

A new method for extracting the weak phase γ from $B \rightarrow DK^{(*)}$ decays

Ji-Ho Jang, * and Pyungwon Ko †

*Department of Physics, Korea Advanced Institute of Science and Technology,
Taejon 305-701, Korea*

Abstract

A new method to extract the weak phase γ is suggested by exploiting $B \rightarrow DK^{(*)}$ decay modes that are not Cabibbo suppressed, using the isospin relations, and ignoring the annihilation diagram as usual. Assuming $3 \times 10^8 B\bar{B}$ pair at B factories, one can determine γ with $3 - \sigma$ accuracy for $80^\circ \lesssim \gamma \lesssim 150^\circ$ using $B \rightarrow DK$ modes and for $50^\circ \lesssim \gamma \lesssim 170^\circ$ using $B \rightarrow DK^*$ modes.

*e-mail : jhjang@chep6.kaist.ac.kr

†e-mail : pko@charm.kaist.ac.kr

One of the goals of B factories which will launch its mission at the end of 1999 is to test the KM paradigm for the CP violation by verifying that the unitary triangle (UT) can be constructed in a consistent manner [1]. Namely, one measures three sides and three angles of the unitarity triangle in all possible ways, and examine if a single triangle emerges from various different measurements. If so, one can verify that CP violations in $K_L \rightarrow \pi\pi$ and B decays all result from a single KM phase in the CKM matrix [2]. Otherwise, there should be some new physics which is more exciting from the particle physics point of view. Still, it is utmostly important to test the KM picture from B decays. As of today, the least known quantities of the UT are $|V_{ub}/V_{cb}|$, and its three angles [3] :

$$|V_{ub}/V_{cb}| = 0.080 \pm 0.020, \quad (1)$$

$$-1.0 \leq \sin 2\alpha \leq 1.0, \quad (2)$$

$$0.30 \leq \sin 2\beta \leq 0.88, \quad (3)$$

$$0.27 \leq \sin^2 \gamma \leq 1.0. \quad (4)$$

The first quantity is determined from charmless semileptonic B decays (both inclusive and exclusive), and suffers from intrinsic theoretical uncertainties such as breakdown of the heavy quark mass expansion near the phase space boundary, or the poorly known $B \rightarrow \pi$ (or ρ) semileptonic form factors. One can estimate the uncertainty in $|V_{ub}/V_{cb}|$ as $\sim 25\%$ conservatively. Three angles of the UT can be loosely bounded from various low energy phenomenology. The angle $\beta(\equiv \phi_1)$ can be measured in the gold-plated mode, $B_d \rightarrow J/\psi K_S$ without any hadronic uncertainty. The angle $\alpha(\equiv \phi_2)$ can be measured from $B \rightarrow \pi\pi$, but there is some penguin contamination that cannot be too small considering the recent observation of $B \rightarrow K\pi$ at the level of branching ratio of $\sim 1.4 \times 10^{-5}$ by CLEO Collaboration [4]. Still, one can hope to perform the isospin analysis and remove the penguin contribution, thereby being able to extract the $\alpha(\phi_2)$ with a reasonable accuracy [5]. The most difficult to measure is the angle $\gamma(\equiv \phi_3)$. There have been a lot of suggestions and discussions about how to measure this quantity at B factories (running at the top of $\Upsilon(4S)$ resonance) [6,7]. Unfortunately there is no best way to determine γ from B decays up to now. Any method suggested so far has some weak points, e.g., involving measurements of decay modes that have too low branching ratios. In Ref. [6], the authors proposed to extract γ using the independent measurements of $B \rightarrow D^0 K$, $B \rightarrow \bar{D}^0 K$ and $B \rightarrow D_{CP} K$. However the charged B meson decay mode $B^- \rightarrow \bar{D}^0 K^-$ is experimentally difficult to measure. The reason is that the final \bar{D}^0 meson should be identified using $\bar{D}^0 \rightarrow K^+ \pi^-$, but it is difficult to distinguish it from doubly Cabibbo suppressed $D^0 \rightarrow K^+ \pi^-$ following color and CKM allowed $B^- \rightarrow D^0 K^-$. There are some variant methods to overcome these difficulties. In Ref. [8], Atwood *et al.* used different final states into which the neutral D meson decays to extract information of γ . In Ref. [9], Gronau proposed that the angle γ is determined only using the color allowed decay modes, $B^- \rightarrow D^0 K^-$, $B^- \rightarrow D_{CP} K^-$ and their charge conjugation modes.

