ leads to writing the $SU(6) \times O(3)$ states, with the help of Clebsch-Gordan coefficients, as

$$\begin{aligned}
|JJ_3; II_3(\ell, S = I + \rho)\rangle &= \sum_{m_\ell, m_1, \alpha_1, \eta} \begin{pmatrix} \ell & S & J \\ m_\ell & m & J_3 \end{pmatrix} \begin{pmatrix} S_c & 1/2 & S \\ m_1 & m_2 & m \end{pmatrix} \begin{pmatrix} I_c & 1/2 & I \\ \alpha_1 & \alpha_2 & I_3 \end{pmatrix} c_{\rho, \eta} \\
&\times |S_c = I_c = I + \eta/2; m_1, \alpha_1\rangle \otimes |1/2; m_2, \alpha_2\rangle \otimes |\ell, m_\ell\rangle \quad (1.1)
\end{aligned}$$

Here, the m 's are angular momentum projections, the α 's are isospin projections. Note that $S_c = I_c$ since the (nonstrange) core is symmetric in $SU(6)$ indices, and we have written $I_c = I + \eta/2$, where $\eta = \pm 1$. Note also that $\rho \equiv S - I = \pm 1, 0$. States with strangeness are defined analogously, except that $SU(3)$ Clebsch-Gordan coefficients appear in that case. For reasons of simplicity, we have restricted to nonstrange baryons in this work.

For $S = I \pm 1$, the notation simplifies since $c_{\pm, \pm} = 1$ and $c_{\pm, \mp} = 0$. For $S = I$,

$$c_{0+} = +\sqrt{\frac{S(N_c + 2(S + 1))}{N_c(2S + 1)}} \quad \text{and} \quad c_{0-} = -\sqrt{\frac{(S + 1)(N_c - 2S)}{N_c(2S + 1)}}. \quad (1.2)$$

(The orthogonal combination gives the totally symmetric SU(6) state.) The explicit form of the state allows us to calculate analytically the matrix elements for any operator for arbitrary N_c .

For the physical case of $N_c = 3$, the above expressions admit the 7 nonstrange states of the **70**-plet. Strictly speaking, the label **70**-plet refers only to the mixed symmetry baryons appearing at $N_c = 3$; for large N_c , the representations tend to be much larger. In the **70**-plet, the nonstrange states consist of two isospin-3/2 states, $\Delta_{1/2}$ and $\Delta_{3/2}$, and five isospin-1/2 states, $N_{1/2}$, $N'_{1/2}$, $N_{3/2}$, $N'_{3/2}$, and $N'_{5/2}$. The subscript indicates total baryon spin; unprimed states have quark spin 1/2 and primed states have quark spin 3/2. In partial wave notation, the 2 deltas and 5 nucleons are labeled S_{31} , D_{33} , S_{11} (twice), D_{13} (twice), and D_{15} , respectively.

Section II explains the operator analysis and gives matrix elements of a complete set of operators for the baryon states. Section III contains our analysis of physical masses and mixing angles. Closing comments are made in Sec. IV.

II. OPERATORS AND MATRIX ELEMENTS

Since the physically observed baryons are assigned to irreducible representations of the symmetry group $SU(6) \times O(3)$, it is natural to write all possible mass operators in terms of the generators of this group. $O(3)$ is the group of spatial rotations and is generated by the orbital angular momentum operator ℓ^i , while the spin-flavor group $SU(6)$ has spin S^i , flavor T^a , and combined spin-flavor G^{ia} generators. In the two-flavor case, these operators are defined by

$$S^i \equiv q^\dagger \left(\frac{\sigma^i}{2} \otimes \mathbb{1} \right) q,$$

$$T^a \equiv q^\dagger \left(\mathbb{1} \otimes \frac{\tau^a}{2} \right) q,$$

$$G^{ia} \equiv q^\dagger \left(\frac{\sigma^i}{2} \otimes \frac{\tau^a}{2} \right) q, \quad (2.1)$$

where σ^i and τ^a are the usual Pauli matrices. The field operators q , q^\dagger , which we term “quarks,” are not the dynamical quarks, but rather eigenstates of the spin-flavor group such that an appropriately symmetrized collection of N_c of them have the quantum numbers of the physical baryons. The collection of operators constructed with these fields completely spans all possible physical mass operators. Only if the quarks are heavy can the fields q be identified with the dynamical valence quarks.

