

EXCLUSIVE NONLEPTONIC DECAYS OF HEAVY MESONS IN QCD ^a

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We use operator product expansion and QCD light-cone sum rules to estimate the amplitude of the decay $B \rightarrow J/\psi K$ taking into account leading nonfactorizable contributions. The result is very similar to the estimate obtained earlier from a four-point QCD sum rule. We discuss applications of this method to other nonleptonic B decay modes.

1 Introduction

Exclusive nonleptonic decays of B and D mesons are complicated processes influenced by strong interactions at small and large distances. Theoretically, the decay amplitudes are determined from an effective weak Hamiltonian in terms of short-distance Wilson coefficients and matrix elements of local four-quark operators. While the short-distance effects can be calculated in QCD perturbation theory using renormalization group methods, it is extremely difficult to obtain reliable and accurate estimates for the hadronic matrix elements. Here we report on an estimate of the matrix element for the decay $B \rightarrow J/\psi K$ using QCD sum rule techniques.

The decay amplitude for $B \rightarrow J/\psi K$ is determined by the matrix element

$$\begin{aligned} \langle K J/\psi | H_W | B \rangle = & \frac{G_F}{\sqrt{2}} V_{cb} V_{cs}^* \left[\left(c_2(\mu) + \frac{c_1(\mu)}{3} \right) \langle K J/\psi | O_2(\mu) | B \rangle \right. \\ & \left. + 2c_1(\mu) \langle K J/\psi | \tilde{O}_2(\mu) | B \rangle \right], \end{aligned} \quad (1)$$

where G_F is the Fermi constant, V_{cb} and V_{cs} are the relevant CKM matrix elements, and $c_1(\mu)$ and $c_2(\mu)$ are the Wilson coefficients of the four-quark operators

$$O_1 = (\bar{s}\Gamma^\rho c)(\bar{c}\Gamma_\rho b), \quad O_2 = (\bar{c}\Gamma^\rho c)(\bar{s}\Gamma_\rho b), \quad (2)$$

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respectively, with $\Gamma_\rho = \gamma_\rho(1 - \gamma_5)$. The operator

$$\tilde{O}_2 = (\bar{c}\Gamma^\rho \frac{\lambda^a}{2} c)(\bar{s}\Gamma_\rho \frac{\lambda^a}{2} b) \quad (3)$$

with $tr(\lambda^a \lambda^b) = 2\delta^{ab}$ originates from the Fierz rearrangement of O_1 .

In the usual factorization approximation, the matrix element of O_2 is split into the product

$$\langle K J/\psi | O_2(\mu) | B \rangle = \langle J/\psi | \bar{c}\Gamma^\rho c | 0 \rangle \langle K | \bar{s}\Gamma_\rho b | B \rangle, \quad (4)$$

involving simpler matrix elements of quark currents:

$$\langle 0 | \bar{c}\Gamma^\rho c | J/\psi(p) \rangle = f_\psi m_\psi \epsilon^\rho, \quad (5)$$

$$\langle K(q) | \bar{s}\Gamma_\rho b | B(p+q) \rangle = 2f^+(p^2)q_\rho + (f^+(p^2) + f^-(p^2))p_\rho. \quad (6)$$

In the above, f_ψ is the J/ψ decay constant, ϵ^ρ is the polarization vector, and $f^\pm(p^2)$ are form factors. The momentum assignment is also indicated. The matrix element of \tilde{O}_2 vanishes in this approximation because of colour conservation.

The short-distance coefficients $c_{1,2}(\mu)$ and the matrix elements entering (1) are scale-dependent, whereas the decay constants and form factors determining the r.h.s. of (4) are physical scale-independent quantities. Therefore, factorization can at best be an approximation valid at a particular scale. As shown in ref. ¹, in the $B \rightarrow J/\psi K$ channel factorization does not work at $\mu = \mathcal{O}(m_b)$. The main nonfactorizable contribution comes from the matrix element of \tilde{O}_2 which is conveniently parametrized¹ in the form

$$\langle K J/\psi | \tilde{O}_2(\mu) | B \rangle = 2f_\psi m_\psi \tilde{f}_{B\psi K}(\mu)(\epsilon \cdot q). \quad (7)$$