In this letter, we suggest another method for extracting γ from Cabibbo allowed $B \rightarrow DK^{(*)}$ decays. We construct three different triangles from various $B \rightarrow DK^{(*)}$ decays, each of which involves decay modes with rather large branching ratios. From these triangles, one can determine the weak phase γ with a reasonable accuracy if one has $3 \times 10^8 B\bar{B}$'s at B factories. Both $B \rightarrow DK$ modes and the self-tagging modes $B \rightarrow DK^*$ are considered with an assumption that the annihilation diagrams are negligible in both cases. This assumption

may be questionable for the $B \rightarrow DK^*$ decays in light of the recent work by Ali *et al.* [10], which claims that the annihilation diagram may not be ignorable in the $B \rightarrow PV$ channel for the case of light pseudoscalar (P) and light vector (V) mesons. This claim is based on the generalized factorization approximation. We leave this as an open question here, with a remark that one can easily test this assumption by measuring the branching ratio for $B^- \rightarrow D^- \bar{K}^{*0}$ and comparing it with other decays we use, such as $B^{-(0)} \rightarrow D_1 \bar{K}^{*(0)}$ and $\bar{B}^0 \rightarrow D_1 \bar{K}^{*0}$.

The recent CLEO measurement of $Br(B^+ \rightarrow \bar{D}^0 K^-) = (2.57 \pm 0.65 \pm 0.32) \times 10^{-4}$ [11] gives light on the determination of one angle γ of unitary triangle. Let us begin with $B \rightarrow DK$ and define their amplitudes as follows,

$$\begin{aligned}
A(B^- \rightarrow D^0 K^-) &= A\lambda^3 B_1 e^{i\delta_1} &= A\lambda^3 (T + C) \\
A(\bar{B}^0 \rightarrow D^+ K^-) &= \frac{1}{2} A\lambda^3 (B_1 e^{i\delta_1} - B_0 e^{i\delta_0}) &= A\lambda^3 T \\
A(\bar{B}^0 \rightarrow D^0 \bar{K}^0) &= \frac{1}{2} A\lambda^3 (B_1 e^{i\delta_1} + B_0 e^{i\delta_0}) &= A\lambda^3 C \\
A(B^- \rightarrow \bar{D}^0 K^-) &= \frac{1}{2} A\lambda^3 R_b e^{-i\gamma} (B'_1 e^{i\delta'_1} + B'_0 e^{i\delta'_0}) &= A\lambda^3 R_b e^{-i\gamma} (C' + A') \\
A(B^- \rightarrow D^- \bar{K}^0) &= \frac{1}{2} A\lambda^3 R_b e^{-i\gamma} (B'_1 e^{i\delta'_1} - B'_0 e^{i\delta'_0}) &= A\lambda^3 R_b e^{-i\gamma} (-A') \\
A(\bar{B}^0 \rightarrow \bar{D}^0 \bar{K}^0) &= A\lambda^3 R_b e^{-i\gamma} B'_1 e^{i\delta'_1} &= A\lambda^3 R_b e^{-i\gamma} C'.
\end{aligned} \tag{5}$$

where we use the Wolfenstein parametrization of CKM matrix elements, and $R_b \equiv \sqrt{\rho^2 + \eta^2}$. $B_I^{(\prime)}$ denotes the amplitude for the isospin I state. The first equalities are written in terms of the isospin amplitudes, whereas the second ones are written in terms of diagrammatic representations (T means a tree diagram, C means a color-suppressed diagram, and so on). The above equations give two isospin relations,

$$\begin{aligned}
A(B^- \rightarrow D^0 K^-) &= A(\bar{B}^0 \rightarrow D^+ K^-) + A(\bar{B}^0 \rightarrow D^0 \bar{K}^0) \\
A(B^- \rightarrow \bar{D}^0 K^-) &= -A(B^- \rightarrow D^- \bar{K}^0) + A(\bar{B}^0 \rightarrow \bar{D}^0 \bar{K}^0).
\end{aligned} \tag{6}$$

