

Electron Self-Energy in Strong Magnetic Field: Summation of Double Logarithmic Terms

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We study the electron self-energy in a strong magnetic field when the parameter $\eta \equiv (\alpha/2\pi) \ln^2(eB/m_0^2) \sim 1$ and explore the transition between the perturbative regime $\eta \ll 1$ and the nonperturbative massless QED regime $\eta \gg 1$ where chiral symmetry is broken spontaneously and electrons acquire the dynamically induced mass.

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I. INTRODUCTION.

Recently, it has been shown that, in the massless QED, the chiral symmetry of the lagrangian is broken spontaneously when the system is placed in an external magnetic field [1–4]. Spontaneous chiral symmetry breaking is associated with the appearance of the gap in the electron spectrum [2]

$$m_{\text{dyn}}(B) \propto \sqrt{eB} \exp \left\{ -\frac{\pi}{2} \sqrt{\frac{\pi}{2\alpha}} \right\}, \quad (1)$$

the formation of the fermion condensate $\langle \bar{e}e \rangle_B \sim eB m_{\text{dyn}}(B)$ [5], and the appearance of the massless “pion”.

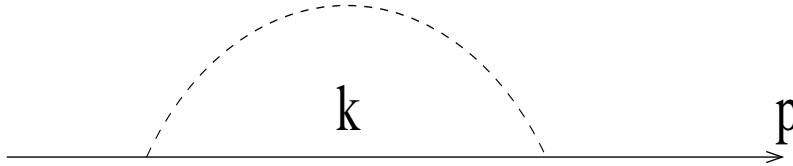


FIG. 1.

On the other hand, in the real World with a finite physical electron mass m_0 and when an external magnetic field is weak enough, we are in the perturbative regime and the shift of the pole of the electron propagator can be found by calculating the one-loop graph in Fig. 1 in the presence of the field. Assuming $eB/m_0^2 \gg 1$, but $(\alpha/2\pi) \log^2(eB/m_0^2) \ll 1$, the result is [6]

$$m(B) = m_0 \left[1 + \frac{\alpha}{4\pi} \log^2 \frac{eB}{m_0^2} + \dots \right] \quad (2)$$

In this paper, we explore the region when the dimensionless parameter $\eta = (\alpha/2\pi) \log^2(eB/m_0^2)$ is of order 1. We perform an accurate summation of all leading double logarithmic contributions $\propto [\alpha \log^2(eB/m_0^2)]^n$ and show that the result of the summation is a nontrivial function which has a singularity at some value of η . This signalizes the breakdown of the perturbation theory and the transition to the nonperturbative regime involving spontaneous breaking of chiral symmetry. We solve then a self-consistent Hartree–Fock equation and find the effective mass dependence in the whole region of the parameter η which interpolates smoothly between perturbative result (2) (and its leading logarithmic version) at small η and the nonperturbative asymptotics (1) at large η .

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II. GROSS – NEVEU MODEL.

Before going over to four dimensions, let us carry out the same program in the two–dimensional Gross–Neveu model [7]. The model is exactly solvable for large number of fermions and its dynamics is very well understood. We will see later that QED_4 in an external magnetic field and the Gross–Neveu model have very much in common and the physics of the transitional region in these two theories is essentially the same.

The lagrangian density of the model reads

$$\mathcal{L} = \sum_i \bar{\psi}_i (i \not{\partial} - m_0) \psi_i + \frac{g^2}{2} \left[\left(\sum_i \bar{\psi}_i \psi_i \right)^2 - \left(\sum_i \bar{\psi}_i \gamma_5 \psi_i \right)^2 \right], \quad (3)$$

where the fermion field carries an additional "color" index $i = 1, \dots, N$. In the absence of bare mass m_0 the lagrangian (3) possesses $U(1)$ chiral symmetry $\psi_i \rightarrow e^{i\theta\gamma_5} \psi_i$. In two dimensions, the coupling g is dimensionless. We will assume it to be small. When the number of colours N is large, the perturbative series involves only the so called daisy or, better to say, "cactus" graphs (see Fig. 2); all other graphs are suppressed as inverse powers of N .

