ELECTROWEAK PHASE TRANSITION IN A STRONG MAGNETIC FIELD

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ABSTRACT

Phase transitions induced by high temperatures and strong magnetic fields are investigated in the Standard model. The consistent effective potential including the one-loop and ring diagram contributions is calculated and investigated for the wide range of the fields, temperatures and Higgs boson mass m_H. All other particles - fermions and bosons - are taken into account with actual values of their masses. This effective potential is real at sufficiently high temperatures . It is shown that symmetry restoration is a first type phase transition for $m_H < 70$ Gev . For heavier Higgs particles and the field strengths $H > 0.1 \cdot 10^{24} G$ the local electroweak minimum could not be realized at all. Hence, the upper limit on the Higgs mass as well as the limit on the field strengths in the phase transition epoch follow.

1.Introduction

The concept of symmetry restoration at high temperature has been intensively used in studying the evolution of the universe at its early stages. Nowadays it gives a possibility to investigate various problems of cosmology and particle physics [1],[2]. In particular, the type of the electroweak phase transition and hence a further evolution of the universe depends on the mass m_H of the Higgs boson. Most investigations of the electroweak phase transition have included into consideration a high temperature environment as the main ingredient [2], [3]. But in recent a few years cogent arguments followed from different approaches in favour of the presence of strong magnetic fields at that stage have appeared [5], [6](recent survey on the magnetic fields in the universe is Ref.[7]). So, the phase transition at high temperature and strong fields has to be of interest . Moreover, at present time when all masses of fundamental particles, except m_H, are known it is possible to investigate in details the phase transition as the function of this parameter and to determine the properties of the vacuum, its structure and to specify a so-called metastability bound on m_H [3],[4].

One of the ways to have strong magnetic fields in the electroweak phase transition epoch was proposed by Vachaspati [5]. From his analysis it follows that under very general conditions the fields $H \sim T_i^2$ in the patches of sub-horizon scales can be generated during a large class of grand unified transitions [6],[7], where T_i is transition temperature. The second one is formation of the Savvidy vacuum magnetic state at high temperature ($H \sim gT^2, g$ is gauge coupling constant) [14],[7],[24]. In latter case only the abelian field configurations could arise spontaneously since they are sourceless. For many problems it is important to estimate the field strengths presented, but it is difficult to realize that without detailed investigations within specific models. Usually, only one type of fields is considered. Therefore, results obtained in such a way give an upper estimate of the field. This remark is relevant to our present investigation. In current literature along with usual magnetic field a hypercharge magnetic field presenting in the restored phase is discussed (see for example [9]). The latter one is converted into ordinary magnetic field during the phase transition.

In what follows, we will consider the case when the magnetic field is presenting in both broken and restored phases. We believe that this is a good approximation which gives possibility to investigate essential features of the phase transition. This scenario assumes that the field has been generated at a GUT scale via the Savvidy mechanism and presented during the phase transition. Such a picture is likely since, as it follows from our consistent calculation of the effective potential which found to be real at sufficiently high temperatures (and, in particular, in the restored phase), constant magnetic field is stable. This is the most important point of the present analysis. In any case, it may have relevance to the description of the phase with broken symmetry at high temperatures and strong fields.

The discussed "primordial" fields are usually considered as seed fields responsible for generation of the observed magnetic fields in galaxies [7].

Various aspects of the phase transitions in magnetic fields at high temperature have been investigated by many authors [8]-[15]. In Refs. [12],[19] the phase transitions derived with the effective potential (EP) of the bosonic part of the Salam-Weinberg model and the vacuum structures of the phases have also been described. In Ref.[15] in addition to the temperature and magnetic fields a chemical potential was incorporated. But the role of the fermions has not been investigated in detail. However, due to a rather heavy t-quark mass, $m_t \simeq 175$ Gev, an unbounded global minimum of the EP is produced in addition to the electroweak local one for not very heavy Higgs scalars [3]. As will be shown, strong magnetic fields influence essentially the phase transition dynamics realized in this case. Another aspect of the electroweak phase transition, which also was not investigated but plays an important role, is the influence of so-called ring diagrams at high temperature and strong field. At zero field it was investigated in Refs.[16],[17] where it has been shown the importance of these diagrams for determining the type of the phase transition. In Ref.[17] the t-quark mass was chosen of order 110Gev. So, taking into account the present day data, it should be considered as a qualitative estimate of the role of ring diagrams even for zero-field case.