Here, the dimensionless quantity $\tilde{f}_{B\psi K}(\mu)$ is the analog of $f^+(m_\psi^2)$ in (4). Note that the nonfactorizable effects in the matrix element of O_1 are subdominant and therefore neglected here. Substitution of (4) and (7) in (1) yields

$$\langle K J/\psi | H_W | B \rangle = \sqrt{2}G_F V_{cb} V_{cs}^* a_2^{B\psi K} f_\psi f^+ m_\psi (\epsilon \cdot q) \quad (8)$$

with

$$a_2^{B\psi K} = c_2(\mu) + \frac{c_1(\mu)}{3} + 2c_1(\mu) \frac{\tilde{f}_{B\psi K}(\mu)}{f^+(m_\psi^2)}. \quad (9)$$

An estimate of the matrix element (7) using four-point QCD sum rules is described in ref. ¹. In this approach, originally suggested in ref. ² for D

decays, gluonic interactions which break factorization are associated with the nonperturbative nature of the QCD vacuum as described by quark and gluon condensates. The nonfactorizable matrix element (7) turns out to be small in comparison to the factorized matrix element (4). Numerically, one finds

$$\tilde{f}_{B\psi K}(\mu) = -(0.045 \text{ to } 0.075) \quad (10)$$

and $f^+(m_\psi^2) = 0.55$. Note that the relevant scale μ in (10) is of order of the Borel mass $M \simeq \sqrt{m_B^2 - m_b^2} \simeq 2.4 \text{ GeV}$. Although the ratio $\tilde{f}_{B\psi K}(M)/f^+(m_\psi^2) \simeq 0.1$ is small, it has a strong impact in (9) because of the enhancement by the large short-distance coefficient $c_1(M) \simeq 1.1$ and because of a partial cancellation in $c_2(M) + c_1(M)/3 \simeq 0.09$. The resulting prediction for a_2 is comparable in size but opposite in sign to the outcome of phenomenological fits^{3,4} assuming a universal coefficient a_2 (and a_1). It should be stressed that in our approach the nonfactorizable effects are expected to be nonuniversal. Furthermore, the nonfactorizable term in (9) tends to cancel the nonleading in $1/N_c$ factorizable term $c_1/3$. This resembles the $1/N_c$ -rule for D decays⁵.

An alternative approach was suggested in ref. ⁶ using operator product expansion (OPE) and absorbing the nonfactorizable gluon interactions in hadronic matrix elements of new effective operators. In ref. ⁶ this method was applied to the decay mode $\bar{B}^0 \rightarrow D^+\pi^-$. In the heavy quark limit, the new operator reduces in this case to the HQET chromomagnetic operator. The matrix element of the latter between B and D states can be reliably estimated. A similar approach can also be applied⁷ to decay modes where the nonfactorizable amplitudes involve hadronic matrix elements between a B and a light meson. Here we describe such an estimate for $B \rightarrow J/\psi K$.

2 The correlation function and OPE

As suggested in ref. ⁶, we replace the J/ψ state in (7) by the generating current $\bar{c}\gamma_\mu c$ and consider the following correlation function:

$$\begin{aligned} A_\mu(p, q) &= i \int d^4x e^{ipx} \langle K(q) | T\{\bar{c}(x)\gamma_\mu c(x), \tilde{O}_2(0)\} | B(p+q) \rangle \\ &= (-p^2 q_\mu + (p \cdot q)p_\mu) A(p^2), \end{aligned} \quad (11)$$

where $(p+q)^2 = m_B^2$ and $q^2 = m_K^2$ is put to zero. Because of current conservation, there is only one invariant amplitude A . For the momentum p of the $\bar{c}\gamma_\mu c$ current we require

$$p^2 \ll 4m_c^2. \quad (12)$$

Inserting a complete set of states with J/ψ quantum numbers between the operators in (11) one gets, schematically,

$$A_\mu(p, q) = \frac{\langle 0 | \bar{c} \gamma_\mu c | J/\psi \rangle \langle K J/\psi | \tilde{O}_2 | B \rangle}{m_\psi^2 - p^2} + \sum_{h=\psi', \dots} \frac{\langle 0 | \bar{c} \gamma_\mu c | h \rangle \langle K h | \tilde{O}_2 | B \rangle}{m_h^2 - p^2}. \quad (13)$$

