

FINAL STATE INTERACTION EFFECTS ON γ FROM $B \rightarrow DK$

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ABSTRACT

The implications of a negligible annihilation contribution in $B \rightarrow DK$ decays are reanalyzed and are shown to lead to no new constraints on the weak phase γ from color-allowed $B^\pm \rightarrow DK^\pm$ decays. A test of negligible annihilation is proposed in $B^+ \rightarrow D^+ K^0$ (or $B^+ \rightarrow D^+ K^{*0}$), and an application is presented in which γ can be determined from these processes (or corresponding $B \rightarrow DK^*$ decays) supplemented with isospin-related neutral B decays.

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Recently one of us proposed a method [1] to constrain the CKM weak phase $\gamma = -\text{Arg}(V_{ub}^* V_{ud}/V_{cb}^* V_{cd})$ from *color-allowed* $B^\pm \rightarrow DK^\pm$ decays [2], when both flavor and CP-eigenstate neutral D mesons are considered. Decays with flavor states have already been observed by CLEO [3] with branching ratio $\mathcal{B}(B^\pm \rightarrow DK^\pm)/\mathcal{B}(B^\pm \rightarrow D\pi^\pm) = 0.055 \pm 0.014 \pm 0.005$, or $\mathcal{B}(B^\pm \rightarrow DK^\pm) \simeq 3 \times 10^{-4}$. Our approach was general and involved no dynamical assumptions about hadronic weak matrix elements and about final state interactions. Subsequently Xing [4] claimed that a certain improvement in this method may be achieved by making the dynamical assumption of a negligible annihilation contribution. While this assumption is reasonable, it may be spoiled by rescattering effects [5, 6] and would have to be tested experimentally. One of the purposes of the present letter is to suggest such a test. Our second purpose is to go over the arguments in [4] and to point out a certain flaw in the treatment of final state interactions. By presenting a correct analysis we will show that, in fact, the assumption of a vanishing annihilation contribution does not lead to any further constraint on γ beyond the one obtained in [1]. Finally, we will present a scheme [7] which involves also *neutral*

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B decays to DK states through which γ can be determined when neglecting annihilation. Since these decay modes involve $B^0 \rightarrow DK^0$, in which the neutral B meson must be flavor-tagged, it would be advantageous to consider instead the corresponding self-tagged decays $B \rightarrow DK^*$. All the considerations applied below to $B \rightarrow DK$ apply also to $B \rightarrow DK^*$.

For completeness, let us recapitulate the results of [1]. Writing

$$A(B^+ \rightarrow \bar{D}^0 K^+) = \bar{a}e^{i\bar{\Delta}} \quad , \quad A(B^+ \rightarrow D^0 K^+) = ae^{i\Delta}e^{i\gamma} \quad , \quad (1)$$

and introducing the two CP-eigenstates, $D_{1,2} = (D^0 \pm \bar{D}^0)/\sqrt{2}$, one considers the two charge-averaged ratios of rates for these states and for the flavor states

$$R_i \equiv \frac{2[\Gamma(B^+ \rightarrow D_i K^+) + \Gamma(B^- \rightarrow D_i K^-)]}{\Gamma(B^+ \rightarrow \bar{D}^0 K^+) + \Gamma(B^- \rightarrow D^0 K^-)} \quad , \quad i = 1, 2 \quad . \quad (2)$$

One finds

$$R_{1,2} = 1 + r^2 \pm 2r \cos \delta \cos \gamma \quad , \quad (3)$$

where $r \equiv a/\bar{a}$, $\delta \equiv \Delta - \bar{\Delta}$. This leads to two inequalities

$$\sin^2 \gamma \leq R_{1,2} \quad , \quad i = 1, 2 \quad , \quad (4)$$

which could potentially imply new constraints on γ in future experiments [1].

