

Flavor violation and $\tan\beta$ in gauge mediated models with messenger-matter mixing

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Abstract

We consider the Minimal Gauge Mediated Model (MGMM) with either fundamental or antisymmetric messenger multiplets and study consequences of messenger-matter mixing. We find that in these models, unlike MGMM without mixing, wide range of $\tan\beta$ is allowed. It is shown that existing experimental limits on processes with lepton flavor violation, and on $K^0 - \bar{K}^0$, $D^0 - \bar{D}^0$ and $B^0 - \bar{B}^0$ - mixing place significant constraints on relevant coupling constants and mixing parameters. On the other hand, the contributions of the messenger-matter mixing to the rates of $\tau \rightarrow e\gamma$, $\tau \rightarrow \mu\gamma$ and $b \rightarrow s\gamma$ may be well below the present experimental limits depending on the value of $\tan\beta$. We also point out the possibility of sizeable slepton oscillations in this model.

1 Introduction

Presently, much attention is paid to flavor physics in supersymmetric theories. Flavor violation naturally emerges in those SUSY models where supersymmetry is broken by supergravity interactions. The corresponding soft-breaking terms are often assumed to be universal at the Planck (string) scale. This universality breaks down due to the renormalization group evolution between the Planck (string) and GUT scales [1]. As a result, sizeable mixing in slepton and squark sectors at low energies is induced. This mixing leads to lepton flavor violation for ordinary leptons ($\mu \rightarrow e\gamma$, etc.) and FCNC processes ($K^0 - \bar{K}^0$ -mixing, etc.) at rates close to existing experimental limits [2].

Another class of SUSY models invokes gauge mediated supersymmetry breaking [3]. In these models, gauge interactions do not lead to flavor violation because the messenger-matter interactions are flavor blind. Nevertheless, there is a way to introduce flavor changing interactions in these theories. This possibility is based on the observation that some of the messenger fields have the same quantum numbers as some of the Standard Model (SM) fields. Therefore, it is natural to consider direct mixing between messenger and matter fields [4]. In such variants of the gauge mediated models, messengers not only transfer SUSY breaking to usual matter, but also generate flavor violation. Another effect of this mixing is the absence of heavy stable charged (and colored) particles (messengers) in the theory [4]. Other known possibilities to solve the latter problem [5] faced the necessity of fine-tuning of the messenger mass parameters.

The purpose of this paper is to show that in a reasonable range of parameters, messenger-matter mixing in the Minimal Gauge Mediated Model (MGMM)[6] can give rise to the observable rates of rare lepton flavor violating processes like $\mu \rightarrow e\gamma$ and $\mu - e$ conversion and can play a significant role in quark flavor physics. Also we study the effect of messenger-matter mixing on radiative electroweak symmetry breaking. We will consider the messengers belonging either to fundamental or to antisymmetric tensor representation of the $SU(5)$ GUT group.

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We will see that the tree level mixing between the Standard Model fermions is small due to see-saw type suppression. The tree level mixing between sparticles is also small in the part of the parameter space which is natural for MGMM. However, we observe that radiative corrections involving interactions with ordinary Higgs sector induce much stronger mixing of the scalars of MSSM. The point is that messengers obtain masses not through interactions with ordinary Higgs fields, but through interactions with hidden sector. So, the overall matrices of trilinear couplings of sleptons (squarks) and messengers with Higgs fields are not proportional to the corresponding mass matrices (unlike in the Standard Model). As a consequence, in the basis of eigenstates of the tree level mass matrix, matrices of trilinear couplings are not diagonal in flavor. The largest trilinear terms involve squarks and sleptons as well as messenger and Higgs superfields. It is these terms that cause slepton and additional squark mixing³ at the one loop level through loops with Higgs fields and messengers.

We find out that, unlike MGMM without mixing (where $\tan \beta$ is large, $\tan \beta \gtrsim 50$ [6]), there is a wide region of allowed $\tan \beta$ in the model with mixing. The value of $\tan \beta$ is determined by the magnitude of the mixing terms. This observation gives rise to an interesting possibility to relate the rates of flavor violating processes to the Higgs sector parameters.

At high $\tan \beta$, messenger masses of order 10^5 GeV and messenger-matter Yukawa couplings of order $10^{-1} - 10^{-3}$, the rates of $\mu \rightarrow e\gamma$ and $\mu - e$ conversion and flavor violating τ decays are comparable to their experimental limits. Similar result applies to additional FCNC processes in quark sector. At low $\tan \beta$ the experimental constraints on the mixing terms are weaker by about an order of magnitude. It turns out that theoretical constraints inherent in the model imply that at low $\tan \beta$ the contribution of messenger-matter mixing to $b \rightarrow s\gamma$ process is negligible and $\tau \rightarrow e\gamma$ and $\tau \rightarrow \mu\gamma$ decay rates are much below the present experimental limits. The latter observation means that the discovery of lepton flavor violation in τ decays will rule out models with messenger-matter mixing and low $\tan \beta$.

In Ref. [7], the lepton mixing was explored in the model with messengers in the fundamental representation of $SU(5)$. In fact, the analysis of Ref. [7] is valid at low $\tan \beta$. Here we study radiative electroweak breaking and extend the previous analysis to the quark sector and antisymmetric messenger representation as well as to high $\tan \beta$.

It is worth mentioning that the constraints coming from the flavor changing processes, as presented here, may be used for qualitative estimates of the mixing parameters not only in MGMM with mixing, but also in more general gauge mediated models.

2 The model

The MGMM contains, in addition to MSSM particles, two messenger multiplets Q_M and \bar{Q}_M which belong to $\mathbf{5}$ and $\bar{\mathbf{5}}$ representations of $SU(5)$. We will consider also an alternative model with the antisymmetric messengers ($\mathbf{10}$ and $\bar{\mathbf{10}}$). Other representations are incompatible with asymptotic freedom of the unified theory. Messengers couple to a MSSM singlet Ξ through the superpotential term

$$\mathcal{W}_{ms} = \lambda \Xi Q_M \bar{Q}_M. \quad (1)$$

³These mixing terms are additional to the ordinary mixing in the squark sector of MSSM appearing due to the Yukawa couplings to the Higgs fields.

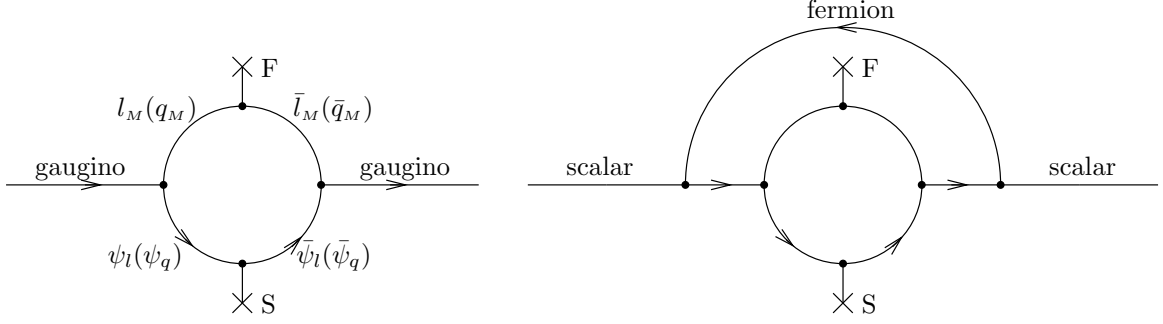


Figure 1: Typical diagrams inducing masses for the MSSM sparticles. Messenger fields run in loops.

Ξ obtains non-vanishing vacuum expectation values F and S via hidden-sector interactions,

$$\Xi = S + F\theta\theta.$$

Gauginos and scalar particles of MSSM obtain masses in one and two loops respectively (see Fig. 1). Their values are [8]

$$M_{\lambda_i} = c_i \frac{\alpha_i}{4\pi} \Lambda f_1(x) \quad (2)$$

for gauginos and

$$\tilde{m}^2 = 2\Lambda^2 \sum_{i=1}^3 c_i C_i \left(\frac{\alpha_i}{4\pi}\right)^2 f_2(x) \quad (3)$$

for scalars. Here α_i are gauge coupling constants of $SU(3) \times SU(2) \times U(1)$, C_i are values of the quadratic Casimir operator for various matter fields: $C_3 = 4/3$ for color triplets (zero for singlets), $C_2 = 3/4$ for weak doublets (zero for singlets), $C_1 = \left(\frac{Y}{2}\right)^2$, where Y is the weak hypercharge. For messengers belonging to the fundamental and antisymmetric representation one has $c_1 = 5/3$, $c_2 = c_3 = 1$ and $c_1 = 5$, $c_2 = c_3 = 3$, respectively.

The two parameters entering eqs. (2) and (3) are

$$\Lambda = \frac{F}{S}$$

and

$$x = \frac{\lambda F}{\lambda^2 S^2}$$

The dependence of the soft masses on x is very mild, as the functions $f_1(x)$ and $f_2(x)$ do not deviate much from 1 [5, 9],

$$f_1(x) = \frac{1}{x^2} [(1+x) \ln(1+x) + (1-x) \ln(1-x)],$$

$$f_2(x) = \frac{1+x}{x^2} \left[\ln(1+x) - 2\text{Li}_2\left(\frac{x}{1+x}\right) + \frac{1}{2}\text{Li}_2\left(\frac{2x}{1+x}\right) \right] + (x \rightarrow -x).$$

Hence, in the absence of mixing between messengers and leptons (and quarks), the predictions of this theory at realistic energies are determined predominantly by the value of Λ , while the value of x is almost unimportant.

Unlike the masses of MSSM particles, the masses of messenger fields strongly depend on x . Namely, the vacuum expectation value of Ξ mixes scalar components of messenger fields and gives them masses

$$M_{\pm}^2 = \frac{\Lambda^2}{x^2}(1 \pm x)$$

It is clear that x must be smaller than 1. Masses of fermionic components of messenger superfields are all equal to $\frac{\Lambda}{x}$.

In fact, the values of x entering eqs. (2) and (3) are different for strongly and weakly interacting sparticles. The reason is that the Yukawa couplings run differently below the GUT scale. The value of Λ remains universal for different matter fields [5]. It has been found that the difference between 'strong' x and 'weak' x does not exceed 30% [10]. This effect is not essential for the values of x not very close to 1 and we will ignore it in what follows.

It has been argued in Ref.[11] that Λ must be larger than 70 TeV, otherwise the theory would generically be inconsistent with mass limits from LEP. The characteristic features of the model without messenger-matter mixing are large $\tan\beta$ [6] (an estimate of Refs.[10, 11] is $\tan\beta \gtrsim 50$) and large squark masses. Parameter μ of the Higgs sector is predicted to be about 500 GeV. There is large mixing between $\tilde{\tau}_R$ and $\tilde{\tau}_L$, proportional to $\tan\beta$ and μ . It results in the mass splitting of τ -sleptons so that the Next Lightest Supersymmetric Particle (NLSP) is a combination of $\tilde{\tau}_R$ and $\tilde{\tau}_L$ [6, 11], the LSP being gravitino. Bino is slightly heavier, but lighter than $\tilde{\mu}_{L,R}$ and $\tilde{e}_{L,R}$.

