

# Formation of Black Holes in First Order Phase Transitions

M.Yu.Khlopov<sup>1,2,3,4\*</sup>, R.V.Konoplich<sup>1,2,3,4†</sup>, S.G.Rubin<sup>2,4‡</sup> and A.S.Sakharov<sup>2,4§</sup>

<sup>1</sup>*Dipartimento di Fisica I Universita' di Roma "La Sapienza", P-le A.Moro,2,I-00185 Rome,Italy*

<sup>2</sup>*Center for CosmoParticle Physics "Cosmion"*

<sup>3</sup>*Institute of Applied Mathematics, Miusskaya Pl.4, 125047 Moscow, Russia*

<sup>4</sup>*Moscow Engineering Physics Institute (Technical University), Kashirskoe Sh.31, 115409 Moscow, Russia*

## Abstract

A new mechanism of black hole formation in a first order phase transition is proposed. In vacuum bubble collisions the interaction of bubble walls leads to the formation of nontrivial vacuum configuration. The consequent collapse of this vacuum configuration induces the black hole formation with high probability. Observational constraints on the spectrum of primordial black holes allow to obtain new nontrivial restrictions on parameters of inflation models with first order phase transitions.

## 1 Introduction

At present time black holes (BH) can be created only by a gravitational collapse of compact objects with mass more than about three Solar mass [1]. However at the early stage of evolution of the Universe there are no limits on the mass of BH formed by several mechanisms. The simplest one is a collapse of strongly inhomogeneous regions just after the end of inflation [2]. Another possible source of BH could be a collapse of cosmic strings [3] that are produced in early phase transitions with symmetry breaking. The collisions of the bubble walls [4, 5] created at phase transitions of the first order can lead to a primordial black hole (PBH) formation.

We discuss here new mechanism of PBH production in the collision of two vacuum bubbles. The known opinion of the BH absence in such processes is based on strict conservation of the original O(2,1) symmetry. Whereas there are ways to break it . Firstly, the radiation of scalar waves indicates the entropy increasing and hence the permanent breaking of the symmetry during

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\*e-mail: mkhlopov@orc.ru

†e-mail: konoplic@orc.ru

‡e-mail: sgrubin@orc.ru

§e-mail: sakhas@landau.ac.ru

the bubble collision. Secondly, the vacuum decay due to thermal fluctuation does not possess this symmetry from the beginning. The simplest example of a theory with bubble creation is a scalar field theory with two non degenerated vacuum states. Being stable at a classical level, the false vacuum state decays due to quantum effects, leading to a nucleation of the bubbles of true vacuum and their subsequent expansion [6]. The potential energy of the false vacuum is converted into a kinetic energy of the bubble walls thus making them highly relativistic in a short time. The bubble expands till it collides with another one. As it was shown in [4, 5] a black hole may be created in the collision of several bubbles. Our investigations show that BH can be created as well with a probability of order unity in the collisions of only two bubbles. It initiates the enormous production of BH that leads to essential cosmological consequences discussed below.

In Section 2 the evolution of the field configuration in the collisions of bubbles is discussed. The BH mass distribution is obtained in Section 3. In Section 4 cosmological consequences of the BH production in bubble collisions at the end of inflation are considered.

## 2 Evolution of field configuration in collisions of true vacuum bubbles

Consider a theory where a probability of false vacuum decay equals  $\Gamma$  and energy difference between the false and true vacuum equals  $\rho_V$ . The vacuum decay proceeds through the nucleation of bubbles of new phase separated from the false vacuum outside by initially unmoving walls. The wall of the bubble increases quickly its velocity up to the speed of light  $v = c = 1$  due to conversion of the false vacuum energy into its kinetic one.

Let us discuss dynamics of collision of two true vacuum bubbles that have been nucleated in points  $(\mathbf{r}_1, t_1), (\mathbf{r}_2, t_2)$  and which are expanding into false vacuum. Following papers [4, 7] let us assume for simplicity that the horizon size is much greater than the distance between the bubbles. Just after collision mutual penetration of the walls up to the distance comparable with its width is accompanied by a significant potential energy increase [8]. Then the walls reflect and accelerate backwards. The space between them is filled by the field in the false vacuum state converting the kinetic energy of the wall back to the energy of the false vacuum state and slowdown the velocity of the walls. Meanwhile the outer area of the false vacuum is absorbed by the outer wall, which expands and accelerates outwards. Evidently, there is an instant when the central region of the false vacuum is separated. Let us note that the FVB does not possess spherical symmetry at the moment of its separation from outer walls but wall tension restores the symmetry during the first oscillation of FVB. As it was shown in [7], the further evolution of FVB consists of several stages:

- 1) FVB grows up to the definite size  $D_M$  until the kinetic energy of its wall become zero;
- 2) After this moment the false vacuum bag begins to shrink up to a minimal size  $D^*$ ;
- 3) Secondary oscillation of the false vacuum bag occurs.