In the present paper we study the electroweak phase transition at high temperatures and constant strong magnetic fields H . We calculate and investigate the one-loop effective potential(EP) and the contributions of ring diagrams. In contrast to previous considerations we include the content of all bosons and fermions with the corresponding masses. So, the only free parameter remains the mass m_H. The role of ring diagrams is of great importance because they contain the terms which cancel out the imaginary part of the one-loop EP. The total EP is real at sufficiently high temperatures and suitable for investigations of symmetry behaviour. As it will be shown, the influence of fermions (mainly heavy quarks) is very essential. For values of mass $m_H < 70 Gev$ and weak field strengths the electroweak phase transition is of the first type one. It is turned out that the magnetic field stimulates either the generation of the electroweak minimum and tends to remove the potential barrier separating this metastable state from the unbounded global minimum produced due to heavy fermions. At any high temperature there exists the corresponding field strength H at which the metastable vacuum could not be produced at all. This property can be used to find an upper limit on H at the transition epoch. The paper is organized as follows. In Sects.2,3 the one-loop contributions of bosons and fermions to the EP$V^{(1)}(T, H, \phi_c)$ are calculated in the form convenient for numerical study. In Sect.4 we compute the contributions of ring diagrams. Further in Sect.5 symmetry behaviour is investigated for a number of values of m_H and H . For comparison, we consider separately the cases when only the one-loop EP is taken into account and when the ring diagrams are included. Discussion of the results obtained is given in Sect.6.

2. Boson contributions to $V^{(1)}(T, H, \phi_c)$

The Lagrangian of the boson sector of the Salam-Weinberg model is

$$L = -\frac{1}{4} F^a_{\mu\nu} F^{\mu\nu}_a - \frac{1}{4} G_{\mu\nu} G^{\mu\nu} + (D_\mu \Phi)^+ (D^\mu \Phi) + \frac{m^2}{2} (\Phi^+ \Phi) - \frac{\lambda}{4} (\Phi^+ \Phi)^2, \tag{1}$$

where the standard notations are introduced

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g \varepsilon^{abc} A^b_\mu A^c_\nu,$$
$$G_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu,$$
$$D_\mu = \partial_\mu + \frac{1}{2} i g A^a_\mu \tau^a + \frac{1}{2} i g' B_\mu. \tag{2}$$

The vacuum expectation value of the field Φ is

$$< \Phi >= \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \phi_c \end{pmatrix}. \tag{3}$$

The fields corresponding to the W-,Z-bosons and photons, respectively, are

$$W^\pm_\mu = \frac{1}{\sqrt{2}} (A^1_\mu \pm i A^2_\mu),$$
$$Z_\mu = \frac{1}{\sqrt{g^2 + g'^2}} (g A^3_\mu - g' B_\mu),$$
$$A_\mu = \frac{1}{\sqrt{g^2 + g'^2}} (g' A^3_\mu + g B_\mu). \tag{4}$$

The external electromagnetic field is introduced by splitting the potential in two parts: $A_\mu = \bar{A}_\mu + A^R_\mu$, where A^R describes a radiation field and $\bar{A} = (0, 0, H x^1, 0)$ corresponds to the constant magnetic field directed along the third axis. We make use of the gauge- fixing conditions [23]

$$\partial_\mu W^{\pm\mu} \pm i e \bar{A}_\mu W^{\pm\mu} \mp i \frac{g^2 \phi^2}{4\xi} \phi^\pm = C^\pm(x), \tag{5}$$

$$\partial_\mu Z^\mu - \frac{i}{\xi'}(g^2 + g'^2)^{1/2}\phi_z = C_z, \tag{6}$$

where $e = gsin\theta_w, tang\theta_w = g'/g, \phi^\pm, \phi_z$ are the Goldstone fields, ξ, ξ' are the gauge fixing parameters, $C^\pm, C_z$ are arbitrary functions and ϕ_c is a scalar condensate value. In what follows, all calculations will be done in the general relativistic renormalizable gauge (5),(6) and after that we set $\xi, \xi' = 0$ choosing the unitary gauge.

