Nucleon structure functions from a chiral soliton*

H. Weigel[a†‡], L. Gamberg[b], H. Reinhardt[a] and O. Schröder[a]

[a]Institute for Theoretical Physics, Tübingen University
Auf der Morgenstelle 14, D–72076 Tübingen, Germany

[b]Department of Physics and Astronomy, University of Oklahoma
440 W. Brooks Ave, Norman, OK 73019–0225, USA

We study nucleon structure functions within the bosonized version of the Nambu–Jona–Lasinio (NJL) model in which the nucleon emerges as the soliton in the chiral field. Upon boosting to the infinite momentum frame and performing the q^2–evolution in the context of the Gottfried sum rule for electron nucleon scattering we determine the intrinsic scale μ^2 of the NJL chiral soliton. We also compute the leading twist contributions of the polarized structure functions g_1 and g_2. We compare these model predictions with experiment by evolving them from μ^2 to the scale where the data are taken. Analogously we analyze the chiral–odd structure functions h_T and h_L. Finally we generalize the treatment to flavor SU(3).

1. INTRODUCTION

The purpose of this investigation is to provide a link between two successful although seemingly unrelated pictures of baryons. On one side we have the quark parton model which describes the scaling behavior of the structure functions in deep inelastic scattering (DIS) processes. The deviations from these scaling laws are computable in the framework of perturbative QCD. On the other side we have the chiral soliton approach motivated by generalizing QCD to an arbitrary number of color degrees of freedom, N_C. For $N_C \to \infty$, QCD is equivalent to an effective meson theory. Although this theory is not explicitly known it can be modeled by assuming that at low energies only the light mesons (pions, kaons, ρ, ω) are relevant. The major building block to model the effective theory is chiral symmetry and its spontaneous breaking. Baryons emerge as non–perturbative (topological) configurations of the meson fields, the so–called solitons. The link between these two pictures can be established by computing structure functions within a chiral soliton model for the nucleon from the hadronic tensor

$$W^{ab}_{\mu\nu}(q) = \frac{1}{4\pi}\int d^4\xi\, \mathrm{e}^{iq\cdot\xi} \times \langle N(P)| \left[J^a_\mu(\xi), J^{b\dagger}_\nu(0)\right] |N(P)\rangle \ , \quad (1)$$

which describes the strong interaction part of the DIS cross–section. In eq (1) $|N(P)\rangle$ refers to the nucleon state with momentum P and $J^a_\mu(\xi)$ to the hadronic current suitable for the process under consideration. In most soliton models the current commutator (1) remains intractable. However, the Nambu and Jona–Lasinio (NJL) model [1] of quark flavor dynamics contains simple current operators since derivatives of the quarks fields only appear in form of a free Dirac Lagrangian. Most importantly, the bosonized [2] version of the NJL–model contains soliton solutions [3, 4]. Here we confine ourselves to presenting the key issues and results of the structure function calculation; details may be traced from our recent papers [5, 6, 7, 8]. For subsequent studies see also [9].

2. THE NUCLEON IN THE NJL CHIRAL SOLITON MODEL

The NJL–model under consideration contains a chirally symmetric quartic quark interaction in the scalar–pseudoscalar channel. After bosonization the action reads [2]

$$\mathcal{A} = \mathrm{Tr}\ln_\Lambda \left(i\partial\!\!\!/ - mU^{\gamma_5}\right) + \frac{m_0 m}{4G}\mathrm{tr}\left(U + U^\dagger - 2\right) . \quad (2)$$

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†Heisenberg–Fellow

‡Address after Sept. 1st 1998: Center for Theoretical Physics, Laboratory for Nuclear Science and Dept. of Physics, Massachusetts Institute of Technology, Cambridge, Mass. 02139.

The pion fields $\boldsymbol{\pi}$ are contained in the chiral field $U = \exp(i\boldsymbol{\tau}\cdot\boldsymbol{\pi}/f_\pi)$. In eq (2) tr denotes discrete flavor trace while Tr also includes the functional trace. The parameters of the model are the coupling constant G, the current quark mass m_0 and the UV cut–off Λ. These are adjusted to the pion mass $m_\pi = 135\text{MeV}$ and decay constant $f_\pi = 93\text{MeV}$. This leaves one parameter undetermined which we express in terms of the constituent quark mass m. It arises as the solution to the Schwinger–Dyson (gap) equation and characterizes the spontaneous breaking of chiral symmetry.