Substitution of (5) and (7) in (13) then leads to the following dispersion relation for the invariant amplitude A :

$$A(p^2) = \frac{2f_\psi^2 \tilde{f}_{B\psi} K}{m_\psi^2 - p^2} + \frac{2f_{\psi'}^2 \tilde{f}_{B\psi'} K}{m_{\psi'}^2 - p^2} + \int_{4m_D^2}^{\infty} \frac{\rho^h(s) ds}{s - p^2}. \quad (14)$$

The J/ψ ground-state contribution in this relation contains the desired matrix element (7). The second term in the r.h.s. represents the contribution of the ψ' resonance, whereas the dispersion integral takes into account all excited charmonium and non-resonant states in J/ψ channel located above the open charm threshold. Possible subtraction terms are not shown for brevity. They will be eliminated later. In the region (12), it is possible to expand the product of operators entering (11) in a series of local operators. A similar expansion has been applied in ref. ⁸ in order to estimate the long-distance effect in $B \rightarrow K^* \gamma$ due to interaction of virtual charmed quarks with soft gluons. As a first step, we contract the c -quark fields in the product of the currents $\bar{c} \gamma_\mu c$ and $\bar{c} \Gamma^\rho \frac{\lambda^a}{2} c$ and use

$$\begin{aligned} \langle 0 | T \{ c(x) \bar{c}(0) \} | 0 \rangle &= i \int \frac{d^4 k}{(2\pi)^4} e^{-ik(x-y)} \left\{ \frac{\not{k} + m_c}{k^2 - m_c^2} + \right. \\ &+ \frac{1}{2} g_s G_{\tau\lambda}^a(0) \frac{\lambda^a}{2} \left[\frac{\epsilon^{\tau\lambda\delta\beta} \gamma_\delta \gamma_5 k_\beta - m_c \sigma^{\tau\lambda}}{(k^2 - m_c^2)^2} \right. \\ &\left. \left. + y^\tau \frac{(k^2 - m_c^2) \gamma^\lambda - 2k^\lambda (\not{k} + m_c)}{(k^2 - m_c^2)^2} \right] \right\}. \quad (15) \end{aligned}$$

Here the first term is the free c -quark propagator and the second term incorporates the one-gluon emission in the fixed-point ($x = 0$) gauge $A_\mu^a(x) = 1/2 x^\tau G_{\tau\mu}^a(0)$. The free-quark loop vanishes because of $Tr \lambda^a = 0$. The lowest order nonvanishing contribution involves a single gluon-field tensor. The corresponding Wilson coefficient can be calculated from the diagrams shown in Fig. 1. The result is :

$$i \int d^4 x e^{ipx} T \{ \bar{c}(x) \gamma_\mu c(x), \bar{c} \Gamma_\nu \frac{\lambda^a}{2} c(0) \} = t_{\mu\nu\alpha\beta} g_s \tilde{G}^{\alpha\beta} I_c(p^2) + \dots \quad (16)$$

with

$$t_{\mu\nu\alpha\beta} = p_\mu p_\alpha g_{\nu\beta} + p_\nu p_\alpha g_{\mu\beta} - p^2 g_{\mu\alpha} g_{\nu\beta} , \quad (17)$$

$$\tilde{G}_{\alpha\beta}^a = \frac{1}{2} \epsilon_{\alpha\beta\sigma\tau} G^{a\sigma\tau} , \quad (18)$$

$$I_c(p^2) = \frac{1}{4\pi^2} \int_0^1 dx \frac{x^2(1-x)}{m_c^2 - p^2 x(1-x)} . \quad (19)$$



Figure 1: Diagrams determining the Wilson coefficients in the OPE (16).

The higher-dimensional terms of the expansion denoted by the ellipses in (16) involve operators with derivatives of the gluon field and/or multiple gluon-field operators. As discussed in ref. ⁸, for the physical b and c masses, these terms are expected to contribute at the level of 10% or less. We neglect them here.

