LIGHT MESONS ELM FORM FACTOR AND RUNNING COUPLING EFFECTS*

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Abstract

The pion and kaon electromagnetic form factors $F_M(Q^2)$ are calculated at the leading order of pQCD using the running coupling constant method. In computations dependence of the mesons distribution amplitudes on the hard scale Q^2 is taken into account. The Borel transform and resummed expression for $F_M(Q^2)$ are found. The effect of the next-to-leading order term in expansion of $\alpha_S(\lambda Q^2)$ in terms of $\alpha_S(Q^2)$ on the pion form factor $F_\pi(Q^2)$ is discussed, comparison is made with the infrared matching scheme's result.

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1. Investigation of the infrared (ir) renormalon effects in various inclusive and exclusive processes is one of the important and interesting problems in the pQCD [1] (and references therein). It is known that all-order resummation of ir renormalons corresponds to the calculation of the one-loop Feynman diagrams with the running coupling constant $\alpha_S(-k^2)$ at the vertices or, alternatively, to calculation of the same diagrams with non-zero gluon mass. Both these approaches are generalization of the Brodsky, Lepage and Mackenzie (BLM) scale-setting method [2] and amount to absorbing certain vacuum polarization corrections appearing at higher-order calculations into the one-loop QCD coupling constant. Studies of ir renormalon problems have opened also new prospects for evaluation of higher twist corrections to processes' characteristics.

Unlike inclusive processes exclusive ones have additional source of ir renormalon contributions [3]-[5]; integration over longitudinal fractional momenta of hadron constituents in the expression of the elm form factor generates such corrections.

2. In the context of pQCD the meson elm form factor has the form,

$$F_M(Q^2) = \int_0^1 \int_0^1 dxdy \phi_M^*(y, \hat{Q}^2) T_H(x, y; Q^2, \alpha_S(\hat{Q}^2)) \phi_M(x, \hat{Q}^2), \qquad (1)$$

where $Q^2 = -q^2$ is the square of the virtual photon's four-momentum. Here ϕ_M is the meson distribution amplitude, containing all non-perturbative hadronic binding effects, whereas T_H is the hard-scattering amplitude of the subprocess $q\overline{q}' + \gamma^* \to q\overline{q}'$ and can be found using pQCD. In (1) $\hat{Q}^2$ is the factorization and renormalization scale, which is taken as the square of the momentum transfer of the exchanged hard gluon in corresponding Feynman diagrams. Such choice for $\hat{Q}^2$ allows one to remove large terms proportional to $\ln(Q^2/\mu_F^2)$ and $\ln(Q^2/\mu_R^2)$ from the next-to-leading order correction to T_H [6].

At the leading order T_H is given by the following expression

$$T_H = \frac{16\pi C_F}{Q^2} \left[\frac{2}{3} \frac{\alpha_S(Q^2(1-x)(1-y))}{(1-x)(1-y)} + \frac{1}{3} \frac{\alpha_S(Q^2 xy)}{xy} \right], \quad C_F = \frac{4}{3}. \qquad (2)$$

One of the important ingredients of our study is the choice of the meson distribution amplitude $\phi_M(x, Q^2)$. In the framework of pQCD it is possible to predict the dependence of $\phi_M(x, Q^2)$ on Q^2 using evolution equation, but not its shape (its dependence on x). In this work for the pion and kaon we use model distribution amplitudes proposed in Refs.[7]-[9]. For the pion they have the following form

$$\phi_\pi(x,\mu_0^2)=\phi_{asy}^\pi(x)\left[a+b(2x-1)^2+c(2x-1)^4\right], \tag{3}$$

where $\phi_{asy}^\pi(x)$ is the pion asymptotic distribution amplitude,

$$\phi_{asy}^\pi(x)=\sqrt{3}f_\pi x(1-x), \tag{4}$$

and $f_\pi=0.093$ GeV is the pion decay constant.

The constants a,b,c in (3) were found by means of the QCD sum rules method at the normalization point $\mu_0=0.5$ GeV and take different values [7],[8],[9] defining the pion's alternative distribution amplitudes. For the kaon we have

$$\phi_K(x,\mu_0^2)=\phi_{asy}^K(x)\left[a+b(2x-1)^2+c(2x-1)^3\right], \tag{5}$$

with $a=0.4, b=3, c=1.25$ and $f_K=0.122$ GeV.