An energy functional for non–perturbative, static field configurations $U(\boldsymbol{r})$ is derived from (2). It can be expressed as a regularized sum of single quark energies ϵ_μ of the associated one–particle Dirac Hamiltonian in the background of $U(\boldsymbol{r})$. The distinct level (v), which is strongly bound is referred to as the valence quark state. Its explicit occupation guarantees unit baryon number. The classical soliton $U_{\rm cl}(\boldsymbol{r})$ is determined by self–consistently minimizing the energy functional. To generate states with nucleon quantum numbers the (unknown) time dependent field configuration is approximated by elevating the zero modes to time dependent collective coordinates

$$U(\boldsymbol{r},t) = A(t)U_{\rm cl}(\boldsymbol{r})A^\dagger(t),\ A(t)\in {\rm SU}(2)\,. \qquad (3)$$

Upon canonical quantization the angular velocities, $\boldsymbol{\Omega} = -i{\rm tr}(\boldsymbol{\tau}A^\dagger\dot{A})$, are replaced by the spin operator $\boldsymbol{J}$ via $\boldsymbol{\Omega} = \boldsymbol{J}/\alpha^2$ with α^2 being the moment of inertia. To compute nucleon properties the action (2) is expanded in powers of $\boldsymbol{\Omega}$ corresponding to an expansion in $1/N_C$. In particular the valence quark wave–function $\Psi_{\rm v}(\boldsymbol{x})$ acquires a linear correction

$$\begin{aligned}\Psi_{\rm v}(\boldsymbol{x},t) &= {\rm e}^{-i\epsilon_{\rm v}t}A(t)\psi_{\rm v}(\boldsymbol{x}) \qquad (4)\\ &= {\rm e}^{-i\epsilon_{\rm v}t}A(t)\left\{\Psi_{\rm v}(\boldsymbol{x}) + \sum_{\mu\ne{\rm v}}\Psi_\mu(\boldsymbol{x})\frac{\langle\mu|\boldsymbol{\tau}\cdot\boldsymbol{\Omega}|{\rm v}\rangle}{2(\epsilon_{\rm v}-\epsilon_\mu)}\right\}.\end{aligned}$$

Here $\psi_{\rm v}(\boldsymbol{x})$ refers to the spatial part of the body–fixed valence quark wave–function with the rotational corrections included.

3. STRUCTURE FUNCTIONS FROM THE SOLITON

In order to extract the leading twist contributions to the structure function one computes the hadronic tensor in the Bjorken limit

$$\begin{aligned} q_0 &= |\boldsymbol{q}| - M_N x \quad \text{with} \quad |\boldsymbol{q}|\to\infty\,,\\ x &= -q^2/2P\cdot q \quad \text{fixed}\,. \end{aligned} \qquad (5)$$

For localized field configurations, such as the soliton, the symmetric part of hadronic tensor (which contains the unpolarized structure functions) then reads [10],

$$\begin{aligned} W^{lm}_{\{\mu\nu\}}(q) &= \zeta\int\frac{d^4k}{(2\pi)^4}\ S_{\mu\rho\nu\sigma}\ k^\rho\ {\rm sgn}\,(k_0)\ \delta\left(k^2\right)\\ &\times\int dt\ {\rm e}^{i(k_0+q_0)t}\int d^3x_1\int d^3x_2\ {\rm e}^{-i(k+q)\cdot(x_1-x_2)}\\ &\times\langle N|\Big\{\bar{\hat{\Psi}}(\boldsymbol{x}_1,t)t_l t_m\gamma^\sigma\hat{\Psi}(\boldsymbol{x}_2,0)\\ &\qquad -\bar{\hat{\Psi}}(\boldsymbol{x}_2,0)t_m t_l\gamma^\sigma\hat{\Psi}(\boldsymbol{x}_1,t)\Big\}|N\rangle. \end{aligned} \qquad (6)$$

Note that the quark spinors are functionals of the soliton. Here $S_{\mu\rho\nu\sigma} = g_{\mu\rho}g_{\nu\sigma} + g_{\mu\sigma}g_{\nu\rho} - g_{\mu\nu}g_{\rho\sigma}$ and $\zeta = 1(2)$ for the structure functions associated with the vector (weak) current and t_m is a suitable isospin matrix. The matrix element between the nucleon states ($|N\rangle$) is taken in the space of the collective coordinates.