The two pseudo-asymmetries

$$\mathcal{A}_i \equiv \frac{\Gamma(B^+ \rightarrow D_i K^+) - \Gamma(B^- \rightarrow D_i K^-)}{\Gamma(B^+ \rightarrow \bar{D}^0 K^+) + \Gamma(B^- \rightarrow D^0 K^-)} \quad , \quad i = 1, 2 \quad , \quad (5)$$

are given by

$$\mathcal{A}_2 = -\mathcal{A}_1 = r \sin \delta \sin \gamma \quad , \quad (6)$$

and together with the two ratios R_i could, in principle, provide sufficient information to determine the three parameters r , δ and γ (up to certain discrete ambiguities). However, since r is suppressed by a smaller than one ratio of CKM factors and by a color factor, one expects $r \approx 0.1$, which would be too small to be measured from the tiny deviation of $(R_1 + R_2)/2$ from unity. Similarly, unless δ is very large, the asymmetries may be too small to permit nonzero measurements.

While the above equations and constraints follow generally from the CKM structure of the weak charged currents, one may try to supplement these equations with assumptions about the dynamics of the above hadronic decays. One such common assumption [8] is the neglect of annihilation diagrams. This assumption was made in [4], where it was claimed to reduce the number of independent parameters by essentially relating r and δ , and consequently to lead to more stringent constraints on γ .

In order to study the implication of this assumption, let us consider the isospin structure of the amplitudes for the decays $B \rightarrow \bar{D}K$ and $B \rightarrow DK$. Since the transition operators for $\bar{b} \rightarrow \bar{c}u\bar{s}$ and $\bar{b} \rightarrow \bar{u}c\bar{s}$ are both pure $\Delta I = 1/2$, these processes can be described in terms of two *independent* pairs of complex amplitudes, corresponding to

the two final mesons being in $I = 0$ and 1 states [9, 10]. Thus, we have for $B \rightarrow \bar{D}K$ from $\bar{b} \rightarrow \bar{c}u\bar{s}$

$$\begin{aligned} A(B^+ \rightarrow \bar{D}^0 K^+) &= \bar{A}_1 e^{i\bar{\delta}_1} = \bar{T} + \bar{C} \quad , \\ A(B^0 \rightarrow D^- K^+) &= \frac{1}{2} \bar{A}_1 e^{i\bar{\delta}_1} - \frac{1}{2} \bar{A}_0 e^{i\bar{\delta}_0} = \bar{T} \quad , \\ A(B^0 \rightarrow \bar{D}^0 K^0) &= \frac{1}{2} \bar{A}_1 e^{i\bar{\delta}_1} + \frac{1}{2} \bar{A}_0 e^{i\bar{\delta}_0} = \bar{C} \quad , \end{aligned} \quad (7)$$

and for $B \rightarrow DK$ from $\bar{b} \rightarrow \bar{u}c\bar{s}$

$$\begin{aligned} A(B^+ \rightarrow D^0 K^+) &= \left[\frac{1}{2} A_1 e^{i\delta_1} + \frac{1}{2} A_0 e^{i\delta_0} \right] e^{i\gamma} = C + A \quad , \\ A(B^+ \rightarrow D^+ K^0) &= \left[\frac{1}{2} A_1 e^{i\delta_1} - \frac{1}{2} A_0 e^{i\delta_0} \right] e^{i\gamma} = -A \quad , \\ A(B^0 \rightarrow D^0 K^0) &= A_1 e^{i\delta_1} e^{i\gamma} = C \quad . \end{aligned} \quad (8)$$

We note that there are four independent CP-conserving phases describing in general the dominantly inelastic rescattering in the two pairs of $I = 0$ and 1 channels. In Ref. [4] (and also in [10]) the corresponding phases in $B \rightarrow \bar{D}K$ and in $B \rightarrow DK$ were assumed to be equal, $\bar{\delta}_i = \delta_i$, $i = 1, 2$. We do not expect this to be the case in general, owing to the different hadronic dynamics following the distinct $\bar{b} \rightarrow \bar{c}u\bar{s}$ and $\bar{b} \rightarrow \bar{u}c\bar{s}$ quark subprocesses as described in the next paragraph.