Substitution of (16) in (11) yields

$$A(p^2) = -g_{BK}(p^2)I_c(p^2) , \quad (20)$$

where the effective form factor g_{BK} is defined by the matrix element

$$t_{\mu\nu\alpha\beta} \langle K(q) | \bar{s} \Gamma^\nu g_s \tilde{G}^{\alpha\beta} b | B(p+q) \rangle = (p^2 q_\mu - (p \cdot q) p_\mu) g_{BK}(p^2) \quad (21)$$

with $\tilde{G}^{\alpha\beta} = \frac{\lambda^a}{2} \tilde{G}^{a\alpha\beta}$. Note that the relevant scale for A is the scale of OPE, that is $\mu \simeq 2m_c$. The calculation of the form factor $g_{BK}(p^2)$ requires some nonperturbative method. In the next section we will estimate it using light-cone sum rules.

3 Light-cone sum rule for the form factor g_{BK}

To evaluate the matrix element (21) we proceed as in the calculation of the $B \rightarrow \pi, K$ form factors in ref. ⁹ (see also the reviews ref. ^{1,10}). Considering the

correlation function

$$\begin{aligned}
F_\mu(p, q) &= it_{\mu\nu\alpha\beta} \int d^4x e^{ipx} \langle K(q) | T\{\bar{s}(x)\Gamma^\nu g_s \tilde{G}^{\alpha\beta}(x)b(x), \bar{b}(0)i\gamma_5 d(0)\} | 0\rangle \\
&= (p^2 q_\mu - (p \cdot q)p_\mu) F(p^2, (p+q)^2) ,
\end{aligned} \tag{22}$$

we insert a complete set of intermediate states with B -meson quantum numbers between the currents. This yields:

$$\begin{aligned}
F_\mu(p, q) &= t_{\mu\nu\alpha\beta} \left(\frac{\langle K | \bar{s}\Gamma^\nu g_s \tilde{G}^{\alpha\beta} b | B\rangle \langle B | \bar{b}i\gamma_5 d | 0\rangle}{m_B^2 - (p+q)^2} \right. \\
&\quad \left. + \sum_{h_B} \frac{\langle K | \bar{s}\Gamma^\nu g_s \tilde{G}^{\alpha\beta} b | h_B\rangle \langle h_B | \bar{b}i\gamma_5 d | 0\rangle}{m_{h_B}^2 - (p+q)^2} \right) ,
\end{aligned} \tag{23}$$

where the ground state contribution contains the matrix element of interest. Using (21) and

$$m_b \langle 0 | \bar{b}i\gamma_5 d | B\rangle = m_B^2 f_B , \tag{24}$$

where f_B is the B meson decay constant, and representing the sum over excited and continuum states by a dispersion integral, one finds

$$F(p^2, (p+q)^2) = \frac{m_B^2 f_B g_{BK}(p^2)}{m_b(m_B^2 - (p+q)^2)} + \int_{s_0^h}^{\infty} \frac{\rho^{h_B}(p^2, s) ds}{s - (p+q)^2} . \tag{25}$$

In the momentum region $(p+q)^2 \ll m_b^2$ and $p^2 \ll 4m_c^2$, the amplitude F can also be calculated with the help of OPE. The result can be brought in the form of a dispersion integral:

$$F_{QCD}(p^2, (p+q)^2) = \frac{1}{\pi} \int_{m_s^2}^{\infty} ds \frac{\text{Im}F_{QCD}(p^2, s)}{s - (p+q)^2} . \tag{26}$$

Furthermore, the spectral density of the higher states in (25) is approximated by

$$\rho^{h_B}(p^2, s)\Theta(s - s_0^h) = \frac{1}{\pi} \text{Im}F_{QCD}(p^2, s)\Theta(s - s_0^B) , \tag{27}$$

as suggested by quark-hadron duality. With (27) it is then straightforward to subtract the contribution of the excited and continuum states in the equation

given by (25) and (26) and solve this equation for g_{BK} . After performing the Borel transformation in $(p+q)^2$, the solution reads:

$$g_{BK}(p^2) = \frac{m_b}{\pi f_B m_B^2} \int_{m_b^2}^{s_0^B} \text{Im} F_{QCD}(p^2, s) \exp\left(\frac{m_B^2 - s}{M^2}\right) ds. \quad (28)$$

