

# Bound $q\bar{q}$ Systems in the Framework of the Different Versions of the 3-Dimensional Reductions of the Bethe-Salpeter Equation

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## Abstract

Bound  $q\bar{q}$ -systems are studied in the framework of different 3-dimensional relativistic equations derived from the Bethe-Salpeter equation with the instantaneous kernel in the momentum space. Except the Salpeter equation, all these equations have a correct one-body limit when one of the constituent quark masses tends to infinity. The spin structure of the confining  $qq$  interaction potential is taken in the form  $x\gamma_1^0\gamma_2^0 + (1-x)I_1I_2$ , with  $0 \leq x \leq 1$ . At first stage, the one-gluon-exchange potential is neglected and the confining potential is taken in the oscillator form. For the systems  $(u\bar{s})$ ,  $(c\bar{u})$ ,  $(c\bar{s})$  and  $(u\bar{u})$ ,  $(s\bar{s})$  a comparative qualitative analysis of these equations is carried out for different values of the mixing parameter  $x$  and the confining potential strength parameter. We investigate: 1) the existence/nonexistence of stable solutions of these equations; 2) the parameter dependence of the general structure of the meson mass spectrum and leptonic decay constants of pseudoscalar and vector mesons. It is demonstrated that none of the 3-dimensional equations considered in the present paper does simultaneously describe even general qualitative features of the whole mass spectrum of  $q\bar{q}$  systems. At the same time, these versions give an acceptable description of the meson leptonic decay characteristics.

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## I. INTRODUCTION

The Bethe-Salpeter (BS) equation provides a natural basis for the relativistic treatment of bound  $q\bar{q}$  systems in the framework of the constituent quark model. However, due to the lack of the probability interpretation of the 4-dimensional (4D) BS amplitude as well as due to serious mathematical diseases which are inherent in the BS approach to the bound-state problem, various 3-dimensional (3D) reduction schemes of the original BS equation are usually used. As it is well known, the simplest version of this sort of reduction immediately arises if the kernel of the BS equation is taken in the instantaneous (static) approximation. As a result, the Salpeter equation is obtained. The Salpeter equation was used for the description of the bound  $q\bar{q}$  system without further approximation in Refs. [1–12], whereas some additional approximations were made in Refs. [13,14]. However, as it is well known, the Salpeter equation itself is not free from some drawbacks. Namely, it does not have a correct one-body limit (the Dirac equation) when the mass of one of the particles tends to infinity. From the general viewpoint, this property is expected to be important for the  $q\bar{q}$  system with one heavy and one light quark. In order to avoid the above difficulty, in Refs. [15,16] the effective noninteracting 3D Green function for two fermions was chosen in the form that guarantees the correct one-body limit of 3D relativistic equations with the static BS kernel. These versions of the 3D equations will be referred hereafter as to the MW and CJ versions, respectively. A new version of the effective free propagator for two scalar particles which also possesses this property was suggested in Ref. [17]. The effective 3D Green function for two noninteracting fermions can be constructed from this propagator in a standard way.

Taking into account the fact that the relativistic effects are important for  $q\bar{q}$  systems with quarks from light and light-heavy sectors, it seems interesting to carry out the investigation of this sort of systems in the framework of the above-mentioned different versions of 3D relativistic equations. This will allow one to shed light on the problem of ambiguity coming from the choice of a particular 3D reduction scheme of the BS equation, and to find the characteristics of the bound  $q\bar{q}$  systems, which are more sensitive to this choice. For the meson mass spectrum this problem was addressed to in Refs. [18,19] where the MW and CJ versions of 3D relativistic equations together with the Salpeter equation (Sal. version) were considered in the configuration space to (partially) avoid the difficulties related to a highly singular behavior of the linear confining potential in the momentum space at the zero momentum transfer. The version of the 3D reduction of the BS equation suggested in Ref. [17] significantly differs from the MW and CJ versions and can be written down only in the momentum space. Consequently, it seems interesting to study together all versions in the momentum space and to investigate a wider class of characteristics of the bound systems, including the decay characteristics of the mesons, which are sensitive to the behavior of the meson wave functions, and the meson mass spectrum. These problems will form the subject of the present paper.

The layout of the present paper is as follows. In Section II, we present different versions of the 3D bound-state equations in the momentum space, and perform the partial-wave expansion of the obtained equations. The numerical solution of these equations with the oscillator-type potential is considered in Section III. The general structure of the meson mass spectra obtained from the solution of these equations is discussed in detail. In Section IV, the leptonic decay characteristics of the pseudoscalar and vector mesons are calculated

using the wave functions obtained from the solution of these equations.

## II. THE RELATIVISTIC 3D EQUATIONS

The relativistic 3D equations for the wave function of the bound  $q\bar{q}$  systems, corresponding to the instantaneous kernel of the BS equation, i.e. when  $K(P; p, p') \rightarrow K_{st}(\vec{p}, \vec{p}')$ , for all versions considered below in the c.m.f. can be written in a common form

$$\tilde{\Phi}_M(\vec{p}) = \tilde{G}_{0eff}(M, \vec{p}) \int \frac{d^3\vec{p}'}{(2\pi)^3} [iK_{st}(\vec{p}, \vec{p}') \equiv \hat{V}(\vec{p}, \vec{p}')] \tilde{\Phi}_M(\vec{p}') \quad (1)$$

where  $M$  is the mass of the bound system, and the equal-time wave function  $\tilde{\Phi}_M(\vec{p})$  is related to the BS amplitude  $\Phi_P(p)$  as

$$\tilde{\Phi}_M(\vec{p}) = \int \frac{dp_0}{2\pi} \Phi_{P=(M, \vec{0})}(p) \quad (2)$$

The effective 3D Green function of two noninteracting-quark system  $\tilde{G}_{0eff}$  is defined as

$$\tilde{G}_{0eff}(M, \vec{p}) = \int \frac{dp_0}{2\pi i} [G_{0eff}(M; p) = g_{0eff}(M; p)(\not{p}_1 + m_1)(\not{p}_2 + m_2)] \quad (3)$$

Here  $g_{0eff}(M; p)$  is the effective propagator of two scalar particles. The operator  $\tilde{G}_{0eff}$  is given in the form

$$\tilde{G}_{0eff}(M, \vec{p}) = \sum_{\alpha_1=\pm} \sum_{\alpha_2=\pm} \frac{D^{(\alpha_1\alpha_2)}(M; p)}{d(M; p)} \Lambda_{12}^{(\alpha_1\alpha_2)}(\vec{p}, -\vec{p}) \gamma_1^0 \gamma_2^0, \quad p \equiv |\vec{p}| \quad (4)$$

where the projection operators  $\Lambda_{12}^{(\alpha_1\alpha_2)}$  are defined by

$$\Lambda_{12}^{(\alpha_1\alpha_2)}(\vec{p}_1, \vec{p}_2) = \Lambda_1^{(\alpha_1)}(\vec{p}_1) \otimes \Lambda_2^{(\alpha_2)}(\vec{p}_2), \quad \Lambda_i^{(\alpha_i)}(\vec{p}_i) = \frac{\omega_i + \alpha_i \hat{h}_i(\vec{p}_i)}{2\omega_i} \quad (5)$$

$$\hat{h}_i(\vec{p}_i) = \gamma_i^0(\vec{\gamma}_i \vec{p}_i) + m_i \gamma_i^0, \quad \omega_i = (m_i^2 + \vec{p}_i^2)^{1/2}$$

and the functions  $D^{(\alpha_1\alpha_2)}(M; p)$  and  $d(M; p)$  are given by the expressions (see Ref. [20])

$$D^{(\alpha_1\alpha_2)} = \frac{(-1)^{\alpha_1+\alpha_2}}{\omega_1 + \omega_2 - (\alpha_1 E_1 + \alpha_2 E_2)}, \quad d = 1$$

$$E_1 + E_2 = M, \quad E_1 - E_2 = \frac{m_1^2 - m_2^2}{M} \equiv b_0 \quad (\text{MW version}) \quad (6)$$

$$D^{(\alpha_1\alpha_2)} = (E_1 + \alpha_1 \omega_1)(E_2 + \alpha_2 \omega_2)$$

$$d = 2(\omega_1 + \omega_2)a, \quad a = E_i^2 - \omega_i^2 = [M^2 + b_0^2 - 2(\omega_1^2 + \omega_2^2)]/4 \quad (\text{CJ version}) \quad (7)$$

Note that for the case of CJ version Eq. (7) is obtained from Eqs. (3) and (4) by using the expression for  $g_{0eff}(M; p)$  defined from the dispersion relation which guarantees the

elastic unitarity. The same condition is satisfied by the expression of  $g_{0eff}(M; p)$  suggested in Ref. [17], (see formula (10) from this paper). According to this condition, the particles in the intermediate states are allowed to go off shell proportionally to their mass, so that when one of the particles becomes infinitely massive, it automatically keeps that particle fully on-mass-shell and the equation is reduced to the one-body equation. Using this expression for  $g_{0eff}(M; p)$  in Eq. (3), we derive the expression for  $\tilde{G}_{0eff}$  having the form (4) where

$$D^{(\alpha_1\alpha_2)} = (E_1 + \alpha_1\omega_1)(E_2 + \alpha_2\omega_2) - \frac{R-b}{2y} \left[ \frac{R-b}{2y} + (E_1 + \alpha_1\omega_1) - (E_2 + \alpha_2\omega_2) \right]$$

$$d = 2RB, \quad R = (b^2 - 4y^2a)^{1/2}, \quad B = \frac{R-b}{2y} \left[ \frac{R-b}{2y} + b_0 \right] + a,$$

$$b = M + b_0y, \quad y = \frac{m_1 - m_2}{m_1 + m_2} \quad (8)$$

Hereafter this version will be referred to as the MNK version (our comments on Ref. [17] see in Ref. [20]).

