

A New Regime for Dense String Networks in the One Scale Model with Friction

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We examine the compatibility between the one scale model with friction for the evolution of cosmic string networks and the Kibble mechanism for string network formation during a second order phase transition. We find a regime which connects (in a dramatically short time $\sim .1t_c$) the dense string network (small network scale $L \sim 1/T_c$) created by the Kibble *mechanism* to the (dilute) Kibble *regime* ($L \sim t^{5/4}$) in which friction dominated strings remain till times $t_* \sim (M_P/T_c)^2 t_c$. The enormous loss of string length implied by this result, for strings formed below the GUT scale, is due to the continued dissipative motion of the string network which follows from the one scale model's identification of the typical curvature radius with the network scale and the fact that the phase transition time $t_c \sim M_{Planck}/T_c^2$ is much larger than the damping time scale $l_f \sim T_c^2/T^3$. The significant implications for string mediated Baryogenesis are also discussed briefly.

I. INTRODUCTION

Topological defects are a natural consequence of the symmetry breaking phase transitions predicted by spontaneously broken gauge theories of particle physics. They are produced in the early universe and can have important consequences for the structure of the present day universe [1].

Recently it has been suggested that the baryon asymmetry in our universe can be generated by strings carrying electroweak flux [2-5]. Such scenarios have the advantage that they can automatically satisfy the out of equilibrium condition even though the electroweak phase transition is of second order. The amplitude of the baryon asymmetry generated is directly dependent on the string density that survives after the phase transition. Since GUT scale cosmic strings can serve as seeds for the formation of galaxies, the evolution of such string networks has been much investigated both analytically and numerically. As we discuss below, the role of friction due to passage of strings through the ambient plasma becomes increasingly important as we go to lower scales of string formation. For GUT scale strings, therefore, friction plays only a limited role. It is perhaps for this reason that the evolution of string networks in the presence of friction has scarcely been addressed [1,6]. Recently, however, an analytic model for string evolution with friction which includes the effects of velocity evolution has been analysed [7,8]. This analysis has made it clear that there are notable differences in the evolution of strings in the presence of friction, which we now recount [1,6].

The frictional force per unit length experienced by the string network due to its scattering of the particles of the ambient plasma has the generic form $\vec{F}_f = -\beta\gamma\vec{v}T^3$. Here \vec{v} is the velocity of the string element relative to the plasma, $\gamma = (1 - v^2)^{-1/2}$, T is the temperature and β a numerical factor $\sim 1 - 10^{-3}$. This frictional force modifies the equation of motion of the string in the expanding universe by replacing the Hubble damping term coefficient $2\dot{a}/a$ by $2\dot{a}/a + \beta T^3 a/\eta^2$, where the dot signifies a conformal time derivative, a is the cosmic scale factor and η is the string scale (vev). From the ratio of these terms it follows that for $T \geq \eta^2/m_P$ the frictional force completely dominates the Hubble term and controls string evolution. This domination extends upto a time $t_* = \sigma^2 t_c$, where $\sigma = m_P/\eta$ and t_c is the time of string formation. For strings created below 10^6 GeV the friction dominated evolution thus persists till the epoch of matter domination, whereas for GUT scale strings friction is clearly much less significant.

The length scale characterizing frictional phenomena is $l_{fr} \sim \eta^2/\beta T^3$. Thus at the time t_c of string formation one has $l_{fr}/t_c \sim 1/(\sigma\beta)$. This simple consideration (that the cosmological time scale t_c is much larger than the friction time scale) makes it clear that frictional phenomena have ample scope to modify the network even via powerlaw evolution of the network correlation length when, as is generically true for non-GUT strings, σ is a large number (and β is not pathologically small). This observation is the basis of the new transient regime uncovered in this paper.

Finally an important consequence of friction is that it prevents the uncontrollable generation of small scale structure that characterizes undamped string evolution. Freely moving strings inevitably exhibit cusping behaviour at some points on the string where the velocity is close to luminal [1]. This feature bedevils the study of GUT scale strings since it prevents the string network from settling into a scaling regime where the network is characterized by a single correlation length that scales with the horizon. The continued importance of motion on the smallest length scales necessitates the use of complicated multiscale models [9]. In contrast, since friction ensures that the velocity of the

string is always far from luminal, small scale structure generation is unlikely to be important in the friction dominated regime. Thus there is good reason to believe [7,8] that even a simple single scale model can capture the gross features of the friction dominated evolution, even at the high initial densities predicted by the Kibble mechanism (see below), provided string velocity evolution is accounted for.