The dependence of the meson distribution amplitude on Q^2 can be obtained by means of the following expression

$$\phi_M(x,Q^2)=\phi_{asy}^M(x)\sum_{n=0}^{\infty}r_n C_n^{3/2}(2x-1)A_n. \tag{6}$$

Here $\left\{C_n^{3/2}(2x-1)\right\}$ are the Gegenbauer polynomials, γ_n is the anomalous dimension and

$$A_n=\left[\frac{\alpha_S(Q^2)}{\alpha_S(\mu_0^2)}\right]^{\gamma_n/\beta_0}$$

$\beta_0=11-2n_f/3$ being the QCD beta-function's one-loop coefficient.

The QCD running coupling constant $\alpha_S(\hat{Q}^2)$ in Eq.(2) suffers from ir singularities associated with the behaviour of the $\alpha_S(\hat{Q}^2)$ in the soft regions $x\to 0,\ y\to 0;\ \ x\to 1,\ y\to 1$. Therefore, $F_M(Q^2)$ can be found after proper regularization of $\alpha_S(\hat{Q}^2)$ in these soft end-point regions. In the framework of the frozen coupling approximation such regularization is achieved by equating $\hat{Q}^2$ to its mean value $Q^2/4$ and removing $\alpha_S(Q^2/4)$ as the constant factor from integrand in (1). As a result, one obtains form factors with the same shape, but different magnitudes depending on a distribution amplitude used in computations. To solve this problem in the context of the running coupling method let us relate the running coupling $\alpha_S(\lambda Q^2)$ in terms of $\alpha_S(Q^2)$ by means of the renormalization group equation [10]

$$\alpha_S(\lambda Q^2) \simeq \frac{\alpha_S}{1+(\alpha_S\beta_0/4\pi)\ln\lambda} - \frac{\alpha_S^2}{4\pi}\frac{\beta_1 \ln[1+(\alpha_S\beta_0/4\pi)\ln\lambda]}{\beta_0[1+(\alpha_S\beta_0/4\pi)\ln\lambda]^2}, \qquad (7)$$

where α_S is the one-loop QCD coupling constant $\alpha_S(Q^2)$ and $\beta_1 = 102 - 38n_f/3$ is the beta-function's two-loop coefficient.

3. As was demonstrated in our works [3]-[5], integration in (1) using (2) and (7) generates ir divergences and as a result for $F_M(Q^2)$ we get a perturbative series with factorially growing coefficients. This series can be resummed using the Borel transformation [11],

$$\left[Q^2F_M(Q^2)\right]^{res} = \frac{(16\pi f_M)^2}{\beta_0}\int_0^\infty du \exp\left(-\frac{4\pi u}{\beta_0\alpha_S}\right) B\left[Q^2F_M\right](u), \qquad (8)$$

where $B[Q^2F_M](u)$ is the Borel transform of the corresponding perturbative series [4], [5].

When we take both of the variables x, y in Eq.(2) as the running ones, for $B[Q^2F_M](u)$ we find

$$B\left[Q^2F_M\right](u) = \sum_{n=1}^{N}\left(\frac{\mathbf{m}_n}{(n-u)^2} + \frac{\mathbf{l}_n}{n-u}\right). \qquad (9)$$

The exact expressions for $\mathbf{m}_n, \mathbf{l}_n$ can be found in Ref.[12].

The Borel transform (9) has double and single poles at $u = n$. They are ir renormalon poles, which are responsible for divergence of the perturbative series for $Q^2F_M(Q^2)$. The resummed expression (8) can be calculated with the help of the principal value prescription [10],[11]

$$\left[Q^2F_M(Q^2)\right]^{res} = \frac{(16\pi f_M)^2}{\beta_0}\sum_{n=1}^{N}\left[-\frac{\mathbf{m}_n}{n} + (\mathbf{l}_n + \mathbf{m}_n\ln\lambda)\frac{li(\lambda^n)}{\lambda^n}\right], \qquad (10)$$

where $li(\lambda)$ is the logarithmic integral [13],

$$li(\lambda) = P.V.\int_0^\lambda \frac{dx}{\ln x}, \quad \lambda = Q^2/\Lambda^2. \qquad (11)$$

In (10) we have taken into account the dependence of the distribution amplitude $\phi_M(x, Q^2)$ on the scale Q^2 (in [12], for simplicity, we have replaced $\phi_M(x, \hat{Q}^2) \to \phi_M(x, Q^2)$) and it is the generalization of our results for the pion and kaon elm form factors [4],[5].