The process of periodical expansions and contractions leads to energy losses of FVB in the form of quanta of scalar field. It has been shown in the [7, 9] that only several oscillations take place. On the other hand, important note is that the secondary oscillations might occur only if the minimal size of the FVB would be larger than its gravitational radius,  $D^* > r_g$ . The opposite case ( $D^* < r_g$ ) leads to the BH creation with the mass about the mass of the FVB. As we will show

later the probability of BH formation is almost unity in a wide range of parameters of theories with first order phase transitions.

### 3 Gravitational collapse of FVB and BH creation

Consider in more details the conditions of converting FVB into BH. The mass  $M$  of FVB can be calculated in a framework of a specific theory and can be estimated in a coordinate system  $K'$  where the colliding bubbles are nucleated simultaneously. The radius of each bubble  $b'$  in this system equals to half of their initial coordinate distance at first moment of collision. Apparently the maximum size  $D_M$  of the FVB is of the same order as the size of the bubble, since this is the only parameter of necessary dimension on such a scale:  $D_M = 2b'C$ . The parameter  $C \leq 1$  is obtained by numerical calculations in the framework of each theory, but its numerical value does not affect significantly conclusions.

One can find the mass of FVB that arises at the collision of two bubbles of radius:

$$M = \frac{4\pi}{3} (Cb')^3 \rho_V \quad (1)$$

This mass is contained in the shrinking area of false vacuum. Suppose for estimations that the minimal size of FVB is of order wall width  $\Delta$ . The BH is created if minimal size of FVB is smaller than its gravitational radius. It means that at least at the condition

$$\Delta < r_g = 2GM \quad (2)$$

the FVB can be converted into BH (where  $G$  is the gravitational constant).

As an example consider a simple model with Lagrangian

$$L = \frac{1}{2} (\partial_\mu \Phi)^2 - \frac{\lambda}{8} (\Phi^2 - \Phi_0^2)^2 - \epsilon \Phi_0^3 (\Phi + \Phi_0). \quad (3)$$

In the thin wall approximation the width of the bubble wall can be expressed as  $\Delta = 2 (\sqrt{\lambda} \Phi_0)^{-1}$ . Using (2) one can easily derive that at least FVB with mass

$$M > \frac{1}{\sqrt{\lambda} \Phi_0 G} \quad (4)$$

should be converted into BH of mass  $M$ . The last condition is valid only in case when FVB is completely contained in the cosmological horizon, namely  $M_H > 1/\sqrt{\lambda} \Phi_0 G$  where the mass of the cosmological horizon at the moment of phase transition is given by  $M_H \cong m_{pl}^3/\Phi_0^2$ . Thus for the potential (3) at the condition  $\lambda > (\Phi_0/m_{pl})^2$  the BH is formed. This condition is valid for any realistic set of parameters of theory. Let us find the mass and velocity distributions of such BHs, supposing its mass is large enough to satisfy the inequality (2). Apparently these distributions depend on coordinates and times of nucleation of the bubbles. The probability of the collision of

the bubbles, which have been nucleated at the distance  $|\mathbf{r}_1 - \mathbf{r}_1|$  from each other has the form:

$$\begin{aligned} dP &= dP_1 \cdot dP_2 \cdot P_-, \\ dP_1 &= \Gamma dt_1 d^3 r, \\ dP_2 &= \Gamma dt_2 4\pi |\mathbf{r}_2 - \mathbf{r}_1|^2 d|\mathbf{r}_2 - \mathbf{r}_1|, \end{aligned} \quad (5)$$

where  $dP_1$  is the probability of the bubble nucleation with coordinates  $(\mathbf{r}_1, t_1)$ ,  $dP_2$  is the probability of nucleation of the second bubble at the distance  $|\mathbf{r}_2 - \mathbf{r}_1| \equiv 2b$  from first one (we have integrated over angles assuming the space isotropy). The factor  $P_- = \exp(-\Gamma\Omega)$  determines the probability, which takes into account the absence of additional bubbles in 4- dimensional region  $\Omega$ . In the following we consider the probability density of the false vacuum decay being a free parameter. The region  $\Omega$  will be calculated below. Integrating(5) and assuming time independence of vacuum decay probability we obtain