Using the properties of the projection operators  $\Lambda_{12}^{(\alpha_1\alpha_2)}$  and Eqs. (4)-(8), the following system of equations can be derived from Eq. (1)

$$[M - (\alpha_1\omega_1 + \alpha_2\omega_2)]\tilde{\Phi}_M^{(\alpha_1\alpha_2)}(\vec{p}) =$$

$$= A^{(\alpha_1\alpha_2)}(M; p) \Lambda_{12}^{(\alpha_1\alpha_2)}(\vec{p}, -\vec{p}) \int \frac{d^3\vec{p}'}{(2\pi)^3} \gamma_1^0 \gamma_2^0 \hat{V}(\vec{p}, \vec{p}') \sum_{\alpha'_1=\pm} \sum_{\alpha'_2=\pm} \tilde{\Phi}_M^{(\alpha'_1\alpha'_2)}(\vec{p}') \quad (9)$$

where  $\tilde{\Phi}_M^{(\alpha_1\alpha_2)}(\vec{p}) = \Lambda_{12}^{(\alpha_1\alpha_2)}(\vec{p}, -\vec{p}) \tilde{\Phi}_M(\vec{p})$  and the functions  $A^{(\alpha_1\alpha_2)}(M; p)$  are given by

$$A^{(\pm\pm)} = \pm 1, \quad A^{(\pm\mp)} = \frac{M}{\omega_1 + \omega_2} \quad (\text{MW version}) \quad (10)$$

$$A^{(\alpha_1\alpha_2)} = \frac{M + (\alpha_1\omega_1 + \alpha_2\omega_2)}{2(\omega_1 + \omega_2)} \quad (\text{CJ version}) \quad (11)$$

$$A^{(\alpha_1\alpha_2)} = \frac{1}{2RB} \left\{ a[M + (\alpha_1\omega_1 + \alpha_2\omega_2)] - \right.$$

$$\left. - [M - (\alpha_1\omega_1 + \alpha_2\omega_2)] \frac{R-b}{2y} \left[ \frac{R-b}{2y} + (E_1 + \alpha_1\omega_1) - (E_2 + \alpha_2\omega_2) \right] \right\} \quad (\text{MNK version}) \quad (12)$$

As to the Salpeter equation, it can be obtained from the MW version by putting  $A^{(\pm\mp)} = 0$  and  $\tilde{\Phi}_M^{(\pm\mp)} = 0$ .

Note that for quarks with the equal masses ( $m_1 = m_2 = m$ ) and  $\omega = (m^2 + \vec{p}^2)^{1/2}$ , from Eqs. (10) and (11) we have

$$A^{(\pm\pm)} = \pm 1, \quad A^{(\pm\mp)} = \frac{M}{2\omega} \quad (\text{MW version}) \quad (13)$$

$$A^{(\pm\pm)} = \frac{M + 2\omega}{4\omega}, \quad A^{(\pm\mp)} = \frac{M}{4\omega} \quad (\text{CJ version}) \quad (14)$$

One observes from Eqs. (13) and (14) that in the limit of the equal-mass quarks the bound-state mass  $M$  enters multiplicatively in the coefficients in front of the mixed-energy components  $\tilde{\Phi}_M^{(\pm\mp)}(\vec{p})$  both in the l.h.s. and r.h.s. of Eq. (9). Consequently, dividing both sides of the equations for these components by  $M$ , one arrives at the (nondynamical) constraints on all components of the wave function which must be considered together with the remaining two dynamical equations for the components  $\tilde{\Phi}_M^{(\pm\pm)}(\vec{p})$ . These equations for the bound state mass  $M$  are linear in the MW version and are nonlinear in the CJ version due to the fact that the r.h.s. of the equations depends on the value of  $M$  one is looking for. In the MNK version with an account of the property of the function  $R$  given by Eq. (8)  $\lim_{m_1 \rightarrow m_2} R/y = \lim_{m_1 \rightarrow m_2} M/y$ , from Eq. (12) we have

$$A^{(\pm\pm)} = \frac{M + 2\omega}{2\omega} \quad A^{(\pm\mp)} = \frac{1}{2} \quad (\text{MNK version}) \quad (15)$$

In this case, from Eq. (9) it follows that characteristic features of the bound-state equations remain unchanged – again it is the system of nonlinear equations in  $M$  and it includes all components of the wave function  $\tilde{\Phi}_M^{(\alpha_1\alpha_2)}(\vec{p})$ , as it is in the case for the quarks with nonequal masses.

Further, we write the unknown function  $\tilde{\Phi}_M^{(\alpha'_1\alpha'_2)}$  in Eq. (9) in the form analogous to that used in Ref. [9], where the bound  $q\bar{q}$  systems were studied in the framework of the Salpeter equation

$$\tilde{\Phi}_M^{(\alpha_1\alpha_2)}(\vec{p}) = N_{12}^{(\alpha_1\alpha_2)}(p) \left( \frac{1}{\alpha_1(\vec{\sigma}_1\vec{p})/(\omega_1 + \alpha_1 m_1)} \right) \otimes \left( \frac{1}{-\alpha_2(\vec{\sigma}_2\vec{p})/(\omega_2 + \alpha_2 m_2)} \right) \chi_M^{(\alpha_1\alpha_2)}(\vec{p}) \quad (16)$$

where

$$N_{12}^{(\alpha_1\alpha_2)}(p) = \left( \frac{\omega_1 + \alpha_1 m_1}{2\omega_1} \right)^{1/2} \left( \frac{\omega_2 + \alpha_2 m_2}{2\omega_2} \right)^{1/2} \equiv N_1^{(\alpha_1)}(p) N_2^{(\alpha_2)}(p) \quad (17)$$

Then, if the  $qq$  interaction potential operator  $\hat{V}(\vec{p}, \vec{p}')$  is taken in the form [9]

$$\hat{V}(x; \vec{p}, \vec{p}') = \gamma_1^0 \gamma_2^0 \hat{V}_{og}(\vec{p} - \vec{p}') + [x\gamma_1^0 \gamma_2^0 + (1-x)I_1 I_2] \hat{V}_c(\vec{p} - \vec{p}'), \quad (0 \leq x \leq 1), \quad (18)$$

the following system of equations for the Pauli  $2 \otimes 2$  wave functions  $\chi_M^{(\alpha_1\alpha_2)}$  can be derived

$$\begin{aligned} & [M - (\alpha_1\omega_1 + \alpha_2\omega_2)] \chi_M^{(\alpha_1\alpha_2)}(\vec{p}) = \\ & = A^{(\alpha_1\alpha_2)}(M; p) \sum_{\alpha'_1=\pm} \sum_{\alpha'_2=\pm} \int \frac{d^3\vec{p}'}{(2\pi)^3} \hat{V}_{eff}^{(\alpha_1\alpha_2, \alpha'_1\alpha'_2)}(\vec{p}, \vec{p}', \vec{\sigma}_1, \vec{\sigma}_2) \chi_M^{(\alpha'_1\alpha'_2)}(\vec{p}') \end{aligned} \quad (19)$$

where

$$\begin{aligned} \hat{V}_{eff}^{(\alpha_1\alpha_2, \alpha'_1\alpha'_2)}(\vec{p}, \vec{p}', \vec{\sigma}_1, \vec{\sigma}_2) = & N_{12}^{(\alpha_1\alpha_2)}(p) \left[ V(1; \vec{p} - \vec{p}') \hat{B}_1^{(\alpha_1\alpha_2, \alpha'_1\alpha'_2)}(\vec{p}, \vec{p}', \vec{\sigma}_1, \vec{\sigma}_2) \right. \\ & \left. + V(x; \vec{p} - \vec{p}') \hat{B}_2^{(\alpha_1\alpha_2, \alpha'_1\alpha'_2)}(\vec{p}, \vec{p}', \vec{\sigma}_1, \vec{\sigma}_2) \right] N_{12}^{(\alpha'_1\alpha'_2)}(p') \end{aligned} \quad (20)$$