The valence quark approximation ignores the vacuum polarization in (6), *i.e.* the quark field operator $\hat{\Psi}$ is substituted by the valence quark contribution (4). For constituent quark mass under consideration $m = 400\text{MeV}$ this approximation is well justified since this level provides the dominant share to static observables [3, 4]. The structure function $F_2(x)$ is obtained from (6) by an appropriate projection[4]. Then eq (6) yields the structure functions in the nucleon rest frame (RF). In order to obtain proper support these are transformed to the infinite momentum frame (IMF) [11, 12]:

$$f_{\rm IMF} = \frac{1}{1-x}\,f_{\rm RF}\left(-\ln(1-x)\right)\,. \qquad (7)$$

Adopting the point of view that the model approximates QCD at the low scale μ^2 we apply a leading order DGLAP evolution to $f_{\rm IMF}$. Demanding a *best agreement* with the data at the experimental scale Q^2 for the linear combination entering the Gottfried sum rule ($F_2^{ep} - F_2^{en}$) determines $\mu^2 = 0.4\text{GeV}^2$. In figure 1 the resulting structure function is compared to the data [13]. Apparently

[4] In the Bjorken limit the Callan–Gross relation $F_2(x) = 2xF_1(x)$ is satisfied.

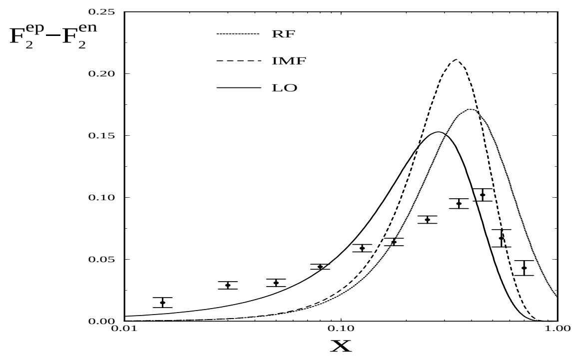

Figure 1. The unpolarized structure function entering the Gottfried sum rule. RF: rest frame, IMF: boosted to the infinite momentum frame, LO: leading order QCD evolution to $Q^2 = 4\text{GeV}^2$.

the gross features are reproduced. Moreover the integral of the Gottfried sum rule

$$S_G = \int_0^\infty \frac{dx}{x} \left(F_2^{ep} - F_2^{en}\right) = \begin{cases} 0.29\ , & m = 400\text{MeV} \\ 0.27\ , & m = 450\text{MeV} \end{cases} \tag{8}$$

agrees reasonably well with the empirical value $S_G = 0.235 \pm 0.026$ [13]. In particular the deviation from the naïve value (1/3) [14] is in the direction demanded by experiment.

In figure 2 we display the analogous results for the polarized structure functions g_1 and g_2 which are obtained from the antisymmetric piece of the hadronic tensor. Details of the calculations are presented in ref [7]. Here we wish to mention that g_2 contains both twist–2 and twist–3 pieces which are treated separately under the QCD evolution. We also stress that the starting point $\mu^2 = 0.4\text{GeV}^2$ of this evolution is no longer a free parameter. Apparently the model reproduces the empirical data quite well, although the associated error bars are sizable. Related quantities are the chiral odd structure functions $h_T(x)$ and $h_L(x)$. These may similarly be defined to the hadronic tensor (1), however, as the correlation between a current and the scalar density $\bar{\Psi}\Psi$. Eventually these structure functions will be obtained from the fragmentation region of DIS. The corresponding NJL soliton calculation is reported in ref [16] and the results are shown in figure 3.

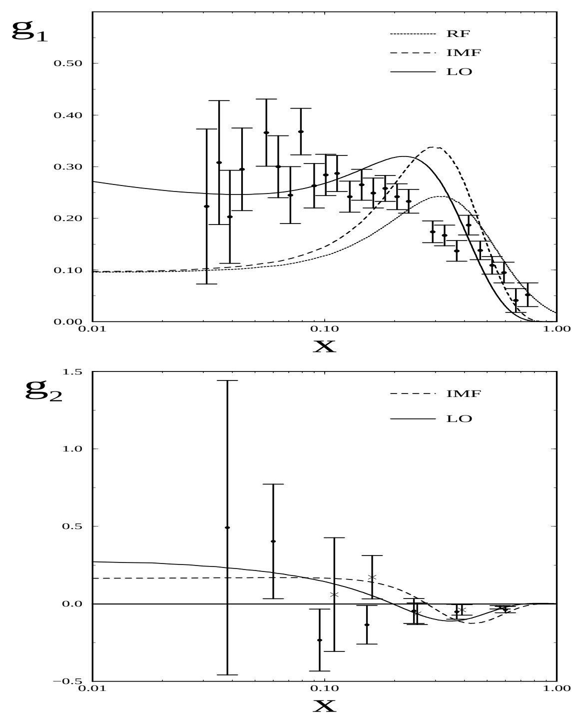

Figure 2. The polarized structure functions g_1 (top) and g_2 (bottom) after projection and QCD evolution. Data from SLAC [15].