Messenger fields may be odd or even under R-parity. For instance, fundamental messengers, depending on their R-parity, have the same quantum numbers as either fundamental matter or fundamental Higgs, so the messenger-matter mixing arises naturally. In the latter case triplet messenger fields will give rise to fast proton decay due to the possible Higgs-like mixing with ordinary fields, unless the corresponding Yukawa couplings are smaller than 10^{-21} [12]. Another way to solve this problem is to assume messenger doublet-triplet splitting. In that case one can try to identify messengers with Higgs fields. Such theories were discussed in Refs.[13, 14]; it was shown that there are serious difficulties with particle spectrum and naturalness.

We will consider messengers which are odd under R-parity. Then the components of the fundamental messengers $Q_M^{(5)} = (l_M, q_M)$ have the same quantum numbers as left leptons and right down-quarks, while components of antisymmetric messengers $Q_M^{(10)}$ have quantum numbers of the right leptons, left quarks and right up-quarks. We assume that there is one generation of messengers, fundamental or antisymmetric, and consider their mixing with ordinary matter separately.

In the case of fundamental representation one can introduce messenger-matter mixing [4]

$$\mathcal{W}_{mm}^{(5)} = H_D L_i Y_{ij}^{(5)} E_j + H_D D_i X_{ij}^{(5)} Q_j \quad (4)$$

where

$$H_D = (h_D, \chi_D), \quad H_U = (h_U, \chi_U)$$

are Higgs doublet superfields,

$$L_{\hat{i}} = (\tilde{l}_{\hat{i}}, l_{\hat{i}}) = \left\{ \begin{array}{ll} (\tilde{l}_{L\hat{i}}, l_{L\hat{i}}) & , \hat{i} = 1, \dots, 3 \\ (\tilde{l}_M, l_M) & , \hat{i} = 4 \end{array} \right\}, \quad D_{\hat{i}} = (\tilde{d}_{\hat{i}}, d_{\hat{i}}) = \left\{ \begin{array}{ll} (\tilde{d}_{R\hat{i}}, d_{R\hat{i}}) & , \hat{i} = 1, \dots, 3 \\ (\tilde{d}_M, d_M) & , \hat{i} = 4 \end{array} \right\}$$

are left doublet superfields and right triplet superfields and

$$E_j = (\tilde{e}_{Rj}, e_{Rj}), \quad Q_j = (\tilde{q}_{Lj}, q_{Lj}) = \left(\begin{array}{c} (\tilde{u}_{Lj}) \\ (\tilde{d}_{Lj}) \end{array} \right), \quad j = 1, 2, 3.$$

are right lepton singlet superfields and left quark doublet superfields, respectively.

Hereafter $\hat{i}, \hat{j} = 1, \dots, 4$ label the three left lepton (and right down-quark) generations and the messenger field, $i, j = 1, \dots, 3$ correspond to the three leptons (quarks) and $Y_{\hat{i}j}^{(5)}, X_{\hat{i}j}^{(5)}$ are the 4×3 matrices of Yukawa couplings,

$$Y_{\hat{i}j}^{(5)} = \begin{pmatrix} Y_e & 0 & 0 \\ 0 & Y_\mu & 0 \\ 0 & 0 & Y_\tau \\ Y_{41}^{(5)} & Y_{42}^{(5)} & Y_{43}^{(5)} \end{pmatrix}, \quad X_{\hat{i}j}^{(5)} = \begin{pmatrix} Y_d & 0 & 0 \\ 0 & Y_s & 0 \\ 0 & 0 & Y_b \\ X_{41}^{(5)} & X_{42}^{(5)} & X_{43}^{(5)} \end{pmatrix}$$

In terms of component fields, the tree level scalar potential and Yukawa terms are

$$\begin{aligned} V^{(5)} = & \lambda^2 S^2 \tilde{l}_M^* \tilde{l}_M + \mu^2 h_D h_D^* + |\lambda S \tilde{l}_M + h_D \tilde{e}_{Rj} Y_{4j}^{(5)}|^2 + |\mu h_U + Y_{\hat{i}j}^{(5)} \tilde{l}_{\hat{i}} \tilde{e}_{Rj} + X_{\hat{i}j}^{(5)} \tilde{d}_{\hat{i}} \tilde{q}_{Lj}|^2 \\ & + |Y_{ij}^{(5)} h_D \tilde{e}_{Rj}|^2 + |Y_{ij}^{(5)} \tilde{l}_{\hat{i}} h_D|^2 + \left(\lambda S l_M \bar{l}_M - \lambda F \tilde{l}_M \tilde{l}_M \right. \\ & \left. + Y_{ij}^{(5)} (h_D l_i e_{Rj} + \chi_D \tilde{l}_i e_{Rj} + \chi_D l_i \tilde{e}_{Rj}) + \mu \chi_D \chi_U + h.c. \right) \\ & + \lambda^2 S^2 \tilde{q}_M^* \tilde{q}_M + |\lambda S \tilde{q}_M + h_D \tilde{q}_{Lj} X_{4j}^{(5)}|^2 + |X_{ij}^{(5)} h_D \tilde{q}_{Lj}|^2 + |X_{ij}^{(5)} \tilde{d}_{\hat{i}} h_D|^2 \\ & + \left(\lambda S q_M \bar{q}_M - \lambda F \tilde{q}_M \tilde{q}_M + X_{ij}^{(5)} (h_D d_i q_{Lj} + \chi_D \tilde{d}_i q_{Lj} + \chi_D d_i \tilde{q}_{Lj}) + h.c. \right), \end{aligned} \quad (5)$$

where μ is the usual parameter of MSSM. Besides these terms, there are soft-breaking terms coming from loops involving messenger fields. In the absence of messenger-matter mixing (4) they have the form (at the SUSY breaking scale, which is of order of Λ)

$$\begin{aligned} \mathcal{V}_{sb} = & \tilde{m}_{Li}^2 \tilde{e}_{Li} \tilde{e}_{Li}^* + \tilde{m}_{Ri}^2 \tilde{e}_{Ri} \tilde{e}_{Ri}^* + \\ & \tilde{m}_{qLi}^2 \tilde{q}_{Li} \tilde{q}_{Li}^* + \tilde{m}_{qRi}^2 \tilde{q}_{Ri} \tilde{q}_{Ri}^* + \tilde{m}_{dLi}^2 \tilde{d}_{Li} \tilde{d}_{Li}^* + \tilde{m}_{dRi}^2 \tilde{d}_{Ri} \tilde{d}_{Ri}^*. \end{aligned} \quad (6)$$

where $\tilde{m}_{Lj}^2, \tilde{m}_{Rj}^2, \tilde{m}_{uLj}^2, \tilde{m}_{dLj}^2, \tilde{m}_{uRj}^2, \tilde{m}_{dRj}^2$ are given by eq. (3). Low energy effective potential can be obtained from eq. (6) by making use of renormalization group equations. It is worth noting that the boundary conditions for soft Higgs mass term $B\mu h_U h_D$ and for scalar trilinear couplings are set to zero in MGMM because their values at the scale Λ are suppressed in comparison with other terms in eq. (6). Messenger-matter mixing modifies eq. (6); we will consider this modification later on.

There is no CP-violation in this theory in lepton sector. Arbitrary phases may be rotated away by redefinition of the lepton fields. On the other hand, there appears CP-violation in quark sector, in addition to the CKM mechanism. There are three phases in the matrix $X^{(5)}$ and only one of them may be set equal to zero by redefinition of the messenger fields.

The same formalism applies to antisymmetric messenger fields. The mixing terms have the following form,

$$\mathcal{W}_{mm}^{(10)} = H_D E_i Y_{ij}^{(10)} L_j + H_D Q_i W_{ij}^{(10)} D_j + H_U U_i X_{ij}^{(10)} Q_j \quad (7)$$

One can make the first term in the Lagrangian (7) real by redefinition of lepton fields. So, CP-violation comes only from the last two terms. In addition to the SM CP-violating phase they contain six new phases in Yukawa couplings⁴ $X_{4j}^{(10)}$ and $X_{j4}^{(10)}$ and two phases in couplings $W_{4j}^{(10)}$.

To summarize, messenger-matter mixing in the leptonic sector occurs through the Yukawa couplings $Y_{4i}^{(5)}$ or $Y_{4i}^{(10)}$ ($i = 1, 2, 3$), depending on the representation of the messenger fields. Likewise, the mixing in the quark sector appears through $X_{4i}^{(5)}$ or $X_{i4}^{(10)}$, $X_{4i}^{(10)}$, $W_{4i}^{(10)}$. In the following sections we sometimes use the collective notation Y_i for the couplings Y_{4i} , X_{4i} , X_{i4} and W_{4i} in statements applicable to all of them.

3 Induced mixing of matter fields.

In this section we consider mixing between the fields of MSSM that appears after messengers are integrated out. It is straightforward to check that fermion mixing terms are small at the tree level (the tree level fermion mass matrices are presented in Appendix **A**). In principle, this mixing (the off-diagonal terms in eqs. (43) and (44)) may lead to lepton and quark flavor violation due to one loop diagrams involving scalars and gauginos [2]. However, these mixing terms are negligible due to see-saw type mechanism: in MGMM one definitely has $\lambda S > 10^4$ GeV, $\tan \beta \gtrsim 1$ and the tree level fermion mixing terms are smaller than 10^{-4} even at $Y_i \sim 1$, see eqs. (43) and (44). The corresponding contributions to flavor violating rates are too small to be observable.

Mixing in the scalar sector is also small at the tree level (the tree level mass matrices of scalars are given in Appendix **B**). After the scalar messengers are integrated out at the tree level, the lepton flavor violating terms in the mass matrix of right sleptons and quark flavor violating terms in the mass matrix of down-squarks are of order

$$Y_i^* Y_j \left(v_D^2 x^2 + \frac{\mu^2 v_U^2 x^2}{\Lambda^2} \right) \quad (8)$$

for generic values of x (not too close to 1). Analogous expression with interchange of v_U and v_D holds for up-squarks in the case of antisymmetric messengers. These terms are smaller than the one loop contributions (see below). The only substantial non-diagonal terms in the slepton mass matrix are $(-\lambda F)$ in the messenger sector and $\tilde{\tau}_R - \tilde{\tau}_L$ mixing proportional to $\tan \beta$. Due to the latter, the NLSP is $\tilde{\tau}$ at $\tan \beta > 25$ [6], while at lower $\tan \beta$ the NLSP is neutralino. The only substantial non-diagonal terms in the squark mass matrix are $(-\lambda F)$ in the messenger sector and $\tilde{b}_R - \tilde{b}_L$ mixing (and $\tilde{t}_R - \tilde{t}_L$ mixing in the case of antisymmetric messengers).

The dominant contributions to mixing in slepton and squark sectors appear through one loop diagrams from trilinear terms in the superpotential, that involve H_D for fundamental

⁴The seventh phase in $X_{44}^{(10)}$ which corresponds to messenger-messenger mixing will not be of interest because its effects on the processes in SM sector are negligible.

messengers, and also H_U for antisymmetric ones. The fact that the one loop mixing terms of scalars are proportional to the large parameter Λ^2 is obvious from eq. (5): say, one of the cubic terms in the scalar potential, $[\lambda SY_{4j}^{(5)} \bar{l}_M^* h_D \tilde{e}_{Rj} + \text{h.c.}]$, contains $(\lambda S) = \frac{\Lambda}{x}$ explicitly.