$$\hat{B}_1^{(\alpha_1\alpha_2\alpha'_1\alpha'_2)} = 1 + \frac{\alpha_1\alpha_2\alpha'_1\alpha'_2(\vec{\sigma}_1\vec{p})(\vec{\sigma}_2\vec{p})(\vec{\sigma}_1\vec{p}')(\vec{\sigma}_2\vec{p}')}{(\omega_1 + \alpha_1m_1)(\omega_2 + \alpha_2m_2)(\omega'_1 + \alpha'_1m_1)(\omega'_2 + \alpha'_2m_2)} \quad (21)$$

$$\hat{B}_2^{(\alpha_1\alpha_2\alpha'_1\alpha'_2)} = \frac{\alpha_1\alpha'_1(\vec{\sigma}_1\vec{p})(\vec{\sigma}_1\vec{p}')}{(\omega_1 + \alpha_1m_1)(\omega'_1 + \alpha'_1m_1)} + \frac{\alpha_2\alpha'_2(\vec{\sigma}_2\vec{p})(\vec{\sigma}_2\vec{p}')}{(\omega_2 + \alpha_2m_2)(\omega'_2 + \alpha'_2m_2)} \quad (22)$$

$$V(x; \vec{p} - \vec{p}') = V_{og}(\vec{p} - \vec{p}') + (2x - 1)V_c(\vec{p} - \vec{p}') \quad (23)$$

Now using the partial-wave expansion

$$\chi_M^{(\alpha_1\alpha_2)}(\vec{p}) = \sum_{LSJM_J} \langle \hat{p} | LSJM_J \rangle R_{LSJ}^{(\alpha_1\alpha_2)}(p), \quad \left( \hat{p} = \frac{\vec{p}}{p} \right) \quad (24)$$

$$V(\vec{p} - \vec{p}') = (2\pi)^3 \sum_{LSJM_J} V^L(p, p') \langle \hat{p} | LSJM_J \rangle \langle LSJM_J | \hat{p}' \rangle \quad (25)$$

where

$$V^L(p, p') = \frac{2}{\pi} \int_0^\infty j_L(pr) j_L(p'r) r^2 dr \quad (26)$$

with  $j_L(x)$  being the spherical Bessel function, the following system of equations can be obtained from Eq. (19) for the radial wave functions  $R_{LSJ}^{(\alpha_1\alpha_2)}(p)$

$$\begin{aligned} & [M - (\alpha_1\omega_1 + \alpha_2\omega_2)] R_{J(\frac{0}{1})J}^{(\alpha_1\alpha_2)}(p) = A^{(\alpha_1\alpha_2)}(M; p) N_{12}^{(\alpha_1\alpha_2)}(p) \times \\ & \times \sum_{\alpha'_1\alpha'_2} \int_0^\infty p'^2 dp' \left\{ \left[ \left( 1 + a_{12\otimes 12}^{(\alpha_1\alpha_2, \alpha'_1\alpha'_2)}(p, p') \right) V^J(1; p, p') + \right. \right. \\ & \left. \left. + a_{\oplus}^{(\alpha_1\alpha_2, \alpha'_1\alpha'_2)}(p, p') V_{\oplus J}^{(0)}(x; p, p') \right] R_{J(\frac{0}{1})J}^{(\alpha'_1\alpha'_2)}(p') - \right. \\ & \left. - \left[ a_{\ominus}^{(\alpha_1\alpha_2, \alpha'_1\alpha'_2)}(p, p') V_{\ominus J}(x; p, p') \right] R_{J(\frac{0}{0})J}^{(\alpha'_1\alpha'_2)}(p') \right\} N_{12}^{(\alpha'_1\alpha'_2)}(p') \\ & [M - (\alpha_1\omega_1 + \alpha_2\omega_2)] R_{J\pm 11J}^{(\alpha_1\alpha_2)}(p) = A^{(\alpha_1\alpha_2)}(M; p) N_{12}^{(\alpha_1\alpha_2)}(p) \times \\ & \times \sum_{\alpha'_1\alpha'_2} \int_0^\infty p'^2 dp' \left\{ \left[ V^{J\pm 1}(1; p, p') + a_{12\otimes 12}^{(\alpha_1\alpha_2, \alpha'_1\alpha'_2)}(p, p') V_{J\pm 11J}(1; p, p') + \right. \right. \\ & \left. \left. + a_{\oplus}^{(\alpha_1\alpha_2, \alpha'_1\alpha'_2)}(p, p') V^J(x; p, p') \right] R_{J\pm 11J}^{(\alpha'_1\alpha'_2)}(p') + \right. \\ & \left. + \left[ a_{12\otimes 12}^{(\alpha_1\alpha_2, \alpha'_1\alpha'_2)}(p, p') \frac{2}{2J+1} V_{\ominus J}(1; p, p') \right] R_{J\mp 11J}^{(\alpha'_1\alpha'_2)}(p') \right\} N_{12}^{(\alpha'_1\alpha'_2)}(p') \end{aligned} \quad (27)$$

where

$$\begin{aligned}
a_{12 \otimes 12}^{(\alpha_1 \alpha_2, \alpha'_1 \alpha'_2)}(p, p') &= a_{12}^{(\alpha_1 \alpha_2)}(p, p) a_{12}^{(\alpha'_1 \alpha'_2)}(p', p') \\
a_{\oplus \ominus}^{(\alpha_1 \alpha_2, \alpha'_1 \alpha'_2)}(p, p') &= a_{11}^{(\alpha_1 \alpha'_1)}(p, p') \pm a_{22}^{(\alpha_2 \alpha'_2)}(p, p') \\
a_{ij}^{(\alpha_i \alpha_j)}(p, p') &= a_i^{\alpha_i}(p) a_j^{\alpha_j}(p'), \quad a_i^{\alpha_i}(p) = \frac{\alpha_i p}{\omega_i + \alpha_i m_i}
\end{aligned} \tag{28}$$

and

$$\begin{aligned}
V_{\oplus J}^{(0)}(x; p, p') &= \frac{1}{2J+1} \left[ \binom{J}{J+1} V^{J-1}(x; p, p') + \binom{J+1}{J} V^{J+1}(x; p, p') \right] \\
V_{\ominus J}(x; p, p') &= \frac{\sqrt{J(J+1)}}{2J+1} [V^{J-1}(x; p, p') - V^{J+1}(x; p, p')] \\
V_{J \pm 11J}(1; p, p') &= \frac{1}{(2J+1)^2} [V^{J \pm 1}(1; p, p') + 4J(J+1)V^{J \mp 1}(1; p, p')] \\
V(x; p, p') &= V_{og}(x; p, p') + (2x-1)V_c(x; p, p')
\end{aligned} \tag{29}$$

The main purpose of the present study is to carry out the comparative qualitative analysis of the different versions of 3D relativistic equations (27), addressing the question of existence of stable solutions for different values of the scalar-vector mixing parameter  $x$  in the confining part of the potential (Eq. (18)). Also, we investigate the general structure of the meson mass spectrum and calculate the leptonic decay characteristics, namely, the pseudoscalar decay constant  $f_P(P \rightarrow \mu \bar{\nu})$  and the vector meson decay width  $\Gamma(V \rightarrow e^- e^+)$ . For these reasons, at the first stage we neglect in (18) the one-gluon exchange potential. A full analysis of the problem will be made in further publications. Further, according to Ref. [21], we use the oscillator form for the confining potential  $V_c(r)$  which is a simplified, but justified form of a more general potential used in Ref. [9] (at least for the quarks from the light and light-heavy sectors, which are considered in the present paper). Namely, we take

$$\begin{aligned}
V_c(r) &= \frac{4}{3} \alpha_s(m_{12}^2) \left( \frac{\mu_{12} \omega_0^2}{2} r^2 - V_0 \right) \\
\mu_{12} &= \frac{m_1 m_2}{m_{12}}, \quad m_{12} = m_1 + m_2, \quad \alpha_s(Q^2) = \frac{12\pi}{33 - 2n_f} \left( \ln \frac{Q^2}{\Lambda^2} \right)^{-1}
\end{aligned} \tag{30}$$

where  $n_f$  denotes the number of flavors. This potential in the momentum space corresponds to the operator (see (26))

$$V_c^L(p, p') = -\frac{4}{3} \alpha_s(m_{12}^2) \left[ \frac{\mu_{12} \omega_0^2}{2} \left( \frac{d^2}{dp'^2} + \frac{2}{p'} \frac{d}{dp'} - \frac{L(L+1)}{p'^2} \right) + V_0 \right] \frac{\delta(p-p')}{pp'} \tag{31}$$