In the commonly accepted picture of string formation suggested by Kibble [10] ϵ_K string lengths of size $1/T_c$ are formed in each correlation volume $\sim T_c^{-3}$ (where ϵ_K is a probability factor $\sim 10^{-1} - 10^{-3}$). This gives the typical scale characterizing the network at formation as $L_i \sim \epsilon_K^{-1/2}/T_c$.

Let us now turn to the details of the one scale model with friction within whose framework we shall conduct our analysis of the the regime that connects the state of the string network at formation (as predicted by the Kibble mechanism) to the characteristic friction dominated regime, also named after Kibble, where the string network is characterized by a terminal velocity due to a balance between frictional and curvature forces.

As its name implies, the one scale model [1] attempts to describe the gross features of the string network in terms of a single length scale L , whose evolution is determined by the values of a few parameters only. Due to this virtue of simplicity it has assumed a paradigmatic value in the study of string networks.

Since the frictional force is velocity dependent it is clear that it's effect can be accounted for correctly only by improving the one-scale model to include the velocity evolution of the string network along with friction. This improvement of the one scale model has recently been presented in [7]. The damped Nambu-Goto equations for a friction dominated string [1] are derived by averaging over the network and making the assumption that the network curvature scale R is the same as the network correlation length L at all times.

The analysis of [7], however, does not cover the case of initial conditions appropriate to the formation of string networks in second order phase transitions via the Kibble mechanism [10]. It is this lacuna that we seek to fill in this letter.

In the one-scale model the (long) string length density n_L is by definition L^{-2} where L is the scale which characterizes both the density and the average curvature of the network. The network evolution in a FRW background is described by the following coupled first order differential equations [1,7,8]

$$\begin{aligned} \frac{dL}{dt} &= HL(1+v^2) + \frac{Lv^2}{2l_f} + \frac{cv}{2}, \\ \frac{dv}{dt} &= (1-v^2) \left[\frac{k}{L} - v \left(2H + \frac{1}{l_f} \right) \right], \end{aligned} \quad (1)$$

where H, l_f are the Hubble parameter and friction length scale respectively.

The friction length l_f for gauge strings interacting with the cosmic plasma is given by $\eta^2/\beta T^3$ where β is typically ~ 1 . In the derivation of [7,8] the parameter k characterizes the ‘‘wiggleness’’ of the strings and can be expected to be ~ 1 when the network motion is well damped by friction for the reasons discussed above. Thus $k \sim 1$ in the epoch when friction is important while it can become small at the late times of the linear scaling regime when the strings are undamped and therefore kinky.

The parameter c characterizes the rate at which the network chops itself into loops by self-intersection. For dense networks reconnection of loops back onto the long string network may be important and we envisage that this effect can at least qualitatively be captured by a lower effective value of c . Such reconnection processes have been observed in numerical simulations [11] albeit in the case of semi-local strings. Since we consider generic strings that do intercommute rather than the special sorts that can only self entangle we shall take $c \sim 1$ as well.

The analysis of [7] uncovers the existence of three sequential regimes: a ‘‘stretching’’ regime ($L \sim L_i(t/t_c)^{1/2}, v \sim t(L_i\sigma)^{-1}$) with very low velocities, followed by the friction dominated or ‘‘Kibble’’ regime where $L \sim (t^5/t_c\sigma^2)^{1/4}, v \sim (t/t_c\sigma^2)^{1/4}$ and finally the well known linear scaling regime where friction is unimportant and L scales with the horizon while v is essentially fixed and quasi relativistic. The duration of the stretching regime becomes shorter as the starting value L_i is lowered (i.e for denser networks). Indeed once $L_i \sim t_c/\sqrt{\sigma}$ the simulations of [7,8] show that the evolution appears to begin at t_c with the Kibble regime itself, omitting the stretching stage. The initial value of L in the Kibble regime is clearly $> t_c/\sqrt{\sigma}$.