To compute the EP $V^{(1)}$ in the background magnetic field let us introduce the proper time,s, representation for the Green functions

$$G^{ab} = -i \int_0^\infty ds \exp(-isG^{-1^{ab}}) \tag{7}$$

and use the method of Ref.[18], allowing in a natural way to incorporate the temperature into this formalism. A basic formula of Ref.[18] connecting the Matsubara Green functions with the Green functions at zero temperature is needed,

$$G_k^{ab}(x, x'; T) = \sum_{-\infty}^{+\infty} (-1)^{(n+[x])\sigma_k} G_k^{ab}(x - [x]\beta u, x' - n\beta u), \tag{8}$$

where $G_k^{ab}$is the corresponding function at $T = 0, \beta = 1/T, u = (0, 0, 0, 1)$, the symbol $[x]$ means an integer part of $x_4/\beta, \sigma_k = 1$ in the case of physical fermions and $\sigma_k = 0$ for boson and ghost fields. The Green functions in the right-hand side of formula (8) are the matrix elements of the operators G_k computed in the states $\mid x', a)$ at $T = 0$, and in the left-hand side the operators are averaged in the states with $T \neq 0$. The corresponding functional spaces U^0 and U^T are different but in the limit of Tß0 U^T transforms into U^0.

The one-loop contribution into EP is given by the expression

$$V^{(1)} = \frac{1}{2} Tr \log G^{ab}, \tag{9}$$

where G^{ab} stands for the propagators of all the quantum fields $W^\pm, \phi^\pm, ...$ in the background magnetic field H. In the s- representation the calculation of the trace can be done in accordance with formula [21]

$$Tr \log G^{ab} = -\int_0^\infty \frac{ds}{s} tr \exp(-isG_{ab}^{-1}) \tag{10}$$

Details of calculations based on the s-representation and the formula (8) can be found, for example,in Refs.[18],[19],[24]. The terms with $n = 0$ in Eqs.(8), (9) give zero temperature expressions for Green's functions and effective potential $V^{(1)}$, respectively. They are the only terms possessing divergences. To eliminate them and uniquely fix the potential we use the following renormalization conditions at $H, T = 0$[19]:

$$\frac{\partial^2 V(\phi, H)}{\partial H^2} \mid_{H=0,\phi=\delta(0)} = \frac{1}{2}, \tag{11}$$

$$\frac{\partial V(\phi, H)}{\partial \phi} \mid_{H=0,\phi=\delta(0)} = 0, \tag{12}$$

$$\frac{\partial^2 V(\phi, H)}{\partial \phi^2} \mid_{H=0,\phi=\delta(0)} = \mid m^2 \mid, \tag{13}$$

where $V(\phi, H) = V^{(0)} + V^{(1)} + \cdots$ is the expansion in a number of loops and $\delta(0)$ is the vacuum value of a scalar field determined in a tree approximation.

It is convenient for what follows to introduce the dimensionless quantities: $h = H/H_0 (H_0 = M_w^2/e), \phi = \phi_c/\delta(0), K = m_H^2/M_w^2$, $B = \beta M_w, \tau = 1/B = T/M_w, \mathcal{V} = V/H_0^2$ and $M_w = \frac{g}{\sqrt{2}}\delta(0)$.

After reparametrization the scalar field potential is directly expressed in terms of the ratio K,

$$\mathcal{V}^{(0)} = \frac{h^2}{2} + K(-\frac{\phi^2}{4} + \frac{\phi^4}{8}). \tag{14}$$

The renormalized one-loop EP is given by the sum of the functions

$$\mathcal{V}_1 = \mathcal{V}^{(0)} + \mathcal{V}^{(1)}(\phi, H, K) + \omega^{(1)}(\phi, h, K, \tau), \tag{15}$$

where $\mathcal{V}^{(1)}$ is the one-loop EP at $T = 0$, which has been studied already in Ref.[23]. It has the form:

$$\mathcal{V}^{(1)} = \mathcal{V}_{w,z}^{(1)} + \mathcal{V}_{\phi}^{(1)}, \tag{16}$$

where

$$\mathcal{V}_{w,z}^{(1)} = \frac{3\alpha}{\pi} \quad [h^2 Log\Gamma_1(\frac{1}{2} + \frac{\phi^2}{2h}) + h^2\zeta'(-1) + \frac{1}{16}\phi^4 - \frac{1}{8}\phi^4 log\frac{\phi^2}{2h} + \frac{1}{24}h^2$$

