

# Electromagnetic Transition Form Factor of Pseudoscalar Meson and $\eta - \eta'$ Mixing

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## Abstract

The electromagnetic transition form factors of  $\eta$  and  $\eta'$  are calculated in the light-cone perturbation theory. We show that it is unreliable to determine the  $\eta - \eta'$  mixing angle without any additional normalization conditions other than their decay widths to two photons. The possible intrinsic  $c\bar{c}$  component in the flavor singlet is investigated. The heavy quark pair has distinct properties from the light ones in electromagnetic transition processes of pseudoscalar mesons. It is possible to explore the size of  $c\bar{c}$  component and our numerical results disfavor a large portion of  $c\bar{c}$  component.

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## I. INTRODUCTION

The recent CLEO experiments related with  $\eta$  and  $\eta'$  supply possibilities to extract the information about the structures of these two pseudoscalar mesons. For example, the CLEO collaboration [1,2] has reported very large branching ratios for inclusive  $\eta'$  production

$$B(B^\pm \rightarrow \eta' X_s; 2.0\text{GeV} \leq P_{\eta'} \leq 2.7\text{GeV}) = (7.5 \pm 1.5 \pm 1.1) \times 10^{-4}, \quad (1)$$

and for the exclusive decay  $B^\pm \rightarrow \eta' K^\pm$

$$B(B^\pm \rightarrow \eta' K^\pm) = (7.8_{-2.2}^{+2.7} \pm 1.0) \times 10^{-5}. \quad (2)$$

To explain the abnormally large production of  $\eta'$  in the standard model, either large portion of intrinsic  $|c\bar{c}\rangle$  component in  $\eta'$  [3,4] or large coupling of  $\eta'$  to  $gg$  [5], even both of them, must be concluded. At the same time, the data of the  $\pi\gamma$ ,  $\eta\gamma$  and  $\eta'\gamma$  transition form factors at higher energies reported by CLEO [6] suggest that  $\eta'$  may have very different non-perturbative properties from  $\pi^0$  and  $\eta$ , while the latter two have similar wave functions. It is reasonable because the physical  $\eta$  and  $\eta'$  states consist dominantly of flavor SU(3) octet  $\eta_8$  and singlet  $\eta_0$ , respectively. The usual mixing scheme reads

$$|\eta\rangle = \cos(\theta)|\eta_8\rangle - \sin(\theta)|\eta_0\rangle, \quad (3a)$$

$$|\eta'\rangle = \sin(\theta)|\eta_8\rangle + \cos(\theta)|\eta_0\rangle. \quad (3b)$$

The mixing angle can be evaluated in various ways but a standard procedure involves the diagonalization of the  $\eta$  and  $\eta'$  mass matrix, which at the lowest order in the chiral perturbation theory yields a mixing angle  $\theta \approx -10^\circ$ . The inclusion of  $\mathcal{O}(p^4)$  corrections [7] to the relation results in significant changes, which yields  $\theta \cong -20^\circ$ .

At the same time, the  $\eta\gamma$  and  $\eta'\gamma$  transition form factors have received extensive theoretical attentions recently. Jakob *et al* [8] extracted the mixing angle from the  $\eta\gamma$  and  $\eta'\gamma$  transition form factors in the modified hard scattering approach [9], which takes into account the transverse degrees of freedom as well as the Sudakov form factor, and got  $\theta = -18^\circ \pm 2^\circ$  with a Gaussian wave function in transverse part. In their approach the chiral anomaly for Goldstone boson is used to determine the decay constants of pseudoscalar mesons, including the  $\eta'$  meson. The parameters related to transverse degree of freedom for all three mesons,  $\pi$ ,  $\eta$  and  $\eta'$ , are assumed to be identical for simplicity. With the same approach, a more general mixing scheme involving two mixing angles was investigated [10] recently. The charm decay constant of the  $\eta'$  meson was estimated to be within the range of  $-65 \text{ MeV} \leq f_{\eta'}^c \leq 15 \text{ MeV}$ . Choi and Ji [11] showed their form factor prediction for both  $\theta = -10^\circ$  and  $-23^\circ$  by using a simple relativistic constituent quark model with a Gaussian wave function motivated from light-cone quantization and the Brodsky-Huang-Lepage (BHL) prescription, which connects the oscillator wave function in the center of mass frame with the light-cone wave function [12]. They fixed the Gaussian parameter by radiative decay of pseudoscalar and vector mesons and found that they are in good agreement with experimental data up to a rather large  $Q^2$ . Anisovich *et al* [13] studied the meson-photon transition form factor by assuming a nontrivial hadron-like  $q\bar{q}$  structure of the photon in the soft region. The data on the  $\pi^0\gamma$  form factor are used to fix the soft photon wave function. Assuming the universality of pseudoscalar meson wave function in the ground-state they found that the

$\eta\gamma$  and  $\eta'\gamma$  transition form factors and the branching ratios for  $\eta \rightarrow \gamma\gamma$  and  $\eta' \rightarrow \gamma\gamma$  are in perfect agreement with the data.