Using the definition of mass eigenstates of D mesons, $D_{1(2)} = \frac{1}{\sqrt{2}}(D^0 \pm \bar{D}^0)$, and neglecting the $D^0 - \bar{D}^0$ mixing, we obtain

$$\begin{aligned}
A(B^- \rightarrow D_1 K^-) &= A(\bar{B}^0 \rightarrow D_1 \bar{K}^0) + \frac{1}{\sqrt{2}} [A(\bar{B}^0 \rightarrow D^+ K^-) - A(B^- \rightarrow D^- \bar{K}^0)] \\
A(B^- \rightarrow D_2 K^-) &= A(\bar{B}^0 \rightarrow D_2 \bar{K}^0) + \frac{1}{\sqrt{2}} [A(\bar{B}^0 \rightarrow D^+ K^-) + A(B^- \rightarrow D^- \bar{K}^0)].
\end{aligned} \tag{7}$$

If we neglect $A(B^- \rightarrow D^- \bar{K}^0)$ which is CKM suppressed and has only the annihilation diagram contribution, we get (from now, we only consider final D_1 states)

$$\begin{aligned}
A(B^- \rightarrow D_1 K^-) &= A(\bar{B}^0 \rightarrow D_1 \bar{K}^0) + \frac{1}{\sqrt{2}} A(\bar{B}^0 \rightarrow D^+ K^-) \\
A(B^+ \rightarrow D_1 K^+) &= A(B^0 \rightarrow D_1 K^0) + \frac{1}{\sqrt{2}} A(B^0 \rightarrow D^- K^+).
\end{aligned} \tag{8}$$

The mixing of the neutral B meson has to be considered in order to obtain the magnitudes of $A(B^0 \rightarrow D_1 K^0)$ and $A(\bar{B}^0 \rightarrow D_1 \bar{K}^0)$. The time dependent decay rate of neutral B mesons whose initial states are B^0 and \bar{B}^0 are given by

$$\Gamma(B_{phys}^0(t) \rightarrow f) = \frac{1}{2}|A_f|^2 e^{-\Gamma t} \left[(1 + |\xi|^2) + (1 - |\xi|^2) \cos(\Delta m t) - 2(\text{Im}\xi) \sin(\Delta m t) \right], \quad (9)$$

$$\Gamma(\bar{B}_{phys}^0(t) \rightarrow f) = \frac{1}{2}|A_f|^2 e^{-\Gamma t} \left[(1 + |\xi|^2) - (1 - |\xi|^2) \cos(\Delta m t) + 2(\text{Im}\xi) \sin(\Delta m t) \right].$$

Here, ξ is

$$\xi = e^{-2i\phi_M} \frac{\bar{A}_f}{A_f}, \quad (10)$$

where $A \equiv A(B^0 \rightarrow f)$, $\bar{A} \equiv A(\bar{B}^0 \rightarrow f)$, $f = D_{1(2)}K_S$, and ϕ_M is the $B^0 - \bar{B}^0$ mixing phase. It is possible to get $|A(\bar{B}^0 \rightarrow D_1\bar{K}^0)|$ and $|A(B^0 \rightarrow D_1K^0)|$ from the coefficients of the constant and $\cos(\Delta m t)$ terms and using $|A(B^0 \rightarrow D_{1(2)}K^0)| = |1/\sqrt{2}A(B^0 \rightarrow D_{1(2)}K_S)|$ and $|A(\bar{B}^0 \rightarrow D_{1(2)}\bar{K}^0)| = |1/\sqrt{2}A(\bar{B}^0 \rightarrow D_{1(2)}K_S)|$.

