

Higher Fock states and power counting in exclusive charmonium decays

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Abstract

The role of higher Fock-state contributions to exclusive charmonium decays will be discussed. It will be argued that the J/ψ (ψ') decays into baryon-antibaryon pairs are dominated by the valence Fock-state contributions. P -wave charmonium decays, on the other hand, receive strong contributions from the $c\bar{c}g$ Fock states since the valence Fock-state contributions are suppressed in these reactions. Numerical results for the decay widths of $J/\psi(\psi') \rightarrow B\bar{B}$ and $\chi_{cJ} \rightarrow P\bar{P}$ will be also presented and compared to data.

1 Introduction

Exclusive charmonium decays have been investigated within perturbative QCD by many authors, e.g. [1]. It has been argued that the dominant dynamical mechanism is $c\bar{c}$ annihilation into the minimal number of gluons allowed by colour conservation and charge conjugation, and subsequent creation of light quark-antiquark pairs forming the final state hadrons ($c\bar{c} \rightarrow ng^* \rightarrow m(q\bar{q})$). The dominance of annihilation through gluons is most strikingly reflected in the narrow widths of charmonium decays into hadronic channels in a mass region where strong decays typically have widths of hundreds of MeV. Since the c and the \bar{c} quarks only annihilate if their mutual distance is less than about $1/m_c$ (where m_c is the c -quark mass) and since the average virtuality of the gluons is of the order of $1 - 2 \text{ GeV}^2$ one may indeed expect perturbative QCD to be at work although corrections are presumably substantial (m_c is not very large).

In hard exclusive reactions higher Fock-state contributions are usually suppressed by inverse powers of the hard scale, Q , appearing in the process ($Q = 2m_c$ for exclusive charmonium decays), as compared to the valence Fock-state contribution. Hence, higher Fock-state contributions are expected to be negligible in most cases. Charmonium decays are particularly interesting because the valence Fock-state contributions are often suppressed for one or the other reason. In such a case higher Fock-state contributions or other peculiar contributions such as power corrections or small components of the hadronic wave functions may become important. Examples of charmonium decays with suppressed valence Fock-state contributions are the hadronic helicity non-conserving processes like $J/\psi \rightarrow \rho\pi$ or $\eta_c \rightarrow B\bar{B}$, the radiative J/ψ decays into light pseudoscalar mesons and, the main topic of this report, the decays of the P -wave charmonium states, χ_{cJ} .

¹Contribution to the Zeuthen Workshop on Loops and Legs in Gauge Theories, Rheinsberg (1998)

2 Velocity scaling

Recently, the importance of higher Fock states in understanding the production and the *inclusive* decays of charmonium has been pointed out [2]. As has been shown the long-distance matrix elements can there be organized into a hierarchy according to their scaling with v , the typically velocity of the c quark in the charmonium. One may apply the velocity expansion to *exclusive* charmonium decays as well [3]. The Fock-state expansions of the charmonia start as

$$\begin{aligned} |J/\psi\rangle &= O(1) |c\bar{c}_1(^3S_1)\rangle + O(v) |c\bar{c}_8(^3P_J)g\rangle + O(v^2) |c\bar{c}_8(^3S_1)gg\rangle + O(v^3) \\ |\chi_{cJ}\rangle &= O(1) |c\bar{c}_1(^3P_J)\rangle + O(v) |c\bar{c}_8(^3S_1)g\rangle + O(v^2). \end{aligned} \quad (1)$$

In [3] it is argued that the amplitude for a two-body decay of a charmonium state satisfies a factorization formula which separates the scale m_c from the lower momentum scales. The decay amplitude is then expressed as a convolution of a hard scattering amplitude that involves the scale m_c , an initial state factor (charmonium wave function) that involves scales of order $m_c v$ and lower, and a final state factor (light hadron wave function) that involves only the scale Λ_{QCD} . If m_c was asymptotically large, the dominant terms in the factorization formula would involve the minimal number of partons in the hard scattering. Terms involving additional partons in the initial state are suppressed by powers of v while terms involving additional partons in the final state are suppressed by powers of Λ_{QCD}/m_c .