It is interesting that an analysis [14] performed in the standard light-cone formalism [15–17] shows that there is still a gap between the theoretical calculations and the data at currently accessible energies. The conclusion is reasonable since only the lowest Fock state and the lowest order contributions (without radiative correction) are taken into account. To fit the data, some efforts must be made to account for the higher Fock state and higher order contributions. The soft photon wave function adopted in Ref. [13], which is extracted from the  $\pi^0\gamma$  data, may be considered as a good phenomenological description for higher order contributions presenting at the soft photon vertex. The soft photon may behave much like a hadron whose wave function evolves by exchanging gluons between the quark-antiquark pair. On the other hand, the normalization and the parameters of the pionic wave function are fixed when only the valence Fock state is taken into account in the light-cone Fock state expansion. The obtained expression is for the valence Fock state. If we go beyond the light-cone Fock state expansion by giving up the constraint from  $\pi \rightarrow \mu\nu$ , to which only the  $|q\bar{q}\rangle$  component contributes, and adjust the parameters in the wave function in a reasonable region, a perfect agreement with the data can be obtained. The same parameter fits the data of the  $\eta\gamma$  and  $\eta'\gamma$  transition form factors, too. A light-cone constituent quark model calculation [11] also justified that some of the higher Fock state contributions in the light-cone perturbation theory may be expressed by a light-cone constituent quark model with only  $q\bar{q}$  being taken into account.

With the aids of models accounting for the higher Fock state and higher order contributions, informations about the wave functions of the  $\eta$  and  $\eta'$  mesons can be extracted from the data of the  $\eta\gamma$  and  $\eta'\gamma$  transition from factors. In section II, a brief review on the  $\pi\gamma$  transition will be given to explain some subtle aspects, including the Melosh rotation connecting the light-cone wave function and the ordinary equal-time wave function, the parameter fixing and the possible corrections. In Sec. III, analyses on the  $\eta\gamma$  and  $\eta'\gamma$  transitions will be presented. By introducing a  $SU_f(3)$  broken wave function, it is found that the theoretical predictions fit the data very well. At the same time, our results are not sensitive to the mixing angle because similar mixing patterns present in both the form factors and the  $\eta(\eta') \rightarrow \gamma\gamma$  amplitudes, while the latter are used as normalization conditions. Without additional normalization conditions being used, determining the mixing angle by fitting the data of the  $\eta\gamma$  and  $\eta'\gamma$  transition form factors is not reliable. The flavor singlet  $|\eta_0\rangle$  may have large portions of  $|c\bar{c}\rangle$  and  $|gg\rangle$  components. In Sec. IV we discuss the possible intrinsic  $|c\bar{c}\rangle$  contributions, which have distinct  $Q^2$  behavior from the light quark contributions. Our results disfavor a large portion of  $|c\bar{c}\rangle$  component in the  $\eta'$  wave function expected in Refs. [3,4] and such a conclusion is in agreement with a number of recent investigations from other viewpoints [18–21]. The last section contains the conclusions and summary.

## II. THE $\pi\gamma$ TRANSITION FORM FACTOR

The approach adopted here for the  $\eta\gamma$  and  $\eta'\gamma$  transition form factors will be very similar to that of  $\pi\gamma$  [14,15] A brief review of the  $\pi\gamma$  transition form factor will help to reveal some subtle aspects in this approach.