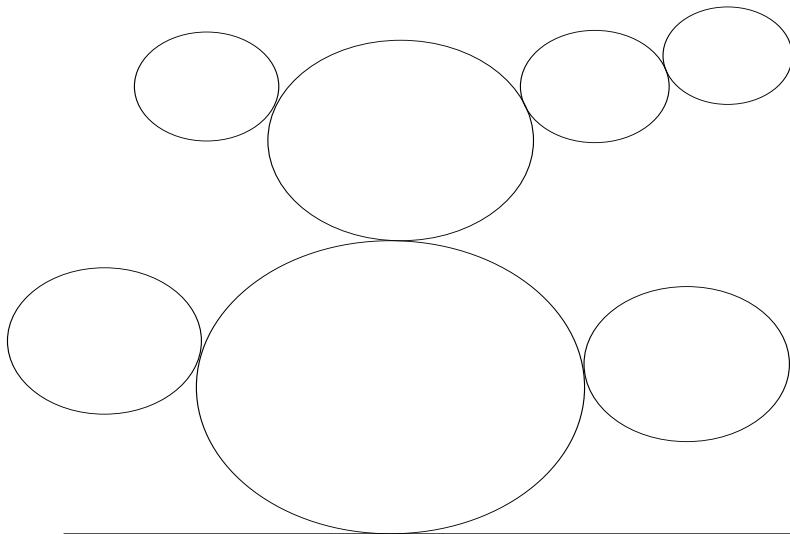


FIG. 2. A cactus graph.

The 1–loop correction to the bare fermion mass m_0 is given by the graph in Fig. 3a. A trivial calculation gives

$$m = m_0 \left[1 + \alpha \log \frac{\Lambda}{m_0} + o(\alpha) \right], \quad (4)$$

where $\alpha = g^2 N / \pi$ and Λ is an ultraviolet cutoff. Next, we can sum up all the leading logarithmic terms $\sim [\alpha \log(\Lambda/m_0)]^n$. They are given by the graphs in Fig. 3b. The result is

$$m^{LL} = \frac{m_0}{1 - \alpha \log \frac{\Lambda}{m_0}}. \quad (5)$$

The leading logarithmic expression blows up at a finite cutoff which signalizes the breakdown of the perturbation theory even in its improved form. In this simple case, however, we can sum up not only the leading logarithmic cactus graphs, but just *all* of them if writing down the self–consistent gap equation

$$m = m_0 + \alpha m \log \frac{\Lambda}{m}, \quad (6)$$

The solution of this equation is a smooth function of Λ . When $\alpha \log(\Lambda/m_0)$ is small, it coincides with the perturbative unimproved, and further, improved results (4), (5). When $\alpha \log(\Lambda/m_0) \gg 1$, we find ourselves in a nonperturbative regime. In this case, one can neglect the bare mass altogether, and the solution of Eq.(6) is just

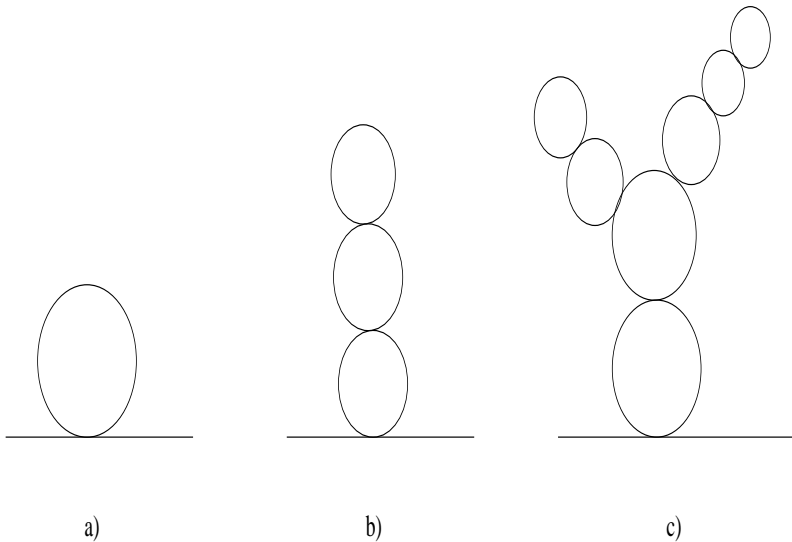


FIG. 3. Graphs describing mass renormalization in the Gross–Neveu model in *a)* one loop, *b)* leading logarithmic approximation, *c)* next-to-leading logarithmic approximation.