The relative strength of various contributions to specific decay processes can easily be estimated. Combining the fact that a hard scattering amplitude involving a P -wave $c\bar{c}$ pair is of order v , with the Fock-state expansion (1), one finds for the amplitudes of χ_{cJ} decays into, say, a pair of pseudoscalar mesons the behaviour

$$M(\chi_{cJ} \rightarrow P\bar{P}) \sim a\alpha_s^2 v + b\alpha_s^2 (v\sqrt{\alpha_s^{\text{soft}}}) + O(v^2), \quad (2)$$

where a and b are constants and α_s^{soft} comes from the coupling of the Fock-state gluon to the hard process. For the decay reaction $J/\psi \rightarrow B\bar{B}$, on the other hand, one has

$$M(J/\psi \rightarrow B\bar{B}) \sim a\alpha_s^3 + b\alpha_s^3 v (v\sqrt{\alpha_s^{\text{soft}}}) + c\alpha_s^3 v^2 \alpha_s^{\text{soft}} + O(v^3). \quad (3)$$

Thus, one sees that in the case of the χ_{cJ} the colour-octet contributions are not suppressed by powers of either v or $1/m_c$ as compared to the contributions from the valence Fock states [3]. Hence, the colour-octet contributions have to be included for a consistent analysis of P -wave charmonium decays. The situation is different for J/ψ decays into baryon-antibaryon pairs: Higher Fock state contributions first start at $O(v^2)$. Moreover, there is no obvious enhancement of the corresponding hard scattering amplitudes, they appear with at least the same power of α_s as the valence Fock state contributions. Thus, despite of the fact that m_c is not very large and v not small ($v^2 \simeq 0.3$), it seems reasonable to expect small higher Fock-state contributions to the baryonic decays of the J/ψ .

3 The modified perturbative approach

Recently, a modified perturbative approach has been proposed [4] in which transverse degrees of freedom as well as Sudakov suppressions are taken into account in contrast to the standard approach of Brodsky and Lepage [5]. The modified perturbative approach possesses the advantage of strongly suppressed end-point regions. In these regions perturbative QCD cannot be applied. Moreover, the running of α_s and the evolution of the hadronic wave function can be taken into account in this approach properly.

Within the modified perturbative approach an amplitude for a $^{2S+1}L_J$ charmonium decay into two light hadrons, h and h' , is written as a convolution with respect to the usual momentum

fractions, x_i , x'_i and the transverse separation scales, \mathbf{b}_i , \mathbf{b}'_i , canonically conjugated to the transverse momenta, of the light hadrons. Structurally, such an amplitude has the form

$$M^{(c)}(2S+1 L_J \rightarrow hh') = f^{(c)}(2S+1 L_J) \int [dx][dx'] \int \frac{[d^2\mathbf{b}]}{(4\pi)^2} \frac{[d^2\mathbf{b}']}{(4\pi)^2} \times \hat{\Psi}_h^*(x, \mathbf{b}) \hat{\Psi}_{h'}^*(x', \mathbf{b}') \hat{T}_H^{(c)}(x, x', \mathbf{b}, \mathbf{b}', t) \exp[-S(x, x', \mathbf{b}, \mathbf{b}', t)], \quad (4)$$

where $x^{(i)}$, $\mathbf{b}^{(i)}$ stand for sets of momentum fractions and transverse separations characterizing the hadron $h^{(i)}$. Each Fock state provides such an amplitude (marked by the upper index c)². $\hat{\Psi}_h$ denotes the transverse configuration space light-cone wave function of a light hadron. $f^{(c)}$ is the charmonium decay constant. Because the annihilation radius is substantially smaller than the charmonium radius this is, to a reasonable approximation, the only information on the charmonium wave function required. $\hat{T}_H^{(c)}$ is the Fourier transform of the hard scattering amplitude to be calculated from a set of Feynman graphs relevant to the considered process. t represents a set of renormalization scales at which the α_s appearing in $\hat{T}_H^{(c)}$, are to be evaluated. The t_i are chosen as the maximum scale of either the longitudinal momentum or the inverse transverse separation associated with each of the virtual partons. Finally, $\exp[-S]$ represents the Sudakov factor which takes into account gluonic corrections not accounted for in the QCD evolution of the wave functions as well as a renormalization group transformation from the factorization scale μ_F to the renormalization scales. The gliding factorization scale to be used in the evolution of the wave functions is chosen to be $\mu_F = 1/\tilde{b}$ where $\tilde{b} = \max\{b_i\}$. The b space form of the Sudakov factor has been calculated in next-to-leading-log approximation by Botts and Sterman [4].