An analysis on the  $\pi\gamma$  transition in the light-cone perturbation theory, based on light-cone quantization and the light-cone Fock state expansion, was first investigated by Brodsky and Lepage [15]. Factorization into a soft part (the pionic wave function  $\psi(x, k_i)$ ) and a hard part (the hard scattering amplitude  $T_H$ ) is verified since soft interactions between initial and final quarks in  $T_H$  all cancel due to the fact that hadron states are color singlets. Thus the form factor can be written in a factorization form as

$$F_{\pi\gamma}(Q^2) = 2\sqrt{n_c}(e_u^2 - e_d^2) \int_0^1 dx \int \frac{d^2k_\perp}{16\pi^3} \psi(x, k_\perp) T_H(x, k_\perp, Q), \quad (4)$$

where the hard scattering amplitude  $T_H$  is given by

$$T_H(x, k_\perp, Q) = \frac{q_\perp \cdot (x_2 q_\perp + k_\perp)}{q_\perp^2 (x_2 q_\perp + k_\perp)^2} + (1 \leftrightarrow 2), \quad (5)$$

with  $x_2 = 1 - x$  and  $q_\perp^2 = Q^2$ . When  $Q^2$  is large, the quark transverse momentum  $k_\perp$  can be omitted in  $T_H$  compared with  $q_\perp$ . The integral over  $k_\perp$  results in a distribution amplitude (DA),  $\phi(x, Q) = \int \frac{d^2k_\perp}{16\pi^3} \psi(x, k_\perp)$ , which evolves to the asymptotic form,  $\sqrt{3}f_\pi x(1-x)$ , as  $Q^2 \rightarrow \infty$ . Thus we get the asymptotic prediction for the  $\pi\gamma$  transition form factor,  $F_{\pi\gamma}(Q^2 \rightarrow \infty) = \frac{2f_\pi}{Q^2}$ . However, the transverse momentum is not negligible at currently accessible energies to be compared with the experimental data [14]. Taking into account the Melosh rotation [22,23], which connects the equal-time spin and the light-cone spin, the light-cone wave function of a pseudoscalar meson can be written as [24–26]

$$\Psi_R(x, k_\perp) = \frac{1}{\sqrt{2(m^2 + k^2)}} \left[ m (\chi_1^\uparrow \chi_2^\downarrow - \chi_1^\downarrow \chi_2^\uparrow) - (k_1 + ik_2) \chi_1^\downarrow \chi_2^\downarrow - (k_1 - ik_2) \chi_1^\uparrow \chi_2^\uparrow \right] \psi(x, k_\perp), \quad (6)$$

where the index 1 (2) means the quark (antiquark) and  $\uparrow$  ( $\downarrow$ ) is the light-cone helicity. The Melosh rotation is one of the most important ingredients of the light-cone quark model and it can be applied to explain the “proton spin puzzle” [27]. The spacial wave function  $\psi(x, k_\perp)$  is modeled as [12]

$$\psi(x, k_\perp) = A \exp \left[ -b^2 \frac{k_\perp^2 + m^2}{x(1-x)} \right] \quad (7)$$

with the Brodsky-Huang-Lepage prescription which connects the oscillator wave function in the equal-time frame with that in the light-cone frame. Such an exponential ansatz for the wave function have a simple form, direct physical explanation (oscillator form in center of mass frame), and good end-point behaviors. From the exponential form (7) follows the distribution amplitude

$$\phi(x) = x(1-x) \frac{A}{16\pi^2 b^2} \exp \left[ -b^2 \frac{m^2}{x(1-x)} \right], \quad (8)$$

which is very close to the asymptotic form [24]. It is well known that the DA evolves very slowly. For a DA close to the asymptotic form, such as eq. (8), the evolution makes little difference. Furthermore, it is difficult to take into account the evolution while the

transverse momentum, and thus the wave function but not DA, is involved. We will just neglect the evolution in the following calculations for simplicity. Many recent analyses on pionic non-perturbative properties [8,24,28,29] favor the asymptotic form of DA rather than the Chernyak-Zhitnitsky (CZ) form [30]. Therefore, the BHL model for the wave function is favored by fitting experimental data.

In the pionic case two important constraints have been derived [12] from the  $\pi \rightarrow \mu\nu$  and  $\pi \rightarrow \gamma\gamma$  decay amplitudes:

$$\int_0^1 dx \int \frac{d^2k_\perp}{16\pi^3} \psi(x, k_\perp) = \frac{f_\pi}{2\sqrt{3}}, \quad (9)$$