Now the system of equations (27) can be reduced to the system of equations with the following structure:

$$[M - (\alpha_1 \omega_1 + \alpha_2 \omega_2)] R_{J(0)J}^{(\alpha_1 \alpha_2)}(p) = -\frac{4}{3} \alpha_s(m_{12}^2) A^{(\alpha_1 \alpha_2)}(M; p) \sum_{\alpha'_1 \alpha'_2} \left\{ V_0 \left[ B_{\oplus}^{(\alpha_1 \alpha_2, \alpha'_1 \alpha'_2)}(p) + \right. \right.$$

$$\begin{aligned}
& + (2x - 1)A_{\oplus}^{(\alpha_1\alpha_2, \alpha'_1\alpha'_2)}(p) \left] R_{J(\frac{0}{1})J}^{(\alpha'_1\alpha'_2)}(p') + \frac{\mu_{12}\omega_0^2}{2} \left[ \left( \hat{D}_B^{(\alpha_1\alpha_2, \alpha'_1\alpha'_2)}(p) + (2x - 1)\hat{D}_A^{(\alpha_1\alpha_2, \alpha'_1\alpha'_2)}(p) - \right. \right. \\
& - \frac{1}{p^2} \left( B_{\oplus}^{(\alpha_1\alpha_2, \alpha'_1\alpha'_2)}(p)J(J+1) + (2x - 1)A_{\oplus}^{(\alpha_1\alpha_2, \alpha'_1\alpha'_2)}(p) \binom{J(J+1)+2}{J(J+1)} \right) \left. \right) R_{J(\frac{0}{1})J}^{(\alpha'_1\alpha'_2)}(p') - \\
& \left. - (2x - 1) \left( \frac{2\sqrt{J(J+1)}}{p^2} A_{\ominus}^{(\alpha_1\alpha_2, \alpha'_1\alpha'_2)}(p) \right) R_{J(\frac{0}{1})J}^{(\alpha'_1\alpha'_2)}(p) \right] \Big\}, \\
& [M - (\alpha_1\omega_1 + \alpha_2\omega_2)]R_{J_{\pm 11}J}^{(\alpha_1\alpha_2)}(p) = -\frac{4}{3}\alpha_s(m_{12}^2)A^{(\alpha_1\alpha_2)}(M; p) \sum_{\alpha'_1\alpha'_2} \left\{ V_0 \left[ B_{\oplus}^{(\alpha_1\alpha_2, \alpha'_1\alpha'_2)}(p) + \right. \right. \\
& + (2x - 1)A_{\oplus}^{(\alpha_1\alpha_2, \alpha'_1\alpha'_2)}(p) \left. \right] R_{J_{\pm 11}J}^{(\alpha_1\alpha_2)}(p) + \frac{\mu_{12}\omega_0^2}{2} \left[ \left( \hat{D}_B^{(\alpha_1\alpha_2, \alpha'_1\alpha'_2)}(p) + (2x - 1)\hat{D}_A^{(\alpha_1\alpha_2, \alpha'_1\alpha'_2)}(p) - \right. \right. \\
& - \frac{1}{p^2} \left( B_{\oplus}^{(\alpha_1\alpha_2, \alpha'_1\alpha'_2)}(p) \left( J(J+1) + 1 \pm \frac{1}{2J+1} \right) \pm \frac{4J(J+1)}{2J+1} B_{\ominus}^{(\alpha_1\alpha_2, \alpha'_1\alpha'_2)}(p) + \right. \\
& \left. \left. + (2x - 1)A_{\oplus}^{(\alpha_1\alpha_2, \alpha'_1\alpha'_2)}(p)J(J+1) \right) \right] R_{J_{\pm 11}J}^{(\alpha_1\alpha_2)}(p) + \\
& \left. + \frac{1}{p^2} \frac{2\sqrt{J(J+1)}}{(2J+1)^2} \left( B_{\oplus}^{(\alpha_1\alpha_2, \alpha'_1\alpha'_2)}(p) - B_{\ominus}^{(\alpha_1\alpha_2, \alpha'_1\alpha'_2)}(p) \right) R_{J_{\mp 11}J}^{(\alpha_1\alpha_2)}(p) \right\} \quad (32)
\end{aligned}$$

where  $A_{\oplus}^{(\alpha_1\alpha_2, \alpha'_1\alpha'_2)}$  and  $B_{\oplus}^{(\alpha_1\alpha_2, \alpha'_1\alpha'_2)}$  are a given functions of  $p$ , and  $\hat{D}_{A(B)}^{(\alpha_1\alpha_2, \alpha'_1\alpha'_2)}$  are certain second order differential operators in  $p$ .

### III. MESON MASS SPECTRUM

In order to solve the system of equations (32) for the bound state mass  $M$ , the unknown radial wave functions  $R_{LSJ}^{(\alpha_1\alpha_2)}(p)$  are expanded in the basis of the radial wave functions  $R_{nL}(p)$  being the solutions of the Shrödinger radial equation with the oscillator potential in the momentum space (31), as it was done in Refs. [9,21]

$$R_{LSJ}^{(\alpha_1\alpha_2)}(p) = (2M(2\pi)^3)^{1/2} \sum_{n=0}^{\infty} C_{LSJn}^{(\alpha_1\alpha_2)} R_{nL}(p) \quad (33)$$

where the multiplier  $(2M(2\pi)^3)^{1/2}$  is introduced for appropriate normalization of the wave function and

$$\begin{aligned}
R_{nL}(p) &= p_0^{-\frac{3}{2}} \left[ \left( \frac{2\Gamma(n+1+\frac{3}{2})}{\Gamma(n+1)} \right)^{1/2} \frac{z^L \exp(-\frac{z^2}{2})}{\Gamma(L+\frac{3}{2})} {}_1F_1(-n, L+\frac{3}{2}; z^2) \equiv \bar{R}_{nL}(z) \right] \\
z &= \frac{p}{p_0}, \quad p_0 = \left( \mu_{12}\omega_0 \left( \frac{4}{3}\alpha_s(m_{12}^2) \right)^{1/2} \right)^{1/2} \quad (34)
\end{aligned}$$

Substituting the expression (33) into the system of differential equations (32), the following system of algebraic equations for the coefficients  $C_{LSJn}^{\alpha_1\alpha_2}$  can be obtained



$$MC_{LSJn}^{(\alpha_1\alpha_2)} = \sum_{\alpha'_1\alpha'_2} \sum_{L'S'n'} H_{LSJn,L'S'n';J}^{(\alpha_1\alpha_2,\alpha'_1\alpha'_2)}(M) C_{L'S'Jn'}^{(\alpha'_1\alpha'_2)} \quad (35)$$

It is necessary to note here that the matrix  $H_{\alpha\beta}$  explicitly depends (except the Sal. version) on the meson mass  $M$  we are looking for. Consequently, the system of equations (35) is nonlinear in  $M$ . Note also that for the quarks with equal masses ( $m_1 = m_2 = m$ ) part of the equations from the system (35), corresponding to  $(\alpha_1\alpha_2) = (\pm\mp)$  for the MW and CJ versions, transforms into the nondynamical constraints between all coefficients  $C_{LSJn}^{(\alpha_1\alpha_2)}$  which should be considered together with the remaining dynamical equations corresponding to  $(\alpha_1\alpha_2) = (\pm\pm)$ .

The numerical algorithm for the solution of the system of nonlinear equations (35) in the case of nonequal mass quarks was discussed in Ref. [20] where the systems  $u\bar{s}$  ( ${}^3S_1, {}^1P_1, {}^3P_0, {}^3P_1, {}^3P_2, {}^1D_2, {}^3D_1, {}^3D_3$ ),  $c\bar{u}$  and  $c\bar{s}$  ( ${}^1S_0, {}^3S_1, {}^1P_1, {}^3P_2$ ) were considered. In brief, the infinite set of equations (35) is truncated at some fixed value  $n = N_{max}$  and the eigenvalue problem is solved for the finite-range matrix  $H$ . Increasing then  $N_{max}$ , one checks the stability of the resulting spectrum with respect to the variation of  $N_{max}$ . Since the r.h.s. of Eq. (35) depends on  $M$ , the solutions are obtained iteratively, starting from some value of  $M$ . In Ref. [20], the existence of stable solutions of the system of equations (35) was investigated for different values of the mixing parameter  $x$  in the confining potential (18). It was found that the existence/nonexistence of stable solutions of Eq. (35) critically depended on the value of  $x$ , on the value of confining interaction strength parameter  $\omega_0$  (31), and on the particular state under consideration. This dependence is different for the different versions of 3D reduction of the BS equation. The instability is primarily caused by the presence of the mixed  $(+-, -+)$  energy components of the wave function in the equations for the  $q\bar{q}$  bound system. Namely, for the parameter  $\omega_0=710$  MeV that leads to a reasonable description of the meson mass spectrum in the framework of the Salpeter equation [9], stable solutions for CJ, MNK and Sal. versions simultaneously exist for the values of the parameter  $x$  from the interval  $0.3 \leq x \leq 0.6 - 0.7$ . For the MW version, in order to provide the existence of stable solutions in the same interval of  $x$ ,  $\omega_0$  must be set to a smaller value (450 MeV). However, in this case the values of masses for all states under consideration turn out to be smaller than the experimental ones. Keeping in mind that the calculated values of masses will further decrease after adding the one-gluon interaction potential, we conclude that the MW version seems to poorly describe the meson mass spectrum. For this reason, along with the Sal. version, below we consider only the CJ and MNK versions, both having a similar theoretical foundation: the effective Green function (3) in these versions is constructed from the elastic unitarity condition.