From Eq. (5), we get other relations for $A(B^- \rightarrow D_1K^-)$ and its charge conjugate,

$$A(B^- \rightarrow D_1K^-) = \frac{1}{\sqrt{2}}A(B^- \rightarrow D^0K^-) + \frac{A\lambda^3 R_b}{2\sqrt{2}}Be^{i(\delta-\gamma)} \quad (11)$$

$$A(B^+ \rightarrow D_1K^+) = \frac{1}{\sqrt{2}}A(B^- \rightarrow D^0K^-) + \frac{A\lambda^3 R_b}{2\sqrt{2}}Be^{i(\delta+\gamma)}$$

where B is given by $Be^{i\delta} \equiv B'_1 e^{i\delta'_1} + B'_0 e^{i\delta'_0}$ and the last terms in Eq.(11) are only $\frac{1}{\sqrt{2}}A(B^- \rightarrow \bar{D}^0K^-)$ and $\frac{1}{\sqrt{2}}A(B^+ \rightarrow D^0K^+)$.

The strategy to determine γ is as follows :

- Using the first equation of Eq.(6), we can draw a triangle and fix $\frac{1}{\sqrt{2}}A(B^- \rightarrow D^0K^-)$. The bottom side of the triangle is $\frac{1}{\sqrt{2}}|A(\bar{B}^0 \rightarrow D^+K^-)| = \frac{1}{\sqrt{2}}|A(B^0 \rightarrow D^-K^+)|$.
- Using Eq.(8), we can draw two triangles whose bottom side is $\frac{1}{\sqrt{2}}|A(\bar{B}^0 \rightarrow D^+K^-)| = \frac{1}{\sqrt{2}}|A(B^0 \rightarrow D^-K^+)|$ and fix $A(B^- \rightarrow D_1K^-)$ and $A(B^+ \rightarrow D_1K^+)$.
- Using three fixed amplitudes, $\frac{1}{\sqrt{2}}A(B^- \rightarrow D^0K^-)$, $A(B^- \rightarrow D_1K^-)$ and $A(B^+ \rightarrow D_1K^+)$, the angle 2γ is determined by Eq.(11) up to some discrete ambiguities (see Fig. 1).

In Fig. 1, we show three triangles that can be constructed from our strategy. The thick solid sides are exactly what were problematic in the GLW method [6], since it is almost impossible to experimentally measure those sides. In our case, we use only B decay modes with relatively large branching ratios so that one can avoid the difficulties encountered in the GLW method. The question still remains whether one can extract 2γ from Fig. 1 with a reasonable accuracy by measuring various sides of three triangles, which we would like to address in the following.

Let us estimate the uncertainty in the determination of the weak phase γ by the method suggested in this letter, assuming that 3×10^8 and 10^{11} $B\bar{B}$'s are available at the $\Upsilon(4S)$ resonance (B factories using e^+e^- annihilation) and hadron colliders (such as BTeV or LHCb), respectively. The observed number of events for each mode is

$$N_{\text{obs}} = N_{\text{tot}} \times Br \times f \times \epsilon,$$

where N_{tot}, Br, f and ϵ are the total number of $B - \bar{B}$ events, branching ratios, observation fractions and detector efficiencies, respectively. From N_{obs} one can determine the uncertainty ΔN_{obs} of the branching ratio. The K^0 is identified by the $K_S \rightarrow \pi^+\pi^-$ mode and using the fact that the half of K_S is K^0 . The K^{*0}, \bar{K}^{*0} mesons are distinguished using $K^{*0} \rightarrow K^+\pi^-$ and $\bar{K}^{*0} \rightarrow K^-\pi^+$ modes and they are also used for self-tagging of B^0 and \bar{B}^0 respectively. In D^0, \bar{D}^0 meson tagging, we add the $D^0 \rightarrow K^-\pi^+\pi^0$ mode to the $D^0 \rightarrow K^-\pi^+, D^0 \rightarrow K^-\pi^+\pi^+\pi^-$ modes to increase the observation rate. In Table 1, the tagging modes, observation rate and detector efficiencies are summarized. For each collider, we assume the same detection efficiencies as Dunietz's work [7]. From the experimental value for $Br(B^+ \rightarrow D^0 K^-) \sim 2.6 \times 10^{-4}$, one can extract the size of $|T + C|$. Assuming that $|C/T| \approx |C'/T| \approx \lambda = 0.22$ as in Ref. [12], and allowing C and C' to have phases δ_C and $\delta_{C'}$ relative to the T amplitude, one can estimate the branching ratios of other decay modes that participate in the triangles shown in Fig. 1. Then the uncertainty of the amplitude $A = \sqrt{Br}$ is determined by $\Delta A/A = (1/2)(\Delta Br/Br) \approx 1/(2\sqrt{N_{\text{obs}}})$. Using this information, we investigate the possibility of the determination of γ from three triangles and its uncertainty.