The goal of the large N_c analysis is to organize operators by their effects on a given observable (in this case, the masses) in a systematic expansion in powers of N_c . Factors of N_c originate either as coefficients of operators in the Hamiltonian, or through matrix elements of those operators. For example, the unit operator $\mathbb{1}$ contributes at $O(N_c^1)$, since each quark contributes coherently in the matrix element. The spin of the baryon S^2 is known to contribute to the masses at $O(1/N_c)$ [7], because the matrix elements of S^i are of $O(N_c^0)$ for baryons that have spins of order unity as $N_c \rightarrow \infty$. Similarly, matrix elements of T^a are $O(N_c^0)$ in the two-flavor case since the baryons considered have isospin of $O(N_c^0)$, but the operator G^{ia} has matrix elements on this subset of states of $O(N_c^1)$. This means that the contributions of the N_c quarks add incoherently in matrix elements of the operator S^i or T^a but coherently for G^{ia} . Note that matrix elements of a given operator are not necessarily homogeneous in N_c ; for example, values such as $N_c + 3$ can occur.

In this work, the generators S_c^i , T_c^a , G_c^{ia} are reserved to mean those acting upon the core, while separate SU(6) generators s^i , t^a , and g^{ia} are defined for the single excited-quark system. Including the operator ℓ^i completes the list of building blocks for the necessary mass operators. Strictly speaking, the naive symmetry group for this operator basis is $SU(6) \times SU(6) \times O(3)$, although actually only the diagonal subgroup $SU(6) \times O(3)$ truly acts on the baryon states. Given this enlarged notation, it is possible to construct a large number of operators; however, many are linearly dependent and may be discarded.

Since the excited system consists of only one quark, at most one generator from among

s , t , or g appears in any operator. A similar but stronger statement may be made for the core: Since we ultimately perform phenomenological analysis on cores with two quarks, at most two of the set $\{S_c, T_c, G_c\}$ are needed. However, operator reduction rules exist [8] that significantly reduce the number of core operators that must be considered. Finally, since we are interested here in $\ell = 1$ baryons, only operator combinations of $\Delta\ell = 0, 1, 2$ need be considered. Hence, only up to two factors of ℓ^i are required. Indeed, when two ℓ 's appear, it is convenient to use $\ell_{ij}^{(2)}$, the rank two tensor combination of ℓ^i and ℓ^j given by

$$\ell_{ij}^{(2)} = \frac{1}{2}\{\ell_i, \ell_j\} - \frac{\ell^2}{3}\delta_{ij}. \quad (2.2)$$

The explicit power of N_c for a given operator is determined by using the usual large N_c counting of $1/\sqrt{N_c}$ for each quark-quark-gluon coupling, and considering the minimal number of exchanged gluons necessary to generate a given operator. To be specific, we decompose an n -body operator \mathcal{O} as follows:

$$\mathcal{O} = X_* \prod_{i=1}^{n-1} X_i, \quad (2.3)$$

where X_* represents all operators acting on the excited quark, including factors of ℓ^i , and each X_i represents an $SU(6)$ generator acting on the core. The physical realization of such an operator requires exchanging a minimum of $n-1$ gluons between different quarks, leading to a suppression of $1/N_c^{n-1}$, which we henceforth include in the definition of the operators. If $X_* = \mathbb{1}$, the result is maintained as written if n rather than $n-1$ operators act on the core.

One then considers each core operator X_i to determine whether its matrix elements are coherent for the baryon states under consideration; a factor of N_c is included for each coherent operator. Thus, the full large N_c counting of the matrix element is $O(N_c^{1-n+m})$, where m is the number of coherent X_i . In the two-flavor case, only G_c^{ia} has coherent matrix elements. The order of the matrix element thus obtained determines whether or not we retain the operator in computing a given process to a desired order in the $1/N_c$ expansion.

With this counting, one finds 22 potentially independent, time-reversal even, isosinglet operators: One ($\mathbb{1}$) with matrix element of $O(N_c^1)$, two at $O(N_c^0)$, ten at $O(1/N_c)$, and nine

at $O(1/N_c^2)$ or higher. This counting does not fully take into account numerous relations between the matrix elements of the operators evaluated on the nonstrange $\ell = 1$ baryons. One reduction that has been included uses the observation that, for the nonstrange mixed symmetry states,

$$\frac{1}{N_c} \langle gG_c \rangle = -\frac{N_c + 1}{16N_c} + \delta_{S,I} \frac{I(I + 1)}{2N_c^2}. \quad (2.4)$$

This operator naively produces $O(N_c^0)$ matrix elements, but both the $O(N_c^0)$ and $O(1/N_c)$ parts are the same for all baryons in the multiplet, and thus may be absorbed into matrix elements of $\mathbb{1}$, with the remainder being demoted to $O(1/N_c^2)$. Similarly, both $\langle \ell s \rangle$ and $\langle \ell t G_c \rangle$ are $O(N_c^0)$, but it may be observed that $\langle \ell s + 4\ell t G_c / (N_c + 1) \rangle$ is $O(1/N_c)$, so only $\langle \ell s \rangle$ truly represents an independent $O(N_c^0)$ operator. The full set of operator reductions will be presented in a future publication [18].