After diagonalizing the messenger mass matrix we obtain the diagrams contributing to slepton mixing to the order $(\lambda S)^2$, which are shown in Fig. 2. Similar diagrams contribute to

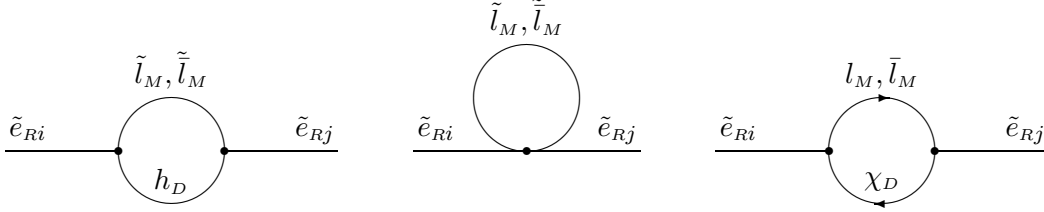


Figure 2: The diagrams dominating the slepton mixing matrix.

the mass matrix for left squarks. In the antisymmetric case we obtain from similar diagrams mixing terms in the mass matrices of left sleptons and squarks. If supersymmetry were unbroken, the sum of these diagrams would be equal to zero. In our case of broken supersymmetry the resulting contributions to the mass matrices of MSSM scalars are

for fundamental messengers

$$\text{right sleptons} \quad \delta m_{ij}^2 = -\frac{\Lambda^2}{8\pi^2} Y_{4i}^{(5)} Y_{4j}^{(5)} f_3(x) \quad (9)$$

$$\text{left (up and down) squarks} \quad \delta m_{ij}^2 = -\frac{\Lambda^2}{16\pi^2} X_{4i}^{(5)*} X_{4j}^{(5)} f_3(x) \quad (10)$$

for antisymmetric messengers

$$\text{left sleptons (selectrons and sneutrino)} \quad \delta m_{ij}^2 = -\frac{\Lambda^2}{16\pi^2} Y_{4i}^{(10)} Y_{4j}^{(10)} f_3(x) \quad (11)$$

$$\text{right up squarks} \quad \delta m_{ij}^2 = -\frac{\Lambda^2}{8\pi^2} X_{i4}^{(10)*} X_{j4}^{(10)} f_3(x) \quad (12)$$

$$\text{left (up and down) squarks} \quad \delta m_{ij}^2 = -\frac{\Lambda^2}{16\pi^2} X_{4i}^{(10)*} X_{4j}^{(10)} f_3(x) \quad (13)$$

$$\text{right down squarks} \quad \delta m_{ij}^2 = -\frac{\Lambda^2}{8\pi^2} W_{4i}^{(10)*} W_{4j}^{(10)} f_3(x) \quad (14)$$

where

$$f_3(x) = \frac{1}{x^2} \left\{ -\ln(1-x^2) - \frac{x}{2} \ln\left(\frac{1+x}{1-x}\right) \right\} \quad (15)$$

These terms were obtained to the zeroth order in the Higgs masses. The higher order contributions are suppressed by the squared gauge coupling constants or by small ratio $\frac{\mu}{\Lambda}$. Notice that mixing terms (10), (12)-(14) violate CP.

Since scalars get negative shifts in squared masses, these expressions for the soft terms immediately imply theoretical bounds on Yukawa couplings Y_i which come from the requirement [4] that none of the scalar squared masses becomes negative (see below).

Now, let us see that the contributions (9) – (14) are much larger than the tree level mixing terms. As an example, at small x one has for the one loop terms

$$\delta m_{ij}^2 \simeq -\frac{a}{6} \frac{\Lambda^2}{16\pi^2} Y_i^* Y_j x^2 \quad (16)$$

where $a = 1$ or 2 depending on the $SU(2)$ representation of the fields. These terms dominate over the tree level ones (8) provided the following inequalities are satisfied,

$$v_D^2 + \frac{\mu^2 v_U^2}{\Lambda^2} < \frac{\Lambda^2}{96\pi^2}, \quad v_U^2 + \frac{\mu^2 v_D^2}{\Lambda^2} < \frac{\Lambda^2}{96\pi^2}.$$

In MGMM one has $\Lambda > 10$ TeV and $\mu \simeq 400 \div 500$ GeV, so these inequalities indeed hold.

In the case of very small values of x the dominant contributions to δm_{ij}^2 come from two loops rather than one loop. The two-loop contributions appear from the same diagrams as those shown in Fig. 2 but with one-loop enhanced vertices. The result for the diagonal mass terms was found in Ref. [15] and has the following form,

$$\delta m_i^2 = \frac{d_i}{8\pi^2} \frac{|Y_{4i}|^2}{4\pi} \left(\frac{D}{2} \frac{|Y_{4i}|^2}{4\pi} - C\alpha \right) \Lambda^2. \quad (17)$$

Here $C = \sum C_i$ and $D = \sum d_i$ where the sums run over all superfields participating in the mixing interactions; C_i denote quadratic Casimir operators and d_i are the numbers of fields circulating in the Yukawa loop. The off-diagonal terms are similar. One can compare this result to eqs. (9) – (14) and find the region of parameters where the contributions (17) become significant,

$$x^2 < \frac{3}{\pi} \frac{d_i}{a} \left| \frac{D}{2} \frac{|Y_{4i}|^2}{4\pi} - C\alpha \right| \quad (18)$$

Requirement of positivity of the scalar masses implies that the first term in eq. (18) has to be smaller than the second one, so the two-loop corrections become essential in the region $x \sim \sqrt{\alpha}$.

4 Electroweak breaking and squark masses

It was already mentioned that radiative electroweak breaking in MGMM without messenger-matter mixing leads to large values of $\tan\beta \gtrsim 50$. The usual way to avoid this limit is to assume some extra soft contribution to the Higgs sector of the theory. In this section we consider electroweak breaking in the model with messenger-matter mixing. In particular, we show that wide range of values of $\tan\beta$ is now allowed without any additional parameters in the Higgs sector of the model. For definiteness, we concentrate on the case of fundamental messengers.

Minimization of the Higgs potential results in the following two equations,

$$\begin{aligned}\sin 2\beta &= \frac{-2B\mu}{m_{h_U}^2 + m_{h_D}^2 + 2\mu^2} \\ \mu^2 &= \frac{m_{h_D}^2 - m_{h_U}^2 \tan^2 \beta}{\tan^2 \beta - 1} - \frac{1}{2}M_Z^2\end{aligned}\quad (19)$$

The parameter B characterizes the magnitude of the soft mixing term in the Higgs sector, $B\mu h_U h_D$. At the two loop level it is equal to [6]

$$B = M_{\lambda_2}(-0.12 + 0.17Y_t^2), \quad (20)$$

where M_{λ_2} is given by eq. (2).

In MGMM without messenger-matter mixing the value of the soft mass $m_{h_D}^2$ is given by eq. (3) while $m_{h_U}^2$ receives additional large negative one-loop correction due to large Yukawa coupling between H_U and t-quark,

$$\delta m_{h_U}^2 = -\frac{3Y_t^2}{4\pi^2}m_{\tilde{t}}^2 \ln\left(\frac{\Lambda}{xm_{\tilde{t}}}\right). \quad (21)$$

It is natural to expect large value of $\tan\beta$ in such a situation. Indeed, if the two Higgs fields were not mixed and only one of them (h_U in our case) obtained the negative mass squared then $\tan\beta \equiv \frac{v_U}{v_D}$ would be equal to infinity. Of course, there is mixing between h_U and h_D in SUSY theories due to μ -term. However, it follows from eqs. (3) and (21) that $\delta m_{h_U}^2 \gg m_{h_U}^2, m_{h_D}^2$ so that eq. (19) takes the following simple form

$$\mu^2 \simeq -\delta m_{h_U}^2, \quad \sin 2\beta \simeq -\frac{2B}{\mu} \quad (22)$$

It is clear from eqs. (20) and (22) that $\sin 2\beta \ll 1$, and, therefore, $\tan\beta \gg 1$. This simple estimate gives

$$\tan\beta \gtrsim 50 \quad (23)$$

At so large values of $\tan\beta$ other corrections to Higgs masses (e.g., the corrections to $m_{h_D}^2$ due to the interaction with b-quark analogous to eq. (21)) have to be taken into account. However, detailed analysis [10, 11] in which these corrections are included, confirms the estimate (23).

In the presence of messenger-matter mixing, $m_{h_D}^2$ receives additional negative contributions from the diagrams analogous to those shown in Fig. 2. The purpose of this section is to demonstrate that these contributions can make $\tan\beta$ to be as low as $\tan\beta \sim 1$. The simplified analysis along the lines outlined above is sufficient for this purpose, as additional contributions (e.g., due to b-quark) are negligible at not too large $\tan\beta$. Certainly, our results become qualitative at $\tan\beta \gtrsim 30$, when these corrections are significant.

In our case of fundamental messengers, the contribution to $m_{h_D}^2$ due to messenger-matter mixing is equal to

$$\delta m_{h_D}^2 = -d_D^{(5)} \frac{\Lambda^2}{16\pi^2} f_3(x), \quad (24)$$

where

$$d_D^{(5)} = \sum_{i=1}^3 \left(|Y_{4i}^{(5)}|^2 + 3|X_{4i}^{(5)}|^2 \right) \quad (25)$$

Depending on the Yukawa coupling constants, the mass splitting $\delta m_{h_D}^2$ may be of the same order as $\delta m_{h_U}^2$. Therefore the value of $\sin 2\beta$ gets modified as compared to eq. (22). Instead, one has the estimate,

$$\mu^2 \simeq \delta m_{h_U}^2, \quad \sin 2\beta \simeq \frac{-2B\mu}{\mu^2 - \delta m_{h_D}^2} \quad (26)$$

Therefore, messenger-matter mixing reduces $\tan \beta$. The result of numerical solution of eq. (19) in the theory with mixing is shown in Fig. 3, where we set $x = 1$ in the argument of the logarithm in eq. (21)⁵.