$$m = \Lambda e^{-\frac{1}{\alpha}}. \quad (7)$$

A finite gap (7) signalizes the spontaneous breakdown of the chiral symmetry in the massless Gross–Neveu model in the leading order in $1/N$ expansion [7]¹.

It is instructive to rewrite the transcendental equation (6) in a differential form

$$\frac{dy}{d\xi} = \frac{\alpha y^2}{1 + \alpha y}, \quad y(0) = 1, \quad (8)$$

where we introduced $y = m/m_0$ and $\xi = \log \Lambda/m_0$. This is nothing else as the standard renormalization group equation for a mass. If we now expand the right hand side of (8) keeping only the leading in α term, we come at the singular solution (5).

In the next order in α we have the equation which takes care of all the leading and next-to-leading logarithmic terms:

$$y' = \alpha y^2 - \alpha^2 y^3. \quad (9)$$

In the diagram language, it corresponds to the summation of a particular infinite set of cactus graphs with one chain (like in Fig. 3b) or with two or more chains branched out of one and the same link where a logarithm is lost (see Fig. 3c). The solution tends to a "fixed point" $y_* = 1/\alpha$ when $\xi \rightarrow \infty$.

In the order $\sim \alpha^3$, the equation summing leading, next-to-leading, and next-to-next-to-leading logarithms reads

$$y' = \alpha y^2 - \alpha^2 y^3 + \alpha^3 y^4, \quad (10)$$

In the graph language that corresponds to taking into account also the graphs with 2 branchings. The solution of Eq. (10) blows up at a finite ξ like it did in the leading order. In the still next order $\sim \alpha^4$, we obtain again the equation with (the same !) fixed point etc.

The solution to the exact equation (8) does not have any fixed point, neither does it blow up at a finite ξ . We plotted it (for $\alpha = .1$) together with the perturbative approximations [Eq.(5)] and the solution to the equation (9)] and the nonperturbative asymptotics (7) [$y_{as}(\xi) = \exp(\xi - 1/\alpha)$ in our variables] in Fig. 4.

¹ The Mermin-Wagner-Coleman theorem [8] forbids the spontaneous breakdown of continuous symmetries in two dimensions. The fermion dynamical mass m is an order parameter of chiral symmetry breaking only in leading order in $1/N$. In the next-to-leading order approximation, spontaneous chiral symmetry breaking is washed out by interactions of would-be Nambu-Goldstone (NG) bosons. In fact, in this model we have the realization of the Berezinsky–Kosterlitz–Thouless (BKT) phase: while chiral symmetry is not broken in this phase, the fermions acquire dynamical mass m , and the would-be NG boson transforms into a BKT gapless excitation [9].

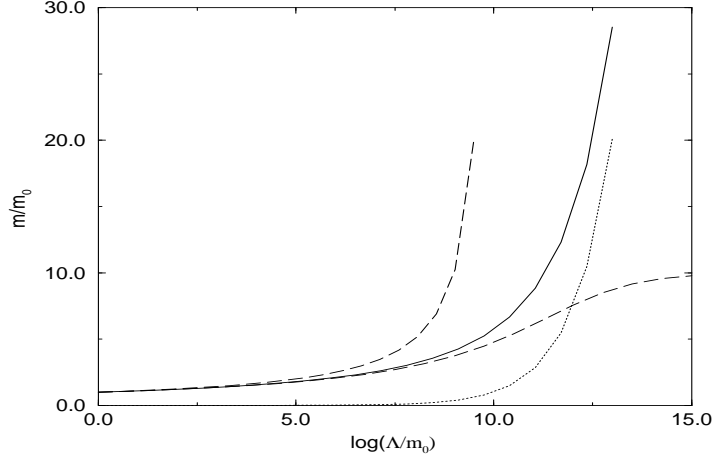


FIG. 4. Fermion mass in the Gross–Neveu model (solid line). The upper and lower dashed lines are the result of summation of, correspondingly, the leading and the next-to-leading perturbative logarithms and the dotted line is the nonperturbative asymptotics.