As mentioned before, exclusive charmonium decays have been analysed several times before, e.g. [1]. New refined analyses are however necessary for several reasons: in previous analyses the standard hard scattering approach has been used with the running of α_s and the evolution of the wave function ignored. In the case of the χ_{cJ} also the colour-octet contributions have been disregarded. Another very important disadvantage of some of the previous analyses is the use of light hadron distribution amplitudes (DAs), representing wave functions integrated over transverse momenta, that are strongly concentrated in the end-point regions. Despite of their frequent use, such DAs were always subject to severe criticism. In the case of the pion, they lead to a clear contradiction to the recent CLEO data [6] on the $\pi\gamma$ transition form factor, $F_{\pi\gamma}$. As detailed analyses unveiled, the $F_{\pi\gamma}$ data require a DA that is close to the asymptotic form $\propto x(1-x)$ [7, 8]. Therefore, these end-point region concentrated DAs should be discarded for the pion and perhaps also for other hadrons like the nucleon [9].

4 Results for $J/\psi (\psi') \rightarrow B\bar{B}$

According to the statements put forward in Sects. 1 and 2, higher Fock-state contributions are neglected in this case.

From the permutation symmetry between the two u quarks and from the requirement that the three quarks have to be coupled in an isospin 1/2 state it follows that there is only one independent scalar wave function in the case of the nucleon if the 3-component of the orbital angular momentum is assumed to be zero. Since $SU(3)_F$ is a good symmetry, only mildly broken by quark mass effects, one may also assume that the other octet baryons are described by one scalar wave function. It is parameterized as

$$\Psi_{123}^{B_s}(x, \mathbf{k}_\perp) = \frac{f_{B_s}}{8\sqrt{3!}} \phi_{123}^{B_s}(x) \Omega_{B_s}(x, \mathbf{k}_\perp) \quad (5)$$

² In higher Fock-state contributions additional integration variables may appear.

channel	$p\bar{p}$	$\Sigma^0\bar{\Sigma}^0$	$\Lambda\bar{\Lambda}$	$\Xi^-\bar{\Xi}^+$	$\Delta\bar{\Delta}$	$\Sigma^*\bar{\Sigma}^*$
Γ_{3g}	174	113	117	62.5	65.1	40.8
$\Gamma_{\text{exp}} [11]$	188 ± 14	110 ± 15	117 ± 14	78 ± 18	96 ± 26	45 ± 6

Table 1: Results for $J/\psi \rightarrow B\bar{B}$ decay widths (in [eV]) taken from [10]. The three-gluon contributions are evaluated with $m_c = 1.5$ GeV, $\Lambda_{\text{QCD}} = 210$ MeV and $a_{B_{10}} = 0.85$ GeV $^{-1}$. The widths for the decuplet baryon channels are given for one charge state.

in the transverse momentum space. The set of indices 123 refers to the quark configuration $u_+ u_- d_+$; the wave functions for other quark configurations are to be obtained from (5) by permutations of the indices. The transverse momentum dependent part Ω is parameterized as a Gaussian in $k_{\perp i}^2/x_i$ ($i = 1, 2, 3$). The transverse size parameter, a_{B_8} , appearing in that Gaussian, as well as the octet-baryon decay constant, f_{B_8} , are assumed to have the same value for all octet baryons. In [9] these two parameters as well as the nucleon DA have been determined from an analysis of the nucleon form factors and valence quark structure functions at large momentum transfer ($a_{B_8} = 0.75$ GeV $^{-1}$; $f_{B_8} = 6.64 \times 10^{-3}$ GeV 2 at a scale of reference $\mu_0 = 1$ GeV). The DA has been found to have the simple form