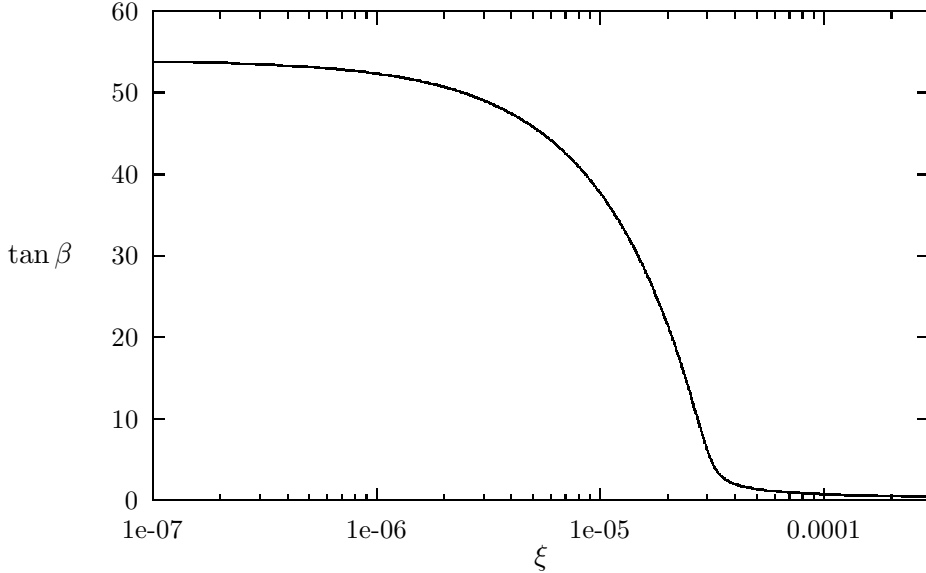


Figure 3: $\tan \beta$ as function of $\xi = \delta m_{h_D}^2 / \Lambda^2 = -d_D^{(5)} \frac{f_3(x)}{16\pi^2}$.

We will see in section 5 that experimental bounds coming from flavor physics constrain not the individual couplings Y_i but the products $Y_i Y_j$. So, the only bound on $\delta m_{h_D}^2$ is theoretical one, related to the requirement of positivity of masses of squarks and sleptons. Namely, with loop corrections (9) – (14) to the scalar mass matrix included, two of its eigenvalues remain the same and the third one decreases. Its value is still positive only if

$$\tilde{m}^2 + \sum_{j=1}^3 \delta m_{jj}^2 > 0. \quad (27)$$

In the case of fundamental messengers one obtains for slepton couplings

$$\sum |Y_{4i}^{(5)}|^2 < \frac{80\pi^2}{3} \frac{f_2(x)}{f_3(x)} \left(\frac{\alpha_1}{4\pi}\right)^2 \quad (28)$$

which at small x reduces to

$$\sum |Y_{4i}^{(5)}|^2 x^2 < 10^{-3}. \quad (29)$$

⁵The extension to small x is straightforward. The results for $\tan \beta$ change only slightly because of weak logarithmic dependence on x .

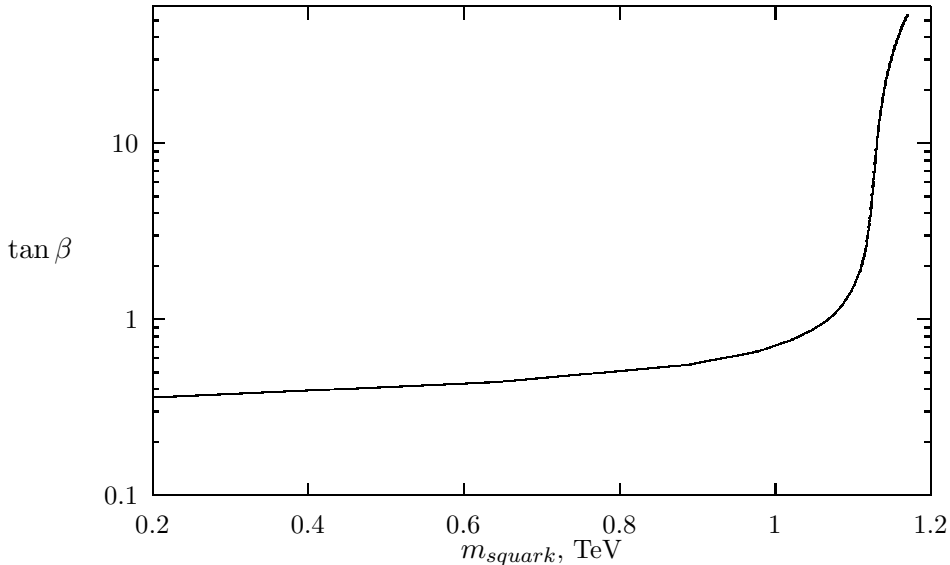


Figure 4: $\tan \beta$ as function of the lightest squark mass m_{squark} .

In squark sector one finds

$$\sum |X_{4i}^{(5)}|^2 < \frac{128\pi^2}{3} \frac{f_2(x)}{f_3(x)} \left(\frac{\alpha_3}{4\pi}\right)^2 \quad (30)$$

which in the case of small x reduces to

$$\sum |X_{4i}^{(5)}|^2 x^2 < 0.1 .$$

These inequalities give $\delta m_{h_D}^2/\Lambda^2 < 3 \times 10^{-4}$. Hence, all values of ξ shown in Fig. 3 are allowed; the value of $\tan \beta$ strongly depends on the mixing parameter d and may actually be rather small. One can show that this conclusion survives if higher order corrections are taken into account.

On the other hand, the parameter μ weakly depends on mixing and grows rapidly only at $\tan \beta \lesssim 1$.

It is worth pointing out the correspondence between the mass spectrum and the value of $\tan \beta$ in MGMM with mixing. The theoretical limit on $Y_{4i}^{(5)}$, eq. (29), implies that these couplings are always too small to alter the value of $\tan \beta$ and one can neglect their contribution in eq. (25). Consequently, small $\tan \beta$ in this model is correlated with large corrections to the mass matrix of left squarks. As was already mentioned, these corrections lead to the significant decrease of the mass of one of the squark doublets.

The relation between $\tan \beta$ and the lightest squark mass at $\Lambda = 100$ TeV is presented in Fig. 4. There are two distinct regions on this plot. The first region corresponds to rapidly changing $\tan \beta$ ($50 \div 1$) and slowly changing m_{squark} ($1.2 \div 1.0$ TeV). In fact, there exists also squark mass splitting due to the mixing between left and right squarks proportional to $\tan \beta$, which was not taken into account above. It emerges even without messenger-matter mixing, in the same way as $\tilde{\tau}_R - \tilde{\tau}_L$ mass splitting mentioned in section 2, and is of order $0.1 \div 0.2$ TeV

at large $\tan\beta$. Hence, the mass range of the lightest squark $m_{squark} \sim 1.2 \div 1.0$ TeV is not a peculiarity of the model with messenger-matter mixing. On the other hand, the second region ($\tan\beta \sim 1, > m_{squark} = 250 \div 1000$ GeV, where the lower bound is the experimental limit on the squark mass) provides a distinctive signature of the model with messenger-matter mixing. The observation of one and only one pair of relatively light left up and down squarks would be a strong indication of the gauge mediation scenario with fundamental messengers; in this case $\tan\beta$ is predicted to be quite low and its value is correlated to the mass of the lightest squark.

In the case of antisymmetric messengers, messenger-matter mixing contributes to $m_{h_D}^2$ and $m_{h_U}^2$,

$$\delta m_{h_D}^2 = -d_D^{(10)} \frac{\Lambda^2}{16\pi^2} f_3(x) \quad , \quad \delta m_{h_U}^2 = -d_U^{(10)} \frac{\Lambda^2}{16\pi^2} f_3(x) \quad (31)$$

where

$$d_D^{(10)} = \sum_{i=1}^3 \left(|Y_{4i}^{(10)}|^2 + 3|W_{4i}^{(10)}|^2 \right) \quad , \quad d_U^{(10)} = \sum_{i=1}^3 \left(3|X_{i4}^{(10)}|^2 + 3|X_{4i}^{(5)}|^2 \right) .$$

The requirement of positivity for squared scalar masses gives

$$\begin{aligned} \sum |Y_{4i}^{(10)}|^2 &< 72\pi^2 \frac{f_2(x)}{f_3(x)} \left(\frac{\alpha_2}{4\pi} \right)^2 , \\ \sum |W_{4i}^{(10)}|^2 &< 64\pi^2 \frac{f_2(x)}{f_3(x)} \left(\frac{\alpha_3}{4\pi} \right)^2 , \\ \sum |X_{i4}^{(10)}|^2 &< 64\pi^2 \frac{f_2(x)}{f_3(x)} \left(\frac{\alpha_3}{4\pi} \right)^2 , \\ \sum |X_{4i}^{(10)}|^2 &< 128\pi^2 \frac{f_2(x)}{f_3(x)} \left(\frac{\alpha_3}{4\pi} \right)^2 , \end{aligned}$$

which in the case of small x reduce to

$$\begin{aligned} \sum |Y_{4i}^{(10)}|^2 x^2 &< 3 \cdot 10^{-2} , \\ \sum |W_{4i}^{(10)}|^2 x^2 &< 0.2 , \\ \sum |X_{4i}^{(10)}|^2 x^2 &< 0.2 , \\ \sum |X_{i4}^{(10)}|^2 x^2 &< 0.4 . \end{aligned} \quad (32)$$

These inequalities give $\xi_D = \delta m_{h_D}^2 / \Lambda^2 < 6 \cdot 10^{-4}$ and $\xi_U = \delta m_{h_U}^2 / \Lambda^2 < 2 \cdot 10^{-3}$. The small values of $\tan\beta$ are also possible, if ξ_D is large enough so that $m_{h_D}^2$ and $m_{h_U}^2$ are of the same order. This happens when some of the Yukawa couplings $W_{4i}^{(10)}$ are sufficiently large. Large $W_{4i}^{(10)}$ induce large corrections to the mass matrix of right down squarks, $\delta \tilde{m}_{ij}^2 \propto W_{4i}^{(10)} W_{4j}^{(10)} x^2$. Consequently, small $\tan\beta$ is correlated with light right down squark in the case of antisymmetric messengers.

5 Flavor violation

Let us now consider the effects of scalar mixing on the usual leptons and quarks. In what follows we neglect flavor violating amplitudes coming not from slepton and squarks matrices given by eqs. (9) – (14) but from renormalization of gauge couplings by messengers. These effects become significant at the same range of x as the two-loop corrections, eq. (18), and play a similar role. Namely, they prevent Y_i to be arbitrarily large at small x . The point is that all limits obtained from eqs. (9) – (14) contain the products $Y_i^* Y_j f_3(x) \sim Y_i^* Y_j x^2$ while two-loop corrections (18), as well as the corrections originating from the gauge coupling renormalization provide limits on Yukawa couplings themselves.

Also, we make further simplification. Various mixing terms (including Standard Model ones) provide additive contributions to the amplitudes of the processes under consideration. Nevertheless, we obtain the constraints by considering every contribution separately, i.e., by setting all others to zero and neglecting possible interference. This will be sufficient for understanding the allowed magnitudes of the Yukawa couplings that induce mixing⁶. We again point out that we consider fundamental and antisymmetric cases separately, so we do not discuss contributions which are proportional to $Y^{(5)} Y^{(10)}$ in spite of their presence in theories with various messengers belonging to different representations. More accurate consideration of these points is straightforward.

As shown in the previous section, the wide range of $\tan \beta$ is allowed in this model, depending on the mixing terms. We will consider the cases of high $\tan \beta \sim 50$ and low $\tan \beta \sim 1$ separately. The reason is that two different contributions to the amplitudes dominate in these two regimes. The first one⁷ is proportional to $\tan \beta$ and dominates at high $\tan \beta$. The second one is independent of $\tan \beta$ and becomes significant at low $\tan \beta$.