III. QED IN A MAGNETIC FIELD

To begin with, let us spend some time to explain how the leading logarithmic 1-loop result (2) can be derived. The experience and insights acquired will be useful when handling higher-loop contributions.

To calculate the graph in Fig. 1, we need to know the electron Green function in a magnetic field. The explicit expression has been found by Schwinger [10]. First of all, we can write

$$G_B(x, y) = \exp \left\{ i e \int_y^x A_\mu(\xi) d\xi_\mu \right\} \int \frac{d^4 k}{(2\pi)^4} e^{-ik(x-y)} G_B(k), \quad (11)$$

where the integral in the phase factor is done along the straight line connecting x and y . Generally, the momentum space fermion propagator $G_B(k)$ is given by a complicated integral over the proper time. It can be expanded in terms of Landau levels (the magnetic field is directed along the third axis) [11]

$$G_B(k) = i \exp \left(-\frac{k_\perp^2}{eB} \right) \sum_{n=0}^{\infty} (-1)^n \frac{D_n(eB, k)}{k_\parallel^2 - m_0^2 - 2eBn} \quad (12)$$

with

$$D_n(eB, k) = (\hat{k}_\parallel + m_0) \left[(1 - i\gamma^1\gamma^2) L_n \left(\frac{2k_\perp^2}{eB} \right) - (1 + i\gamma^1\gamma^2) L_{n-1} \left(\frac{2k_\perp^2}{eB} \right) \right] + 4\hat{k}_\perp L_{n-1} \left(\frac{2k_\perp^2}{eB} \right), \quad (13)$$

where L_n^α are the generalized Laguerre polynomials, k_\parallel and k_\perp are longitudinal and transverse momentum components, $\hat{k}_\parallel = k^0\gamma^0 - k^3\gamma^3$, $\hat{k}_\perp = k^1\gamma^1 + k^2\gamma^2$, $k_\parallel^2 = k_0^2 - k_3^2$, $k_\perp^2 = k_1^2 + k_2^2$.

Fortunately, as we will see later, we do not need the full expression (12) for our purposes, but only its asymptotic form in the region of small momenta $k \ll \sqrt{eB}$. In this region, it suffices to retain only the lowest Landau levels (LLL) in the spectral decomposition of the Green function which acquires then a very simple form

$$G_B(k) = i e^{-k_\perp^2/eB} \frac{\hat{k}_\parallel + m_0}{k_\parallel^2 - m_0^2} (1 - i\gamma^1\gamma^2). \quad (14)$$

Basically, this Green's function describes free motion in the longitudinal direction of the states with $\sigma = \mathbf{\Sigma}\mathbf{B}/|\mathbf{B}| = -1$ (the factor $1 - i\gamma^1\gamma^2$ filters out the states with wrong sign of the spin projection σ). The LLL approximation is

justified when both k_{\perp}^2 and k_{\parallel}^2 are much smaller than eB . Thereby, it is more consistent to set the form factor $e^{-k_{\perp}^2/eB}$ (appearing due to a finite size of the Landau orbits) to 1, but we should keep in mind that all the momenta integrals should be cut off at $k_{\perp}^2, k_{\parallel}^2 \lesssim eB$.

