

# Hyperons as collective excitations of chiral solitons\*

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According to the large  $N_C$  limit of QCD baryons are considered as soliton solutions in effective meson theories. While the classical solitons dwell in the isospin subgroup of flavor SU(3) hyperon states are generated by canonical quantization of the collective coordinates which describe the flavor orientation of the soliton. The resulting Hamiltonian is diagonalized exactly allowing one to discuss the dependence of various baryon properties on flavor symmetry breaking. In particular axial charges, baryon magnetic moments and radiative decay widths are considered.

## 1. INTRODUCTION

The generalization of QCD to an arbitrary large number ( $N_C$ ) of color degrees of freedom indicates that for  $N_C \rightarrow \infty$  QCD becomes equivalent to a non-linear effective theory  $\mathcal{A}$  of weakly interacting mesons, as the associated meson coupling constants vanish in this limit. The action  $\mathcal{A}$  may contain soliton solutions and the dependences of their properties on  $N_C$ , inherited from the meson coupling constants, is that of baryons in large  $N_C$  QCD. Hence the soliton solutions are identified as baryons [ 1]. The building block to construct  $\mathcal{A}$  for low energies is the concept of chiral symmetry and its spontaneous breaking. Therefore the chiral field, *i.e.* the non-linear realization of the pseudoscalar fields ( $\Phi^a = \pi, k, \eta$ )

$$U = \exp(i\lambda^a \Phi^a / f_a), \quad (1)$$

is the relevant degree of freedom. Here  $\lambda^a$  are the SU(3) Gell-Mann matrices and  $f^a$  denote pertinent decay constants. The most prominent chirally invariant model containing solitons is the so-called Skyrme model [ 2]. This model can be generalized to the three flavor case [ 3] with the explicit breaking flavor symmetry included (for a review see [ 4]). The essential ingredient is that the strange current quark mass is smaller than  $\Lambda_{\text{QCD}}$ .

To minimize the soliton mass,  $M_{\text{cl}}$  in the three flavor model the static hedgehog must be embedded in the isospin subspace:

$$U_H(\vec{r}) = \exp\left(i \sum_{i=1}^3 \hat{r}_i \lambda^i F(r)\right). \quad (2)$$

Subsequently the zero-modes of  $U_H$  are assumed to be time dependent:

$$U(\vec{r}, t) = A(t) U_H(\vec{r}) A^\dagger(t). \quad (3)$$

The quantum mechanical treatment of the collective coordinates  $A(t) \in \text{SU}(3)$  leads to states which are identified as the low-lying  $\frac{1}{2}^+$  and  $\frac{3}{2}^+$  baryons. This treatment is called the rigid rotator approach (RRA) to include strangeness.

## 2. BARYON STATES WITH FLAVOR SYMMETRY BREAKING

Substituting the *ansatz* (3) yields the Lagrangian for the collective coordinates  $A$

$$L(A, \dot{A}) = -M_{\text{cl}} + \frac{1}{2}\alpha^2 \sum_{i=1}^3 \Omega_i^2 + \frac{1}{2}\beta^2 \sum_{\alpha=4}^7 \Omega_\alpha^2 - \frac{\sqrt{3}}{2}\Omega_8 - \frac{1}{2}\gamma(1 - D_{88}) + \dots \quad (4)$$

The angular velocities  $\Omega_a$  and the adjoint representation  $D_{ab}$  are defined as

$$A^\dagger \dot{A} = \frac{i}{2} \sum_{a=1}^8 \lambda^a \Omega_a, \quad D_{ab} = \frac{1}{2} \text{tr}(\lambda_a A \lambda_b A^\dagger). \quad (5)$$

The coefficients  $\alpha^2, \beta^2$  and  $\gamma$  are functionals of the soliton. Their actual values depend on the details

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of the considered meson theory [4]. The contribution linear in  $\Omega_8$  stems from the Wess–Zumino action which mocks up the axial anomaly in the effective meson theory [5]. The term including the SU(3)–D–function is due to flavor symmetry breaking. The ellipsis in eq (4) refer to symmetry breaking terms which are subject to the specific model.