$$\phi_{123}^N(x, \mu_0) = 60x_1x_2x_3 [1 + 3x_1]. \quad (6)$$

It behaves similar to the asymptotic form, only the position of the maximum is shifted slightly. For the hyperon and decuplet baryon DAs suitable generalizations of the nucleon DA are used. The J/ψ decay constant $f_{J/\psi}$ ($= f^{(1)}({}^3S_1)$) is obtained from the electronic J/ψ decay width; it amounts to 409 MeV. Except in phase space factors, the baryon masses are ignored and $M_{J/\psi}$ is replaced by $2m_c$ for consistency since the binding energy is an $O(v^2)$ effect.

Calculating the hard scattering amplitude from the Feynman graphs for the elementary process $c\bar{c} \rightarrow 3g^* \rightarrow 3(q\bar{q})$ and working out the convolution (4), one obtains the widths for the J/ψ decays into $B\bar{B}$ pairs listed and compared to experimental data in Table 1. As can be seen from that table a rather good agreement with the data is obtained. In addition to the three-gluon contribution there is also an isospin-violating electromagnetic one generated by the subprocess $c\bar{c} \rightarrow \gamma^* \rightarrow 3(q\bar{q})$. According to [10] this contribution seems to be small.

The extension of the perturbative approach to the baryonic decays of the ψ' is now a straightforward matter. One simply has to rescale the J/ψ widths by the ratio of the corresponding electronic widths

$$\Gamma(\psi' \rightarrow B\bar{B}) = \frac{\rho_{\text{p.s.}}(m_B/M_{\psi'})}{\rho_{\text{p.s.}}(m_B/M_{J/\psi})} \frac{\Gamma(\psi' \rightarrow e^+e^-)}{\Gamma(J/\psi \rightarrow e^+e^-)} \Gamma(J/\psi \rightarrow B\bar{B}), \quad (7)$$

where $\rho_{\text{p.s.}}(z) = \sqrt{1 - 4z^2}$ is the phase space factor. In contrast to previous calculations the ψ' and the J/ψ widths do not scale as $(M_{J/\psi}/M_{\psi'})^8$ since the hard scattering amplitude is evaluated with $2m_c$ instead with the bound-state mass. Results for the baryonic decay widths of the ψ' are presented in Table 2. Again good agreement between theory and experiment is found with the exception of the $\Sigma^0\bar{\Sigma}^0$ channel.

5 Results for $\chi_{cJ} \rightarrow P\bar{P}$, $J = 0, 2$

Let me first discuss the color-singlet contribution to the $\pi\pi$ channel since this is an essentially parameter-free contribution. The pion wave function is parameterized analogue to (5) with $\Phi_\pi = \Phi_{\text{AS}} = 6x(1 - x)$. The pion decay constant, f_π , is 131 MeV and the transverse size parameter is given by $a_\pi = 1/(\sqrt{8}\pi f_\pi)$ which automatically satisfies the $\pi_0 \rightarrow \gamma\gamma$ constraint

channel	$p\bar{p}$	$\Sigma^0\bar{\Sigma}^0$	$\Lambda\bar{\Lambda}$	$\Xi^-\bar{\Xi}^+$	$\Delta\bar{\Delta}$	$\Sigma^*\bar{\Sigma}^*$
Γ_{3g}	76.8	55.0	54.6	33.9	32.1	24.4
Γ_{exp} [12]	76 ± 14	26 ± 14	58 ± 12	23 ± 9	25 ± 8	16 ± 8
[11]	53 ± 15					

Table 2: The three-gluon contributions to the $\psi' \rightarrow B\bar{B}$ decay widths (in [eV]) [10]. The widths for the decuplet baryon channels are given for one charge state.