Substituting Eqs.(11) and (14) in the Feynman integral, taking some care when handling the phase factors $\propto \exp\{ie \int A_{\mu} d\xi_{\mu}\}$, and performing the Wick rotation, the 1-loop expression for the euclidean mass operator acquires the form ²

$$M_1(p_{\parallel}^2) = \frac{\alpha m_0}{2\pi} \int^{eB} \frac{d^2 k_{\parallel}}{k_{\parallel}^2 + m_0^2} \int^{eB} \frac{dk_{\perp}^2}{k_{\perp}^2 + (\mathbf{k}_{\parallel} - \mathbf{p}_{\parallel})^2}. \quad (15)$$

We assumed here that the external electron momentum p is purely longitudinal. One can show that the dependence of $M(p_{\parallel}^2, p_{\perp}^2)$ on p_{\perp}^2 is weak when $p_{\perp}^2 \ll eB$.

The integral (15) is double logarithmic receiving a main contribution from the region $m_0^2 \ll k_{\parallel}^2 \ll k_{\perp}^2 \ll eB$. That justifies the use of the approximate LLL expression (14). For $k \gtrsim \sqrt{eB}$, the LLL approximation is not valid, but the corresponding contribution in the mass operator would not involve a large logarithm $\log(eB/m_0^2)$. To find the mass shift (2), it suffices to set $p_{\parallel}^2 = m_0^2$ in Eq.(15) and calculate the integral with double logarithmic accuracy. In the following, we will need, however, also the off-shell mass operator with arbitrary p_{\parallel}^2 . An important remark is that, if $p_{\parallel}^2 \gg m_0^2$, the structure of the integrand in Eq. (15) depends on the relation between k_{\parallel} and p_{\parallel} . There are two distinct regions, both of them providing a double logarithmic contribution in $M_1(p_{\parallel})$.

A) $k_{\parallel}^2 \gg p_{\parallel}^2$. In that case, the integral is reduced to

$$M_1^A(p_{\parallel}^2) \approx \frac{\alpha m_0}{2\pi} \int_{p_{\parallel}^2}^{eB} \frac{dk_{\parallel}^2}{k_{\parallel}^2} \int_{k_{\parallel}^2}^{eB} \frac{dk_{\perp}^2}{k_{\perp}^2} = \frac{\alpha m_0}{4\pi} \log^2 \frac{eB}{p_{\parallel}^2} \quad (16)$$

B) $m_0^2 \ll k_{\parallel}^2 \ll p_{\parallel}^2$. We have

$$M_1^B(p_{\parallel}^2) \approx \frac{\alpha m_0}{2\pi} \int_{m_0^2}^{p_{\parallel}^2} \frac{dk_{\parallel}^2}{k_{\parallel}^2} \int_{p_{\parallel}^2}^{eB} \frac{dk_{\perp}^2}{k_{\perp}^2} = \frac{\alpha m_0}{2\pi} \log \frac{p_{\parallel}^2}{m_0^2} \log \frac{eB}{p_{\parallel}^2} \quad (17)$$

The sum of two contributions (16) and (17) is

$$M_1(p_{\parallel}^2) = \frac{\alpha m_0}{2\pi} \left[L l - \frac{1}{2} L^2 \right], \quad L = \log \frac{eB}{p_{\parallel}^2}, \quad l = \log \frac{eB}{m_0^2}. \quad (18)$$

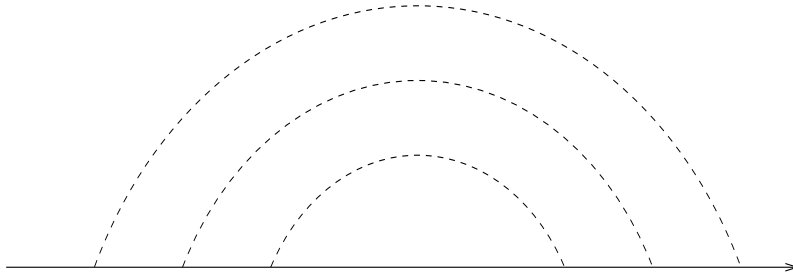


FIG. 5. A rainbow graph.

²We have chosen the Feynman gauge, but the mass defined as the position of the pole of electron propagator at zero momentum should not depend on the gauge and it does not [1,2,12].