5.1 Lepton sector

Let us first consider the case of fundamental messengers. There are two types of non-diagonal elements in the slepton mass matrix. The first one is the flavor diagonal left-right mixing, coming from the tree level potential (5) and proportional to $\mu m_f \tan \beta$, where m_f is the mass of the corresponding fermion flavor. The second type is the flavor violating mixing (9) in the sector of right sleptons.

The mass matrix of right sleptons can be diagonalized by an orthogonal rotation; let us denote the corresponding orthogonal 3×3 matrix by V_{ij} . As a result, one of the masses of right sleptons (without loss of generality we denote the corresponding slepton as $\tilde{\tau}_R$) receives negative contribution equal to

$$\Delta m_{R,3}^2 = -\frac{\Lambda^2}{8\pi^2} f_3(x) \Delta^2 ,$$

where

$$\Delta^2 = (Y_{41}^{(5)})^2 + (Y_{42}^{(5)})^2 + (Y_{43}^{(5)})^2 .$$

There are two sources of flavor violation after this rotation, namely, the interactions between neutralino, right sleptons and leptons and left-right mixing. The situation here is completely

⁶It worth mentioning, however, that in case of the lepton mixing there are no Standard Model contributions and our limits are exact in this sense.

⁷This contribution was missed in Ref. [7].

analogous to the lepton flavor violation in the SUSY $SU(5)$ model with universal soft terms at the Planck scale, which was studied in detail in Ref. [2].

The resulting rate of $\mu \rightarrow e\gamma$ decay is equal to [2]

$$\Gamma(\mu \rightarrow e\gamma) = \frac{\alpha}{4} m_\mu^3 \left(|F_2^{(a)}|^2 + |F_2^{(b)}|^2 \right), \quad (33)$$

where

$$F_2^{(a)} = \frac{\alpha_1}{4\pi M_{Bino}} \mu m_\mu \tan \beta V_{23} V_{13} \left[G_2 \left(\tilde{m}_L^2, \tilde{m}_R^2 - \frac{\Lambda^2}{8\pi^2} f_3(x) \Delta^2 \right) - G_2 \left(\tilde{m}_L^2, \tilde{m}_R^2 \right) \right], \quad (34)$$

$$F_2^{(b)} = \frac{\alpha_1}{4\pi} m_\mu V_{23} V_{13} \left[G_1 \left(\tilde{m}_R^2 - \frac{\Lambda^2}{8\pi^2} f_3(x) \Delta^2 \right) - G_1 \left(\tilde{m}_R^2 \right) \right] \quad (35)$$

with

$$G_2(m_1^2, m_2^2) = \frac{g_2 \left(\frac{m_1^2}{M_{Bino}^2} \right) - g_2 \left(\frac{m_2^2}{M_{Bino}^2} \right)}{m_1^2 - m_2^2}, \quad g_2(r) = \frac{1}{2(r-1)^3} [r^2 - 1 - 2r \ln r],$$

$$G_1(m^2) = \frac{1}{M_{Bino}^2} g_1 \left(\frac{m^2}{M_{Bino}^2} \right), \quad g_1(r) = -\frac{1}{6(r-1)^4} [2 + 3r - 6r^2 + r^3 + 6r \ln r].$$

At large $\tan \beta$ (say, $\tan \beta \sim 50$), the leading contribution to $\mu \rightarrow e\gamma$ decay is given by the term (34) which comes from the diagram shown in Fig. 5a with left-right slepton mixing insertion. (We will comment on the validity of this approximation later on.) This contribution

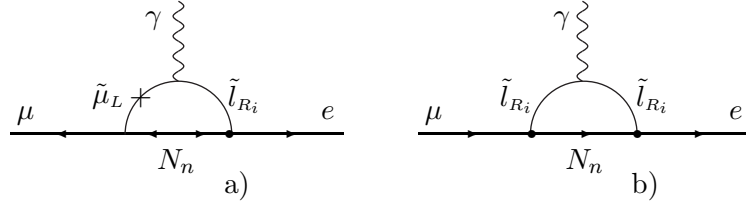


Figure 5: Diagrams contributing to $\mu \rightarrow e\gamma$ decay

is enhanced by a factor of order $\frac{M_{Bino} \mu \tan \beta}{\tilde{m}_L^2} \sim 30$ in comparison with the term (35), coming from the diagram of Fig. 5b without chirality flip. In Fig. 5, N_n denotes combinations of bino and higgsino. In fact, the dominant effect comes from bino, as higgsino is significantly heavier in MGMM.

The elements of the rotating matrix V_{ij} can be found explicitly in our case. The relevant matrix elements are

$$V_{i3} = \frac{Y_{4i}^{(5)}}{\Delta}.$$

Hence, the functions (34), (35) can be written in the following form,

$$F_2^{(a)} = \frac{\alpha_1}{4\pi M_{Bino}} \mu m_\mu \tan \beta \frac{Y_{41}^{(5)} Y_{42}^{(5)}}{\Delta^2} \left[G_2 \left(\tilde{m}_L^2, \tilde{m}_R^2 - \frac{\Lambda^2}{8\pi^2} f_3(x) \Delta^2 \right) - G_2 \left(\tilde{m}_L^2, \tilde{m}_R^2 \right) \right] \approx \quad (36)$$

$$\frac{\alpha_1}{4\pi} \mu m_\mu \tan \beta \delta m_{12}^2 \frac{\partial G_2 \left(\tilde{m}_L^2, \tilde{m}_R^2 \right)}{\partial \tilde{m}_R^2},$$

	$\tan \beta \sim 50$	$\tan \beta \sim 1$
$\mathbf{5} + \overline{\mathbf{5}}$:	$\left Y_{41}^{(5)} Y_{42}^{(5)} \right ^{1/2} x < 6 \cdot 10^{-4}$	$\left Y_{41}^{(5)} Y_{42}^{(5)} \right ^{1/2} x < 0.003$
$\mathbf{10} + \overline{\mathbf{10}}$:	$\left Y_{41}^{(10)} Y_{42}^{(10)} \right ^{1/2} x < 0.002$	$\left Y_{41}^{(10)} Y_{42}^{(10)} \right ^{1/2} x < 0.03$

Table 1: The constraints on the Yukawa couplings coming from the $\mu \rightarrow e\gamma$ decay at $\Lambda = 100$ TeV for fundamental and $\Lambda = 50$ TeV for antisymmetric messengers in the cases of high and low $\tan \beta$ and $x \lesssim 0.8$.

$$F_2^{(b)} = \frac{\alpha_1}{4\pi} m_\mu \frac{Y_{41}^{(5)} Y_{42}^{(5)}}{\Delta^2} \left[G_1 \left(\tilde{m}_R^2 - \frac{\Lambda^2}{8\pi^2} f_3(x) \Delta^2 \right) - G_1(\tilde{m}_R^2) \right] \approx \frac{\alpha_1}{4\pi M_{Bino}} m_\mu \delta m_{12}^2 \frac{\partial G_1(\tilde{m}_R^2)}{\partial \tilde{m}_R^2}, \quad (37)$$

where the approximate equalities hold at small Δ and correspond to mass insertion approximation with respect to mixing of right sleptons. Equations (36) and (37) and experimental bounds [16] on the rate $\Gamma(\mu \rightarrow e\gamma)$ give limits on the parameters $Y_{41}^{(5)}$, $Y_{42}^{(5)}$ and $Y_{43}^{(5)}$ which appear in the combinations $Y_{41}^{(5)} Y_{42}^{(5)}$ and Δ . The maximal allowed value of the product $Y_{41}^{(5)} Y_{42}^{(5)}$ corresponds to the regime $Y_{41}^{(5)} = Y_{42}^{(5)}$ and $Y_{43}^{(5)} = 0$. In this regime $\Delta^2 = 2Y_{41}^{(5)} Y_{42}^{(5)}$; we will see that Δ is small there, so that approximate equalities in eqs. (36) and (37) indeed hold. This means that conservative upper limit on the product $Y_{41}^{(5)} Y_{42}^{(5)}$ can be obtained by making use of the mass insertion with respect to mixing of the right sleptons from the very beginning. The corresponding limits on the product $\sqrt{Y_{41}^{(5)} Y_{42}^{(5)}} f_3(x) \sim \sqrt{Y_{41}^{(5)} Y_{42}^{(5)}} x$ at x not very close to one is shown in Table 1. One can see that constraints on the mixing terms in the case of high $\tan \beta$ are stronger.

At $Y_{41}^{(5)} \neq Y_{42}^{(5)}$ and/or $Y_{43}^{(5)} \neq 0$, one has $\Delta^2 > 2Y_{41}^{(5)} Y_{42}^{(5)}$ and it follows from the explicit forms of G_1 and G_2 that limits on the product $\sqrt{Y_{41}^{(5)} Y_{42}^{(5)}} x$ are stronger than those presented in Table 1. The deviation from the mass insertion results (approximate equalities in eqs. (36) and (37)) is not very strong, however, so Table 1 still gives an idea of the allowed values of Yukawa couplings.

The slepton mixing in the gauge-mediated models gives rise also to $\mu \rightarrow e$ conversion. The dominant contribution to its rate $\Gamma(\mu \rightarrow e)$ is given by penguin-type diagrams, while box diagrams are suppressed by squark masses. So, there is a simple relation between the rates of $\mu \rightarrow e$ conversion and $\mu \rightarrow e\gamma$ [7],

$$\frac{\Gamma(\mu \rightarrow e)}{\Gamma(\mu \rightarrow e\gamma)} = 2.8 \cdot 10^{-2},$$

while the ratio of experimental limits [16] is

$$\frac{\Gamma(\mu \rightarrow e)_{exp}^{lim}}{\Gamma(\mu \rightarrow e\gamma)_{exp}^{lim}} = 1.1 \cdot 10^{-1}.$$

Hence, the existing limit on $\mu - e$ - conversion gives weaker (by a factor of 1.4) bound on the product of Yukawa couplings $|Y_{41}^{(5)} Y_{42}^{(5)}|^{1/2}$.