When quantizing the collective coordinates one identifies the right generators of SU(3):  $R_a = -\partial L/\partial\Omega_a$ . The first three generators actually are (up to a sign) the spin–operators. The constraint  $R_8 = \sqrt{3}/2$  requires half–integer spin eigenstates. The Hamiltonian  $H = -\sum_{a=1}^8 R_a\Omega_a - L = H_0 + H_{SB}$  is conveniently separated into flavor symmetric ( $H_0$ ) and symmetry breaking ( $H_{SB}$ ) pieces. For  $H_{SB} = 0$  the eigenstates are members of a certain SU(3) representation, *e.g.* the octet for the  $\frac{1}{2}^+$  baryons. Due to flavor symmetry breaking these states acquire contributions from higher dimensional representations [6], *e.g.*

$$|N\rangle = |N, \mathbf{8}\rangle + 0.075\gamma\beta^2|N, \overline{\mathbf{10}}\rangle + 0.049\gamma\beta^2|N, \mathbf{27}\rangle + \dots \quad (6)$$

$$|\Lambda\rangle = |\Lambda, \mathbf{8}\rangle + 0.060\gamma\beta^2|\Lambda, \mathbf{27}\rangle + \dots \quad (7)$$

for the nucleon and the  $\Lambda$  hyperon. The higher order perturbation pieces have been indicated. Similarly the shift of the baryon mass due to symmetry breaking is computed. For the nucleon one finds

$$\delta M_N = \frac{-1}{2\beta^2} \left[ 0.3\gamma\beta^2 + 0.029(\gamma\beta^2)^2 + \dots \right]. \quad (8)$$

Obviously the product  $\gamma\beta^2$  is the effective symmetry breaker rather than only  $\gamma$ . Typical values are  $\gamma\beta^2 \approx 3.0 - 4.0$ . Actually the collective Hamiltonian can be diagonalized exactly by means of an ‘‘Euler angle’’ representation for the collective coordinates  $A$  [7]. A typical result for the baryon spectrum is displayed in table 1. The mass differences are reasonably well reproduced, on the 10% level. Many static properties can be computed once the baryon states have been constructed from  $H$ . In particular the variation with flavor symmetry breaking can be studied. An example is provided in figure 1 where various axial current matrix elements are displayed. Although the flavor changing axial current matrix elements, which enter the Cabibbo model for hyperon beta decay, vary only moderately with symmetry breaking the

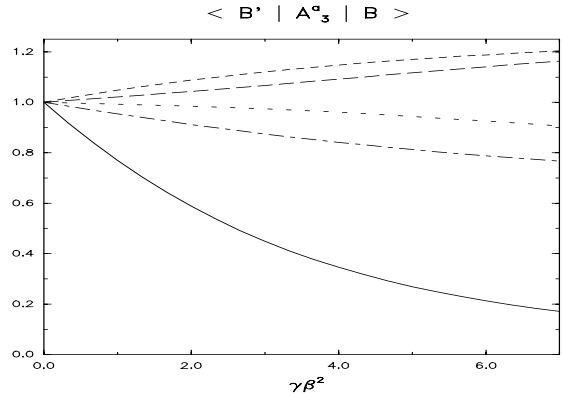


Figure 1. Axial current matrix elements as a function of the effective symmetry breaking parameter  $\gamma\beta^2$ . Full line:  $\langle p|\bar{s}\gamma_3\gamma_5s|p\rangle$ ; dashed dotted line:  $\langle p|\bar{u}\gamma_3\gamma_5s|\Lambda\rangle$ ; dotted line:  $\langle n|\bar{u}\gamma_3\gamma_5s|\Sigma^->$ ; long dashed line:  $\langle \Lambda|\bar{u}\gamma_3\gamma_5s|\Xi^->$ ; dashed line:  $\langle p|\bar{u}\gamma_3\gamma_5d|n\rangle$ ; These matrix elements, which are taken from refs [8] and [9], are normalized to the flavor symmetric values.

diagonal matrix element  $\langle p|\bar{s}\gamma_3\gamma_5s|p\rangle$  gets reduced to approximately half its flavor symmetric value. Hence SU(3) is eventually a good symmetry to relate various beta decay matrix elements, however, it seems dangerous to assume SU(3) symmetry to extract  $\langle p|\bar{s}\gamma_3\gamma_5s|p\rangle$  from data on hyperon beta decay. This has frequently been done in the context of the *proton spin puzzle* and yielded an unexpectedly large amount of polarized strange quarks in the nucleon.