Let us consider now higher loop contributions. The leading logarithmic terms come from the rainbow graphs depicted in Fig. 5. The n-th order contribution for the mass operator is obtained from the contribution of the (n-1)-th order by the virtue of the recurrent relations

$$M_n(p_{\parallel}^2) = \frac{\alpha}{2\pi} \int^{eB} M_{n-1}(k_{\parallel}^2) \frac{d^2 k_{\parallel}}{k_{\parallel}^2 + m_0^2} \int^{eB} \frac{dk_{\perp}^2}{k_{\perp}^2 + (\mathbf{k}_{\parallel} - \mathbf{p}_{\parallel})^2}. \quad (19)$$

This is nothing else, of course, as the n-th approximant of the integral equation

$$M(p_{\parallel}^2) = m_0 + \frac{\alpha}{2\pi} \int^{eB} M(k_{\parallel}^2) \frac{d^2 k_{\parallel}}{k_{\parallel}^2 + m_0^2} \int^{eB} \frac{dk_{\perp}^2}{k_{\perp}^2 + (\mathbf{k}_{\parallel} - \mathbf{p}_{\parallel})^2} \quad (20)$$

[$M(p_{\parallel}^2) = m_0 + M_1(p_{\parallel}^2) + \dots$].

Again, the leading logarithmic contributions in Eqs. (19), (20) come from two distinct kinematic regions: $eB \gg k_{\perp}^2 \gg k_{\parallel}^2 \gg p_{\parallel}^2 \gg m_0^2$ and $eB \gg k_{\perp}^2 \gg p_{\parallel}^2 \gg k_{\parallel}^2 \gg m_0^2$. Actually, as far as the leading logarithmic terms are concerned, the equation (20) can be greatly simplified:

$$M(p_{\parallel}^2) = m_0 + \frac{\alpha}{2\pi} \left[\log \frac{eB}{p_{\parallel}^2} \int_{m_0^2}^{p_{\parallel}^2} \frac{dk_{\parallel}^2}{k_{\parallel}^2} M(k_{\parallel}^2) + \int_{p_{\parallel}^2}^{eB} \frac{dk_{\parallel}^2}{k_{\parallel}^2} \log \frac{eB}{k_{\parallel}^2} M(k_{\parallel}^2) \right]. \quad (21)$$

This equation is easy to solve. One way to do it is to find the subsequent approximants

$$\begin{aligned} M_2(p_{\parallel}^2) &= \left(\frac{\alpha}{2\pi}\right)^2 m_0 \left[\frac{1}{24} L^4 - \frac{1}{6} L^3 l + \frac{1}{3} L l^3 \right], \\ M_3(p_{\parallel}^2) &= \left(\frac{\alpha}{2\pi}\right)^3 m_0 \left[-\frac{1}{720} L^6 + \frac{1}{120} L^5 l - \frac{1}{18} L^3 l^3 + \frac{2}{15} L l^5 \right], \dots \end{aligned} \quad (22)$$

and to sum them over. The shift of the electron pole mass [it is defined as the solution of the dispersive equation $m = M(m^2)$, but to the leading logarithmic order, we may just set $m = M(m_0^2) = M(L = l)$] is then given by the series

$$m = m_0 \left[1 + \frac{\alpha}{4\pi} \log^2 \frac{eB}{m_0^2} + \frac{5}{24} \left(\frac{\alpha}{2\pi} \log^2 \frac{eB}{m_0^2} \right)^2 + \frac{61}{720} \left(\frac{\alpha}{2\pi} \log^2 \frac{eB}{m_0^2} \right)^3 + \dots \right]. \quad (23)$$

A more nice and easy way to handle Eq.(21) is to notice that it is equivalent to the following second order differential equation:

$$\frac{d}{dp_{\parallel}^2} \left(p_{\parallel}^2 \frac{dM(p_{\parallel}^2)}{dp_{\parallel}^2} \right) + \frac{\alpha}{2\pi} \frac{M(p_{\parallel}^2)}{p_{\parallel}^2} = 0, \quad (24)$$

with the boundary conditions

$$\left. \frac{dM(p_{\parallel}^2)}{dp_{\parallel}^2} \right|_{p_{\parallel}^2=m_0^2} = 0, \quad M(p_{\parallel}^2) \Big|_{p_{\parallel}^2=eB} = m_0. \quad (25)$$