Analogously, one can find limits coming from flavor violating τ decays. One finds, that for these decays the limits in the regimes when one of the Yukawas is equal to zero and $\tan \beta \sim 50$ are also strong enough for the mass insertion approximation with respect to Δ^2 to be valid. The corresponding limits on Yx are shown in Table 2. Another situation occurs at low $\tan \beta$. In

	$\tau \rightarrow \mu\gamma$	$\tau \rightarrow e\gamma$
5 + $\bar{5}$:	$ Y_{42}^{(5)} Y_{43}^{(5)} ^{1/2} x < 0.01$	$ Y_{41}^{(5)} Y_{43}^{(5)} ^{1/2} x < 0.01$
10 + $\bar{10}$:	$ Y_{42}^{(10)} Y_{43}^{(10)} ^{1/2} x < 0.04$	$ Y_{41}^{(10)} Y_{43}^{(10)} ^{1/2} x < 0.04$

Table 2: The constraints on the Yukawa couplings coming from flavor violating τ decays at $\Lambda = 100$ TeV for fundamental and $\Lambda = 50$ TeV for antisymmetric messengers, $x \lesssim 0.8$ and $\tan \beta = 50$.

the case of flavor violating τ decays large values of Δ even at $Y_{41}^{(5)} = 0$ (or $Y_{42}^{(5)} = 0$ depending on the type of τ decay) are not restricted experimentally and mass insertion technique is not valid. The upper bounds on the relevant combination of Yukawa couplings derived from the experimental limits on $\Gamma(\tau \rightarrow \mu\gamma)$ and $\Gamma(\tau \rightarrow e\gamma)$ [16] are weaker than theoretical constraints inherent in this model (see section 4.1). At $\tan \beta \sim 1$ the experimental limits on the rates of flavor changing τ decays are at least by order magnitude larger than maximum rates allowed in this model. It is worth noting that this value of suppression factor corresponds to the extreme limit $Yx \gtrsim 1$. This factor scales as $(Yx)^{-4}$ at smaller value of mixing. Hence, self-consistence of this model at low $\tan \beta$ requires that the rates $\Gamma(\tau \rightarrow e\gamma)$ and $\Gamma(\tau \rightarrow \mu\gamma)$ are lower than the present experimental limits.

There is another type of flavor violating processes, namely, oscillations of charged sleptons and sneutrinos. Pair production of sleptons (which decay into leptons and bino) in $e^+ - e^-$ annihilation at the Next Linear Collider will result in acoplanar $l_i^+ - l_i^-$ events with missing energy. Recall, that the NLSP in the model with low $\tan \beta$ is neutralino \tilde{N} , while in model with high $\tan \beta$ the NLSP is $\tilde{\tau}$, so there will be additional τ -leptons coming from bino decays in the latter case. In the presence of slepton mixing, the slepton oscillations leading to lepton flavor violating $l_i^\pm - l_j^\mp$ events, are possible [17].

For example, let us consider the case of fundamental messengers, low $\tan \beta$ and $Y_{43}^{(5)} = 0$. Oscillations of $\tilde{\mu}_R$ and \tilde{e}_R are characterized by the corresponding mixing angle, which in this model can be found from eq. (9),

$$\tan 2\phi = 2 \frac{|Y_{41}^{(5)} Y_{42}^{(5)}|}{|Y_{41}^{(5)}|^2 - |Y_{42}^{(5)}|^2} \quad (38)$$

If $Y_{41}^{(5)} \sim Y_{42}^{(5)}$ then the slepton mixing is close to maximal. The cross section of $e^+ e^- \rightarrow e^\pm \mu^\mp + 2\tilde{N}$ may, however, be suppressed even at large mixing if the lifetimes of

$\tilde{\mu}_R$ and \tilde{e}_R are small compared to the period of oscillations. The absence of such a suppression requires [17]

$$2\Gamma_{sl}M_{e_R} < |M_{e_R}^2 - M_{\mu_R}^2| \quad (39)$$

where M_{e_R} and M_{μ_R} denote the true slepton masses. For slepton decay width Γ_{sl} we have

$$\Gamma_{sl} = \frac{\alpha_1}{2}\tilde{m}_R \left(1 - \frac{M_{bino}^2}{\tilde{m}_R^2}\right)^2, \quad (40)$$

where \tilde{m}_R is an average slepton mass. By making use of eqs. (2), (3) and (9), it is straightforward to translate the condition (39) into a condition imposed on Yukawa couplings $Y_{41}^{(5)}$ and $Y_{42}^{(5)}$. For example, at small x one has

$$(|Y_{41}^{(5)}|^2 + |Y_{42}^{(5)}|^2)x^2 > 5 \cdot 10^{-6} \quad (41)$$

This relation and limits shown in Table 1 imply that at low $\tan\beta$ there is a fairly wide range of parameters in which $\mu \rightarrow e\gamma$ decay and slepton oscillations are both allowed. Note, that unlike $\mu \rightarrow e\gamma$ rate, slepton oscillation parameters $\sin 2\phi$ and $\frac{2\Gamma_{sl}M_{e_R}}{|M_{e_R}^2 - M_{\mu_R}^2|}$ are independent of Λ . Analogously, oscillations of $\tilde{\tau}$ into other sleptons are also possible in the regime $Y_{43}^{(5)} \neq 0$. In model with high $\tan\beta$ the constraints on Yukawa couplings are significantly stronger and slepton oscillations are suppressed in comparison with the case of low $\tan\beta$.

In the case of antisymmetric messengers, the analysis is similar to one presented above. The only difference is that flavor violation originates from the sector of left sleptons. In this case mixing in vertices with wino and zino arises in addition to mixing in vertices with bino discussed above. Corresponding amplitudes are smaller than bino mediated ones, because superpartners of weak bosons are significantly heavier than bino in MGMM⁸. Since our purpose is to estimate the maximal allowed values of the mixing parameters it is sufficient to take into account only bino mediated diagrams.

We present our results in Tables 1, 2, where the limits on the Yukawa couplings at $x \lesssim 0.8$ and $\Lambda = 100$ TeV for fundamental messengers and $\Lambda = 50$ TeV for antisymmetric ones are given⁹. The experimental limits on the rates of the rare processes [16] are given for convenience in Table 7 of Appendix C. Let us make a few remarks that apply to both cases of fundamental and antisymmetric messengers. As one can see from eq. (34), these bounds increase approximately linearly with Λ . Contributions to amplitudes coming from diagrams with chirality flip are proportional to $\tan\beta$ and dominate at high $\tan\beta$. Therefore corresponding bounds are inversely proportional to $\sqrt{\tan\beta}$ at large $\tan\beta$. In similarity to the case of fundamental messengers, for antisymmetric ones the rates of flavor changing τ decays are at the level of current experiments in case of high $\tan\beta$ and are forbidden at low $\tan\beta$ regime. At $\tan\beta \sim 1$ the maximum rates of flavor changing τ decays in model with antisymmetric messengers are at least by order of magnitude smaller than the current experimental limits.

Finally, it is worth noting that there are corrections to eq. (34) due to the diagrams with larger number of left-right mass insertions. These corrections are proportional to $\tan\beta$ and can

⁸This suppression survives even after taking into account that $\alpha_2 > \alpha_1$, because gaugino masses are proportional to corresponding α_i .

⁹We take smaller value of Λ for antisymmetric messengers, because superpartners are heavier in this case at the same value of Λ .

be significant in high $\tan\beta$ region. To the leading order in m_τ/m_e , m_τ/m_μ these corrections are proportional to $V_{33} = \frac{Y_{43}^{(5)}}{\Delta}$. Consequently, in the case of $\mu \rightarrow e\gamma$ decay they are not essential at $Y_{43}^{(5)} = 0$ and the results given in Table 1 are not modified. In the case of flavor changing τ decays these corrections make the limits, presented in Table 2 slightly stronger. However, numerical analysis shows that the limits get modified very modestly even at large splitting, when very light slepton appears. For example, for fundamental messengers at $\Lambda = 100$ TeV (which corresponds to $\tilde{m}_{R1} \sim 150$ GeV; we use the spectrum of MGMM found in Ref. [11]) the exact amplitude is only 1.4 times larger than one obtained by making use of eq. (33) when the lightest slepton mass reaches its present experimental limit of 60 GeV.

5.2 Quark sector

We have found the bounds on various products of Yukawa couplings of messengers with quarks coming from the requirement that corresponding mixing is consistent with the present experimental limits on the rare processes. We take into account only gluino mediated contributions to the rare processes. It is worth noting that there are also chargino and photino contributions to flavor changing processes. These contributions will make the limits presented below stronger by a factor of order one and they are not significant as far as semi-quantitative estimates of the allowed values of Yukawa couplings Y_i are concerned. The experimental limits [16] are summarized in Table 7 of Appendix C.

To calculate these bounds we make use of the results of Ref. [18], where flavor violation in the MSSM with general mass matrix was studied in the mass insertion approximation in δm_{ij}^2 . As we have seen in the previous section, mass insertion approximation with respect to chirality conserving terms works well up to very large mass splitting; in the latter regime exact calculations typically lead to slightly more stringent limits.

However, it is worth noting that at high $\tan\beta$, in analogy to the flavor changing lepton decays, it may be insufficient to take into account only one flavor changing insertion. The point is that additional left-right mixing insertion can significantly enhance some of the amplitudes. This is not the case for box diagrams. Only this type of diagrams contributes to amplitudes with $\Delta F = 2$ and, consequently, one can directly apply the results of Ref. [18] for $\Delta F = 2$ processes. The limits on the Yukawa couplings coming from $\Delta F = 2$ processes are basically independent of $\tan\beta$.

In the case of $\Delta F = 1$ processes ($b \rightarrow s\gamma$ decay and CP-violation in K^0 decays) penguin diagrams give contributions as well, and additional left-right mixing mass insertion is essential. Following Ref. [18], we take it into account by introducing an effective left-right mixing flavor changing insertion

$$(\delta\tilde{m}_{ij}^2)_{eff} = \delta\tilde{m}_{ij}^2 \times \frac{m_q \mu \tan\beta}{\tilde{m}_q^2}.$$

Here $\delta\tilde{m}_{ij}^2$ is the original chirality conserving mass insertion, m_q and \tilde{m}_q are masses of the corresponding quark and squark. Hence, $\Delta F = 1$ processes depend on the value of $\tan\beta$.

We present our results in Tables 3,4 and 6, where the limits on the Yukawa couplings for x not very close to one ($x \lesssim 0.8$) and $\Lambda = 100$ TeV are shown.

There are also limits on CP-violating terms (see Tables 3, 5) coming from the analysis of $K^0 - \bar{K}^0$ system and $K \rightarrow \pi\pi$ decays. At high (low) $\tan\beta$ the limits from the latter process are

	$K^0 - \bar{K}^0, \Delta F = 2$	$K^0 - \bar{K}^0, \epsilon, \Delta F = 2$
$\mathbf{5} + \bar{\mathbf{5}} :$	$\left \text{Re} \left(X_{41}^{(5)*} X_{42}^{(5)} \right)^2 \right ^{1/4} x < 0.08$	$\left \text{Im} \left(X_{41}^{(5)*} X_{42}^{(5)} \right)^2 \right ^{1/4} x < 0.02$
$\mathbf{10} + \bar{\mathbf{10}} :$	$\left \text{Re} \left(W_{41}^{(10)*} W_{42}^{(10)} \right)^2 \right ^{1/4} x < 0.1$	$\left \text{Im} \left(X_{41}^{(10)*} X_{42}^{(10)} \right)^2 \right ^{1/4} x < 0.05$
$\mathbf{10} + \bar{\mathbf{10}} :$	$\left \text{Re} \left(X_{41}^{(10)*} X_{42}^{(10)} \right)^2 \right ^{1/4} x < 0.2$	$\left \text{Im} \left(W_{41}^{(10)*} W_{42}^{(10)} \right)^2 \right ^{1/4} x < 0.04$
$\mathbf{10} + \bar{\mathbf{10}} :$	$\left \text{Re} X_{41}^{(10)*} X_{42}^{(10)} W_{41}^{(10)*} W_{42}^{(10)} \right ^{1/4} x < 0.04$	$\left \text{Im} \left(X_{41}^{(10)*} X_{42}^{(10)} W_{41}^{(10)*} W_{42}^{(10)} \right) \right ^{1/4} x < 0.01$

Table 3: The constraints on Yukawa couplings coming from $K^0 - \bar{K}^0$ mixing at $\Lambda = 100$ TeV for fundamental messengers and at $\Lambda = 50$ TeV for antisymmetric ones, $x \lesssim 0.8$.