In any case, there are only 9 observables (masses and mixing angles) in the nonstrange $\ell = 1$ system, and so only 9 independent operators are required. An independent basis is presented in Table I: All 3 occurring up to $O(N_c^0)$ (first considered in Ref. [17]), and a selection of 6 at $O(1/N_c)$ whose matrix elements, when combined with the first 3, are seen to be independent for $N_c = 3$. With these operators denoted by $\mathcal{O}_1, \mathcal{O}_2, \dots, \mathcal{O}_9$, respectively, the nine independent mass matrix elements are given by

$$M_j = \sum_{i=1}^9 c_i \langle \mathcal{O}_i \rangle_j \quad (j = 1 \dots 9). \quad (2.5)$$

The coefficients c_i are independent of N_c at leading order, given our choice of operator normalization. These operator coefficients encapsulate all unknown strong interaction physics unspecified by the large N_c spin-flavor analysis. In Table I we present the matrix elements $\langle \mathcal{O}_i \rangle_j$. Explicit spin and flavor indices are suppressed when their contraction is unambiguous.

III. RESULTS

In addition to the nonstrange mixed symmetry states defined in Sec. I, two mixing angles are necessary to specify the $S = 1/2$ and $S = 3/2$ nucleon mass eigenstates. We define

$$\begin{bmatrix} N(1535) \\ N(1650) \end{bmatrix} = \begin{bmatrix} \cos \theta_{N1} & \sin \theta_{N1} \\ -\sin \theta_{N1} & \cos \theta_{N1} \end{bmatrix} \begin{bmatrix} N_{1/2} \\ N'_{1/2} \end{bmatrix} \quad (3.1)$$

and

$$\begin{bmatrix} N(1520) \\ N(1700) \end{bmatrix} = \begin{bmatrix} \cos \theta_{N3} & \sin \theta_{N3} \\ -\sin \theta_{N3} & \cos \theta_{N3} \end{bmatrix} \begin{bmatrix} N_{3/2} \\ N'_{3/2} \end{bmatrix}, \quad (3.2)$$

as in Ref. [13]. The mass eigenvalues and mixing angles can be expressed in terms of the coefficients c_i of the operators presented in the previous section.

Since we have found an operator basis that completely spans the 9-dimensional space of observables, we can solve for the c_i given the experimental data. For each baryon mass, we assume that the central value corresponds to the midpoint of the mass range quoted in the *Review of Particle Properties* [19]; we take the one standard deviation error as half of the stated range. To determine the off-diagonal mass matrix elements, we use the mixing angles extracted from the analysis of strong decays given in Ref. [13], $\theta_{N1} = 0.61 \pm 0.09$ and $\theta_{N3} = 3.04 \pm 0.15$. These values are consistent with those obtained in [15] from radiative decays. Solving for the operator coefficients, we obtain the values shown in Table II.

Naively, one expects the c_i to be of comparable size. Using the value of c_1 as a point of comparison, it is clear that there are no operators with anomalously large coefficients. Thus, we find no conflict with the naive $1/N_c$ power counting rules. It is interesting that a number of the operators appear to be unimportant in describing the experimental data (presumably due to the underlying dynamics). For example, of the two operators that contribute to the masses at $\mathcal{O}(1)$, the operator $\mathcal{O}_2 = \ell s$ has a coefficient which is suppressed relative to $\mathcal{O}_3 = \ell^{(2)} g G_c / N_c$ by more than factor of 10; in effect, this operator is no more important than a typical $O(1/N_c^2)$ correction. Of the operators that contribute to the masses at order $1/N_c$, only the operator $\mathcal{O}_6 = S_c^2 / N_c$ contributes as much as one would expect, with the next largest corrections coming from the operators $\mathcal{O}_4 = \ell s + 4\ell t G_c / (N_c + 1)$ and $\mathcal{O}_5 = \ell S_c / N_c$.