The results are shown in Fig. 2 (a) and (b), where the horizontal and the vertical axes represent γ and $\Delta\gamma$ (in degrees), respectively. Since the phases δ_C and $\delta_{C'}$ are unknown, we considered four different cases with a fixed $\delta_C = 10^\circ$: (i) $\delta_{C'} = \delta_C$ (the real curve), (ii) $\delta_{C'} = -2\delta_C$ (the dashed curve), (iii) $\delta_{C'} = 0$ (the dotted curve), and (iv) $\delta_{C'} = +2\delta_C$ (the dashed-dotted curve). We observe some dependence of $\Delta\gamma$ on the phases δ_C and $\delta_{C'}$ through the branching ratios. Our method can provide a good determination of γ for $80^\circ \lesssim \gamma \lesssim 150^\circ$ or so. One can achieve an accuracy of better than 3- σ for this range of γ . For small γ , our method fails and one has to resort to other methods.

Now, let us repeat the same analysis for the self-tagging modes, $B \rightarrow DK^*$. The advantage of these self-tagging modes is that the number of available B decays become doubled compared to the $B \rightarrow DK$ modes and the time dependent analysis is unnecessary. As before, we can define several amplitudes similarly to Eqs. (5) by $K \rightarrow K^*$. Then the same equations as (6),(8) and (11) can be obtained. One thing to be kept in mind is the adequacy of neglecting the amplitude for $B^- \rightarrow D^- \bar{K}^{*0}$ and its charge conjugate, which are Cabibbo suppressed and generated by the annihilation diagram at the quark level. Usually such annihilation diagrams are neglected, since they are suppressed by f_B/m_B relative to other diagrams. However this may not be true for the case of $B \rightarrow PV$ modes as recently discussed by Ali *et al.* in the context of the generalized factorization approach [10]. They claim that the annihilation branching ratio might be an order of magnitude higher than that of the penguin diagram only. One can verify the usual assumption of neglecting the annihilation diagram in $B \rightarrow DK^*$ only through the experimental measurement of the branching ratio for $B^- \rightarrow D^- \bar{K}^{*0}$.

With this point kept in mind, we may proceed as before to construct three triangles as shown in Fig. 1, and determine γ . For the estimate of the uncertainties, let us assume that the branching ratio for $B^0 \rightarrow D^- K^{*+}$ is about 4×10^{-4} adopting the results of Neubert and Stech based on the factorization approximation for B decays into heavy and light mesons [13]. Then, assuming the same relation between T, C and C' and their relative phases as before ($B \rightarrow DK$), one can estimate the uncertainties in the sides of three triangles in Fig. 1 (with $K \rightarrow K^*$) and the weak phase γ . The results are shown in Fig. 2 (c) and (d). As before, we can determine γ with 3- σ precision if $50^\circ \lesssim \gamma \lesssim 170^\circ$. This range of γ covers

substantial part of γ that is allowed in Eq. (4). The uncertainty in γ is about $5^\circ - 20^\circ$ in this range of γ , and we achieve a better determination of γ from the self-tagging $B \rightarrow DK^*$ decay modes.