	$B^0 - \bar{B}^0, \Delta F = 2$	$D^0 - \bar{D}^0, \Delta F = 2$
$\mathbf{5} + \bar{\mathbf{5}} :$	$\left \text{Re} \left(X_{41}^{(5)*} X_{43}^{(5)} \right)^2 \right ^{1/4} x < 0.1$	$\left \text{Re} \left(X_{41}^{(5)*} X_{42}^{(5)} \right)^2 \right ^{1/4} x < 0.1$
$\mathbf{10} + \bar{\mathbf{10}} :$	$\left \text{Re} \left(X_{41}^{(10)*} X_{43}^{(10)} \right)^2 \right ^{1/4} x < 0.3$	$\left \text{Re} \left(X_{41}^{(10)*} X_{42}^{(10)} \right)^2 \right ^{1/4} x < 0.3$
$\mathbf{10} + \bar{\mathbf{10}} :$	$\left \text{Re} \left(W_{41}^{(10)*} W_{43}^{(10)} \right)^2 \right ^{1/4} x < 0.2$	$\left \text{Re} \left(X_{14}^{(10)*} X_{24}^{(10)} \right)^2 \right ^{1/4} x < 0.2$
$\mathbf{10} + \bar{\mathbf{10}} :$	$\left \text{Re} X_{41}^{(10)*} X_{43}^{(10)} W_{41}^{(10)*} W_{43}^{(10)} \right ^{1/4} x < 0.1$	$\left \text{Re} X_{41}^{(10)*} X_{42}^{(10)} X_{14}^{(10)*} X_{24}^{(10)} \right ^{1/4} x < 0.1$

Table 4: The constraints on Yukawa couplings coming from $B^0 - \bar{B}^0$ mixing and $D^0 - \bar{D}^0$ mixing at the same values of parameters as in Table 3.

stronger (weaker) than those coming from $K^0 - \bar{K}^0$ system at the level of current experiments.

A typical constraint on Yukawa couplings in the quark sector is $Yx \lesssim 0.1$. It is clear from Tables 3 – 6 that different experiments are sensitive, generally speaking, to different combinations of Yukawa couplings and CP-violating phases. However, one may notice that $K^0 - \bar{K}^0$ system is presently a particularly good probe of the messenger-matter mixing in the quark sector.

It turns out that the contribution to $Br(b \rightarrow s\gamma)$ due to the messenger-matter mixing at low $\tan\beta$ is about 10^{-6} which is 10^2 times smaller than current experimental uncertainties in the region of parameters allowed by theoretical bounds. Meanwhile there are contributions of order 10^{-4} to $Br(b \rightarrow s\gamma)$ in gauge mediated models without mixing (see, e.g., Refs.[19, 11]). Correspondingly, messenger-matter mixing is not significant for this process in the region of low $\tan\beta$.

	$\epsilon'/\epsilon, \tan\beta \sim 50$	$\epsilon'/\epsilon, \tan\beta \sim 1$
$\mathbf{5} + \overline{\mathbf{5}}$:	$\left \text{Im} X_{41}^{(5)*} X_{42}^{(5)} \right ^{1/2} x < 0.05$	$\left \text{Im} X_{41}^{(5)*} X_{42}^{(5)} \right ^{1/2} x < 0.3$
$\mathbf{10} + \overline{\mathbf{10}}$:	$\left \text{Im} X_{41}^{(10)*} X_{42}^{(10)} \right ^{1/2} x < 0.09$	$\left \text{Im} X_{41}^{(10)*} X_{42}^{(10)} \right ^{1/2} x < 0.9$
$\mathbf{10} + \overline{\mathbf{10}}$:	$\left \text{Im} W_{41}^{(10)*} W_{42}^{(10)} \right ^{1/2} x < 0.06$	$\left \text{Im} W_{41}^{(10)*} W_{42}^{(10)} \right ^{1/2} x < 0.6$

Table 5: The constraints on Yukawa couplings coming from $\Delta F = 1$ CP-violating processes at high and low $\tan\beta$. The parameters are the same as in Table 3.

Limits coming from $b \rightarrow s\gamma$ decay in the region of high $\tan\beta$ are shown in Table 6.

	$b \rightarrow s\gamma, \Delta F = 1$
$\mathbf{5} + \overline{\mathbf{5}}$:	$\left X_{42}^{(5)*} X_{43}^{(5)} \right ^{1/2} x < 0.2$
$\mathbf{10} + \overline{\mathbf{10}}$:	$\left X_{42}^{(10)*} X_{43}^{(10)} \right ^{1/2} x < 0.3$
$\mathbf{10} + \overline{\mathbf{10}}$:	$\left \text{Re} \left(W_{42}^{(10)*} W_{43}^{(10)} \right)^2 \right ^{1/4} x < 0.2$

Table 6: The constraints on Yukawa couplings coming from $B^0 - \bar{B}^0$ mixing and $b \rightarrow s\gamma$ decay at the same values of parameters as in Table 3; $\tan\beta=50$.

Bounds on Y coming from $\Delta F = 2$ processes scale as $\sqrt{\Lambda}$. This scaling comes from the fact that corresponding four-fermion operators in the effective Hamiltonian originate from diagrams with two mixing insertions δm_{ij}^2 and the coefficients in front of these operators are proportional to $\frac{Y^4}{\Lambda^2}$. Analogously, the limits coming from $\Delta F = 1$ processes scale as Λ . Bounds on Y from $\Delta F = 1$ processes at high $\tan\beta$ are inversely proportional to $\sqrt{\tan\beta}$.

6 Concluding remarks

We have considered mixing between the usual matter and messengers belonging to either the fundamental or antisymmetric complete $SU(5)$ multiplets in the Minimal Gauge Mediated Model. Limits on the corresponding coupling constants coming from various flavor violating processes in lepton and quark sector and CP violating processes in quark sector have been found.

These limits depend on the ratio x of the supersymmetry breaking parameter Λ and messenger scale. The final results were presented for relatively small ratio: $0 < x \lesssim 0.8$. It is straightforward to extend this analysis to x close to 1, and the results do not change drastically.

We have seen that small value of $\tan\beta$ naturally appears in MGMM with messenger-matter mixing. This fact may help to construct a realistic $SU(5)$ Grand Unified Theory, as models with low $\tan\beta$ (with incomplete messenger multiplets) are unifiable with long proton lifetime [20],

unlike models with large¹⁰ $\tan\beta$.

Another consequence of small $\tan\beta$ is the reduction of the mixing between $\tilde{\tau}_R$ and $\tilde{\tau}_L$. As a result, now the NLSP can be photino, rather than the right slepton and, consequently, the predictions of this model for collider experiments can be significantly different. For example, this case is more suitable for explaining the CDF event [22].

We have seen in this paper that in the case of large enough mixing in the MGMM with fundamental messengers, there appears one and only one pair of light left squarks. This fact provides an interesting signature of MGMM with messenger-matter mixing. For $m_{light} < 1$ TeV one has $\tan\beta \simeq 1$ in this model. One expects that similar phenomenon exists also in models with more complicated messenger content.

There are two types of constraints on the allowed region in the space of messenger-matter Yukawa couplings. All experimental bounds coming from the flavor physics limit only the products of different Yukawa couplings $|Y_i Y_j|^{1/2}$ but not Y_i separately. On the other hand, theoretical bounds, coming from the requirement of positivity of the scalar masses, correspond to spherical regions $\sum |Y_i|^2 < \text{const}$ in the space of Yukawa couplings.

The rates of some rare processes depend crucially on the value of $\tan\beta$. At high $\tan\beta \sim 50$ and $x \gtrsim 0.1$, experimentally accessible values of most of the messenger-matter Yukawa couplings are in the interesting range $10^{-3} \div 10^{-1}$. A particularly sensitive probe of messenger-matter mixing is muon flavor violation. For example, in the case of fundamental messengers the present limit on $\mu \rightarrow e\gamma$ decay rate implies $|Y_{41}^{(5)} Y_{42}^{(5)}| x^2 < 4.0 \times 10^{-7}$ while the bound from the $\mu \rightarrow e$ conversion is weaker by a factor of 2. Future experiments on $\mu \rightarrow e\gamma$ decay and $\mu \rightarrow e$ conversion are quite promising from the point of view of MGMM with messenger-matter mixing.

The case of low $\tan\beta$ is somewhat different. The experimentally accessible values of most of the messenger-matter Yukawa couplings are one order of magnitude higher than in previous case, with the exception of $\Delta F = 2$ processes whose sensitivity remains the same. The present limit on $\mu \rightarrow e\gamma$ decay rate implies $|Y_{41}^{(5)} Y_{42}^{(5)}| x^2 < 10^{-5}$. There are basically no experimental constraints coming from $\tau \rightarrow e\gamma$, $\tau \rightarrow \mu\gamma$ decays and the corresponding mixing terms are limited by the self-consistency conditions inherent in the theory. Even at the extreme values $Y_i x \sim 1$ branching ratios of the lepton flavor violating τ decays must be order of magnitude smaller than existing experimental bounds. Hence, at low $\tan\beta$ this model forbids flavor violating τ decays at the level of the next generation experiments. There are essentially no constraints coming from $b \rightarrow s\gamma$ as well, depending on the type of Yukawa couplings, the contributions of messenger-matter mixing to the $b \rightarrow s\gamma$ rate are two orders of magnitude smaller than experimental uncertainties in the case of low $\tan\beta$.

Finally, it is worth noting that the estimates of the allowed range of the mixing parameters presented in this paper are expected to remain qualitatively the same in more general gauge mediated models than MGMM.

¹⁰In the case of complete messenger multiplets, low $\tan\beta$ does not save proton from fast decay in the framework of $SU(5)$ GUT [21]

7 Acknowledgments

The authors are indebted V.A.Rubakov for stimulating interest and helpful suggestions. We thank F.L.Bezrukov, M.L.Libanov, P.G.Tinyakov and S.V.Troitsky for useful discussions. This work is supported in part by Russian Foundation for Basic Research grant 96-02-17449a, by the INTAS grant 96-0457 within the research program of the International Center for Fundamental Physics in Moscow and by ISSEP fellowships.