The solution of Eqs.(24),(25) is

$$M(p_{\parallel}^2) = m_0 \frac{\cos \left(\sqrt{\frac{\alpha}{2\pi}} \log \frac{p_{\parallel}^2}{m_0^2} \right)}{\cos \left(\sqrt{\frac{\alpha}{2\pi}} \log \frac{eB}{m_0^2} \right)}. \quad (26)$$

The pole mass of the electron [defined as $m = M(m_0^2)$] is

$$m = \frac{m_0}{\cos \left(\sqrt{\frac{\alpha}{2\pi}} \log \frac{eB}{m_0^2} \right)}. \quad (27)$$

The expansion of these expressions in α reproduces, of course, Eqs. (22) and (23). Note that Eq.(27) is different from an analogous formula obtained in [13] which had the form

$$m = m_0 \exp \left(\frac{\alpha}{4\pi} \log^2 \frac{eB}{m_0^2} \right). \quad (28)$$

The terms $\sim \alpha$ in Eqs.(27) and (28) coincide with each other and with the Jancovici result (2), but the two-loop coefficients are already different: we have obtained 5/24 in the expansion (23) instead of 1/8 in [13]. Also the behaviour of the functions (27) and (28) at large values of the argument is completely different. When m_0 tends to zero, the electron mass Eq.(27), in contrast to Eq.(28), runs into singularity at some finite m_0 signaling the breakdown of perturbation theory [cf. Eq. (5) !].

The experience with the Gross–Neveu model taught us how to handle this problem. We have to write a self-consistent gap equation which sums up effectively all the relevant graphs. To this end, we just have to replace the mass m_0 in the integrand in Eq. (21) and, correspondingly, in the argument of the cosine function in Eq. (27) by the total mass $m(B)$. We then come to the equation ³

$$m \cos \left(\sqrt{\frac{\alpha}{2\pi}} \log \frac{eB}{m^2} \right) = m_0. \quad (29)$$

The solution of this equation is nonsingular in the limit $m_0 \rightarrow 0$ giving a nonzero dynamical mass (1) of massless electron in a magnetic field [2].

Like in the Gross–Neveu case, we can rewrite Eq.(29) in a differential renormalization-group-like form. Introducing the variable $y = m/m_0$ and differentiating over $\xi = \log(eB/m_0^2)$ we obtain

$$y' = \frac{\sqrt{\frac{\alpha}{2\pi}} y \sqrt{y^2 - 1}}{1 + \sqrt{\frac{2\alpha}{\pi}} \sqrt{y^2 - 1}}, \quad y(0) = 1. \quad (30)$$

We can consider the right hand side of Eq.(30) as an analog of β -function for the "charge" m/m_0 . The analysis is exactly parallel to what we have done for the Gross–Neveu model. In the lowest order in $\sqrt{\alpha}$, Eq.(30) is reduced to

$$y' = \sqrt{\frac{\alpha}{2\pi}} y \sqrt{y^2 - 1} \quad (31)$$

with the solution

$$y = \frac{1}{\cos \left(\sqrt{\frac{\alpha}{2\pi}} \log \frac{eB}{m_0^2} \right)}, \quad (32)$$

which coincides with (27) and blows up at some finite m_0 (or B). In the next order in $\sqrt{\alpha}$ we get

$$y' = \sqrt{\frac{\alpha}{2\pi}} y \sqrt{y^2 - 1} \left[1 - \sqrt{\frac{2\alpha}{\pi}} \sqrt{y^2 - 1} \right]. \quad (33)$$

The right hand side of the last equation has a "fixed point" at $y_0 = \sqrt{1 + \pi/2\alpha}$ to which the solution of Eq.(33) tends as $\log eB/m_0^2 \rightarrow \infty$. If keeping the term of order $(\sqrt{\alpha})^3$ in the expansion of right hand side of (30) the "fixed point" disappears while reappearing again in order $(\sqrt{\alpha})^4$. On the other hand, the exact equation (30) does not have any fixed points (other than the zero field point $y = 1$) and its right hand side behaves as $y/2$ at large y what means that the total electron mass behaves as $m \propto \sqrt{eB}$ in a strong magnetic field. The solution to the full Hartree–Fock equation (30), its perturbative approximants, and the nonperturbative asymptotics (1) are plotted in Fig. 6.