$$\int_0^1 dx \psi(x, k_\perp = 0) = \frac{\sqrt{3}}{f_\pi}. \quad (10)$$

Firstly, it is notable that  $m$  in the non-perturbative wave function, i.e., Eqs. (6) and (7), is the constituent quark mass while contributions from higher helicity states ( $\lambda = \lambda_1 + \lambda_2 = \pm 1$ ) will be proportional to the current quark mass in the corresponding hard scattering amplitude and thus can be ignored due to the fact that helicity must be flipped at one vertex. However, it is not the case for the heavy quark whose current quark mass is almost the same as the constituent quark mass. Secondly, the gauge invariance requires that the  $|q\bar{q}\rangle$  Fock state in pion contributes exactly one half to the full decay amplitude of  $\pi \rightarrow \gamma\gamma$  [12]. Therefore, higher Fock states must have contributed the other half to  $F_{\pi\gamma}$  as  $Q^2 \rightarrow 0$ . While  $Q^2$  increases, the contributions from higher Fock states reduce. It is not strange that the numerical results in Ref. [14] are below the data. At currently accessible energy scale, the  $|q\bar{q}\rangle$  component contributes about 80 ~ 90% to the  $\pi\gamma$  transition form factor. Thirdly, the one-loop radiative correction was calculated in Ref. [31]. If the asymptotic distribution amplitude is used, the size of the one-loop correction is independent of the factorization scale  $\mu$  and less than 15% for  $Q^2 > 3$  GeV. Otherwise, different non-perturbative wave functions result in different sizes of corrections. An appropriate choice of scale  $\mu$  may reorganize the expansion series to reduce the one-loop correction. Numerical analyses [29] show that  $\mu = Q$  provides a good choice of the factorization scale. It is accompanied by small one-loop corrections even for a broad distribution amplitude of CZ type. Since the correction is small, we will not go beyond the lowest order calculation to avoid such a complexity in the following. Finally, taking into account the Melosh rotation in the pionic wave function changes the light-cone perturbation prediction only for a small amount, because the constraint (9) should be changed as

$$\int_0^1 dx \int \frac{d^2k_\perp}{16\pi^3} \psi(x, k_\perp) \sqrt{\frac{m^2}{m^2 + k_\perp^2}} = \frac{f_\pi}{2\sqrt{3}}, \quad (11)$$

and constraint (10) remains unchanged. As a consequence, the parameters in the wave function should be different. The numerical results are shown in Fig. 1 with parameters  $b^2 = 0.848$  GeV<sup>-2</sup>,  $A = 32.7$  GeV<sup>-1</sup> from the constraints (9) and (10) (the dashed line) and  $b^2 = 0.414$  GeV<sup>-2</sup>,  $A = 25.6$  GeV<sup>-1</sup> from the constraints (11) and (10) (the solid line). The quark mass  $m$  does not affect a lot and we adopt  $m = 300$  MeV here.

### III. THE $\eta\gamma$ AND $\eta'\gamma$ FORM FACTORS IN $SU_F(3)$

The transition form factor at zero momentum transfer is connected with the two-photon decay width by

$$F_{R\gamma}(0) = \sqrt{\frac{4}{\alpha^2\pi M_R^3}}\Gamma(R \rightarrow \gamma\gamma), \quad (12)$$

where  $R$  represents a pseudoscalar meson. From the axial anomaly in the chiral limit of QCD, we have

$$\lim_{Q^2 \rightarrow 0} F_{\gamma R}(Q^2) = \frac{1}{4\pi^2 f_R} \quad (13)$$

for  $\pi^0$  and  $\eta$ . This prediction may not hold for  $\eta'$  due to the larger mass of s-quark. In addition, it might be broken because  $\eta'$  is an unlikely candidate for the Goldstone boson. We will not relate the wave function with decay constant in this work. To be consistent with the experimental analysis, we use the same values adopted in Ref. [6]:

$$\begin{aligned} \Gamma(\pi \rightarrow \gamma\gamma) &= 7.74 \text{ eV}, & f_\pi &= 92.3 \text{ MeV}; \\ \Gamma(\eta \rightarrow \gamma\gamma) &= 0.463 \text{ keV}, & f_\eta &= 97.5 \text{ MeV}; \\ \Gamma(\eta' \rightarrow \gamma\gamma) &= 4.3 \text{ keV}. \end{aligned}$$

Unlike the pion decay, only the constraint (10) is available to normalize the amplitude for the  $\eta\gamma$  and  $\eta'\gamma$  transitions, i.e. to determine two parameters in Eq. (16) with the mixing angle  $\theta$  as an input. The CLEO collaboration reported their pole fit results as  $\Lambda_\pi = 776 \pm 10 \pm 12 \pm 16$  MeV,  $\Lambda_\eta = 774 \pm 11 \pm 16 \pm 22$  MeV, and  $\Lambda_{\eta'} = 859 \pm 9 \pm 18 \pm 20$  MeV. The L3 collaboration [32] also presented their pole mass,  $\Lambda_{\eta'} = 900 \pm 46 \pm 22$  MeV. It suggests that the non-perturbative properties of  $\pi$  and  $\eta$  are very similar. It is also consistent with the physical intuition since both  $\pi$  and  $\eta$  are in  $SU_f(3)$  octet and are pseudo-Goldstone particles. It is a natural choice to set  $b_8 = b_\pi$ . The singlet should have different properties. But for simplicity, we will let  $b_0 = b_\pi$  at first. Different choices for  $b_0$  will be discussed, too.