8 Appendix

A Fermion mass matrix

Here we present the explicit forms of fermion mass matrices in the model with fundamental messengers. The lepton mass matrix that includes left and right fermionic messengers has the following form

$$\mathcal{M}_f^l = \mathcal{U}_{fL}^l \mathcal{D}_f^l \mathcal{U}_{fR}^l \quad (42)$$

where

$$\mathcal{U}_{fL}^l = \begin{pmatrix} 1 & 0 & 0 & -\frac{y_e y_1^*}{\lambda^2 S^2} \\ 0 & 1 & 0 & -\frac{y_\mu y_2^*}{\lambda^2 S^2} \\ 0 & 0 & 1 & -\frac{y_\tau y_3^*}{\lambda^2 S^2} \\ \frac{y_e y_1}{\lambda^2 S^2} & \frac{y_\mu y_2}{\lambda^2 S^2} & \frac{y_\tau y_3}{\lambda^2 S^2} & 1 \end{pmatrix} \quad (43)$$

and

$$\mathcal{U}_{fR}^l = \begin{pmatrix} 1 - \frac{|y_1|^2}{2\lambda^2 S^2} & -\frac{y_1^* y_2}{2\lambda^2 S^2} & -\frac{y_1^* y_3}{2\lambda^2 S^2} & -\frac{y_1^*}{\lambda S} \\ -\frac{y_1 y_2^*}{2\lambda^2 S^2} & 1 - \frac{|y_2|^2}{2\lambda^2 S^2} & -\frac{y_2^* y_3}{2\lambda^2 S^2} & -\frac{y_2^*}{\lambda S} \\ -\frac{y_1 y_3^*}{2\lambda^2 S^2} & -\frac{y_2 y_3^*}{2\lambda^2 S^2} & 1 - \frac{|y_3|^2}{2\lambda^2 S^2} & -\frac{y_3^*}{\lambda S} \\ \frac{y_1}{\lambda S} & \frac{y_2}{\lambda S} & \frac{y_3}{\lambda S} & 1 - \frac{|y_1|^2 + |y_2|^2 + |y_3|^2}{2\lambda^2 S^2} \end{pmatrix} \quad (44)$$

are mixing matrices to the leading order in $\frac{y}{\lambda S}$. Here v_U and v_D are the Higgs expectation values,

$$y_i = Y_{4i} v_D, \quad y_{e,\mu,\tau} = Y_{e,\mu,\tau} v_D$$

and

$$\mathcal{D}_f^l = \text{diag} \left(y_e \left(1 - \frac{y_e}{\lambda S} \left| \frac{y_1}{\lambda S} \right|^2 \right), y_\mu \left(1 - \frac{y_\mu}{\lambda S} \left| \frac{y_2}{\lambda S} \right|^2 \right), y_\tau \left(1 - \frac{y_\tau}{\lambda S} \left| \frac{y_3}{\lambda S} \right|^2 \right), \right. \\ \left. \lambda S \left(1 + \left| \frac{y_1 y_e}{\lambda^2 S^2} \right|^2 + \left| \frac{y_2 y_\mu}{\lambda^2 S^2} \right|^2 + \left| \frac{y_3 y_\tau}{\lambda^2 S^2} \right|^2 \right) \right) \quad (45)$$

is the matrix of mass eigenvalues.

For down-quark-like fermions one has

$$\mathcal{M}_f^q = \mathcal{U}_{fL}^q \mathcal{D}_f^q \mathcal{U}_{fR}^q, \quad (46)$$

where

$$\mathcal{U}_{fL}^q = \begin{pmatrix} 1 & 0 & 0 & -\frac{y_d y_1^*}{\lambda^2 S^2} \\ 0 & 1 & 0 & -\frac{y_s y_2^*}{\lambda^2 S^2} \\ 0 & 0 & 1 & -\frac{y_b y_3^*}{\lambda^2 S^2} \\ \frac{y_d y_1}{\lambda^2 S^2} & \frac{y_s y_2}{\lambda^2 S^2} & \frac{y_b y_3}{\lambda^2 S^2} & 1 \end{pmatrix} \quad (47)$$

and

$$\mathcal{U}_{fR}^q = \begin{pmatrix} 1 - \frac{|y_1|^2}{2\lambda^2 S^2} & -\frac{y_1^* y_2}{2\lambda^2 S^2} & -\frac{y_1^* y_3}{2\lambda^2 S^2} & -\frac{y_1^*}{\lambda S} \\ -\frac{y_1 y_2^*}{2\lambda^2 S^2} & 1 - \frac{|y_2|^2}{2\lambda^2 S^2} & -\frac{y_2^* y_3}{2\lambda^2 S^2} & -\frac{y_2^*}{\lambda S} \\ -\frac{y_1 y_3^*}{2\lambda^2 S^2} & -\frac{y_2 y_3^*}{2\lambda^2 S^2} & 1 - \frac{|y_3|^2}{2\lambda^2 S^2} & -\frac{y_3^*}{\lambda S} \\ \frac{y_1}{\lambda S} & \frac{y_2}{\lambda S} & \frac{y_3}{\lambda S} & 1 - \frac{|y_1|^2 + |y_2|^2 + |y_3|^2}{2\lambda^2 S^2} \end{pmatrix} \quad (48)$$

are mixing matrices to the leading order in $\frac{y}{\lambda S}$. Here

$$y_i = Y_{4i} v_D, \quad y_{d,s,b} = Y_{d,s,b} v_D$$

$$\mathcal{D}_f^q = \text{diag}\left(y_d \left(1 - \frac{y_d}{\lambda S} \left|\frac{y_1}{\lambda S}\right|^2\right), y_s \left(1 - \frac{y_s}{\lambda S} \left|\frac{y_2}{\lambda S}\right|^2\right), y_b \left(1 - \frac{y_b}{\lambda S} \left|\frac{y_3}{\lambda S}\right|^2\right), \lambda S \left(1 + \left|\frac{y_1 y_d}{\lambda^2 S^2}\right|^2 + \left|\frac{y_2 y_s}{\lambda^2 S^2}\right|^2 + \left|\frac{y_3 y_b}{\lambda^2 S^2}\right|^2\right)\right). \quad (49)$$

The mass matrices in the model with antisymmetric messengers have similar structure.

B Scalar mass matrix

The tree level mass term of sleptons including the scalar messengers with the quantum numbers of left leptons has the following form

$$\mathcal{V}_{sc}^l = s^l \mathcal{M}_{sc}^l s^{l\dagger}$$

where

$$s^l = (\tilde{e}_L, \tilde{\mu}_L, \tilde{\tau}_L, \tilde{e}_R^*, \tilde{\mu}_R^*, \tilde{\tau}_R^*, l, \bar{l}^*) \quad (50)$$

are the scalar fields (we use the same notation for left selectrons as for full left doublets), and

$$\mathcal{M}_{sc}^l = \begin{pmatrix} \tilde{m}_{eL}^2 & 0 & 0 & \mu Y_e v_U & 0 & 0 & y_e y_1^* & 0 \\ 0 & \tilde{m}_{\mu L}^2 & 0 & 0 & \mu Y_\mu v_U & 0 & y_\tau y_2^* & 0 \\ 0 & 0 & \tilde{m}_{\tau L}^2 & 0 & 0 & \mu Y_\tau v_U & y_\tau y_3^* & 0 \\ \mu Y_e v_U & 0 & 0 & \tilde{m}_{eR}^2 & y_1^* y_2 & y_1^* y_3 & \mu Y_1^* v_U & \lambda S y_1^* \\ 0 & \mu Y_\mu v_U & 0 & y_1 y_2^* & \tilde{m}_{\mu R}^2 & y_2^* y_3 & \mu Y_2^* v_U & \lambda S y_2^* \\ 0 & 0 & \mu Y_\tau v_U & y_1 y_3^* & y_2 y_3^* & \tilde{m}_{\tau R}^2 & \mu Y_3^* v_U & \lambda S y_3^* \\ y_e y_1 & y_\mu y_2 & y_\tau y_3 & \mu Y_1 v_U & \mu Y_2 v_U & \mu Y_3 v_U & \lambda^2 S^2 & -\lambda F \\ 0 & 0 & 0 & \lambda S y_1 & \lambda S y_2 & \lambda S y_3 & -\lambda F & \lambda^2 S^2 \end{pmatrix}$$

The mass matrix of scalar fields with quantum numbers of down quarks may be written as follows

$$\mathcal{V}_{sc}^q = s^q \mathcal{M}_{sc}^q s^{q\dagger},$$

where

$$s = (\tilde{d}_R^*, \tilde{s}_R^*, \tilde{b}_R^*, \tilde{d}_L, \tilde{s}_L, \tilde{b}_L, q, \bar{q}^*) \quad (51)$$

are scalar fields, and

$$\mathcal{M}_{sc} = \begin{pmatrix} \tilde{m}_{dL}^2 & 0 & 0 & \mu Y_d v_U & 0 & 0 & y_d y_1^* & 0 \\ 0 & \tilde{m}_{sL}^2 & 0 & 0 & \mu Y_s v_U & 0 & y_b y_2^* & 0 \\ 0 & 0 & \tilde{m}_{bL}^2 & 0 & 0 & \mu Y_b v_U & y_b y_3^* & 0 \\ \mu Y_d v_U & 0 & 0 & \tilde{m}_{dR}^2 & y_1^* y_2 & y_1^* y_3 & \mu Y_1^* v_U & \lambda S y_1^* \\ 0 & \mu Y_s v_U & 0 & y_1 y_2^* & \tilde{m}_{sR}^2 & y_2^* y_3 & \mu Y_2^* v_U & \lambda S y_2^* \\ 0 & 0 & \mu Y_b v_U & y_1 y_3^* & y_2 y_3^* & \tilde{m}_{bR}^2 & \mu Y_3^* v_U & \lambda S y_3^* \\ y_d y_1 & y_s y_2 & y_b y_3 & \mu Y_1 v_U & \mu Y_2 v_U & \mu Y_3 v_U & \lambda^2 S^2 & -\lambda F \\ 0 & 0 & 0 & \lambda S y_1 & \lambda S y_2 & \lambda S y_3 & -\lambda F & \lambda^2 S^2 \end{pmatrix}$$

C Experimental data

Experimental results used in this paper are summarized in Table 7.

$K^0 - \bar{K}^0$	$\Delta m_K = (3.491 \pm 0.009) \times 10^{-12} \text{ MeV}$
$D^0 - \bar{D}^0$	$\Delta m_D < 1.38 \times 10^{-10} \text{ MeV}$
$B^0 - \bar{B}^0$	$\Delta m_B = (3.12 \pm 0.21) \times 10^{-10} \text{ MeV}$
$\mu \rightarrow e\gamma$	$\text{Br}(\mu \rightarrow e\gamma) < 4.9 \times 10^{-11}$
$\tau \rightarrow e\gamma$	$\text{Br}(\tau \rightarrow e\gamma) < 2.7 \times 10^{-6}$
$\tau \rightarrow \mu\gamma$	$\text{Br}(\tau \rightarrow \mu\gamma) < 3.0 \times 10^{-6}$
$b \rightarrow s\gamma$	$\text{Br}(b \rightarrow s\gamma) = (4.2 \pm 1.0) \times 10^{-4}$
$\mu \rightarrow e \text{ conversion}$	$\Gamma(\mu^- Ti \rightarrow e^- Ti) / \Gamma(\mu^- Ti \rightarrow \text{all}) < 4 \times 10^{-12}$
$CP - \text{violation}$	$\epsilon = (2.268 \pm 0.019) \times 10^{-3}$
$CP - \text{violation}$	$\epsilon' / \epsilon = (1.5 \pm 0.8) \times 10^{-3}$

Table 7: The experimental data on the flavor and CP-violating processes.

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