4. EXTENSION TO FLAVOR SU(3)

The soliton picture has the celebrated feature that it can be extended to flavor SU(3) (for a review see [17]) by generalizing to $A(t) \in \text{SU}(3)$ in eq (3). A detailed discussion of the three flavor NJL chiral soliton model is given in ref [18]. The resulting structure function $g_1(x)$ for the proton as calculated in [8] is shown in figure 4. Apparently the differences to flavor SU(2) are only minor and (after projection and evolution) the data are reasonably reproduced. Moreover this model allows a projection to the strangeness contribution $g_1^{(s)}$. Also this is shown in figure 4. Apparently the smallness of the first moment of $g_1^{(s)}$ is due to cancellations between positive and negative pieces.

5. CONCLUSIONS

We have presented a calculation of nucleon structure functions within a chiral soliton model. We have argued that the soliton approach to the bosonized version of the NJL–model is most suitable since (formally) the required current operator is identical to the one in a free Dirac theory. Hence

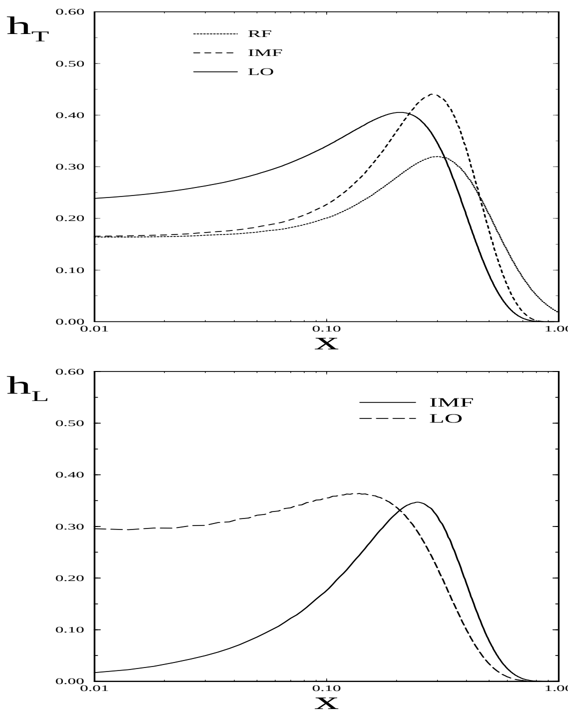

Figure 3. Same as fig 2 for the chiral odd structure functions, evolved to $Q^2 = 4\mathrm{GeV}^2$.

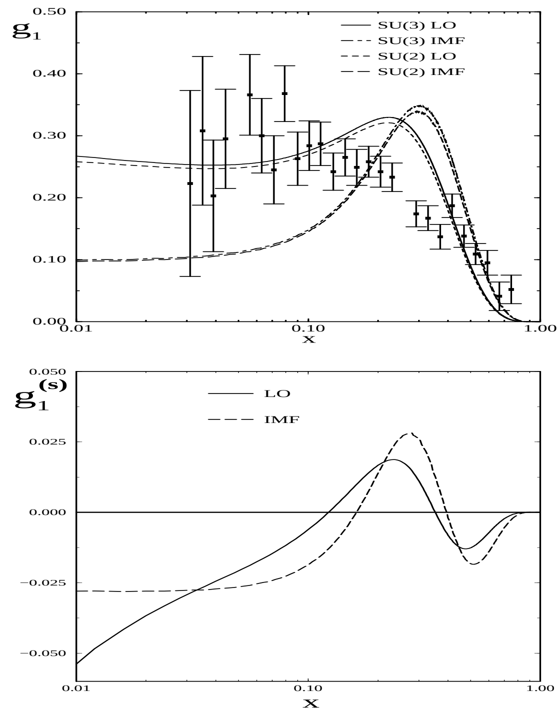

Figure 4. The polarized structure functions g_1 in the three flavor model. Top: Comparison between two and three flavor model. Bottom: Strangeness contribution.

there is no need to approximate the current operator by *e.g.* performing a gradient expansion. Although the calculation contains a few (well–motivated) approximations it reproduces the gross features of the data after taking projection and evolution into account. This happens to be the case for both the polarized as well as the unpolarized structure functions.

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