The results of calculations are given in Figs. 1,2,3, from which one can see that the level ordering is similar for all three versions under consideration. Further, at  $x = 0.5$  the states  ${}^3P_0, {}^3P_1, {}^1P_1$  are degenerate and spin-orbit splitting exists only between the degenerate  ${}^3P_0, {}^3P_1$  states and the  ${}^3P_2$  state. For  $x \neq 0.5$  this degeneration is removed and the calculated level ordering agrees with the experiment for the value  $x = 0.3$ , except the  ${}^3P_2$  state. For the D - states ( $u\bar{s}$ ), experimentally there is degeneration between  ${}^1D_2, {}^3D_3$  states. In our calculations we do not have this degeneration, but for the MNK and CJ versions at  $x = 0.3$  the splitting is very small and increases for  $x = 0.5$  and  $x = 0.7$ . Note, however, that only for these values of  $x$  the sequence of the  ${}^3D_1$  state and other two D-states agrees with the

experiment. For the  $q\bar{q}$  states with the quarks from light-heavy sectors ( $c\bar{u}, c\bar{s}$ ) the spin-dependence of the energy levels for all values of  $x$  is much weaker than the experimental one, but at the same time for  $x \geq 0.5$  the level ordering agrees with experimental data.

As to the pseudoscalar  $q\bar{q}$  systems (the  $^1S_0$  state) with the quarks from the light sector ( $u\bar{s}$ ), the calculated masses in the model under consideration are much larger than the experimental ones, as demonstrated in Fig 1. This might serve as an indication of the fact that if the constituent quark model is used for the description of this sort of systems, the chiral symmetry breaking effect should be taken into account, at least in a phenomenological manner, e.g. by the inclusion of the t'Hooft interaction in the kernel of the Salpeter equation (see Ref. [8]). In order to take into account a full content of global QCD symmetries in a systematic way, a coupled set of Dyson-Schwinger and BS equations should be considered (see, e.g. [22]).

Note that the number of terms ( $N_{max}$ ) in the expansion (33), which is necessary to get stable solutions of the system of equations (35), varies with the constituent quark masses, with the value of the mixing parameter  $x$ , with the state under consideration and is different for the CJ, MNK and Sal. versions. Namely, when the quark masses increase,  $N_{max}$  decreases. The convergence of the numerical procedure used in the calculations is better for  $x \leq 0.5$  and worse for  $x > 0.5$ . For all values of  $x$  the convergence is better for the Sal. version than for the CJ and MNK versions.

We have also calculated the mass spectrum of  $q\bar{q}$  systems with the equal quark masses from the light quark sector ( $u\bar{u}, s\bar{s}$ ). This problem was the subject of our primary interest in the present study. The results of calculations are shown in Figs. 4 and 5. Note that for this sort of systems the convergence of numerical algorithm used in the calculations appears to be not so good for the values of the parameters  $\omega_0=710$  MeV and  $x > 0.5$ . The convergence becomes better for smaller values of  $\omega_0$ . Namely, for the ( $u\bar{u}$ ) system for  $\omega_0=710$  MeV and  $x = 0.6$ , stable solutions in the MNK version for the  $^3S_1$  state do not exist. In the CJ version such a situation holds for other states ( $^3P_2, ^3D_1, ^3D_3$ ) as well. For smaller values of the potential strength parameter, e.g.  $\omega_0=550$  MeV, the stable solutions exist for all states (just these results are shown in fig. 4). Further, in this case the sequence of the energy levels corresponding to the  $^3P_J$  states (the spin-orbit splitting) agrees with the experiment at  $x > 0.5$  in all versions, and in the  $^3D_1$  and  $^3D_3$  states the agreement appears to occur at  $x < 0.5$ .

Consequently, on the basis of the above analysis one can conclude that none of the 3D equations with the simple oscillator kernel considered in the present paper, does simultaneously describe even general features of the mass spectrum of all  $q\bar{q}$  systems under study.

#### IV. SOME DECAY CHARACTERISTICS OF MESONS

For the investigation of the meson decay properties, the normalization condition for the wave function  $\tilde{\Phi}_M(\vec{p}) = \sum_{(\alpha_1\alpha_2)} \tilde{\Phi}_M^{(\alpha_1\alpha_2)}(\vec{p})$  is needed. For the Salpeter wave function this condition is well known [23]

$$\int \frac{d^3\vec{p}}{(2\pi)^3} \left[ |\tilde{\Phi}_M^{(++)}(\vec{p})|^2 - |\tilde{\Phi}_M^{(--)}(\vec{p})|^2 \right] = 2M \quad (36)$$

As to the MW, CJ and MNK versions, the normalization condition for the corresponding wave functions can be obtained with the use of the fact that the effective Green operators (4) in Eq. (1) can be inverted. As a result, the corresponding full 3D Green operators  $\tilde{G}_{0eff}$  can be defined analogously to the 4D case

$$\tilde{G}_{eff}^{-1}(M; \vec{p}, \vec{p}') = (2\pi)^3 \delta^{(3)}(\vec{p} - \vec{p}') \tilde{G}_{0eff}^{-1}(M; \vec{p}) - \hat{V}(\vec{p}, \vec{p}') \quad (37)$$

Since  $\hat{V}(\vec{p}, \vec{p}')$  does not depend on  $M$ , the normalization condition reads

$$\int \frac{d^3 \vec{p}}{(2\pi)^3} \left[ \bar{\Phi}_M(\vec{p}) = \tilde{\Phi}_M^+(\vec{p}) \gamma_1^0 \gamma_2^0 \right] \left[ \frac{\partial}{\partial M} \tilde{G}_{0eff}^{-1}(M; \vec{p}) \right] \tilde{\Phi}_M(\vec{p}) = 2M. \quad (38)$$

Using Eqs. (4)-(8), from Eq. (38) one obtains

$$\sum_{\alpha_1 \alpha_2} \int \frac{d^3 \vec{p}}{(2\pi)^3} \tilde{\Phi}_M^{+(\alpha_1 \alpha_2)}(\vec{p}) f_{12}^{(\alpha_1 \alpha_2)}(M; p) \tilde{\Phi}_M^{(\alpha_1 \alpha_2)}(\vec{p}) = 2M \quad (39)$$

where

$$f_{12}^{(\alpha_1 \alpha_2)} = \frac{\alpha_1 E_1 + \alpha_2 E_2}{M} \quad (\text{MW version}) \quad (40)$$

$$f_{12}^{(\alpha_1 \alpha_2)} = \frac{\omega_1 + \omega_2}{M} \frac{\alpha_1 \omega_1 E_2 + \alpha_2 \omega_2 E_1}{(E_1 + \alpha_1 \omega_1)(E_2 + \alpha_2 \omega_2)} \quad (\text{CJ version}) \quad (41)$$

$$\begin{aligned} f_{12}^{(\alpha_1 \alpha_2)} = & \frac{2}{D^{(\alpha_1 \alpha_2)}} \left\{ \left[ \frac{M}{R} B(1 - y^2) + \frac{M^2}{2} - 2 \left( \frac{R - M}{2y} \right)^2 \right] - \right. \\ & - \frac{B}{D^{(\alpha_1 \alpha_2)}} \left[ \left( M + \frac{\alpha_1 \omega_1 + \alpha_2 \omega_2}{2} R - \frac{M^2}{2} - 2 \left( \frac{R - M}{2y} \right)^2 + \right. \right. \\ & \left. \left. + (\alpha_1 \omega_1 - \alpha_2 \omega_2) \left( \frac{R - M}{2y} + \frac{M}{2} y \right) \right] \right\} \quad (\text{MNK version}) \quad (42) \end{aligned}$$