Using these observations, we can attempt to fit the data using judiciously chosen subsets of the original 9 operators. We fit to the seven mass eigenvalues as well as the two mixing

angles θ_{N1} and θ_{N3} . The operator set we consider first are \mathcal{O}_1 , \mathcal{O}_2 , and \mathcal{O}_3 ; these yield mass predictions accurate to order 1 in the $1/N_c$ expansion, and thus present the first nontrivial spin-flavor symmetry-breaking corrections. We show the result of this fit in Table III. In short, the lowest order operators fail in reproducing the experimental data. Notably, the mixing angles are far off the mark, and the $J = 1/2$ Δ state is predicted to be heavier than the $J = 3/2$ Δ state. The difficulty in obtaining a good fit from a leading order analysis is an outcome that perhaps could have been anticipated: Naively, one might expect that the lowest nontrivial mass corrections are roughly a factor of $N_c = 3$ smaller than the mean baryon mass, or approximately 500 MeV. The largest splitting within our set of seven baryons is ≈ 180 MeV, for example, in the case of the N(1700)-N(1520) mass difference. Thus, we might have concluded *a priori* that $1/N_c$ corrections are necessary in order to reproduce the detailed features of the mass spectrum.

In the remaining fits, we include $1/N_c$ corrections. The 6 parameter fit shown in Table IV includes all the subleading operators that appear to be significant in Table II; the operators included are \mathcal{O}_1 through \mathcal{O}_6 . The resulting fit is in extremely good agreement with the experimental data, with no predicted mass more than 0.4 standard deviations from the corresponding experimental central value, and a χ^2 per degree of freedom of 0.1.

More strikingly, Table II implies that we can discard additional operators and still obtain a reasonable fit. Notice that the smallness of the coefficients c_2 , c_4 , and c_5 renders the corresponding operators numerically unimportant, and thus they can be neglected if we are only interested in working to order $1/N_c$. In Table V we give a fit retaining the remaining three operators, \mathcal{O}_1 , \mathcal{O}_3 , and \mathcal{O}_6 . The χ^2 per degree of freedom for this fit is 1.87, which is not bad considering that we have only included two nontrivial operators. Notice that the particular operator choice for this fit leads to a degeneracy between the $\Delta_{1/2}$ and $\Delta_{3/2}$ which is lifted by the corrections that we have discarded. We do not display the fits corresponding to all possible choices for the subleading operators. It suffices to point out that these additional fits interpolate between the 6 and 3 parameter fits that we have presented in Tables IV and V. One may also fit to the mass eigenvalues and predict the mixing angles. These fits

are not qualitatively different from the ones given here, and will be presented in a longer publication [18]. The crucial observation is that the subleading operator $\mathcal{O}_6 = S_c^2/N_c$ is the most significant ingredient in taking us from the poor fit shown in Table III to the good fits in Tables IV and V.

IV. CONCLUSIONS

The value of the large N_c approach to baryon phenomenology is that it provides an organizing principle in constructing the baryon effective field theory. Studies of the excited baryon mass spectrum in the formative days of SU(6) found numerous operators [20], but in that period there was no organizing principle available to select among them. Beginning with a complete operator basis that spans the space of any desired set of observables, large N_c power counting rules tell us which operators may be discarded if we wish to obtain predictions to a desired level of accuracy. In this sense, our results for the masses and mixing angles of the nonstrange $\ell = 1$ baryons are completely consistent with the large N_c picture. We find no operator with a coefficient that is larger than what one would expect from the naive large N_c power counting rules.

More interesting, however, is that at any given order in $1/N_c$, only some of the operators are of phenomenological relevance. The $O(N_c^0)$ coefficients c_i that we have defined in our effective theory parametrize the long-distance physics that we cannot calculate. If we had found that these coefficients were of comparable size, we might have concluded that large N_c counting arguments alone are sufficient to explain the detailed features of the mass spectrum. Quite the contrary, we find that only a few of our original set of operators are needed to reproduce the experimental data, most notably, the operators S_c^2 and $\ell^{(2)}gG_c$. It is tempting to speculate that the importance of the operator S_c^2 can be understood by considering the explicit nonrelativistic reduction of a one-gluon exchange interaction; the second operator, however, has a nontrivial flavor structure that does not correspond to the usual tensor interaction $\ell^{(2)}sS_c$ that one derives in this approach [21]. A fit analogous to

that of Table V replacing $\ell^{(2)}gG_c/N_c$ with $\ell^{(2)}sS_c/N_c$ increases the χ^2 per degree of freedom from 1.87 to 2.46. Why the underlying dynamics should prefer these operators is a much more difficult question which goes beyond what can be addressed in the effective field theory approach.