$$
\begin{aligned}
&- \frac{1}{24} h^2 log(2h)] \\
&+ \frac{\alpha}{2\pi}[-2h^2 + (h^2 + h\phi^2) log(h+\phi^2) + (h^2 - h\phi^2) log \mid h - \phi^2 \mid] \\
&+ i\frac{1}{2}\alpha h(\phi^2 - h)\theta(h-\phi^2),
\end{aligned}
\tag{17}
$$

$$
\begin{aligned}
\mathcal{V}^{(1)}_{\phi} = & \; K \quad sin^2\theta_w(-\phi^2 + \frac{1}{2}\phi^4) \\
&+ \frac{3\alpha}{4\pi}(1 + \frac{1}{2cos^2\theta})(\frac{1}{2}\phi^4 log\phi^2 - \frac{3}{4}\phi^4 + \phi^2) \\
&+ \frac{\alpha K^2}{32\pi}[(\frac{9}{2}\phi^4 - \frac{3}{4}\phi^2 + \frac{1}{2}) log \mid \frac{3\phi^2 - 1}{2} \mid - \frac{27}{4}\phi^4 + \frac{21}{2}\phi^2]
\end{aligned}
\tag{18}
$$

and $\omega^{(1)}$ is the temperature dependent contribution to the EP determined by the terms of formulae (8),(9) with $n \neq 0$.

We outline the used calculation procedure considering the W-boson contribution as an example [24],

$$
\omega^{(1)}_w = \frac{\alpha}{2\pi} \int\limits_0^{\infty} \frac{ds}{s^2}\; e^{-is(\phi^2/h)} \Big[\frac{1 + 2\cos 2s}{\sin s}\Big] \sum_1^{\infty} \exp(ihB^2 n^2/4s). \tag{19}
$$

As Eq.(17), this expression contains an imaginary part for $h > \phi^2$ appeared due to the tachyonic mode $\varepsilon^2 = p_3^2 + M_w^2 - eH$ in the W-boson spectrum [23]. It can be explicitly calculated by means of an analytic continuation taking into account the shift sß$s - i0$ in the s- plane. Fixing $\phi^2/h > 1$ one can rotate clockwise the integration contour in the s-plane and direct it along the negative imaginary axis. Then, using the identity

$$
\frac{1}{\sinh s} = 2\sum_{p=0}^{\infty} e^{-s(2p+1)} \tag{20}
$$

and integrating over s in accordance with the standard formula

$$
\int\limits_0^{\infty} ds s^{n-1} \exp(-\frac{b}{s} - as) = 2(\frac{a}{b})^{n/2} K_n(2\sqrt{ab}), \tag{21}
$$

$a, b > 0$, one can represent the expression (19) in the form

$$Re\omega_w^{(1)} = -4\frac{\alpha}{\pi}\frac{h}{B}(3\omega_0 + \omega_1 - \omega_2), \tag{22}$$

where

$$\omega_0 = \sum_{p=0}^{\infty}\sum_{n=1}^{\infty}\frac{x_p}{n}K_1(nBx_p); x_p = (\phi^2 + h + 2ph)^{1/2} \tag{23}$$

$$\omega_1 = \sum_{n=1}^{\infty}\frac{y}{n}K_1(nBy), y = (\phi^2 - h)^{1/2} \tag{24}$$

and in the region of parameters $\phi^2 < h$ after analytic continuation

$$\omega_1 = -\frac{\pi}{2}\sum_{n=1}^{\infty}\frac{\mid y \mid}{n}Y_1(nB \mid y \mid), \tag{25}$$

$$\omega_2 = \sum_{n=1}^{\infty}\frac{z}{n}K_1(nBz), z = (\phi^2 + h)^{1/2}, \tag{26}$$

and $K_n(x), Y_n(x)$ are the Macdonald and Bessel functions, respectively. The imaginary part of $\omega_w^{(1)}$ is given by the expression

$$Im\omega_2 = -2\alpha\frac{h}{B}\sum_{n=1}^{\infty}\frac{\mid y \mid}{n}J_1(nB \mid y \mid), \tag{27}$$

$J_n(x)$is Bessel function. As is well known, the imaginary part of EP is signaling the instability of a system [21]. In what follows we shall consider mainly the symmetry behaviour described by the real part of the EP . As the imaginary part is concern, it will be cancelled in consistent calculation including the one-loop and ring diagram contributions to the EP.