Note that for  $m_1 = m_2 = m$  the normalization condition (39) for the MW version is reduced to (36) which can be written in the form of Eq. (39) where

$$f_{12}^{(\alpha_1 \alpha_2)} = \frac{\alpha_1 + \alpha_2}{2} \quad (\text{Sal. version}) \quad (43)$$

From the normalization condition (39) for the wave function given by Eqs. (16), (17) one obtains the normalization condition for the wave functions  $R_{LSJ}^{(\alpha_1 \alpha_2)}(p)$  with the use of the partial-wave expansion (24)

$$\int_0^\infty \frac{p^2 dp}{(2\pi)^3} \sum_{\alpha_1 \alpha_2} f_{12}^{(\alpha_1 \alpha_2)}(M; p) \left[ R_{LSJ}^{(\alpha_1 \alpha_2)}(p) \right]^2 = 2M \quad (44)$$

The functions  $f_{12}^{(--)}$  (40,41) have second order poles at  $p = p_s$ , where

$$a(p_s) = E_i^2 - \omega_i^2(p_s) = 0, \quad p_s = \frac{1}{2} (M^2 + b_0^2 - 2(m_1^2 + m_2^2))^{1/2} \quad (45)$$

The functions  $f_{12}^{(\pm\mp)}$  turn out to be finite ( $f_{12}^{(++)}(p_s)$  is apparently finite). Consequently, the normalization condition (44) for the CJ and MNK versions involves a singular integral of the type

$$I(x_0) = \int_0^\infty \frac{f(x)dx}{(x-x_0)^2} \quad (46)$$

which taking account of the conditions  $f(0) = 0 = f(\infty)$  valid in our case, can be regularized as

$$\int_0^\infty \frac{f(x)dx}{(x-x_0)^2} = \int_0^{2x_0} \frac{[f'(x) - f'(x_0)]dx}{(x-x_0)} + \int_{2x_0}^\infty \frac{f'(x)dx}{x-x_0} \quad (47)$$

Now we can calculate the pseudoscalar ( $S = L = J = 0$ ) decay constant  $f_P(P \rightarrow \mu\bar{\nu})$  and the leptonic decay width of the vector ( $l = 0, S = J = 1$ ) meson  $\Gamma(V \rightarrow e^-e^+)$  (the corresponding decay constant is denoted by  $f_V$ ). In these calculations, instead of the functions  $\tilde{\Phi}_M^{(\alpha_1\alpha_2)}(\vec{p})$  (10) given as a column with the components  $\tilde{\Phi}_{aa}^{(\alpha_1\alpha_2)}$ ,  $\tilde{\Phi}_{ab}^{(\alpha_1\alpha_2)}$ ,  $\tilde{\Phi}_{ba}^{(\alpha_1\alpha_2)}$ ,  $\tilde{\Phi}_{bb}^{(\alpha_1\alpha_2)}$ , it is convenient to introduce the wave function  $\Psi^{(\alpha_1\alpha_2)}$  written in the form (see Ref. [23])

$$\Psi^{(\alpha_1\alpha_2)} = \begin{pmatrix} \tilde{\Phi}_{aa}^{(\alpha_1\alpha_2)} & \tilde{\Phi}_{ab}^{(\alpha_1\alpha_2)} \\ \tilde{\Phi}_{ba}^{(\alpha_1\alpha_2)} & \tilde{\Phi}_{bb}^{(\alpha_1\alpha_2)} \end{pmatrix} (C = i\gamma^2\gamma^0) = \begin{pmatrix} \tilde{\Phi}_{aa}^{(\alpha_1\alpha_2)}\sigma_y & \tilde{\Phi}_{ab}^{(\alpha_1\alpha_2)}\sigma_y \\ \tilde{\Phi}_{ba}^{(\alpha_1\alpha_2)}\sigma_y & \tilde{\Phi}_{bb}^{(\alpha_1\alpha_2)}\sigma_y \end{pmatrix} \quad (48)$$

where  $C$  is the charge conjugation operator. Then, the decay constants  $f_P$  and  $f_V$  are given by the expressions

$$\delta_{\mu 0} M f_P = \sqrt{3} T r \left[ \Psi_{000}(\vec{r}=0) \gamma^\mu (1 - \gamma_5) \right] \quad (49)$$

$$f_V(\lambda) = \sqrt{3} T r \left[ \Psi_{011\lambda}(\vec{r}=0) \gamma^\mu \right] \varepsilon^\mu(\lambda = 0, \pm 1) \quad (50)$$

Here, the factor  $\sqrt{3}$  stems from the color part of the wave functions,  $\varepsilon^\mu(\lambda)$  is the polarization vector of the meson and

$$\Psi_{LSJM_J}(\vec{r}=0) = \int \frac{d^3\vec{p}}{(2\pi)^3} \left[ \Psi_{LSJM_J}(\vec{p}) = \sum_{\alpha_1\alpha_2} \Psi_{LSJM_J}^{(\alpha_1\alpha_2)}(\vec{p}) \right] \quad (51)$$

Using Eqs. (16), (17), (24), (48) and (51), from Eqs. (49) and (50) one obtains

$$f_P = \frac{\sqrt{24\pi}}{M} \int_0^\infty \frac{p^2 dp}{(2\pi)^3} \sum_{\alpha_1\alpha_2} \left[ N_1^{(\alpha_1)}(p) N_2^{(\alpha_2)}(p) - \alpha_1 \alpha_2 N_1^{(-\alpha_1)}(p) N_2^{(-\alpha_2)}(p) \right] R_{000}^{(\alpha_1\alpha_2)}(p) \quad (52)$$

$$f_V(\lambda) = \left\{ -\sqrt{24\pi} \int_0^\infty \frac{p^2 dp}{(2\pi)^3} \sum_{\alpha_1\alpha_2} \left[ N_1^{(\alpha_1)}(p) N_2^{(\alpha_2)}(p) + \frac{\alpha_1 \alpha_2}{3} N_1^{(-\alpha_1)}(p) N_2^{(-\alpha_2)}(p) \right] R_{011}^{(\alpha_1\alpha_2)}(p) \right\} \delta_{\lambda 0} \quad (53)$$

For a given  $f_V$  (53) the leptonic decay width of the vector meson is given by

$$\Gamma(V \rightarrow e^- e^+) = \frac{\alpha_{eff}^2}{4\pi M^3} \frac{1}{3} \sum_{\lambda=0,\pm} |f_V(\lambda)|^2 \quad (54)$$

where

$$\alpha_{eff}^2 = \frac{1}{137} \left( \frac{1}{2}, \frac{1}{18}, \frac{1}{9} \right)$$

for  $\rho^0, \omega$  and  $\varphi$  mesons, respectively.

Finally, using Eqs. (33), (34), (44), (52), (53) and (54), we obtain

$$f_P = \frac{\sqrt{6} p_0^{\frac{3}{2}}}{\pi \sqrt{M}} \left| \sum_{\alpha_1 \alpha_2} \int_0^\infty z^2 dz \left[ N_1^{(\alpha_1)}(p_0, z) N_2^{(\alpha_2)}(p_0, z) - \alpha_1 \alpha_2 N_1^{(-\alpha_1)}(p_0, z) N_2^{(-\alpha_2)}(p_0, z) \right] \bar{R}_{000}^{(\alpha_1 \alpha_2)}(z) \right| \quad (55)$$

$$\Gamma(V \rightarrow e^- e^+) = \frac{4\alpha_{eff}^2 p_0}{(2\pi)^3 M^2} \left| \sum_{\alpha_1 \alpha_2} \int_0^\infty z^2 dz \left[ N_1^{(\alpha_1)}(p_0, z) N_2^{(\alpha_2)}(p_0, z) + \frac{\alpha_1 \alpha_2}{3} N_1^{(-\alpha_1)}(p_0, z) N_2^{(-\alpha_2)}(p_0, z) \right] \bar{R}_{011}^{(\alpha_1 \alpha_2)}(z) \right|^2 \quad (56)$$

where the functions

$$\bar{R}_{LSJ}^{(\alpha_1 \alpha_2)}(z) = \sum_{n=0}^{\infty} C_{LSJ}(\alpha_1 \alpha_2) \bar{R}_{nL}(z) \quad (57)$$

satisfy the normalization condition

$$\sum_{\alpha_1 \alpha_2} \int_0^\infty z^2 dz f_{12}^{(\alpha_1 \alpha_2)}(M; p_0, z) \left[ \bar{R}_{LSJ}^{(\alpha_1 \alpha_2)}(z) \right]^2 = 1 \quad (58)$$

The results of numerical calculations of the quantities  $f_P$  and  $\Gamma$  defined by Eqs. (55) and (56), are given in Table I. We see from Table I that the calculated values of  $f_P$  in the MNK and CJ versions, as a rule, are smaller than in the Sal. version and this fact is related to the presence of the contributions from the "+-" and "-+" components of the wave function (contribution from the "--" component is negligibly small). Further, the calculated value of  $f_P$  is larger in the CJ version than in the MNK version. The calculated value of the quantity  $\Gamma(V \rightarrow e^- e^+)$  weakly depends on the particular choice of the 3D reduction scheme of the BS equation. With the increase of the mixing parameter  $x$  both the quantities  $f_P$  and  $\Gamma(V \rightarrow e^- e^+)$  slightly increase. The calculated values of  $f_P$  and  $\Gamma(V \rightarrow e^- e^+)$  are smaller than the experimental ones.