The right-hand-sides of Eqs. (7) and (8) consist of *equivalent* expressions in terms of a graphical description of amplitudes, where overall signs follow from a specific phase convention for meson states [11]. \bar{T} is a *tree* amplitude involving the subprocess $\bar{b} \rightarrow \bar{c}u\bar{s}$ in which the $u\bar{s}$ produced by the weak current materializes into a single meson in a color-favored manner. \bar{C} (C) is a *color-suppressed* amplitude for $\bar{b} \rightarrow \bar{c}u\bar{s}$ ($\bar{b} \rightarrow \bar{u}c\bar{s}$), where the $u\bar{s}$ ($c\bar{s}$) pairs produced by the weak current end up in different mesons; and A describes *annihilation* of the \bar{b} and the u in a decaying B^+ into a weak current, which then materializes into a pair of mesons. The processes $B \rightarrow \bar{D}K$ are written in terms of \bar{T} and \bar{C} , while $B \rightarrow DK$ are given by C and A .

The assumption $A = 0$ implies equalities between the magnitudes and phases of the two isospin amplitudes in $B \rightarrow DK$, $A_1 = A_0$, $\delta_1 = \delta_0$, and consequently

$$r = \frac{A_1}{\bar{A}_1} = \left| \frac{C}{\bar{T} + \bar{C}} \right| \quad , \quad \delta = \delta_1 - \bar{\delta}_1 \quad . \quad (9)$$

Clearly A_1/\bar{A}_1 and $\delta_1 - \bar{\delta}_1$ are two independent parameters. They remain independent also when assuming factorization for the ratios of amplitudes C/\bar{T} and \bar{C}/\bar{T} . To calculate r using generalized factorization [12] for color-allowed (\bar{T}) and color-suppressed (\bar{C} , C) amplitudes, one would need information about the relative strong phase between \bar{T} and \bar{C} . In the absence of information about the interference between the two terms, one can obtain an approximate estimate by disregarding the smaller \bar{C} contribution. Thus one finds $r \approx |C/\bar{T}| \approx |V_{ub}^* V_{cs}/V_{cb}^* V_{us}| (a_2/a_1) \approx 0.1$, where a value of 0.4 is taken for the CKM ratio [13] and the color suppression factor $a_2/a_1 \approx 0.26$ is taken from a study of $B \rightarrow \bar{D}\pi$ decays [14]. On the other hand, δ remains arbitrary.

This situation is to be contrasted with the arguments presented in [4], where $\delta_i = \bar{\delta}_i$ was assumed, and consequently r and δ were found to be related to each other when factorization was assumed. We stress again that, in general, no such phase relation is expected. The analysis of [4] is clearly expected to hold in the limit in which all strong phases are assumed to vanish; however in reality these phases could be sizable. In the case of vanishing phase differences, Eqs. (3) imply (without assuming $A = 0$) a simple relation

$$\cos \gamma = \frac{R_1 - R_2}{4r} \quad , \quad (10)$$

which would permit a determination of γ from R_1 and R_2 once r is known.

Final state phases in $B \rightarrow \bar{D}K$ can be studied experimentally. Similar studies were carried out in $B \rightarrow \bar{D}\pi$, and an upper limit on the corresponding final state phase-difference was obtained at a level of 20° [15]. The three amplitudes in Eqs. (7) obey a triangle relation [11],

$$A(B^0 \rightarrow D^- K^+) + A(B^0 \rightarrow \bar{D}^0 K^0) = A(B^+ \rightarrow \bar{D}^0 K^+) \quad , \quad (11)$$

shown in Fig. 1, where amplitudes are denoted by the flavor of B and D . The dashed-dotted line (of length $\bar{A}_0/2$) divides $A(B^+ \rightarrow \bar{D}^0 K^+)$ into two equal segments each of length $\bar{A}_1/2$ and forms an angle $\bar{\delta}_0 - \bar{\delta}_1$ with this amplitude. The rate of $B^0 \rightarrow \bar{D}^0 K^0$ would require tagging the flavor of the initial neutral B to avoid interference with $\bar{B}^0 \rightarrow \bar{D}^0 \bar{K}^0$. (Self-tagged $B \rightarrow \bar{D}K^*$ are advantageous in this respect). A similar study of $\delta_0 - \delta_1$, using the triangle formed by the three amplitudes of Eqs. (8), is inhibited by the difficulty of measuring the amplitude of $B^+ \rightarrow D^0 K^+$, where D^0 is identified by a Cabibbo-allowed decay. This amplitude interferes strongly with $B^+ \rightarrow \bar{D}^0 K^+$, where \bar{D}^0 decays to the same state in a doubly Cabibbo-suppressed manner. With a very large number of B mesons produced in dedicated hadronic B production experiments [16], the magnitude of this amplitude can be determined by observing two different neutral D meson final states [17].