As the case of  $\pi\gamma$ , the light-cone predictions for the  $\eta\gamma$  and  $\eta'\gamma$  transition form factors are smaller than the data while only the lowest Fock state and the lowest order contributions are taken into account. To compare them with the data, we assume that higher Fock state and higher order contributions have similar fraction sizes in the transition form factors of all three pseudoscalar particles. These contributions can be estimated from the  $\pi\gamma$  form factor and included into the  $\eta\gamma$  and  $\eta'\gamma$  form factors. The pole form

$$F_{\pi\gamma}^{pole}(Q^2) = \frac{F_{\pi\gamma}(0)}{1 + Q^2/\Lambda_\pi^2} \quad (14)$$

is used as the experimental value. The  $\eta\gamma$  and  $\eta'\gamma$  form factors, after this correction, are obtained as

$$F_{R\gamma}(Q^2) = F_{R\gamma}^{LC}(Q^2) \frac{F_{\pi\gamma}^{pole}(Q^2)}{F_{\pi\gamma}^{LC}(Q^2)}, \quad (15)$$

where  $F_{R\gamma}^{LC}(Q^2)$  is the theoretical prediction in the light-cone calculation with only the lowest Fock state and the lowest order contributions being taken into account.

For certain circumstances,  $SU_f(3)$  is not a good symmetry due to the large value of the  $s$  quark mass. The  $SU_f(3)$  broken form of wave functions for flavor singlet  $\eta_0$  and octet  $\eta_8$  can be modeled as

$$\eta_0 = A_0 \frac{1}{\sqrt{3}} \left[ \exp\left(-b_0^2 \frac{m_q^2 + k_\perp^2}{x_1 x_2}\right) (u\bar{u} + d\bar{d}) + \exp\left(-b_0^2 \frac{m_s^2 + k_\perp^2}{x_1 x_2}\right) s\bar{s} \right], \quad (16a)$$

$$\eta_8 = A_8 \frac{1}{\sqrt{6}} \left[ \exp\left(-b_8^2 \frac{m_q^2 + k_\perp^2}{x_1 x_2}\right) (u\bar{u} + d\bar{d}) - 2 \exp\left(-b_8^2 \frac{m_s^2 + k_\perp^2}{x_1 x_2}\right) s\bar{s} \right] \quad (16b)$$

from the BHL model.

The transition form factor of  $SU(3)$  singlet or octet is

$$F_{i\gamma}^{LC}(Q^2) = 2\sqrt{2n_c} \sum_{q=u,d,s} C_i^q \int_0^1 dx \int \frac{d^2 k_\perp}{16\pi^3} \sqrt{\frac{m_q^2}{m_q^2 + k_\perp^2}} \psi_i^q(x, k_\perp) T_H^q(x, k_\perp, Q), \quad (17)$$

where  $i = 0$  or  $8$  presents the singlet or octet. The charge factors are  $C_8^u = e_u^2/\sqrt{6}$ ,  $C_8^d = e_d^2/\sqrt{6}$ ,  $C_8^s = -2e_s^2/\sqrt{6}$ ,  $C_0^u = e_u^2/\sqrt{3}$ ,  $C_0^d = e_d^2/\sqrt{3}$ , and  $C_0^s = e_s^2/\sqrt{3}$ . The hard scattering amplitude  $T_H^q$  is the same as Eq. (5) and  $\psi_i^q(x, k_\perp)$  is the BHL wave function, Eq. (7), for each flavor quark with the corresponding  $A_i$ ,  $m_q$  and  $b_i$  as input parameters. The quark mass  $m_q$  in the factor from the Melosh rotation will be adopted as  $m_{u,d} = 300$  MeV or  $m_s = 450$  MeV, depending on the flavor in  $\psi_i^q(x, k_\perp)$ .