$$dP/V = 32\pi\Gamma^2 e^{-\Gamma\Omega} b^2 dt_1 dt_2 db. \quad (6)$$

Here  $V$  is the volume of cosmological horizon at the moment of the phase transition. In the following we choose the  $K'$  system mentioned above. The velocity of the system is  $v = (t_1 - t_2) / 2b$  and evidently  $v$  is also the velocity of FVB (or BH). The radius of the colliding bubbles is given by  $b' = b/\gamma$ ,  $\gamma = (1 - v^2)^{-1/2}$ . By using of (1) and (6), it is easy now to obtain the FVB distribution in terms of new variables  $M, v, t$ , where  $M$  is the mass of FVB (or BH) created in the bubble collision, is its velocity and  $t$  is the first moment of the bubble contact:

$$dP/V dv dM = \int \frac{64\pi}{3} \Gamma^2 e^{-\Gamma\Omega} \gamma^4 \left( \frac{M}{C\rho_v} \right)^{1/3} \frac{1}{C\rho_v} dt \quad (7)$$

In order to determine the 4-dimensional area  $\Omega$  we will use some reasonable approximation. Namely, let us assume that every bubble that has reached the sphere of radius  $b'$  with center in the point O at the moment  $t'$  of first bubbles contact prevents the creation of FVB. With this assumption the area  $\Omega$  is determined as

$$\Omega = \int_0^{t'} d\tau' d^3 \mathbf{r}' \theta(r' + \tau' - b' - t') = \frac{\pi}{3} \{(b' + t')^4 - b'^4\} \quad (8)$$

Parameter  $b'$  is related with the mass  $M$  according to (1), the time  $t$  in the  $K'$  system equals to  $\gamma t$ . Therefore, after integration over time the distribution of FVB in mass and velocity takes the form

$$\begin{aligned} dP/V dv dM &= \frac{64\pi}{3} \Gamma^2 \exp \left[ -\Gamma \frac{\pi}{3} \left( \frac{M}{C\rho_v} \right)^{4/3} \right] \gamma^4 \left( \frac{M}{C\rho_v} \right)^{1/3} \frac{1}{C\rho_v} I, \\ I &= \int_{t_-}^{\infty} d\tau \exp \left\{ -\frac{\pi}{3} \Gamma \left[ \left( \frac{M}{C\rho_v} \right)^{1/3} + \gamma\tau \right]^4 \right\}, \\ t_- &= (1 + v) \gamma \left( \frac{M}{C\rho_v} \right)^{1/3} \end{aligned} \quad (9)$$

Let us compare a volume  $V_{bag}$  containing one FVB and a volume  $V_{bubble}$  of one bubble at the end of the phase transition. After numerical integration of expression (9) we get

$$V_{BH} \cong V_{bag} \cong 3.9\Gamma^{-3/4}. \quad (10)$$

On the other hand, the average volume per one bubble is

$$V_{bubble} = \frac{4}{3}\pi \left(\frac{3}{\pi}\right)^{3/4} \Gamma^{-3/4} \cong 4.0\Gamma^{-3/4}, \quad (11)$$

where we assume the bubbles have a spherical form. The expected equality  $V_{bag} = V_{bubble}$  is fulfilled to conclude that our approximations are correct. The distribution (9) can be rewritten in more convenient form in terms of non dimensional mass  $\mu \equiv \left(\frac{\pi}{3}\Gamma\right)^{1/4} \left(\frac{M}{C\rho_v}\right)^{1/3}$ :

$$\frac{dP}{\Gamma^{-3/4}Vdv d\mu} = 64\pi \left(\frac{\pi}{3}\right)^{1/4} \mu^3 e^{\mu^4} \gamma^3 J(\mu, v), \quad (12)$$

$$J(\mu, v) = \int_{\tau}^{\infty} d\tau e^{-\tau^4}, \tau_- = \mu [1 + \gamma^2 (1 + v)].$$

The numerical integration of (12) revealed that the distribution (9) has gaussian like shape with narrow maximum. For example the number of BH with mass 30 times greater than the average one is suppressed by factor  $10^5$ . Average value of the non dimensional mass is equal to  $\mu = 0.32$ . It allows to relate the average mass of BH and volume containing the BH at the moment of the phase transition:

$$\langle M_{BH} \rangle = \frac{C}{4} \mu^3 \rho_v \langle V_{BH} \rangle \simeq 0.012 C \rho_v \langle V_{BH} \rangle, \quad (13)$$

Remind that the constant  $C$  is the model dependent value less then or order unity.