Evidence for a small final state phase difference $\delta_0 - \delta_1$ in $B \rightarrow DK$ can also be obtained by an experimental confirmation of a very small rate for $B^+ \rightarrow D^+ K^0$ given by $|A|^2$. Assuming a characteristic hierarchy of amplitudes $|A| \sim 0.2|C|$ [11, 18], one expects with no rescattering $|A(B^+ \rightarrow D^+ K^0)| \sim 0.2|A(B^+ \rightarrow D^0 K^+)| \sim 0.02|A(B^+ \rightarrow \bar{D}^0 K^+)|$. Consequently, using the measured rate for $B^+ \rightarrow \bar{D}^0 K^+$ [3], one estimates $\mathcal{B}(B^+ \rightarrow D^+ K^0) \sim 10^{-7}$. Such a rate measurement (or an upper limit at this level) is attainable in an upgrade version of CESR [19], PEP-II [20], or KEK-B [21], as long as 300 million $B^+ B^-$ pairs can be produced, and in proposed hadronic experiments [16]. A much larger rate would indicate significant rescattering effects. These effects could occur through much less suppressed intermediate states such as $D_s^+ \pi^0$ and $D_s^+ \eta(\eta')$ [5], the branching ratios of which are expected to be larger than the above by a factor of about $(0.2)^{-4} \approx 600$ according to the same hierarchy.

Assuming that A is small and can be neglected relative to C (i.e. that a branching ratio $\mathcal{B}(B^+ \rightarrow D^+ K^0) \sim 10^{-7}$ or smaller is measured), one can gain knowledge of γ by supplementing information from color-allowed $B^\pm \rightarrow DK^\pm$ with rates of isospin-related neutral B decays [7]. Using the isospin relation Eq. (11) and the approximate equality

(neglecting A)

$$A(B^+ \rightarrow D^0 K^+) \approx A(B^0 \rightarrow D^0 K^0) \quad , \quad (12)$$

one finds

$$A(B^0 \rightarrow D^- K^+) + \sqrt{2}A(B^0 \rightarrow D_1^0 K^0) \approx \sqrt{2}A(B^+ \rightarrow D_1^0 K^+) \quad . \quad (13)$$

A similar triangle relation

$$A(\bar{B}^0 \rightarrow D^+ K^-) + \sqrt{2}A(\bar{B}^0 \rightarrow D_1^0 \bar{K}^0) \approx \sqrt{2}A(B^- \rightarrow D_1^0 K^-) \quad , \quad (14)$$

holds for the charge conjugate amplitudes, obtained from Eqs. (7) and (8) by replacing γ with $-\gamma$. (The triangle (11) is unchanged by charge conjugation). The three triangles (11)(13) and (14), shown in Fig. 1, share a common base $A(B^0 \rightarrow D^- K^+) = A(\bar{B}^0 \rightarrow D^+ K^-)$ and are fixed, up to discrete ambiguities, by seven rate measurements. The angle between the two broken lines connecting the apex of (11) to the apexes of the two other triangles is 2γ . Measurement of the lengths of these two lines, which requires very high statistics to separate Cabibbo-allowed D^0 decays in $B^+ \rightarrow D^0 K^+$ from doubly Cabibbo-suppressed \bar{D}^0 decays in $B^+ \rightarrow \bar{D}^0 K^+$ [17], would provide self-consistency checks.