Summing over all flavors, We obtain the transition form factors of physical  $\eta$  and  $\eta'$  states from Eq. (4)

$$F_{\eta\gamma}^{LC}(Q^2) = \cos(\theta) F_{8\gamma}^{LC}(Q^2) - \sin(\theta) F_{0\gamma}^{LC}(Q^2), \quad (18a)$$

$$F_{\eta'\gamma}^{LC}(Q^2) = \sin(\theta) F_{8\gamma}^{LC}(Q^2) + \cos(\theta) F_{0\gamma}^{LC}(Q^2), \quad (18b)$$

which, to ensure the feasibility of perturbation theory, should be valid only for large  $Q^2$ . While fitting the data, we will choose only the data points with  $Q^2 \geq 2.94$  GeV<sup>2</sup>. It is interesting to note that the form factors  $F_{\eta\gamma}^{LC}(Q^2)$  and  $F_{\eta'\gamma}^{LC}(Q^2)$  follow the same pattern of mixing as the state, Eq. (3).

As  $Q^2 \rightarrow 0$ , the gauge invariance requires that Eq. (18) contributes exact one half to  $F_{R\gamma}(0)$ . Inputting a mixing angle  $\theta$ ,  $A_1$  and  $A_8$  can be fixed by  $F_{R\gamma}(0)$ . The theoretical predictions obtained from Eq. (15) are shown in Fig. 2. The best fit value of  $\theta$  is  $-24^\circ$ . However, we find that the differences between different choices of  $\theta$  are so small that it is in fact unreliable to determine the mixing angle, e.g.  $\theta = -14^\circ$  can not be excluded by the data. The reason is that  $F_{R\gamma}(0)$ , which we use as the normalization condition, shares the same mixing mechanism as the form factor  $F_{R\gamma}(Q^2)$ . The only difference comes from the different contributions between the  $s$  quark and the  $u(d)$  quark at different  $Q^2$ . If  $m_s = m_{u(d)}$ , the mixing will be meaningless in this approach. The same conclusion also holds as  $Q^2 \rightarrow \infty$ , while all quarks can be treated as massless. Since the perturbation calculations are only valid for large  $Q^2$ , this minor difference is negligible. At present, neither the data nor the theoretical approach has reached the accuracy to distinguish the  $s$  or  $u(d)$  quark in the transition form factors of pseudoscalar mesons. Therefore, unless

other normalization conditions (e.g.  $D_s \rightarrow \eta l \nu / \eta' l \nu$ ,  $\eta' \rightarrow \omega \gamma$  and  $\omega \rightarrow \eta \gamma$ , and so on.) are involved, it is unreliable to determine the mixing angle from the data of the transition form factors. Inclusion of other normalization conditions brings additional uncertainties and goes beyond the present work.

#### IV. THE INTRINSIC $c\bar{c}$ COMPONENT

The ‘‘intrinsic quark’’ is one of the novel properties for hadrons and there have been many examples where the non-valence ‘‘intrinsic quark’’ components seem important [33]. The flavor singlet meson  $\eta_0$  may have a large portion of  $c\bar{c}$  component and strong coupling to gluons through QCD axial anomaly. Using operator product expansion and QCD low energy theorems, the non-perturbative intrinsic charm content of the  $\eta'$  meson was evaluated semi-quantitatively in Ref. [3] to be  $f_{\eta'}^c = 50 - 180$  MeV, which suffices to explain the large  $\eta'$  production in  $B$  decay reported by CLEO. Furthermore, Ref. [4] suggested  $f_{\eta'}^c \sim -50$  MeV to account for the data if  $c\bar{c}$  pair is in color octet. Especially, the sign of  $f_{\eta'}^c$  was fixed as minus in their work. A similar size,  $f_{\eta'}^c = 40$  MeV, was argued in Ref. [34], too.

The gluonic components of  $\eta_0$  play no role in electromagnetic transition form factors because the coupling of two photons to two gluons is very small. They are invisible here. However, the  $c\bar{c}$  pair has distinct behavior from the light quark pairs. Unlike the  $s\bar{s}$  pair, different sizes of  $c\bar{c}$  component change the form factor a lot. In the following, the current mass and constituent mass of  $c$  quark are treated as the same and we adopt  $m_c = 1.5$  GeV. The hard scattering amplitude for the heavy quark pair is

$$T_H(x, k_\perp, Q) = \frac{q_\perp \cdot (x_2 q_\perp + k_\perp)}{q_\perp^2 ((x_2 q_\perp + k_\perp)^2 + m_c^2)} + (1 \leftrightarrow 2). \quad (19)$$

It is noted that the helicity-flip amplitude can not be ignored since it is proportional to current quark mass. A direct calculation gives the higher helicity contributions as

$$F_c^{\lambda=\pm 1}(Q^2) = 2\sqrt{2n_c}e_c^2 \int_0^1 dx \int \frac{d^2 k_\perp}{16\pi^3} \psi_c(x, k_\perp) \frac{1}{\sqrt{m_c^2 + k_\perp^2}} \left( \frac{m_c q \cdot k}{q_\perp^2 ((x_2 q_\perp + k_\perp)^2 + m_c^2)} + (1 \leftrightarrow 2) \right). \quad (20)$$