In contrast to conventional sum rules based on the short-distance OPE in terms of local operators, here an expansion in terms of nonlocal operators near the light-cone, $x^2 \simeq 0$, is used to calculate the invariant amplitude F_{QCD} . Contraction of the b -quark fields in the correlation function (22) and use of the free b -quark propagator leads to the diagram Fig. 2a. In this approximation, F_{QCD} is given by a set of bilocal matrix elements $\langle K | \bar{s}(x) \Gamma_a g_s G_{\alpha\beta}(x) d(0) | 0 \rangle$ where Γ_a denotes a combination of Dirac matrices. These matrix elements are parametrized in terms of three-particle light-cone wave functions of different twist:

$$\begin{aligned} \langle K | \bar{s}(x) \sigma_{\mu\nu} \gamma_5 g_s G_{\alpha\beta}(x) d(0) | 0 \rangle &= i f_{3K} [(q_\alpha q_\mu g_{\beta\nu} - q_\beta q_\mu g_{\alpha\nu}) \\ &\quad - (q_\alpha q_\nu g_{\beta\mu} - q_\beta q_\nu g_{\alpha\mu})] \int \mathcal{D}\alpha_i \varphi_{3K}(\alpha_i) e^{iqx(\alpha_1 + \alpha_3)}, \end{aligned} \quad (29)$$

$$\begin{aligned} \langle K | \bar{s}(x) \gamma_\mu \gamma_5 g_s G_{\alpha\beta}(x) d(0) | 0 \rangle &= f_K \left[q_\beta \left(g_{\alpha\mu} - \frac{x_\alpha q_\mu}{qx} \right) \right. \\ &\quad \left. - q_\alpha \left(g_{\beta\mu} - \frac{x_\beta q_\mu}{qx} \right) \right] \int \mathcal{D}\alpha_i \varphi_{\perp K}(\alpha_i) e^{iqx(\alpha_1 + \alpha_3)} \\ &\quad + f_K \frac{q_\mu}{qx} (q_\alpha x_\beta - q_\beta x_\alpha) \int \mathcal{D}\alpha_i \varphi_{\parallel K}(\alpha_i) e^{iqx(\alpha_1 + \alpha_3)}, \end{aligned} \quad (30)$$

$$\begin{aligned} \langle K | \bar{s}(x) \gamma_\mu g_s \tilde{G}_{\alpha\beta}(x) d(0) | 0 \rangle &= i f_K \left[q_\beta \left(g_{\alpha\mu} - \frac{x_\alpha q_\mu}{qx} \right) \right. \\ &\quad \left. - q_\alpha \left(g_{\beta\mu} - \frac{x_\beta q_\mu}{qx} \right) \right] \int \mathcal{D}\alpha_i \tilde{\varphi}_{\perp K}(\alpha_i) e^{iqx(\alpha_1 + \alpha_3)} \\ &\quad + i f_K \frac{q_\mu}{qx} (q_\alpha x_\beta - q_\beta x_\alpha) \int \mathcal{D}\alpha_i \tilde{\varphi}_{\parallel K}(\alpha_i) e^{iqx(\alpha_1 + \alpha_3)} \end{aligned} \quad (31)$$

with $\mathcal{D}\alpha_i = d\alpha_1 d\alpha_2 d\alpha_3 \delta(1 - \alpha_1 - \alpha_2 - \alpha_3)$. The wave function $\varphi_{3K}(\alpha_i) = \varphi_{3K}(\alpha_1, \alpha_2, \alpha_3)$ has twist 3, while $\varphi_{\perp K}$, $\varphi_{\parallel K}$, $\tilde{\varphi}_{\perp K}$ and $\tilde{\varphi}_{\parallel K}$ are all of twist

4. In the following, we consider the $SU(3)$ -symmetry limit where $f_{3K} = f_{3\pi}$, $\varphi_{3K} = \varphi_{3\pi}$, $\varphi_{\perp K} = \varphi_{\perp\pi}$, etc. The explicit expressions for the pion wave functions can be found in ref. ¹. Higher Fock-space components of the K meson wave function are neglected as well as perturbative corrections of the kind shown in Fig. 2b. Omitting further details of the calculation we present the final result:

$$\begin{aligned}
F(p^2, (p+q)^2) &= 2f_{3K}(qp) \int \mathcal{D}\alpha_i \frac{\varphi_{3K}(\alpha_i)}{m_b^2 - (p + (\alpha_1 + \alpha_3)q)^2} \\
&+ 2f_K m_b \int_0^1 \frac{du}{m_b^2 - (p+uq)^2} \left\{ \int_0^u d\alpha_3 (3\tilde{\varphi}_{\perp K}(\alpha_i) - \tilde{\varphi}_{\parallel K}(\alpha_i))_{\substack{\alpha_1=u-\alpha_3 \\ \alpha_2=1-u}} \right. \\
&+ \left. \frac{1}{u} \left(\frac{m_b^2 - p^2}{m_b^2 - (p+uq)^2} - 1 \right) \int_0^u dv \int_0^v d\alpha_3 (\tilde{\varphi}_{\perp K}(\alpha_i) + \tilde{\varphi}_{\parallel K}(\alpha_i))_{\substack{\alpha_1=v-\alpha_3 \\ \alpha_2=1-v}} \right\}. \quad (32)
\end{aligned}$$