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TABLES

	$\langle \mathbb{1} \rangle$	$\langle ls \rangle$	$\frac{1}{N_c} \langle \ell^{(2)} g G_c \rangle$	$\langle ls + \frac{4}{N_c+1} lt G_c \rangle$	$\frac{1}{N_c} \langle \ell S_c \rangle$
$N_{1/2}$	N_c	$-\frac{1}{3N_c}(2N_c - 3)$	0	$+\frac{2}{N_c+1}$	$-\frac{1}{3N_c^2}(N_c + 3)$
$N'_{1/2}$	N_c	$-\frac{5}{6}$	$-\frac{5}{48N_c}(N_c + 1)$	0	$-\frac{5}{3N_c}$
$N'_{1/2} - N_{1/2}$	0	$-\frac{1}{3} \sqrt{\frac{N_c+3}{2N_c}}$	$-\frac{5}{48N_c} \sqrt{\frac{N_c+3}{2N_c}}(2N_c - 1)$	$-\frac{1}{N_c+1} \sqrt{\frac{N_c+3}{2N_c}}$	$+\frac{1}{3N_c} \sqrt{\frac{N_c+3}{2N_c}}$
$N_{3/2}$	N_c	$+\frac{1}{6N_c}(2N_c - 3)$	0	$-\frac{1}{N_c+1}$	$+\frac{1}{6N_c^2}(N_c + 3)$
$N'_{3/2}$	N_c	$-\frac{1}{3}$	$+\frac{1}{12N_c}(N_c + 1)$	0	$-\frac{2}{3N_c}$
$N'_{3/2} - N_{3/2}$	0	$-\frac{1}{6} \sqrt{\frac{5(N_c+3)}{N_c}}$	$+\frac{1}{96N_c} \sqrt{\frac{5(N_c+3)}{N_c}}(2N_c - 1)$	$-\frac{1}{2(N_c+1)} \sqrt{\frac{5(N_c+3)}{N_c}}$	$+\frac{1}{6N_c} \sqrt{\frac{5(N_c+3)}{N_c}}$
$N'_{5/2}$	N_c	$+\frac{1}{2}$	$-\frac{1}{48N_c}(N_c + 1)$	0	$+\frac{1}{N_c}$
$\Delta_{1/2}$	N_c	$+\frac{1}{3}$	0	0	$-\frac{4}{3N_c}$
$\Delta_{3/2}$	N_c	$-\frac{1}{6}$	0	0	$+\frac{2}{3N_c}$

	$\frac{1}{N_c} \langle S_c^2 \rangle$	$\frac{1}{N_c} \langle t T_c \rangle$	$\frac{1}{N_c} \langle \ell^{(2)} s S_c \rangle$	$\frac{1}{N_c^2} \langle \ell^i g^{ia} \{ S_c^j, G_c^{ia} \} \rangle$
$N_{1/2}$	$+\frac{1}{2N_c^2}(N_c + 3)$	$-\frac{1}{4N_c^2}(N_c + 3)$	0	$-\frac{1}{24N_c^3}(5N_c - 1)(N_c + 3)$
$N'_{1/2}$	$+\frac{2}{N_c}$	$-\frac{1}{N_c}$	$+\frac{5}{6N_c}$	$+\frac{5}{24N_c^2}(N_c + 1)$
$N'_{1/2} - N_{1/2}$	0	0	$+\frac{5}{12N_c} \sqrt{\frac{N_c+3}{2N_c}}$	$+\frac{1}{24N_c^2} \sqrt{\frac{N_c+3}{2N_c}}(2N_c - 1)$
$N_{3/2}$	$+\frac{1}{2N_c^2}(N_c + 3)$	$-\frac{1}{4N_c^2}(N_c + 3)$	0	$+\frac{1}{48N_c^3}(5N_c - 1)(N_c + 3)$
$N'_{3/2}$	$+\frac{2}{N_c}$	$-\frac{1}{N_c}$	$-\frac{2}{3N_c}$	$+\frac{1}{12N_c^2}(N_c + 1)$
$N'_{3/2} - N_{3/2}$	0	0	$-\frac{1}{24N_c} \sqrt{\frac{5(N_c+3)}{N_c}}$	$+\frac{1}{48N_c^2} \sqrt{\frac{5(N_c+3)}{N_c}}(2N_c - 1)$
$N'_{5/2}$	$+\frac{2}{N_c}$	$-\frac{1}{N_c}$	$+\frac{1}{6N_c}$	$-\frac{1}{8N_c^2}(N_c + 1)$
$\Delta_{1/2}$	$+\frac{2}{N_c}$	$+\frac{1}{2N_c}$	0	$+\frac{1}{6N_c^2}(N_c + 1)$
$\Delta_{3/2}$	$+\frac{2}{N_c}$	$+\frac{1}{2N_c}$	0	$-\frac{1}{12N_c^2}(N_c + 1)$