The total contributions of all helicity components can be expressed as Eq. (17) by changing the hard scattering amplitude as

$$T_H(x, k_\perp, Q) = \frac{q_\perp \cdot (x_2 q_\perp + 2k_\perp)}{q_\perp^2 ((x_2 q_\perp + k_\perp)^2 + m_c^2)} + (1 \leftrightarrow 2). \quad (21)$$

The spacial wave function is written as

$$\psi_c(x, k_\perp) = A_c \exp \left[ -b_0^2 \frac{k_\perp^2 + m_c^2}{x(1-x)} \right]. \quad (22)$$

The  $c\bar{c}$  contributions to the transition form factor have very different  $Q^2$  behavior from light quark ones because of the higher helicity contributions, the heavy quark propagator, and

the different wave function. Comparison with light quark contributions is shown in Fig. 3 with their relative sizes at zero momentum transfer normalized to unity. This difference may result in observable effects in the transition form factors.

Now we explore the form factor with the interested value  $f_{\eta'}^c = -50$  MeV. The decay constant is connected with the wave function as the pionic case [15] as

$$f_R^q = \langle 0 | \bar{q} \gamma^+ (1 - \gamma^5) q | R \rangle = 2\sqrt{2n_c} \int_0^1 dx \int \frac{d^2k}{16\pi^3} \psi^{q\bar{q}}(x, k_\perp) . \quad (23)$$

The above decay constant is multiplied by a factor of  $\sqrt{2}$  comparing with the previous sections to be consistent with Refs. [3,4,34].

$A_c$  is easily obtained by the definition of the decay constant, Eq. (23), with the input  $f_{\eta'}^c$  and the mixing angle  $\theta$ . The mixing angle is fixed at  $\theta = -20^\circ$  because it is, by our numerical results, still not sensitive even when  $c\bar{c}$  is included. At the same time, we neglect the difference between  $s$  quarks and  $u(d)$  quarks in order to concentrate on  $c$  quarks, i.e., let  $m_s = m_{u(d)} = 300$  MeV. This approximation is valid since the difference is minor, which can be seen in last section. The obtained form factor, comparing with  $f_{\eta'}^c = 0$ , is shown in Fig. 4. The corresponding decay constants are

$$\begin{aligned} f_{\eta'}^{u(d)} &= 56.8 \text{ MeV} , & f_{\eta'}^s &= 105.4 \text{ MeV} , & f_{\eta'}^c &= -50.0 \text{ MeV} ; \\ f_{\eta}^{u(d)} &= 71.1 \text{ MeV} , & f_{\eta}^s &= -62.5 \text{ MeV} , & f_{\eta}^c &= -18.2 \text{ MeV} . \end{aligned}$$

These values for light quarks are similar to that with  $f_{\eta'}^c = 0$  because the  $c\bar{c}$  contributions at zero momentum transfer are very small and change the normalization condition only slightly. They are not far from the naive expectation in  $SU_f(3)$  limit at  $\theta = -20^\circ$ :

$$f_{\eta'}^u \sim f_{\eta'}^d \sim f_{\eta'}^s/2 \sim f_\pi/\sqrt{6} = 54 \text{ MeV} , \quad (24)$$

$$f_{\eta}^u \sim f_{\eta}^d \sim -f_{\eta}^s \sim f_\pi/\sqrt{3} = 77 \text{ MeV} . \quad (25)$$

Since the flavor singlet has different non-perturbative properties from that of the octet, the parameter  $b_0$  is not necessarily the same as  $b_8$ . In fact, with  $f_{\eta'}^c = -50$  MeV, a very good fit can be obtained by adjusting  $b_0^2$  to  $0.3 \text{ GeV}^{-2}$  rather than  $b_0^2 = b_\pi^2 = 0.414 \text{ GeV}^{-2}$ . However, the decay constants will increase apparently for light quarks (about 30% for  $u$  and  $d$  quark):

$$\begin{aligned} f_{\eta'}^{u(d)} &= 74.4 \text{ MeV} , & f_{\eta'}^s &= 119.1 \text{ MeV} , & f_{\eta'}^c &= -50.0 \text{ MeV} ; \\ f_{\eta}^{u(d)} &= 73.3 \text{ MeV} , & f_{\eta}^s &= -48.8 \text{ MeV} , & f_{\eta}^c &= -18.2 \text{ MeV} . \end{aligned}$$