The incompatibility of the Kibble mechanism and the Kibble regime is now evident from the fact that the typical value of L_i dictated by the Kibble mechanism is $\sim t_c\epsilon_K^{-1/2}\sigma^{-1}$ which is smaller than the value at the onset of the Kibble regime by a factor of $\sigma^{-1/2}$. Thus, for typical values of $k, c \sim 1$, there appears to be no regime which carries the network from the dense state predicted by the Kibble Mechanism of string formation to the (dilute) initial states implied by the Kibble regime.

II. NEW TRANSIENT REGIME

As explained above the difficulty (and therefore its resolution) lies in the short period between the phase transition and the onset of the Kibble regime. Consider the evolution of L and v in the period immediately following the phase transition i.e when $\hat{t} = t - t'_c \ll t'_c$ (say), where $t'_c \sim 10t_c$. The crucial observation is that at this time the friction time scale l_f is smaller than the cosmological time scale by a factor $\sim \sigma$. Thus a small cosmological time interval contains many friction time periods so that friction can be very effective in a “short” time. The existence of the fast decay regime that emerges from our analysis is not affected by the precise value of the of $t'_c \sim t_c$ at which the evolution of the one scale model is taken to begin. This can be checked by taking t'_c to be any other multiple of t_c in the range (say) $t_c \leq t'_c \leq 100t_c$. The natural length scale initially is $\eta \sim T_c$ so we scale \hat{t} and L by this length to define a dimensionless time $x = \eta \hat{t}$ and network scale $l = \eta L$. Note that this amounts to measuring time in units of the damping time scale and avoids the introduction of large numbers in the evolution equations which result if one uses t_c as the unit of time. Then the condition $\hat{t} = t - t'_c \ll t_c$ translates to $x \ll \sigma$. Since even for GUT scale strings σ is $\sim 10^4$ the variable x can change by many orders of magnitude before the approximation becomes invalid. Expanding the terms in the evolution equation around t'_c one finds that $HL \sim l/\sigma$ while the friction term is $\sim lv^2$ and the chopping term is $\sim v$. Thus as long as $l < \sigma^{1/2}$ and $v > \sigma^{-1/2}$ the Hubble term is completely dominated by the velocity dependent terms. The evolution equations to leading order in are simply

$$\begin{aligned} 2\frac{dl}{dx} &= lv^2\beta + cv, \\ \frac{dv}{dx} &= (1 - v^2) \left(\frac{k}{l} - v\beta \right). \end{aligned} \quad (2)$$

We wish to find the behaviour of l and v for large x beginning from natural initial conditions (Kibble Mechanism) where $l_i \sim \epsilon_K^{-1/2}$. From the structure of the equation for l it is clear that l always increases. While v will increase when $k \geq v\beta l$ and decrease in the opposite case. So if v is initially very small or zero it will increase rapidly and then again fall off as l increases; while if it is ~ 1 then it will rapidly decrease. Regardless of the initial value of v the evolution locks on to a trajectory where v is very closely approximated by $k/\beta l$. This allows one to solve the equation for l to obtain the power law behaviours

$$\begin{aligned} l &= \left(\frac{(k+c)k}{\beta} \right)^{1/2} x^{1/2}, \\ v &= \left(\frac{k}{\beta(k+c)} \right)^{1/2} x^{-1/2}. \end{aligned} \quad (3)$$

It is easy to check by substitution into eqn(1) that this regime persists until the growth of the Hubble term and the decrease of the velocity dependent term makes them comparable i.e till $x \sim 10^{-1}\sigma$ (for typical values of k, c) by when $t \sim t'_c + .1t_c$. After this the effect of the Hubble terms causes a shift in the power-law exponents and the Kibble regime begins. Note that this is compatible with the results of [7,8] who found that for higher initial network densities (i.e as they varied $\tilde{L}_i = L_i/t_c$ from 1 to $\sigma^{-1/2}$ - the minimum initial network scale considered by them) their “stretching regime” became shorter and shorter till at the highest densities they considered, the Kibble regime replaced the stretching regime to become the initial regime. Since, however, the minimum initial network length scale they considered was still $\sigma^{1/2}$ times larger than the characteristic scale of a network formed via the Kibble mechanism i.e $L \sim 1/\eta \sim 1/T_c$ a regime connecting these two very different scales was needed. The new regime we have pointed out precisely fills the gap.