### 3. THE SLOW ROTATOR

Although the exact diagonalization of the collective Hamiltonian includes the effects of flavor symmetry breaking not all observed deviations from the symmetric formulation are reproduced. An example is given in table 2 where ratios of magnetic moments are compared to experiment. Clearly, the RRA underestimates the observed deviation from unity, which is predicted in the symmetric formulation. This is linked to the inability of the RRA to include the influence of symmetry breaking on the size of the classical soliton. This short–coming is cured within the slow rotator approach (SRA) [10]. Making explicit the strangeness changing angle  $\nu$  in an ‘‘Euler angle’’ parameterization of  $A$  ( $R_2$  are SU(2) matrices):

$$A = R_2(\alpha, \beta, \gamma) e^{-i\nu\lambda^4} R_2(\alpha', \beta', \gamma') e^{-i\rho\lambda^8/\sqrt{3}} \quad (9)$$

Table 1: Baryon mass differences with respect to the nucleon. Data are in MeV. Results are from [ 4].

	$\Lambda$	$\Sigma$	$\Xi$	$\Delta$	$\Sigma^*$	$\Xi^*$	$\Omega$
Model	163	264	388	268	410	545	680
Expt.	177	254	379	293	446	591	733

yields the (static) Lagrangian

$$L_\nu[F] = -M_{cl}[F] - \frac{3}{4}\gamma[F] \sin^2\nu. \quad (10)$$

Minimizing  $L_\nu[F]$  for a given  $\nu$  causes the chiral angle  $F$  to parametrically depend on the strangeness orientation: It decays with the pion mass for  $\nu = 0$  while the configuration which is maximally rotated into strange direction ( $\nu = \pi/2$ ) has the kaon mass entering the Yukawa tail. This additional dependence on the collective coordinates must be taken into account when computing static baryon properties [ 10]. As a result the desired deviation from the symmetric results in the magnetic moments is achieved as shown in table 2.

Table 2: U–spin relations for magnetic moments of  $\frac{1}{2}^+$  baryons.

	$\mu_{\Sigma^+}/\mu_p$	$\mu_{\Xi^0}/\mu_n$	$\mu_{\Xi^-}/\mu_{\Sigma^-}$
RRA	0.98	0.97	0.79
Expt.	0.87	0.66	0.59
SRA	0.85	0.65	0.50

Table 3: Widths for radiative hyperon decays. Results are normalized to the width of  $\Delta \rightarrow \gamma N$ .

	SRA	RRA	SU(3)–sym.
$\Sigma_0^* \rightarrow \gamma \Lambda$	0.509	0.653	0.75
$\Sigma_-^* \rightarrow \gamma \Sigma_-$	0.005	0.007	0
$\Sigma_0^* \rightarrow \gamma \Sigma_0$	0.024	0.035	0.25
$\Sigma_+^* \rightarrow \gamma \Sigma_+$	0.152	0.210	1
$\Xi_-^* \rightarrow \gamma \Xi_-$	0.004	0.011	0
$\Xi_0^* \rightarrow \gamma \Xi_0$	0.204	0.313	1

The predicted baryon radii decrease by about 15% per unit strangeness in agreement with empirical observations [ 11]. In the SRA the widths for the radiative decays of the  $\frac{3}{2}^+$  baryons have been calculated [ 12]. In table 3 the results are compared to the RRA and SU(3) symmetric predictions. Again, sizeable reductions with decreasing strangeness are observed. However, the U–spin predictions [ 13]  $\Gamma(\Sigma_-^* \rightarrow \gamma \Sigma_-) = 0$  and  $\Gamma(\Xi_-^* \rightarrow \gamma \Xi_-) = 0$  are maintained. A similarly strong dependence on strangeness is found for the electric polarizabilities.

The predictions in the magnetic case are small due to the cancellation between dispersive and seagull pieces [ 14].

#### 4. CONCLUSIONS

Considering hyperons as collective excitations of chiral solitons has provided an effective means to study the influence of flavor symmetry breaking on various baryon matrix elements. In particular I have shown that relating  $\langle p | \bar{s} \gamma_3 \gamma_5 s | p \rangle$  to data on hyperon beta decay is suspicious, a result also obtained in other models [ 15]. The pattern of the baryon magnetic moments requires one to include the effect of symmetry breaking in the determination of the soliton profile. This generally yields a sizable decrease of flavor conserving baryon matrix elements with strangeness.

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