This method of measuring γ from $B \rightarrow DK$, or preferably $B \rightarrow DK^*$ [7], demonstrates the power of neglecting the annihilation amplitude. It involves a relatively large number of processes, none of which is suppressed by both V_{ub} and color. All the measured rates are governed by $|V_{cb}V_{us}|$. The rates of the three color-suppressed neutral B decays to neutral D and K mesons are expected to be smaller than the other four rates. Using $|\bar{C}/\bar{T}| \sim 0.2$ [11], one estimates $\mathcal{B}(B^0 \rightarrow \bar{D}^0 K^0) \sim 10^{-5}$. The efficiency for observing D^0 CP-eigenstates is somewhat lower than the one for detecting D^0 or \bar{D}^0 , at a level of a few percent [22]. With 300 million $B^0 \bar{B}^0$ pairs, one may expect the precision in measurements of the smaller amplitudes to be at a level of 10%. (We disregard a tagging efficiency, since the same analysis can be applied to self-tagged $B \rightarrow DK^*$). The errors on the other sides of the triangles are smaller. The neglect of the amplitude A relative to C contributes to a larger error, at a level of 20% (assuming a hierarchy $|A/C| \sim 0.2$ [11]), and is the main source for the error in γ . Carrying out the program of Ref. [17] to measure also the smaller dotted lines representing $A(B^+ \rightarrow D^0 K^+)$ and $A(B^- \rightarrow \bar{D}^0 K^-)$ could reduce this error and resolve the discrete ambiguities in γ . We stress again that neutral B decays to neutral D and K mesons must be flavor-tagged. This can be avoided by studying the corresponding decays $B \rightarrow DK^*$, in which the charged K from $K^* \rightarrow K\pi$ tags the flavor of B .

In summary, we studied the implications on final state phases of a negligible annihilation contribution in $B \rightarrow DK$ decays. Contrary to a claim in [4], we showed that this assumption does not lead to any further constraint on γ from color-allowed $B^\pm \rightarrow DK^\pm$ beyond the ones obtained in [1]. On the other hand, an application of this assumption was demonstrated in which γ can be determined from charged and neutral B decays to DK or DK^* states. A test of a sufficiently small annihilation amplitude was proposed in $B^+ \rightarrow D^+ K^0$ or $B^+ \rightarrow D^+ K^{*0}$ requiring branching ratios of about 10^{-7} or smaller. Conversely, an observation of these decays (and their charge conjugates) with considerably larger branching ratios would provide an early warning of nonnegligible rescattering effects.

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References

- [1] M. Gronau, Phys. Rev. D 58 (1998) 037301.
- [2] For previous suggestions to measure γ , involving also color-suppressed $B \rightarrow D^{(*)}K^{(*)}$ decays, see M. Gronau and D. Wyler, Phys. Lett. B 265 (1991) 172; M. Gronau and D. London, Phys. Lett. B 253 (1991) 483; I. Dunietz, Phys. Lett. B 270 (1991) 75; D. Atwood, G. Eilam, M. Gronau and A. Soni, Phys. Lett. B 341 (1995) 372; D. Atwood, I. Dunietz and A. Soni, Phys. Rev. Lett. 78 (1997) 3257; N. Sinha and R. Sinha, Phys. Rev. Lett. 80 (1998) 3706.
- [3] CLEO Collaboration, M. Athanas *et al.*, Phys. Rev. Lett. 80 (1998) 5493.
- [4] Z-z. Xing, hep-ph/9804434, to be published in Phys. Rev. D.
- [5] B. Blok, M. Gronau and J. L. Rosner, Phys. Rev. Lett. 78 (1997) 3999; 79 (1997) 1167.
- [6] For much recent work on rescattering effects in other processes, $B \rightarrow K\pi$ and $B \rightarrow K\bar{K}$, see A. F. Falk, A. L. Kagan, Y. Nir, and A. A. Petrov, Phys. Rev. D 57 (1998) 4290; M. Gronau and J. L. Rosner, Phys. Rev. D 57 (1998) 6843; hep-ph/9806348, to be published in Phys. Rev. D; M. Neubert, Phys. Lett. B 424 (1998) 152; D. Atwood and A. Soni, Phys. Rev. D 58 (1998) 036005; D. Delépine, J. M. Gérard, J. Pestieau and J. Weyers, Phys. Lett. B 429 (1998) 106; A. J. Buras, R. Fleischer and T. Mannel, hep-ph/9711262; R. Fleischer, hep-ph/9802433, hep-ph/9804319.
- [7] This scheme was pointed out to us by Jang and Ko; J-H. Jang and P. Ko, KAIST-14/98.
- [8] M. Gronau, O. Hernández, D. London and J. L. Rosner, Phys. Rev. D 50 (1994) 4529.
- [9] Y. Koide, Phys. Rev. D 40 (1989) 1685.
- [10] N. G. Deshpande and C. O. Dib, Phys. Lett. B 319 (1993) 313.
- [11] M. Gronau, O. Hernández, D. London and J. L. Rosner, Phys. Rev. D 52 (1995) 6356. We use opposite signs for the phase convention of D^0 and D^+ .
- [12] M. Neubert and B. Stech, hep-ph/9705292, to appear in Heavy Flavours (Second Edition), ed. A. J. Buras and M. Lindner (World Scientific).