On the basis of the analysis of the different versions of the 3D reductions of the bound state BS equation carried out in the present paper, one arrives at the following conclusions:

The existence/nonexistence of stable solutions of the 3D bound-state equations critically depends on the value of the scalar-vector mixing parameter  $x$ . For all 3D versions

(Sal, MNK, CJ) stable solutions coexist for the value of  $x$  from a rather restricted interval  $0.3 \leq x \leq 0.6 - 0.7$ . The level ordering in the mass spectrum is similar for all versions under consideration. However, the sequence of the calculated energy levels agrees with the experiment for some states at  $x < 0.5$  and for other states at  $x > 0.5$ . Consequently, a simultaneous description of even general features of the meson whole mass spectrum turns out not to be possible for a given value of  $x$  from the above-mentioned interval. It is interesting to investigate the dependence of this results on the form of the confining potential. Also, it is interesting to study how it changes when the one-gluon exchange potential is taken into account. This aspect of the problem will be considered at the next step of our investigation. Further, we plan to include the 't Hooft effective interaction in our potential in order to (phenomenologically) take into account the effect of spontaneous breaking of chiral symmetry which is important in the pseudoscalar ( $^1S_0$ ) sector of the constituent model.

The calculated leptonic decay characteristics of mesons are quite insensitive to the particular 3D reduction scheme used and give an acceptable description of experimental data. In future, we also plan to study the validity of this conclusion for a wider class of realistic interquark potentials.

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## TABLE CAPTIONS

### Table I

The pseudoscalar decay constant  $f_P(P \rightarrow \mu\bar{\nu})$  (in MeV) and the leptonic decay width  $\Gamma(V \rightarrow e^-e^+)$  (in KeV) with the allowance for only  $(++)$  and all components of the wave function.

## FIGURE CAPTIONS

### Fig. 1

The mass spectrum (in GeV) of the  $u\bar{s}$  system for the different 3D equations and different values of the scalar-vector mixing parameter  $x$ . The multiplicity of degenerate levels is indicated by the number inside the dash ( $\omega_0 = 710 \text{ MeV}$ ).

### Fig. 2

The same as in Fig. 1 for the  $c\bar{u}$  system.

### Fig. 3

The same as in Fig. 1 for the  $c\bar{s}$  system.

### Fig. 4

The mass spectrum (in GeV) of the  $u\bar{u}$  system for the different 3D equations and different values of the scalar-vector mixing parameter  $x$ . The multiplicity of degenerate levels is indicated by the number inside the dash. The parameter  $\omega_0$  for the  ${}^3S_1$  state in the MNK version and for all states in the CJ version at  $x = 0.6$  is fixed at the value  $550 \text{ MeV}$ , otherwise  $\omega_0 = 710 \text{ MeV}$ .

### Fig. 5

The same as in Fig. 1 for the  $s\bar{s}$  system.



Table I

Decay characteristics		$f_P(P \rightarrow \mu\bar{\nu})$				$\Gamma(V \rightarrow e^-e^+)$			
Meson		$D(cd)$		$D_s(c\bar{s})$		$\varrho(u\bar{u})$		$\varphi(s\bar{s})$	
Versions	$\alpha_1\alpha_2$	$x=0.3$	$x=0.5$	$x=0.3$	$x=0.5$	$x=0.3$	$x=0.5$	$x=0.3$	$x=0.5$
Sal	++	149.	156.	177.	183.	3.94	4.31	0.84	0.90
"	all	148.	155.	176.	182.	3.96	4.38	0.84	0.90
MNK	++	148.	155.	176.	181.	4.44	4.85	0.89	0.95
"	all	141.	145.	168.	172.	4.11	4.20	0.85	0.89
CJ	++	150.	158.	179.	186.	3.98	4.45	0.86	0.94
"	all	149.	152.	176.	180.	3.91	4.23	0.85	0.90
Expt.		< 220		170 ÷ 180		6.8 ± 0.3		1.37 ± 0.05	