In the case with one frozen (for example, y) and one running (x) variables, which corresponds to the choice $\hat{Q}^2 = Q^2(1-x)/2$ and $\hat{Q}^2 = Q^2 x/2$ in (2), for the pion we find (asymptotic distribution amplitude (4))

$$[Q^2 F_\pi(Q^2)]^{res} = \frac{(16\pi f_\pi)^2}{2\beta_0}\left[\frac{li(\tilde{\lambda})}{\tilde{\lambda}} - \frac{li(\tilde{\lambda}^2)}{\tilde{\lambda}^2}\right], \quad \tilde{\lambda} = Q^2/2\Lambda^2. \qquad (12)$$

The ir renormalon analysis carried out (Eqs.(10),(12)) allows one to estimate power corrections to the light mesons' form factor. Another way of such estimation is the ir matching scheme, in the context of which, one explicitly divides power corrections from a full expression by introducing moments of α_S at low scales as new non-perturbative parameters [14]. As an example, let us consider the pion form factor ($\phi_{asy}^\pi(x)$). By freezing one of variables (y) we can express $Q^2 F_\pi$ in terms of moment integrals F_p defined by the formula

$$F_p(Q) = \frac{p}{Q^p}\int_0^Q dk k^{p-1}\alpha_S(k^2).$$

After simple calculations we get

$$Q^2 F_\pi(Q^2) = 64\pi f_\pi^2 \left\{ \left(\frac{\mu}{Q}\right)^2 F_2(\mu) - \left(\frac{\mu}{Q}\right)^4 F_4(\mu) \right.$$

$$\left. + \frac{\alpha_S}{4}\left[1 - 2\Gamma(1, 2z) + \Gamma(1, 4z)\right] + \frac{\alpha_S^2 \beta_0}{32\pi}\left[3 - 4\Gamma(2, 2z) + \Gamma(2, 4z)\right] \right\}, \qquad (13)$$

where $\mu = 2$ GeV is the ir matching scale, $z = \ln(Q/\sqrt{2}\mu)$, $\alpha_S(Q^2/2)$ and $\Gamma(n, x)$ is the incomplete gamma function [13].

4. Results of our numerical calculations are shown in Fig.1. For computation of $Q^2 F_\pi$ in the context of the ir matching scheme, we use $N = 4$ perturbative estimate for moment integrals $F_p(Q)$ (in Eq.(13), expression for $N = 1$ is written down).

It is worth noting that non-perturbative parameters $F_2(\mu)$ and $F_4(\mu)$ can be found using the running coupling method

$$F_2(\mu) = \frac{4\pi}{\beta_0}\frac{li(\lambda)}{\lambda}, F_4(\mu) = \frac{8\pi}{\beta_0}\frac{li(\lambda^2)}{\lambda^2}, \quad \lambda = \mu^2/\Lambda^2.$$

For $\Lambda = 0.2$ GeV we get $F_2(\mu = 2GeV) = 0.421,\ F_4(\mu = 2GeV) = 0.347$. In our calculations we take for $F_2(\mu = 2GeV) \simeq 0.5$ the value deduced from experimental data and for F_4-running coupling prediction 0.347. As is seen, Eq.(12) and ir matching scheme give approximately the same results, excluding a region of small Q^2.

In Fig.2, as an example, the dependence of the ratio $R_K = [Q^2 F_K(Q^2)]^{res}/[Q^2 F_K(Q^2)]^0$ on Q^2, where $[Q^2 F_K(Q^2)]^0$ is the kaon form factor in the frozen coupling approximation, is shown. Here we take into account a dependence of the kaon's distribution amplitude on Q^2 (curve **1**, the dashed curve is taken from Ref.[5], where this dependence was neglected). The ir renormalon corrections can be transferred into the scale of $\alpha_S(Q^2)$ in $[Q^2 F_K(Q^2)]^0$

$$Q^2 \to e^{f(Q^2)} Q^2, \quad f(Q^2) = c_1 + c_2 \alpha_S(Q^2) + c_3 \alpha_S^2(Q^2). \tag{14}$$

Numerical fitting allows us to get (for $n_f = 3$) $c_1 \simeq -1.304, c_2 \simeq -35.604, c_3 \simeq 127.25$ (curve **2**). The similar results can be obtained also for the pion.

It is interesting to clarify an importance of the second term in (7) for considering problem. Investigations demonstrate that an effect of the second term on a whole result is small (see, Ref.[15] for details).