		PDG [11]	BES [12]
$\Gamma[\chi_{c0} \rightarrow \pi^+\pi^-]$ [keV]	45.4	105 ± 47	64 ± 21
$\Gamma[\chi_{c2} \rightarrow \pi^+\pi^-]$ [keV]	3.64	3.8 ± 2.0	3.04 ± 0.73
$\Gamma[\chi_{c0} \rightarrow \pi^0\pi^0]$ [keV]	23.5	43 ± 18	
$\Gamma[\chi_{c2} \rightarrow \pi^0\pi^0]$ [keV]	1.93	2.2 ± 0.6	

Table 3: Results for the χ_{cJ} decay widths into pions ($f_{\chi_c}^{(8)} = 1.46 \times 10^{-3} \text{ GeV}^2$) in comparison with experimental data. The table is taken from [3].

derived in [13]. This form of the pion wave function is strongly constrained by the $\pi\gamma$ transition form factor [7, 8]. The information on the χ_{cJ} wave function needed is the singlet decay constant which is related to the derivative of the non-relativistic χ_{cJ} wave function at the origin

$$f^{(1)}(^3P_0) = \frac{f^{(1)}(^3P_2)}{\sqrt{3}} = \frac{-i}{\sqrt{16\pi m_c}} R'_P(0) \quad (8)$$

In [3] $R'_P(0)$ is chosen to be $0.22 \text{ GeV}^{5/2}$ and, as before, $m_c = 1.5 \text{ GeV}$. The hard scattering amplitude is now computed from the Feynman graphs for the elementary process $c\bar{c} \rightarrow 2g^* \rightarrow 2(q\bar{q})$ and convoluted with the hadronic wave functions according to (4). Evaluating the decays width from the singlet amplitudes, one finds, not surprisingly, values which are too small as compared to experiment [11]: $\Gamma(\chi_{c0(2)} \rightarrow \pi^+\pi^-) = 8.22$ (0.41) keV. Of course, the colour-octet contribution (see (2)) is lacking and one has to estimate it as well and to add it coherently to the colour-singlet amplitude.

For the calculation of the colour-octet amplitude an octet wave function is required. This introduces new degrees of freedom and, for a first estimate, it is advisable to simplify as much as possible in order to keep the number of new free parameters as small as possible. It is important to realize that in the $|c\bar{c}_8g\rangle$ Fock state not only the $c\bar{c}$ pair is in a colour-octet state, but also the three particles, c , \bar{c} and g , are in an S -state. Hence orbital angular momenta are not involved and the transverse degrees of freedom can be integrated over. Therefore, one only has to operate with a DA $\Phi_J^{(8)}(z_1, z_2, z_3)$ and an octet decay constant $f_J^{(8)}$ for each J . It seems reasonable in the spirit of the non-relativistic expansion to approximate the $c\bar{c}g$ DA by $\delta(z_3 - z)$ where $z \simeq \epsilon/(2m_c) \simeq v^2/2$. ϵ is the binding energy of the charmonium and z_3 refers to the momentum fraction the gluon carries. Analogously to the colour-singlet case, the only information on the colour-octet wave function that then enters the final results is the octet decay constant $f_{\chi_c}^{(8)}$ ($= f_0^{(8)} = f_2^{(8)}$). A fit of the unknown decay constant $f_{\chi_c}^{(8)}$ to the experimental data on the decay widths provides the value $f_{\chi_c}^{(8)} = 1.46 \times 10^{-3} \text{ GeV}^2$ which appears to be reasonably large [3]. The resulting decay widths are presented in Table 3. With regard to the large experimental uncertainties fair agreement with experiment is obtained. This analysis can be easily extended to other PP channels, see [3].

6 Conclusions

Exclusive charmonium decays constitute an interesting laboratory for investigating power corrections and higher Fock-state contributions. In particular one can show that in the decays of the χ_{cJ} the contributions from the next-higher charmonium Fock state, $c\bar{c}g$, are not suppressed by powers of m_c or v as compared to the $c\bar{c}$ Fock state and therefore have to be included for a consistent analysis of these decays. For J/ψ (ψ') decays into $B\bar{B}$ pairs the situation is different: Higher Fock-state contributions are suppressed by powers of $1/m_c$ and v . Indeed, as an explicit analysis reveals, with plausible baryon wave functions a reasonable description of the baryonic J/ψ (ψ') decay widths can be obtained alone from the $c\bar{c}$ Fock state while for χ_{cJ} decays into pairs of pseudoscalar mesons agreement with experiment can only be obtained when the color-octet contribution is taken into account.

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