The most remarkable feature of this regime is thus that *the values of L and v very shortly after the phase transition are essentially independent of their initial values.* This is in sharp contrast to the “stretching regime” $L \sim L_c(t/t_c)^{1/2}, v \sim t(L_c\sigma)^{-1}$ found in [7,8] for initial conditions $\tilde{L}_i > \sigma^{-1/2}$.

It is easy to check that the Lyapunov exponents obtained by linearizing the eqns(2) around the solutions eqns(3) are indeed negative so that deviations from these solutions die out exponentially fast as x increases. We have verified the analysis given above by numerically integrating the equations (2) for various initial values of l, v at various values of the parameters k, c, σ . For large x the asymptotic solutions given in eqn.(3) are in excellent agreement with the results of numerical integration of eqns.(2) irrespective of whether the initial velocities are relativistic or very small.

In the Kibble regime the network variables scale as follows ($\tilde{L} = Lt_c^{-1}$):

$$\tilde{L} = \left(\frac{2k(k+c)}{3\sigma} \right)^{1/2} \left(\frac{t}{t_c} \right)^{5/4},$$

$$v = \left(\frac{3k}{2(k+c)\sigma} \right)^{1/2} \left(\frac{t}{t_c} \right)^{1/4} . \quad (4)$$

Matching the onset of the Kibble regime with the end of the stretching regime and imposing the consistency condition that this occurs after the phase transition shows that the stretching regime can occur only if $\tilde{L}_i > \sigma^{-1/2}$. On the other hand for networks arising from the Kibble mechanism ,where $\tilde{L}_i \ll \sigma^{-1/2}$, the new transient regime neatly resolves the inconsistency by rapidly taking the network from the dense state in which it is created to the dilute state consistent with the fact that the minimum value of \tilde{L}_i compatible with the Kibble regime is $\sim \sigma^{-1/2}$ (for typical k, c).

This regime implies a very considerable loss of string length in a very short cosmological time interval $\Delta t \ll t'_c$. The physical reason for this strange phenomenon is simply that near the phase transition time the friction time scale $l_f = \eta^2 \beta / T_c^3 \sim 1/T_c$ is much smaller than the cosmological time $t_c \sim \sigma / T_c$ (for friction dominated gauge strings for which $\sigma \gg 1$). On the other hand the one scale model identifies the average network curvature with the network correlation length and thus enforces curvature driven motion and therefore dissipation. Since the regime takes place over a time interval during which the cosmological scale factor changes very little the energy that is dissipated must come from the string length : which in fact decreases sharply. In [8]the instructive example of the damped motion of a circular string loop is discussed. Such a loop can contract in radius and thus lose string length even while preserving the transversal motion constraint $\dot{\vec{x}} \cdot \vec{x}' = 0$. The total frictional work is exactly the energy loss due to decrease in invariant string length for the loop due to its contraction. Thus one may visualize the growth of the network correlation length and the average curvature radius in tandem as being due to the decaying away of curved and looped regions with high curvature leaving behind a network with less string (larger L) and lower curvature (larger R).

It is perhaps pertinent to point out that this analysis shows that friction is by itself a genuine decay mechanism (independent of any specific interaction like gravitational or electroweak radiation) for both the network and the loops.

III. DISCUSSION

The new regime presented here leads us to the conclusion that the one scale model with friction presents a consistent picture of network evolution for initial network densities and velocities given by the Kibble mechanism. It predicts that the network always emerges (i.e for $t > 10t_c$, say) from the phase transition epoch, almost instantaneously with respect to cosmological time, with a scale $L > (\sigma)^{1/2}/\eta$ and velocity $v < \sigma^{-1/2}$ essentially independently of the values at formation. The drastic increase in string network correlation length implies that the string length density has diminished by a large factor ($1/\sigma$) during a time in which the scale factor is essentially constant. This string loss is due to frictional damping.