³Note that, in contrast to Eq. (6) in the Gross–Neveu model which summed up *all* the perturbative graphs in the large N limit and was just exact, the status of Eq. (29) is somewhat less certain. It is written in the mean field (or Hartree–Fock) approximation neglecting the effects due to momentum dependence of the mass operator in the denominator of the electron propagator. We assume, however, that the mean field analysis is justified here as it is justified in many other physical problems involving gap equations (like standard superconductivity etc). Moreover, a numerical study of the nonlinear gap equation [2] confirms the formula (1).

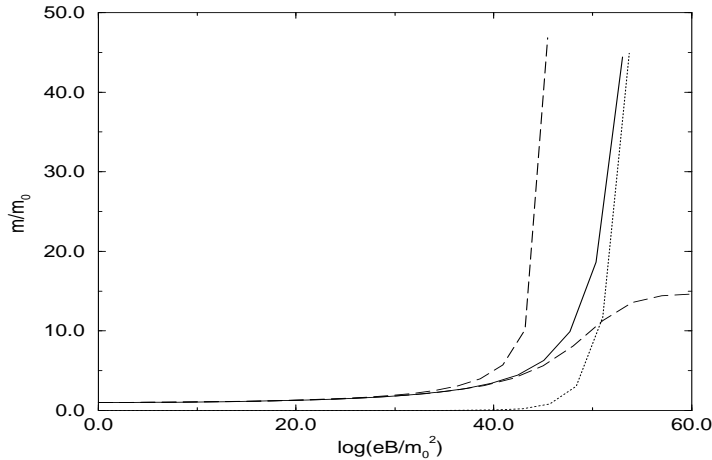


FIG. 6. Electron mass as a function of magnetic field (solid line). The dashed and dotted lines have the same meaning as in Fig. 4.

The main lesson to be learned from our analysis is that, for systems involving spontaneous chiral symmetry breaking, the perturbative methods (even improved by summation of the leading or subleading logarithms) have an intrinsic barrier: one cannot go much further beyond the point $\alpha \log(\Lambda/m_0) \sim 1$ for the Gross–Neveu model or the point $(\alpha/2\pi) \log^2(eB/m_0^2) \sim 1$ for QED_4 in magnetic field: the perturbative approximants either blow up at some small enough value of m_0 or display an unphysical fixed point behaviour. In the region of very large Λ/m_0 for the Gross–Neveu model and eB/m_0^2 for QED , the effects due to explicit breaking of chiral symmetry in the lagrangian become negligible and the correct results can be obtained by solving a self-consistent mean field nonperturbative gap equation.

In the real QED, the value of the parameter $\eta \simeq 1$ is reached for fields of the order $\sim 10^{26}G$. We recall that strong magnetic fields ($B \sim 10^{24}G$) might have been generated during the electroweak phase transition [14]. It has been speculated in [1,2] that the character of electroweak phase transition could be affected by generation of a dynamical electron mass under such strong fields. On the other hand, as is seen from Fig.6, the nonperturbative regime becomes prevailing over the perturbative one for values of η of the order 2.35 what corresponds to magnetic fields $\sim 10^{32}G$. Ambjørn and Olesen [15] have claimed that even larger fields, $\sim 10^{33}G$, would be necessary at early stages of the Universe to explain the observed large-scale galactic magnetic fields.

It was shown in [16] that the critical magnetic field, required for the effect of chiral symmetry breaking to be important, can be substantially decreased by taking into account other interactions, for example Yukawa couplings. It would be interesting to extend the results obtained in present paper to Standard Model.

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