## 4 First order phase transitions in the early Universe

Inflation models ended by a first order phase transition hold a dignified position in the modern cosmology of early Universe (see for example [10, 11]). The interest to these models is due to, that such models are able to generate the observed large-scale voids as remnants of the primordial bubbles for which the characteristic wavelengths are several tens of Mpc. [11]. A detailed analysis of a first order phase transition in the context of extended inflation can be found in [12]. Hereafter we will be interested only in a final stage of inflation when the phase transition is completed. Remind that a first order phase transition is considered as completed immediately after establishing of true vacuum percolation regime. Such regime is established approximately when at least one bubble per unit Hubble volume is nucleated. Accurate computation [12] shows that first order phase transition is successful if the following condition is valid .

$$Q \equiv \frac{4\pi}{9} \left(\frac{\Gamma}{H^4}\right)_{t_{end}} = 1 \quad (14)$$

Here  $\Gamma$  is the bubble nucleation rate. In the framework of first order inflation models the filling of all space by true vacuum takes place due to bubble collisions, nucleated at the final moment of exponential expansion. The collisions between such bubbles occur when they have comoving spatial dimension less or equal to the effective Hubble horizon  $H_{end}^{-1}$  at the transition epoch. If we take  $H_0 = 100hKm/sec/Mpc$  in  $\Omega = 1$  Universe the comoving size of these bubbles is approximately  $10^{-21}h^{-1}Mpc$ . In the standard approach it believes that such bubbles are rapidly

thermalized without leaving a trace in the distribution of matter and radiation. However, in the previous section it has been shown that for any realistic parameters of theory, the collision between only two bubble leads to BH creation with the probability closely to 100% . The mass of this BH is given by (see (13))

$$M_{BH} = \gamma_1 M_{bub} \quad (15)$$

where  $\gamma_1 \leq 10^{-2}$  and  $M_{bub}$  is the mass that could be contained in the bubble volume at the epoch of collision in the condition of a full thermalization of bubbles. The discovered mechanism leads to a new direct possibility of PBH creation at the epoch of reheating in first order inflation models. In standard picture PBHs are formed in the early Universe if density perturbations are sufficiently large, and the probability of PBHs formation from small post- inflation initial perturbations is suppressed exponentially. Completely different situation takes place at final epoch of first order inflation stage; namely collision between bubbles of Hubble size in percolation regime leads to PBHs formation with masses

$$M_0 = \gamma_1 M_{end}^{hor} \quad (16)$$

where  $M_{end}^{hor}$  is the mass of Hubble horizon at the end of inflation. According to (13) the initial mass fraction of this PBHs is given by relationship:

$$\beta_0 = \gamma_1/e \approx 6 \cdot 10^{-3} \quad (17)$$

A suppression  $e^{-1}$  in comparison with (13) is included in order to avoid a possibility of secondary bubbles nucleation inside FVB, which collapses into PBH. The expression (16) can be rewritten in terms of inflation energy scale  $H_{end}$  by the following manner:

$$M_0 = \frac{\gamma_1}{2} \frac{m_{pl}^2}{H_{end}} \quad (18)$$

For example, according to (18) and for typical value of upper limit of  $H_{end} \approx 4 \cdot 10^{-6} m_{pl}$  the initial mass fraction  $\beta$  is contained in PBHs with mass  $M_0 \approx 1g$ .

On the radiation dominated stage the relative contribution of PBHs to total cosmological density grows as a scale factor, and it means that at the moment

$$t_1 \approx \frac{1}{\beta_0^2 H_{end}} \quad (19)$$

over 50% of matter are contained in PBHs. Since PBHs behave as dust-like matter, and the state equation of the Universe has dust like form  $p = 0$  from the moment  $t_1$  . The PBHs dominated dust like stage ends at the moment of full evaporation of PBHs at the moment:

$$t_2 \approx \frac{1}{g_*} \left( \frac{M_0}{m_{pl}} \right)^3 t_{pl} \quad (20)$$

where  $g_*$  is effective number of massless degrees of freedom at this time [13, 14, 2]. In the case of PBHs dominance they could form another ones with greater masses. However the probability of

more massive PBHs formation would be negligible because of very small amplitude of initial post inflation density perturbations  $\delta \simeq 10^{-6}$ .