5. It is evident that ir renormalon effects enhance the ordinary (frozen coupling) perturbative predictions for the pion, kaon elm form factors approximately two times. Our recent studies confirm that neither a dependence of $\phi_M(x, Q^2)$ on Q^2 nor the next-to-leading order term in Eq.(7) changes this picture considerably. This feature of ir renormalons may help one in solution of a contradiction between theoretical interpretations of experimental data for the photon-to-pion transition form factor $F_{\gamma\pi}$ from one side and for the pion elm form factor F_π from another side. Thus in Ref.[16] the authors noted that the scaling and normalization of $F_{\gamma\pi}$ to favor of the pion asymptotic-like distribution amplitude. But then prediction for F_π obtained using the same amplitude is lower than the data by approximately a factor of 2. We think that in this discussion a crucial point is a chosen method of integration in (1). Indeed, unlike F_π the expression for $F_{\gamma\pi}$ at the leading order of pQCD does not contain an integration over $\alpha_S(xQ^2)$. In other words, the running coupling method being applied to Eq.(1) and to the expression for $F_{\gamma\pi}$ (see, Ref.[16]) enhances the perturbative result for the pion elm form factor and at the same time, does not change $F_{\gamma\pi}$. This allows us to suppose that in the context of pQCD the same pion distribution amplitude may explain experimental data for both $F_{\gamma\pi}$ and F_{π}.

FIGURE CAPTIONS

Fig.1 The pion elm form factor calculated using $\phi^{\pi}_{asy}(x)$ (4). The curves correspond to the following computational schemes: **1**- resummed expression (10), **2**-resummed expression (12), dashed-ir matching scheme, dot-dashed-frozen coupling approximation.

Fig.2 The ratio R_K as a function of Q^2.

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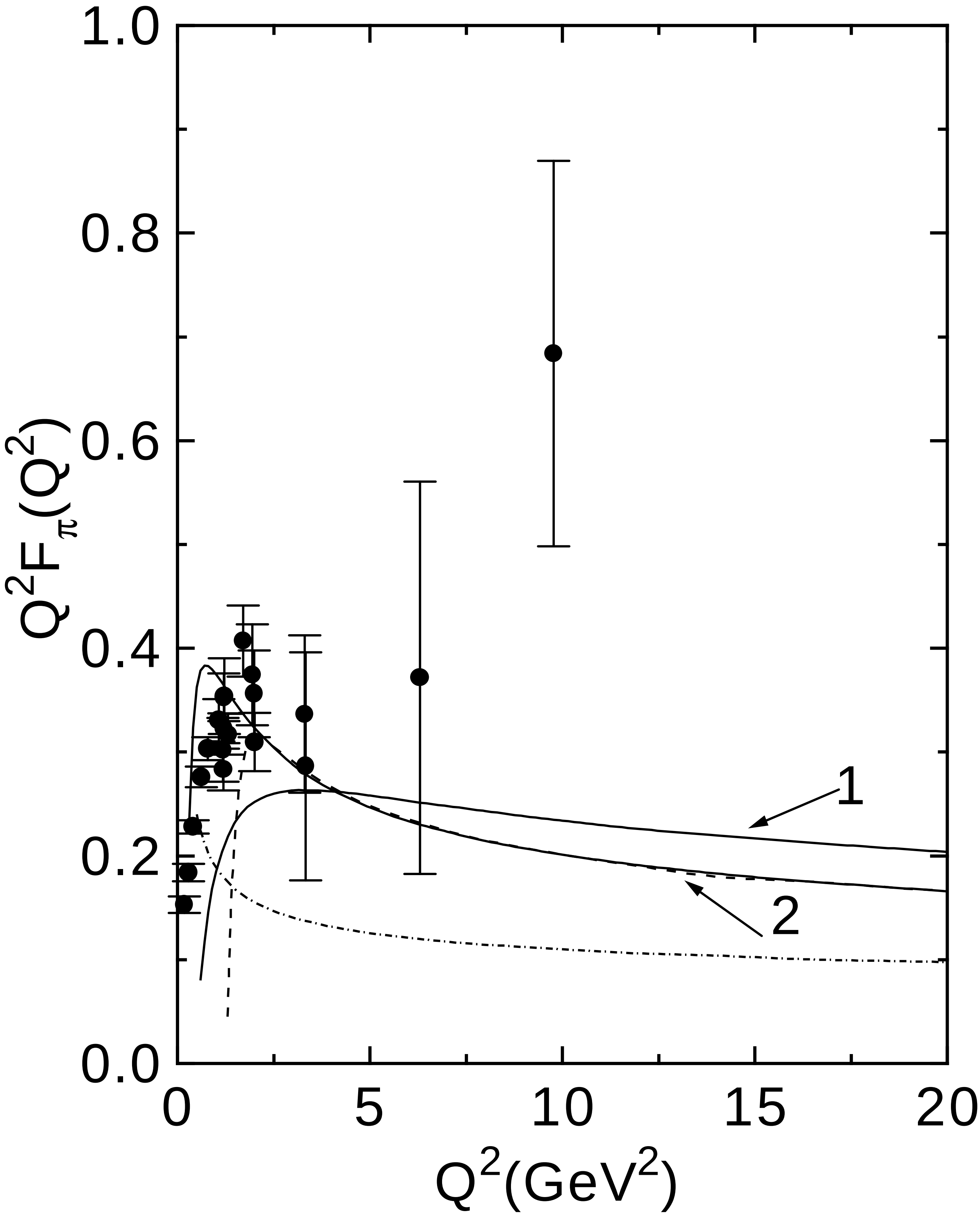