This difference will result in apparent increasement to the radiative decay of mesons, such as  $\rho(\omega, \phi) \rightarrow \eta\gamma$ ,  $\eta' \rightarrow \rho(\omega)\gamma$  and  $\phi \rightarrow \eta'\gamma$ . Choi and Ji [11] have found that the same value  $b_\pi$  for all of these flavor nonet mesons can produce satisfied decay widths for these decay channels. An increasement of 30%, if  $b_0^2 = 0.3 \text{ GeV}^{-2}$  rather than  $b_0 = b_\pi$ , may exceed too much to fit the data of their radiative decays. Therefore the above numerical results disfavor such a choice and thus disfavor a large portion of  $c\bar{c}$  component as  $f_{\eta'}^c = -50$  MeV.

There have been recently a number of investigations [18–21] which support a small  $f_{\eta'}^c$  than that was estimated in Ref. [3]. Our above conclusion is in agreement with these

results from other considerations. For example, in Ref. [21],  $f_{\eta'}^c = -12.3 \sim -18.4$  MeV was suggested. To check the consistency of such a possibility in our approach, we adjust the input to  $f_{\eta'}^c = -15$  MeV and present the result in Fig. 4. The decay constants for different flavor quarks are

$$\begin{aligned} f_{\eta'}^{u(d)} &= 56.0 \text{ MeV} , & f_{\eta'}^s &= 104.6 \text{ MeV} , & f_{\eta'}^c &= -15.0 \text{ MeV} ; \\ f_{\eta}^{u(d)} &= 70.8 \text{ MeV} , & f_{\eta}^s &= -62.7 \text{ MeV} , & f_{\eta}^c &= -5.5 \text{ MeV} , \end{aligned}$$

with  $b_0^2 = b_\pi^2 = 0.414 \text{ GeV}^{-2}$ . We notice that the obtained results are very close to the case of  $f_{\eta'}^c = 0$  and our analysis allow a small  $f_{\eta'}^c$  around  $-15$  MeV. This is consistent with the results in Refs. [18–21].

## V. CONCLUSIONS AND SUMMARY

The electromagnetic transition form factors of  $\eta$  and  $\eta'$  are presented comparing with  $\pi$  in this paper. The numerical results show that there still exists a gap between the data and the light-cone perturbation calculation with the Brodsky-Huang-Lepage wave function as the input for the non-perturbative aspects of the mesons. To model the higher Fock state and higher order contributions, we assume that the fraction size of these contributions are similar to all three mesons: pion,  $\eta$ , and  $\eta'$ . The ratio of the data of the  $\pi\gamma$  transition form factor to the theoretical calculation is multiplied to the light-cone results of the  $\eta\gamma$  and  $\eta'\gamma$  form factors. With such a correction, the obtained transition form factors can be compared with the data. In  $SU_f(3)$ , it is in fact unreliable to determine the mixing angle at present unless additional normalization conditions other than the  $\eta(\eta') \rightarrow \gamma\gamma$  decay widths are included. The reason is that the same mixing mechanism occurs in both the transition form factors and the normalization conditions. Both the experimental (especially for  $\eta'$ ) and theoretical accuracies are not high enough to distinguish the  $u(d)$  and  $s$  quark pairs in these pseudoscalar mesons at high  $Q^2$ .

The heavy quark pair has different  $Q^2$  behavior from the light ones. It is possible to explore the size of the  $c\bar{c}$  component in the flavor singlet. Our results disfavor a large portion of  $c\bar{c}$  component in  $\eta'$ . Such a conclusion is in agreement with a number of recent investigations with a small portion of intrinsic charm in  $\eta'$  from other considerations.

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## FIGURE CAPTION

Fig. 1. The pion-photon transition form factor with different parameters. The dashed line is the form factor without Melosh rotation. The solid line is with Melosh rotation. The dashed-dotted line is the result as  $Q^2 \rightarrow \infty$ , while the evolution of the BHL wave function is neglected.

Fig. 2. The transition form factors with different mixing angles.

Fig. 3. The ratio of the  $c\bar{c}$  contribution to the  $q\bar{q}$  ( $q$  being light quark) contribution to the transition form factor, which is normalized to unity at zero momentum transfer.

Fig. 4. The transition form factors with  $f_{\eta'}^c = 0$  MeV,  $f_{\eta'}^c = -15$  MeV and  $f_{\eta'}^c = -50$  MeV.