There are a number of well-known limits, covering various mass ranges on the maximum allowed mass fraction of PBHs [15, 16, 17, 18, 19, 20, 21]. Some are imposed at the present epoch and some at earlier stages such as nucleosynthesis. These constraints fall into two categories, those from the effect of Hawking radiation and those from the gravitational effects. The evaporation of PBHs via thermal emission has potentially observable astrophysical consequences. Observations have placed limits on the maximum fraction of PBHs allowed at evaporation. PBHs with mass  $M_{ev} \leq 5 \cdot 10^{14}g$  will have evaporated before the present epoch. PBHs more massive than this will not have experienced significant evaporation, and their present density should not close the Universe ( $\Omega_{PBH} < 1$ ).

One generally suppose, that evaporation proceeds until the PBH vanishes completely [21], but there are various arguments against this proposal [22, 23, 24, 25, 26, 27]. If one supposes that BH evaporation leaves a stable relic, it normally is assumed to have a mass of order  $m_{rel} = km_{pl}$ , where  $k \simeq 1 \div 10^3$ . Let us derive the density of PBH relics from PBHs which have been formed at the percolation epoch after first order inflation. Since the probability of such PBHs formation is large, then there are quickly coming on the domination regime, realized the early dust like stage, which is ended at the moment of PBHs evaporation. The mass fraction of the Universe going to relics becomes:

$$\alpha_{rel} = k \frac{m_{pl}}{M_0} \sqrt{\frac{t_{eq}}{t_2}} \quad (21)$$

where  $t_{eq} = 3.2 \cdot 10^{10} h^{-4} s$  is the moment when the densities of matter and radiation become equal. It is clear that matter dominance can not be turned on before  $t_{eq}$ . Consequently the inequality  $\alpha_{rel} < 1/2$  must be valid. By using (18), (20) and (21) we can express this condition in the following form

$$\frac{H_{end}}{m_{pl}} < \frac{0.4\gamma_1}{g_*^{1/5} k^{2/5}} \left( \frac{t_{pl}}{t_{eq}} \right)^{1/5} \leq 7 \cdot 10^{-12} h^{4/5} \frac{\gamma_1}{g_*^{1/5} k^{2/5}} \quad (22)$$

On the other hand the restriction [15] on the relative contribution of PBH with mass  $M < 10^{11}g$  into cosmological density

$$\beta(M) < 10^{-8} \left( \frac{10^{11}g}{M} \right) \quad (23)$$

implies that a significant fraction of the Universe can go to the PBHs with the masses are not exceed  $10^4g$ . Thus, all entropy of our Universe could be produced by such evaporating PBHs. Thereby the PBHs with masses  $M < 10^4g$  should be able to arise with probability of the order unit and without any contradiction with observations. Last speculations imply the following restriction on the first order inflation energy scale:

$$\frac{H_{end}}{m_{pl}} \geq 10^{-9} \gamma_1 \quad (24)$$

Two conditions (22) and (24) are incompatible, and if the relics hypothesis is valid, there is a serious problem for all inflation models with reheating by a first order phase transition.

## 5 Conclusion

In this paper it was shown that BH creation takes place even in the collision of two vacuum bubbles with high probability. The proposed mechanism of BH formation puts severe restrictions on the scenario of the evolution of the Universe. As an example, consider the class of cosmological models, in which an inflation ends by a first order phase transition. The horizon at the end of the inflation is about  $H_{end} > 4 \cdot 10^{-6} m_{pl}$ , what may be estimated by COBE normalized reconstruction of inflation potential. As it was evaluated above, produced black holes have then masses about 1 g.

On the other hand there are arguments that black holes do not evaporate completely leaving at the end of evaporation stable remnants with masses of the order Planck mass.

Putting together these two conjectures one finds the serious trouble for the considered class of inflation models. Evaporation of PBHs, formed in the first order phase transition at the end of inflation, should result in dramatic overproduction of stable remnants. Their contribution into the present cosmological density should correspond to  $10^{-14} g/cm^3$ , being by 15 orders of magnitude above the observational upper limits. So one has to conclude that the effect of the false vacuum bag mechanism of PBH formation makes impossible the coexistence of stable remnants of PBH evaporation with the first order phase transitions at the end of inflation.

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