This generic behaviour we have established qualitatively changes the prevalent picture of network evolution making it consistent with the network at formation. This has drastic implications for Baryogenesis mechanisms that rely on the high initial length density predicted by the Kibble mechanism [2-4]. The length density of string available is reduced by a factor of $1/\sigma$ (i.e 10^{-16}) for strings formed close to the Electroweak transition temperature. A higher scale phase transition does not help since the fast evolution of L in the extended Kibble regime that will occur before the Electroweak transition cuts down the network density by an even greater factor.

It has been proposed that a topological gauge string network formed above the EW transition at a temperature ~ 1 TeV could trap Z flux following the EW transition . For such scenarios, amongst others, the present authors [3,5] have considered the generation of the baryon asymmetry from the string network as given by the Kibble mechanism. Sphalerons [12] have a substantial attractive interaction with Z flux tubes as observed in Ref. [3] , where it was shown that this flux (in particular for double flux quantum strings) may even be sufficient to bind sphalerons on strings. These sphaleron beaded strings then lose string length into loops. These loops decay and in the process release the bound sphalerons.

The sphaleron is a saddlepoint configuration at the top of the instanton potential barrier separating vacua with different winding numbers. It can thus fall into either adjoining vacuum but the CP violation in the Electroweak mass matrix (below the Electroweak symmetry breaking transition temperature) can bias the decay . The net change in winding number that results is translated by the anomaly in the baryon current into a non zero Baryon Asymmetry. The maximum Baryon density one could obtain in this way can be estimated by assuming (M_W is the Electroweak mass scale) that each string length of size $\sim M_W^{-1}$ beads a sphaleron which thereafter decays (at a temperature $\sim M_W$) to yield a baryon number with efficiency ϵ_{CP} (ϵ_{CP} is a dimensionless parameter characterizing the CP violation that biases the sphaleron decay). One obtains (where $n_L(M_W)$ stands for the string length density and L_W for the network scale at $T \sim M_W$)

$$\frac{n_B}{n_\gamma} \sim \epsilon_{CP} m_W^{-2} \times n_L(M_W) \sim \epsilon_{CP} \left(\frac{T_c}{m_W} \right)^2 (T_c L_W)^{-2} \quad (5)$$

In the Kibble regime the minimum value of L_W is $\sim \tilde{L}_i \sim \sigma^{-1/2}$ so that $n_B/n_\gamma \sim \epsilon_{CP}/\sigma$. Due to this dilution of the string length density the Baryon asymmetry produced is thus completely negligible.

A similar mechanism is operative for the twisted string configurations found in ref. [2]; the sphalerons being substituted by twists. Furthermore, order of magnitude estimates indicate that all sphaleron mediated B violation occurring in strings [4] can only attain a maximum efficiency less than the above.

These considerations apply to all scenarios [3,2,4,5] which are based on topological “descendant” strings created above the EW phase transition, capable of trapping EW flux during the EW phase transition. Such mechanisms are thus unlikely to work - as they have lost most of their string length well above the EW transition, that is before sphalerons can bead.

They do not however exclude the possibility that a metastable network formed during the EW transition itself, with sphalerons beaded on strings, may yield an appreciable Baryon asymmetry. This is because of the important difference that in this case the string network is created below the temperature at which sphaleron mediated processes in the bulk are suppressed. Therefore, all string length that is lost, no matter how fast or by what mechanism, contributes to the BA.

In conclusion, we have shown that the evolution of string networks is strongly affected by friction if their scale is well below the Planck scale resulting in a dilution of the string length density by a factor of T_c/M_P . Thus the estimation of the amplitude of the baryon asymmetry produced by electroweak strings should be done as presented in this Letter. As a result, the length density of the string networks produced at the electroweak scale or slightly above it would be too low to explain the observed magnitude of the baryon number in our universe. Similar considerations apply to scenarios that rely on a dense string network to generate a seed magnetic field in the universe [13]. In special circumstances these constraints may however be evaded. Thus if the string hardly interacts with other particles, that is, has a very tiny β , then the frictional force becomes less effective and sufficient string length may survive the phase transition to produce a significant baryon asymmetry. Alternatively if the parameters k, c are very small then again the network may remain sufficiently dense past the Electroweak Ginzburg temperature to allow string based mechanisms to work. Furthermore, since the equations (1) are derived on the basis of the Nambu-Goto approximation which neglects effect of string thickness, they may require correction during the epoch of string formation. These issues require further study.

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