In conclusion, we considered a new method to extract the weak phase γ using the triangles shown in Fig. 1. If one has $3 \times 10^8 B\bar{B}$ pairs at the $\Upsilon(4S)$ resonance, or 10^{11} B 's at hadron colliders, one can determine γ with $3\text{-}\sigma$ precision or better for $80^\circ(50^\circ) \lesssim \gamma \lesssim 150^\circ(170^\circ)$ from $B \rightarrow DK^{(*)}$ decays that are not Cabibbo suppressed. This range of γ covers substantial part of γ that is allowed in Eq. (4). One can also repeat for the $D_2 K^{(*)}$ modes instead of $D_1 K^{(*)}$ modes, which will provide independent informations on γ . For smaller γ , the uncertainty in γ is so large that our method is no longer useful in extracting γ . Also our method fails if there is no/little strong phase difference δ . One has to resort to some other methods in these cases.

Note Added

While we were finishing this paper, we received the paper by M. Gronau and J.L. Rosner [14], who arrive at the same conclusion as our work.

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REFERENCES

- [1] For a review, see the article of H. Quinn in *Particle Data Book*, R.M. Barnett *et al.*, Phys. Rev. **D 54**, 507 (1996).
- [2] M. Kobayashi and T. Maskawa, Prog. Theor. Phys. **49**, 652 (1973).
- [3] A. J. Buras, TUM-HEP-299-97, hep-ph/9711217 ; J.L. Rosner, Nucl. Instrum. Meth. **A 408**, 308 (1998) ; A. Ali, hep-ph/9801270.
- [4] R. Godang *et al.*, Phys. Rev. Lett. **80**, 3456 (1998); J. G. Smith, COLO-HEP-395, hep-ph/9803028 ; J. Roy, Talk at ICHEP98 at Vancouver, Canada.
- [5] M. Gronau and D. London, Phys. Rev. Lett. **65**, 3381 (1990).
- [6] M. Gronau and D. London, Phys. Lett. **B253**, 483 (1991) ; M. Gronau and D. Wyler, Phys. Lett. **B265**, 172 (1991).
- [7] I. Dunietz, Phys. Lett. **B 270**, 75 (1991).
- [8] D. Atwood, I. Dunietz and A. Soni, Phys. Rev. Lett. **78**, 3257 (1997).
- [9] M. Gronau, CALT-68-2159, hep-ph/9802315.
- [10] A. Ali, G. Kramer and C.-D. Lü, hep-ph/9804363.
- [11] M. Athanas *et al.*, Cornell University Report CLNS 98-1541 (1998).
- [12] M. Gronau and J.L. Rosner, Phys. Rev. Lett. **79**, 4333 (1997).
- [13] M. Neubert and Stech, CERN-TH/97-99, HD-THEP-97-23, hep-ph/9705292.
- [14] M. Gronau and J.L. Rosner, EFI-98-29, FERMILAB-PUB-98-227-T, hep-ph/9807447.

TABLES

TABLE I. The tagging and detection efficiencies (f 's and ϵ 's) assumed to estimate the $\Delta\gamma$. See the text for the details. Numerical values are for hadron collider ($\Upsilon(4S)$).

| particles | tagging modes | observation rate (f) | efficiency (ϵ) |
|------------------------|--------------------------------------|---|---------------------------|
| K^0, \bar{K}^0 | $K_S \rightarrow \pi^+\pi^-$ | 1/3 (1/3) | 0.1 (1.0) |
| K^{*0}, \bar{K}^{*0} | $K^{*0} \rightarrow K^+\pi^-$ | 2/3 (2/3) | 0.1 (1.0) |
| $K^{*\pm}$ | $K^{*+} \rightarrow K^0\pi^+$ | 5/9 (5/9) | 0.1 (1.0) |
| | $K^{*+} \rightarrow K^+\pi^0$ | | |
| D^0, \bar{D}^0 | $D^0 \rightarrow K^-\pi^+$ | 0.25 (0.25) | 0.1 (1.0) |
| | $D^0 \rightarrow K^-\pi^+\pi^+\pi^-$ | | |
| | $D^0 \rightarrow K^-\pi^+\pi^0$ | | |
| D^\pm | $D^+ \rightarrow K^-\pi^+\pi^+$ | 0.91 (0.91) | 0.1 (1.0) |
| D_1 | $D^0 \rightarrow \pi^+\pi^-$ | $5.85 \times 10^{-3}(5 \times 10^{-2})$ | 0.1 (1.0) |
| | $D^0 \rightarrow K^+K^-$ | | |