TABLE I. Matrix elements $\langle \mathcal{O}_i \rangle_j$ of 9 operators, labeled as $\mathcal{O}_1, \mathcal{O}_2, \dots, \mathcal{O}_9$, respectively, that are linearly independent for $N_c = 3$. The third and sixth rows correspond to off-diagonal matrix elements.

c_1	c_2	c_3	c_4	c_5	c_6	c_7	c_8	c_9
+0.470	-0.036	+0.369	+0.089	+0.087	+0.418	+0.040	+0.048	+0.012
± 0.017	± 0.041	± 0.208	± 0.203	± 0.157	± 0.085	± 0.074	± 0.172	± 0.673

TABLE II. Operator coefficients in GeV, assuming the complete set of Table I. The vertical divisions separate operators whose contributions to the baryon masses are of orders N_c^1 , N_c^0 and N_c^{-1} , respectively.

Parameters (GeV): $c_1 = 0.542 \pm 0.002$, $c_2 = 0.093 \pm 0.008$, $c_3 = -0.335 \pm 0.039$					
	Fit (MeV)	Exp. (MeV)		Fit (MeV)	Exp. (MeV)
$\Delta(1700)$	1610	1720 ± 50	$N(1520)$	1520	1523 ± 8
$\Delta(1620)$	1657	1645 ± 30	$N(1535)$	1568	1538 ± 18
$N(1675)$	1682	1678 ± 8	θ_{N1}	0.79	0.61 ± 0.09
$N(1700)$	1679	1700 ± 50	θ_{N3}	2.63	3.04 ± 0.15
$N(1650)$	1622	1660 ± 20			

TABLE III. Three parameter fit using operators $\mathcal{O}_{1,2,3}$, giving $\chi^2/\text{d.o.f.} = 23.33/6 = 3.89$. The operators included formally yield the lowest order nontrivial contributions to the masses in the $1/N_c$ expansion.

Parameters (GeV): $c_1 = 0.468 \pm 0.005$, $c_2 = -0.032 \pm 0.045$, $c_3 = 0.327 \pm 0.093$

$c_4 = 0.081 \pm 0.027$, $c_5 = 0.071 \pm 0.042$, $c_6 = 0.413 \pm 0.044$

	Fit (MeV)	Exp. (MeV)		Fit (MeV)	Exp. (MeV)
$\Delta(1700)$	1701	1720 ± 50	$N(1520)$	1523	1523 ± 8
$\Delta(1620)$	1637	1645 ± 30	$N(1535)$	1537	1538 ± 18
$N(1675)$	1678	1678 ± 8	θ_{N1}	0.60	0.61 ± 0.09
$N(1700)$	1712	1700 ± 50	θ_{N3}	3.06	3.04 ± 0.15
$N(1650)$	1662	1660 ± 20			

TABLE IV. Six parameter fit using operators $\mathcal{O}_{1,\dots,6}$, giving $\chi^2/\text{d.o.f.} = 0.31/3 = 0.10$.

Parameters (GeV): $c_1 = 0.461 \pm 0.005$, $c_3 = 0.360 \pm 0.059$, $c_6 = 0.453 \pm 0.030$

	Fit (MeV)	Exp. (MeV)		Fit (MeV)	Exp. (MeV)
$\Delta(1700)$	1683	1720 ± 50	$N(1520)$	1530	1523 ± 8
$\Delta(1620)$	1683	1645 ± 30	$N(1535)$	1503	1538 ± 18
$N(1675)$	1673	1678 ± 8	θ_{N1}	0.45	0.61 ± 0.09
$N(1700)$	1725	1700 ± 50	θ_{N3}	3.04	3.04 ± 0.15
$N(1650)$	1663	1660 ± 20			

TABLE V. Three parameter fit using operators \mathcal{O}_1 , \mathcal{O}_3 , and \mathcal{O}_6 , giving $\chi^2/\text{d.o.f.} = 11.19/6 = 1.87$.