- [13] P. Drell, presented at the XVIII International Symposium on Lepton Photon Interactions, Hamburg, July 1997, Cornell University Report CLNS 97/1521, hep-ex/9711020.
- [14] T. E. Browder, K. Honscheid and D. Pedrini, *Ann. Rev. Nucl. Part. Sci.* 46 (1997) 395.
- [15] CLEO Collaboration, B. Nemati *et al.*, *Phys. Rev. D* 57 (1998) 5363; H. Nelson, private communication.
- [16] S. Stone, presented at Heavy Flavor Physics: A Probe of Nature's Grand Design, Varenna, Italy, July 1997, hep-ph/97090500.
- [17] Atwood, Dunietz and Soni, Ref. [2].
- [18] A somewhat smaller annihilation amplitude was obtained in two model-dependent calculations, Z.-z Xing, *Phys. Rev. D* 53 (1996) 2847, also discussing some final state interaction effects; D.-S. Du, L.-B. Guo and D.-X. Zhang, *Phys. Lett. B* 406 (1997) 110.
- [19] A. Weinstein, in Proceedings of Beauty '97 - Fifth Workshop on *B* Physics at Hadron Machines, University of California at Los Angeles, October 12–17, *Nucl. Instr. Meth. A* 408 (1998) 47.
- [20] PEP II An Asymmetric *B* Factory, SLAC-PUB-5379, June 1993.
- [21] KEKB *B* Factory Design Report, KEK Report 95-7, August 1995.
- [22] A. Soffer, private communication.

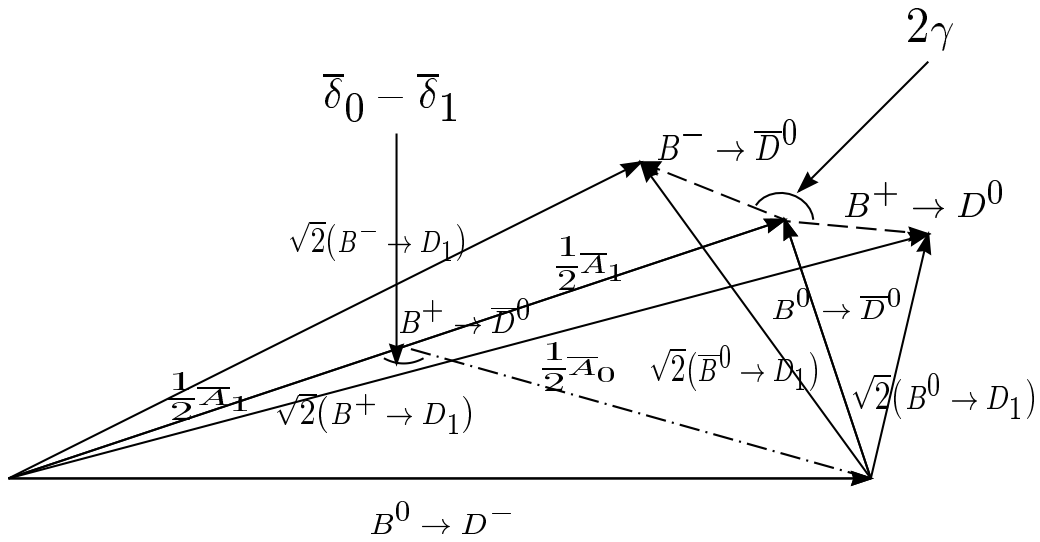


Figure 1: Three triangles representing Eqs. (11)(13) and (14) for $B \rightarrow DK$ amplitudes. Amplitudes are denoted by the flavor of B and D . Dashed-dotted line divides $A(B^+ \rightarrow \bar{D}^0 K^+)$ into two equal segments.