FIGURES

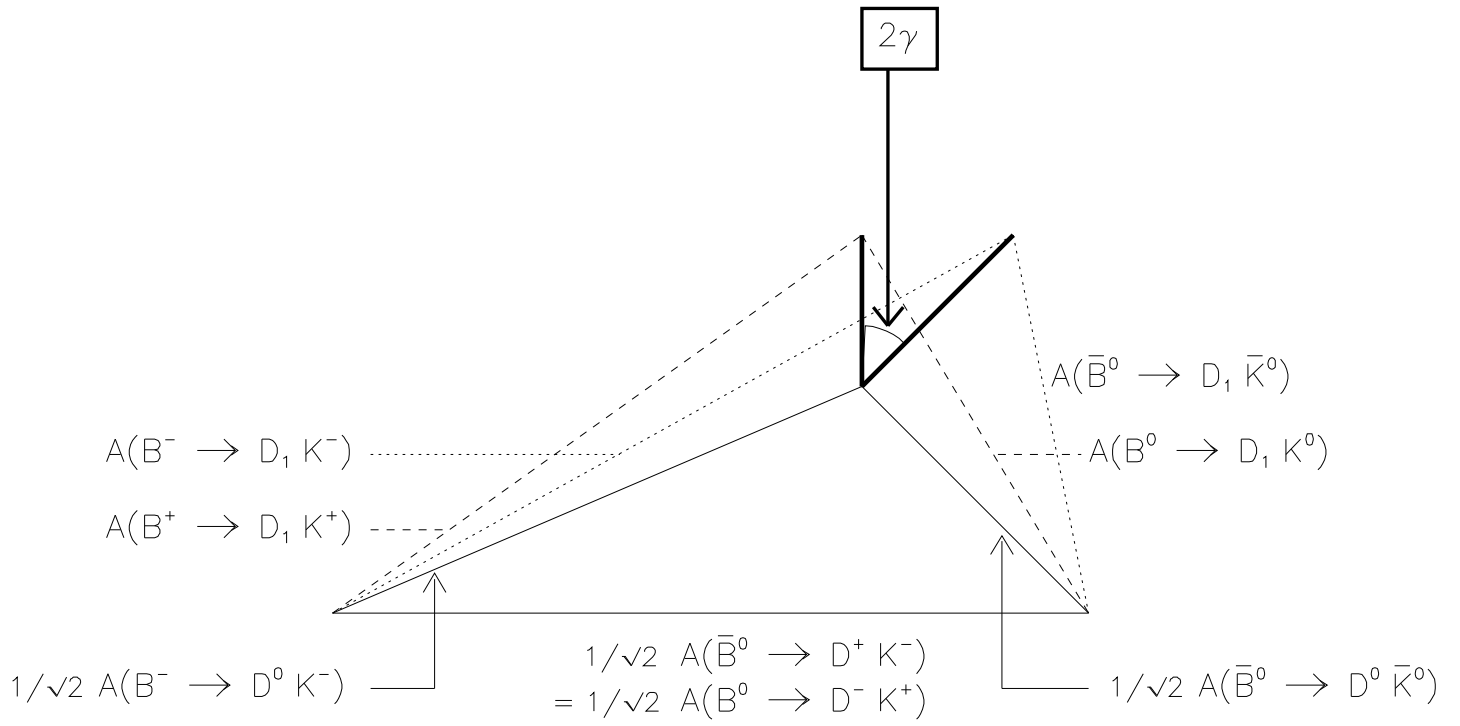


FIG. 1. Three triangles constructed from various $B \rightarrow DK$ modes

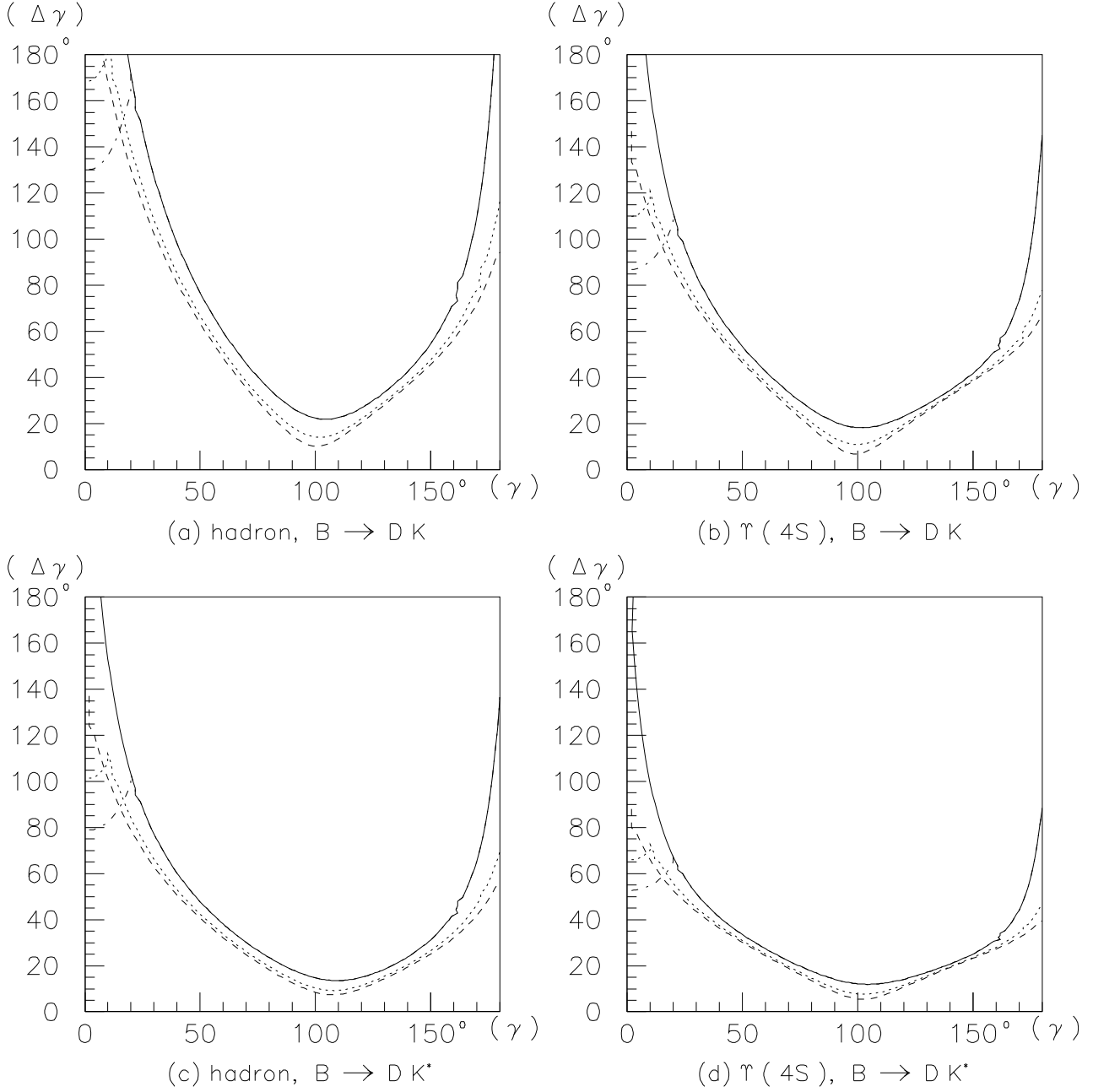


FIG. 2. The $\Delta\gamma$ (error) plot in the determination of γ assuming 10^{11} B 's at the hadron machines and 3×10^8 B 's at B factories : (a) hadron machines and $B \rightarrow DK$, (b) B factories and $B \rightarrow DK$, (c) hadron machines and $B \rightarrow DK^*$ and (d) B factories and $B \rightarrow DK^*$. ($\delta_C = 10^\circ$ fixed, real line : $\delta_{C'} = -2\delta_C$, dashed line : $\delta_{C'} = 0$, dotted line : $\delta_{C'} = \delta_C$, dashed-dotted line : $\delta_{C'} = 2\delta_C$.)