1.0
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Q2Fπ(Q2)
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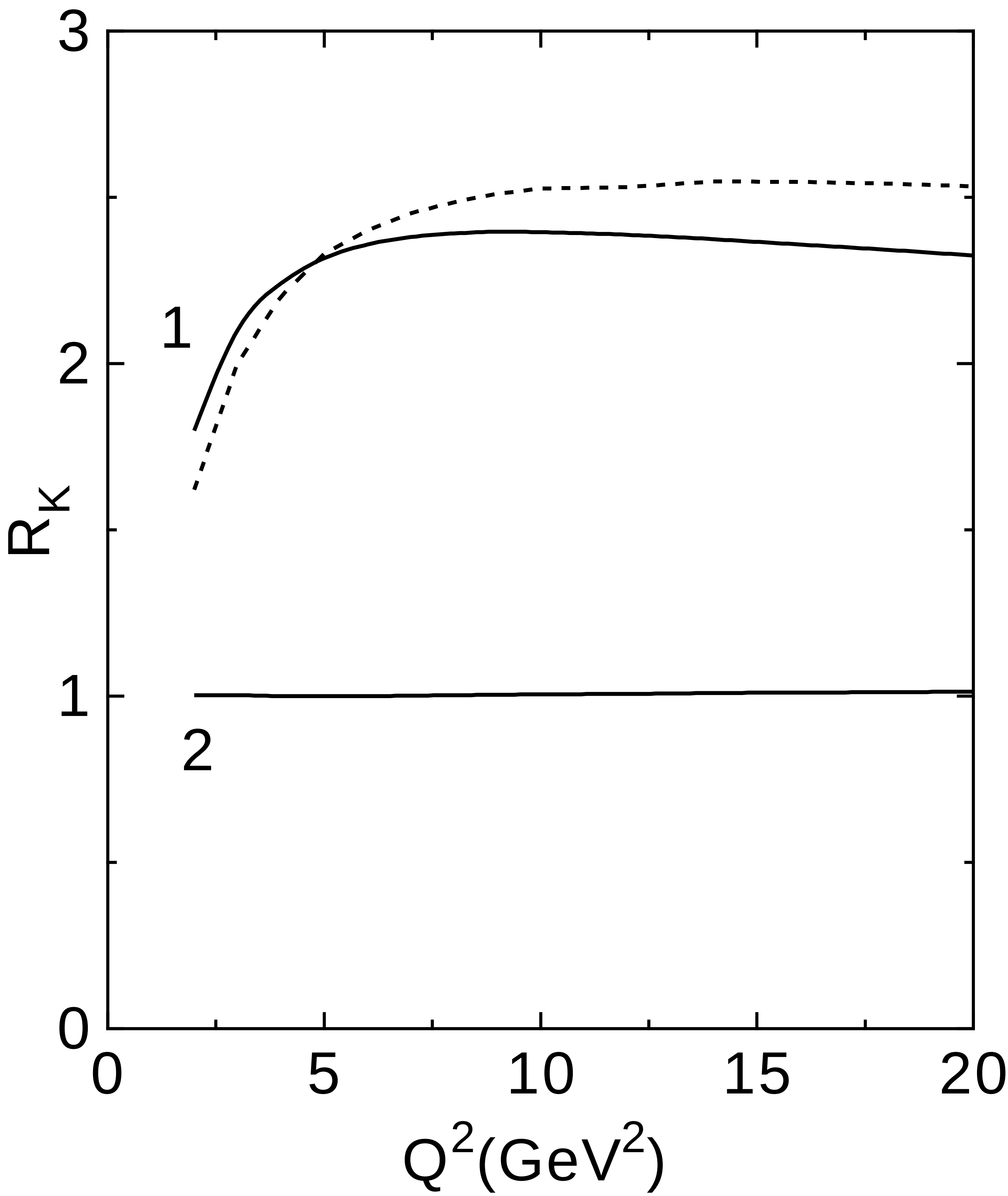

3
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R_K
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