## FIGURES

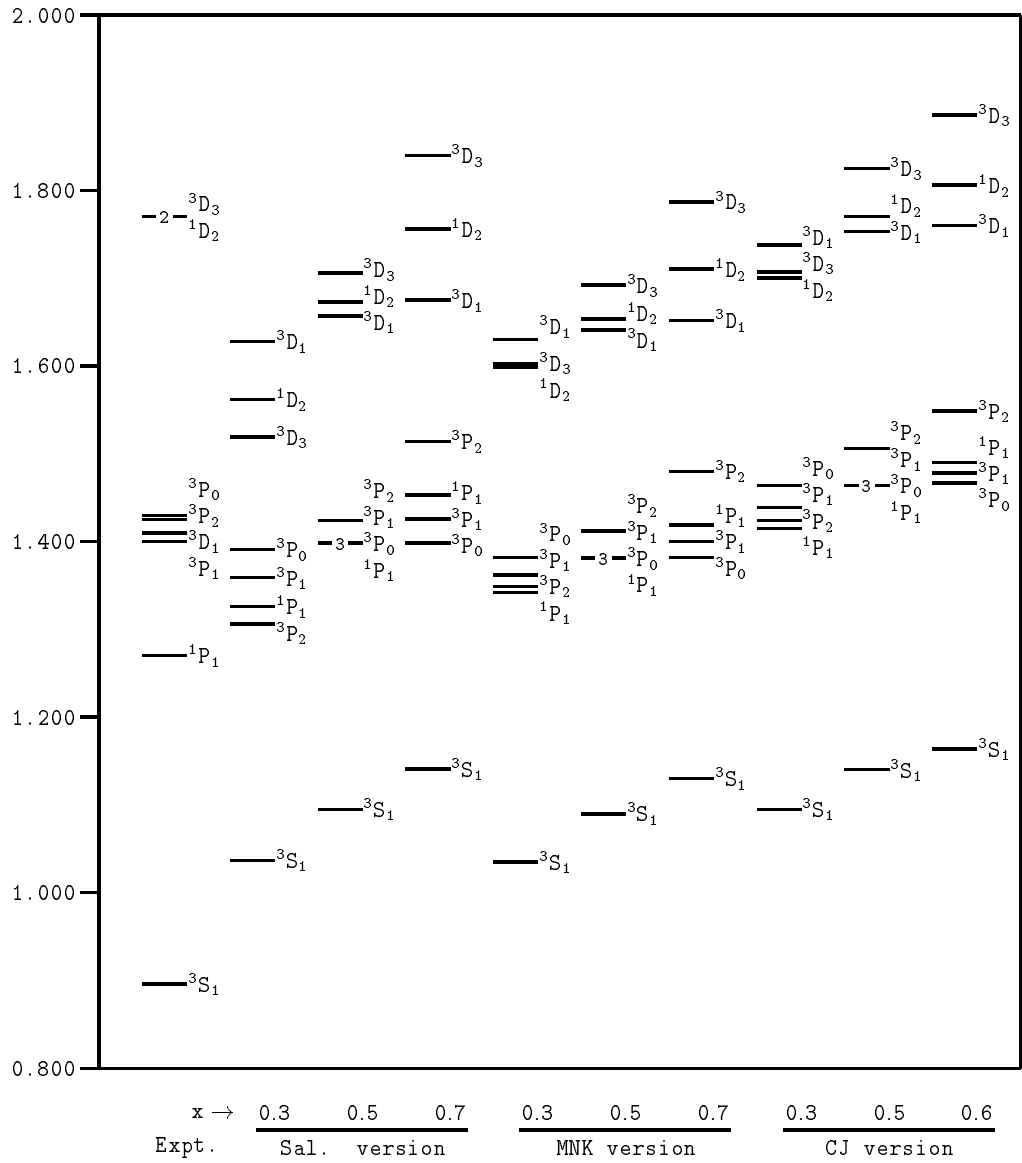


Fig. 1

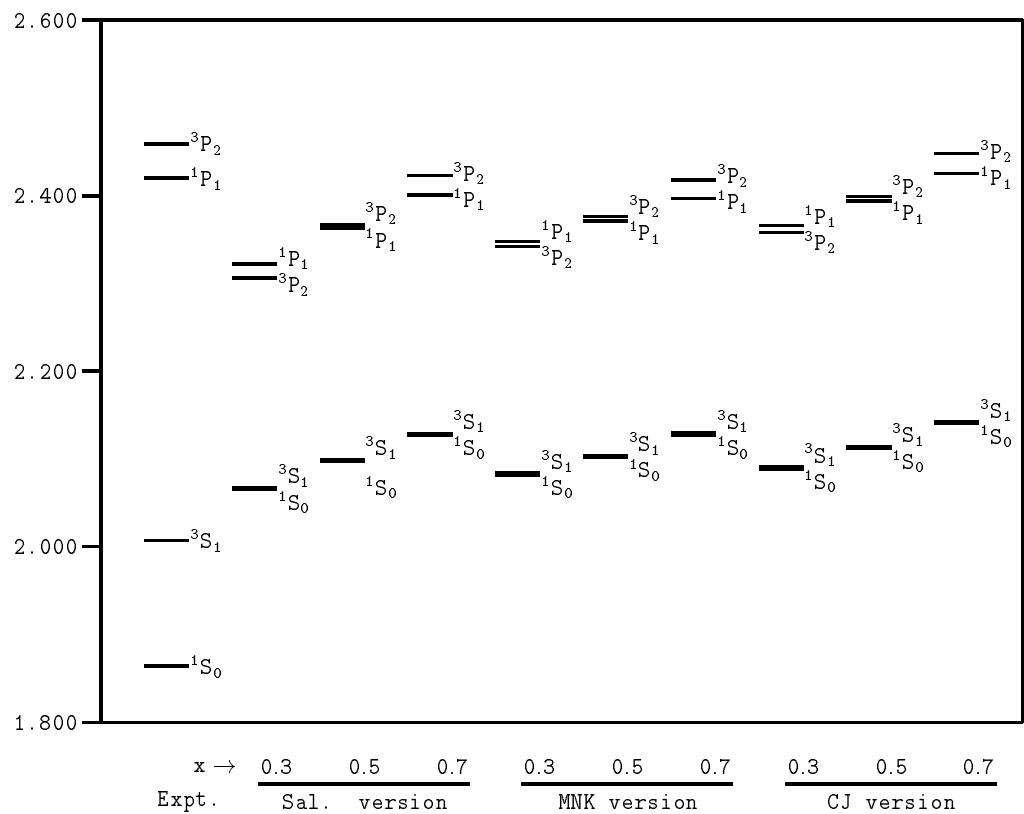


Fig. 2

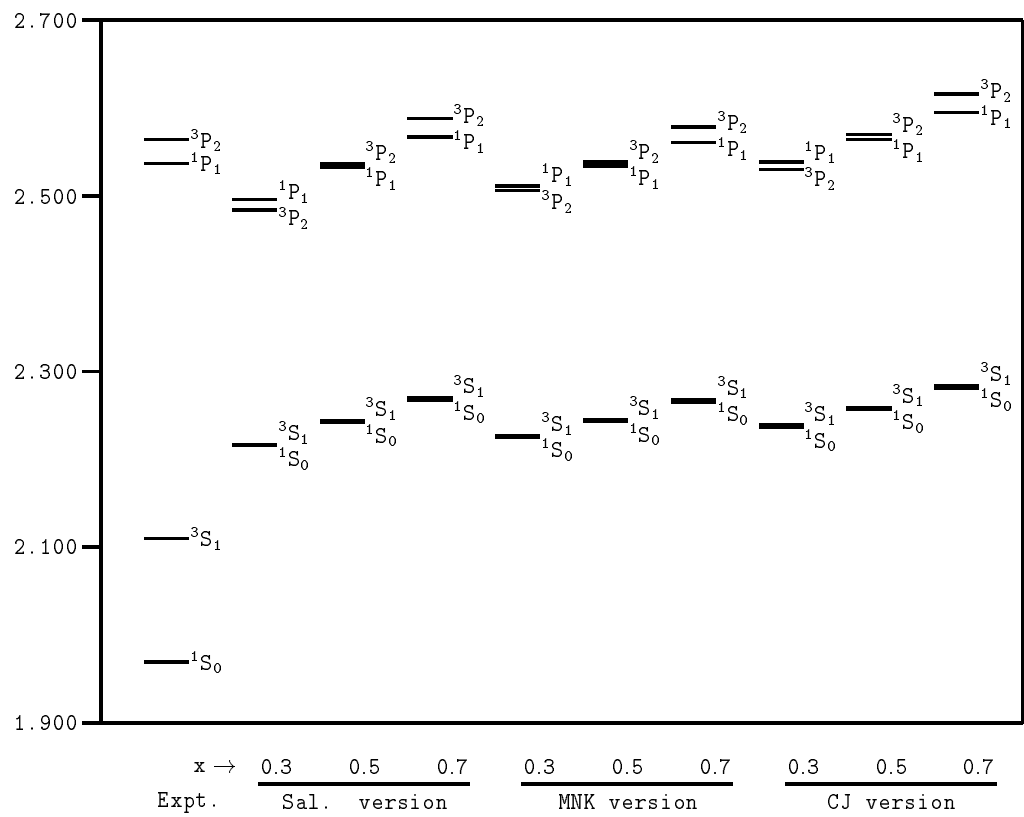


Fig. 3



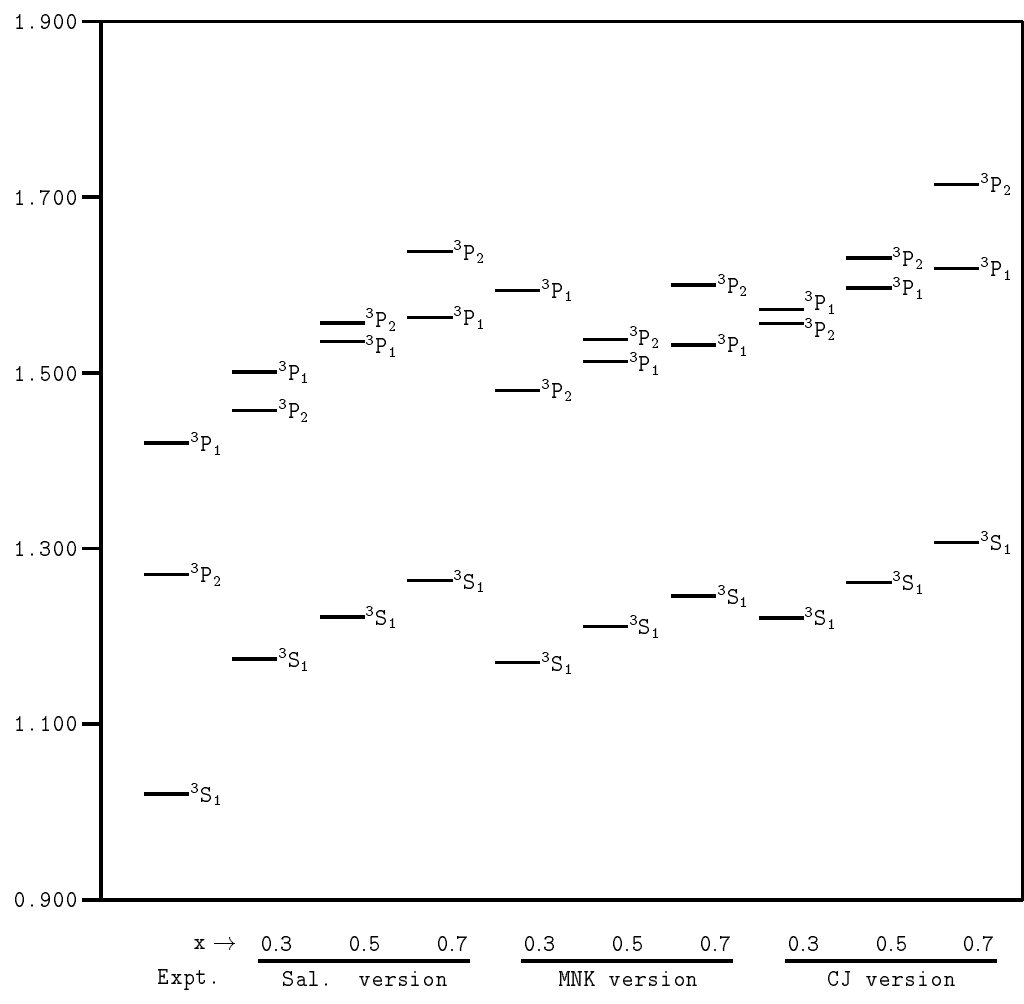


Fig. 5