Substituting the above result in (28) yields the light-cone sum rule:

$$\begin{aligned}
g_{BK}(p^2) &= \frac{m_b^2}{f_B m_B^2} \int_{\Delta}^1 \frac{du}{u} \exp\left(\frac{m_B^2}{M^2} - \frac{m_b^2 - p^2(1-u)}{uM^2}\right) \\
&\times \left\{ \int_0^u d\alpha_3 \left(f_{3K} \frac{m_b^2 - p^2}{m_b u} \varphi_{3K}(\alpha_i) + f_{\pi} (3\tilde{\varphi}_{\perp K}(\alpha_i) - \tilde{\varphi}_{\parallel K}(\alpha_i))_{\substack{\alpha_1=u-\alpha_3 \\ \alpha_2=1-u}} \right) \right. \\
&+ \left. \frac{1}{u} \left(\frac{m_b^2 - p^2}{uM^2} - 1 \right) \int_0^u dv \int_0^v d\alpha_3 (\tilde{\varphi}_{\perp K}(\alpha_i) + \tilde{\varphi}_{\parallel K}(\alpha_i))_{\substack{\alpha_1=v-\alpha_3 \\ \alpha_2=1-v}} \right\}, \quad (33)
\end{aligned}$$

where $\Delta = m_b^2 - p^2/(s_0^B - p^2)$. In the above estimate the scale of the form factor g_{BK} is set by the Borel mass $M \simeq \sqrt{m_B^2 - m_b^2}$.

4 Estimate of the nonfactorizable amplitude in $B \rightarrow J/\psi K$

From (20) and (33) we get an estimate for the invariant amplitude $A(p^2)$ which is related to $\tilde{f}_{B\psi K}$ by (14). In order to suppress the contribution of excited

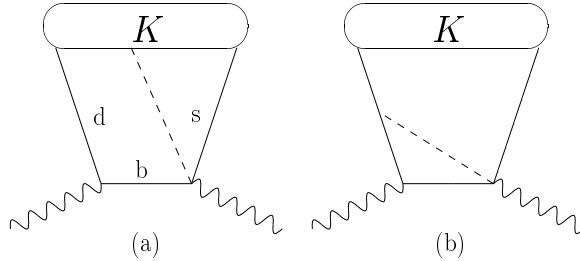


Figure 2: *Diagrammatic representation of the correlation function (22). Solid lines represent quarks, dashed line gluons, wavy lines external currents, and ovals light-cone wave functions of the kaon.*

charmonium and nonresonant states in the latter dispersion relation and to eliminate subtraction terms we take the n -th derivative of (14) at $p^2 = 0$:

$$\begin{aligned} & \tilde{f}_{B\psi K} + \left(\frac{f_{\psi'}}{f_{\psi}}\right)^2 \left(\frac{m_{\psi}}{m_{\psi'}}\right)^{2n+2} \tilde{f}_{B\psi'K} \\ & + \frac{m_{\psi}^{2n+2}}{2f_{\psi}^2} \int_{4m_D^2}^{\infty} \frac{\rho^h(s) ds}{s^{n+1}} = -\frac{m_{\psi}^{2n+2}}{2n!f_{\psi}^2} \frac{d^n}{dp^{2n}} \left[g_{BK}(p^2) I_c(p^2) \right]_{p^2=0}. \end{aligned} \quad (34)$$

Since the influence of higher-dimensional operators neglected in (16) increases with n we restrict the numerical analysis with $n \leq 8$. For the charmed quark mass we take $m_c = 1.3$ GeV and for decay constants the values $f_{\psi} = 405$ MeV, $f_{\psi'}/f_{\psi} \simeq 0.7$ extracted from the measured leptonic widths of the J/ψ and ψ' ¹¹. The values of parameters appearing in the light-cone sum rule for g_{BK} are given in ref.¹.

The contribution from the ψ' to the sum rule (34) is suppressed relative to the J/ψ contribution roughly by a factor 10 at $n = 3$ and a factor 50 at $n = 8$. It is therefore reasonable to neglect the integral over still heavier states in (34) altogether. From the remaining sum rule one can get a first rough estimate of

$\tilde{f}_{B\psi K}$ by taking $n \geq 6$ and dropping the ψ' term. This yields

$$\tilde{f}_{B\psi K}(\mu) = -(0.04, 0.05, 0.07) \text{ at } n = 6, 7, 8, \quad (35)$$

where $\mu \simeq 2m_c = 2.6$ GeV which accidentally coincides with the scale $M \simeq \sqrt{m_B^2 - m_b^2} = 2.4$ GeV of the form factor g_{BK} used to get (35). For a more elaborated estimate one may fit the amplitudes $\tilde{f}_{B\psi K}$ and $\tilde{f}_{B\psi'K}$ to the moments (34) for $n = 2 \div 8$. This gives $\tilde{f}_{B\psi K}(\mu) = -0.06$ and $\tilde{f}_{B\psi'K}(\mu) = +0.3$.

5 Discussion

QCD sum rule techniques together with light-cone wave functions provide new ways to go beyond factorization in exclusive nonleptonic decays of heavy mesons. Within this approach, we have estimated the nonfactorizable amplitude (7) for the decay $B \rightarrow J/\psi K$. The result (35) agrees with an earlier estimate (10) obtained from a four-point sum rule¹. Furthermore, it is interesting to note that the nonfactorizable amplitude $\tilde{f}_{B\psi'K}$ for $B \rightarrow \psi'K$, differs from $\tilde{f}_{B\psi K}$ both in sign and magnitude. This finding underlines the general expectation of nonuniversality of the coefficients a_2 in the sum rule approach.

Although it is encouraging that both methods give similar results, there are still unknown uncertainties. They arise from several sources: neglect of higher-dimensional operators, unknown perturbative corrections, a crude model for the hadronic spectral density of higher states in the J/ψ channel, and numerical uncertainties on the parameters and wave functions. The uncertainty on $\tilde{f}_{B\psi'K}$ is obviously larger than the one on $\tilde{f}_{B\psi K}$, because the former has a much smaller coefficient in (34) than the latter.

Our final comment concerns the original applications in refs.^{6,7} of this method to $B \rightarrow D\pi$. Let us consider the $\bar{B}^0 \rightarrow D^0\pi^0$ mode. The nonfactorizable amplitude

$$\langle D^0\pi^0 | \tilde{\mathcal{O}}'_2 | \bar{B}^0 \rangle \quad (36)$$

with

$$\tilde{\mathcal{O}}'_2 = (\bar{c}\Gamma^\rho \frac{\lambda^a}{2} u)(\bar{d}\Gamma_\rho \frac{\lambda^a}{2} b) \quad (37)$$

has been estimated in ref.⁷ from the correlation function $\langle \pi^0 | T\{\bar{u}\gamma_\mu\gamma_5 c, \tilde{\mathcal{O}}'_2\} | \bar{B}^0 \rangle$, $\bar{u}\gamma_\mu\gamma_5 c$ being the generating current of D^0 . Contraction of all quark fields in the operator product

$$T\{\bar{u}(x)\gamma_\mu\gamma_5 c(x), \bar{c}(0)\Gamma^\rho \frac{\lambda^a}{2} u(0)\} \quad (38)$$

leads to the gluon-field operator, as shown in section 2. However, the OPE of (38) also contains the operator $\bar{u}\frac{\lambda^a}{2}\Gamma_r u$ resulting from contraction of the heavy-quark fields alone. This operator contributes to the sum rule for (36) through matrix elements of effective four-quark operators $\langle \pi^0 | (\bar{u}\frac{\lambda^a}{2}\Gamma_r u)(\bar{d}\Gamma_\rho\frac{\lambda^a}{2}b) | B \rangle$. These contributions which have not been taken into account in refs. ^{6,7} may significantly influence the results for the nonfactorizable amplitudes in $B \rightarrow D\pi$.

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