Carrying out similar calculations for the Z- and Higgs bosons, we obtain [12]:

$$\omega_z = -6\frac{\alpha}{\pi}\sum_{n=1}^{\infty}\frac{\phi^2}{\cos^2\theta_w n^2 B^2}K_2(\frac{nB\phi}{\cos\theta}) \tag{28}$$

$$Re\omega_\phi = \left\{ \begin{array}{c} -2\frac{\alpha}{\pi}\sum\frac{t^2}{B^2n^2}K_2(nBt) \\ \alpha\sum\limits_{n=1}^{\infty}\frac{|t|^2}{n^2B^2}Y_2(nB \mid t \mid) \end{array} \right\}. \tag{29}$$

where the variable $t = [K_w(\frac{3\phi^2-1)}{2})]^{1/2}$ at $3\phi^2 > 1$ and series with the function $Y_2(x)$ has to be calculated at $3\phi^2 < 1$. The corresponding imaginary part is also cancelled as it will be shown below.

The above expressions (16),(22),(28),(29) will be used in numerical studying of the symmetry behaviour at different H, T. There is a cancellation of a number of terms from the zero-temperature contributions given Eqs.(16) and T-depended ones. This fact has a general character and was used in checking of the correctness of calculations.

3. Fermion contributions to $V^{(1)}(H, T, \phi_c)$

To find the explicit form of the fermion contribution to the EP let us consider the standard unrenormalized expression written in the s -representation [22]:

$$V_f^{(1)} = -\frac{1}{8\pi^2}\sum_{n=-\infty}^{\infty}(-1)^n\int_{-\infty}^{+\infty}\frac{ds}{s^3}e^{-(m^2s+\beta^2n^2/4s)}(eHs)cotheHs, \qquad (30)$$

m is a fermion mass. Here, we have incorporated the equation (8) to include a temperature dependence. In what follows, we shall take into account the contributions of all fermions - leptons and quarks - with their masses known at present time. Usually, considering a symmetry behaviour without field one restricts himself by a t-quark contribution only. But in the case of an external field applied this is not a good idea, since the dependence of $V^{(1)}$ on H is a complicated function of the ratio m_f^2/eH. So, at some fixed values of H, T different dependencies on H will contribute for fermions with different masses. Hence, a very complicate dependence on the field takes place in general. We include this in a total, carrying out a numerical calculations and summing up over all the fermions. Now, separating a zero temperature contribution by means of the relation $\sum\limits_{-\infty}^{+\infty} = 1 + 2\sum\limits_{1}^{\infty}$ and introducing the parameter $K_f = m_f^2/M_w^2 = 2G^2_{Yukawa}/g^2$, we obtain for the zero temperature fermion contribution to the dimensionless EP ,

$$\begin{aligned}\mathcal{V}_f(h,\phi) = \;& \tfrac{\alpha}{4\pi}\sum_f K_f^2(-2\phi^2 + \frac{3}{2}\phi^4 - \phi^4 log\phi^2)\\ -\;& \frac{\alpha}{\pi}\sum_f(q_f^2\frac{h^2}{6}\log\frac{2\mid q_f\mid h}{K_f})\end{aligned}$$

$$
\begin{aligned}
&- \frac{\alpha}{\pi}\sum_f [2q_f^2h^2\log\Gamma_1(\frac{K_f\phi^2}{2\mid q_f\mid h}) + (2\zeta'(-1)-\frac{1}{6})q_f^2h^2 \\
&+ \frac{1}{8}K_f^2\phi^4 + (\frac{1}{4}K_f^2\phi^4 - \frac{1}{2}K_f\mid q_f\mid h\phi^2)\log\frac{2\mid q_f\mid h}{K_f\phi^2}] \quad (31)
\end{aligned}
$$

where q_f is a fermion electric charge, the sum $\sum\limits_f = \sum\limits_{f=1}^{3}(leptons) + 3\sum\limits_{f=1}^{3}(quarks)$ counts the contributions of leptons and quarks with their electric charges. The Γ_1 function is defined as follows (see, for example, survey [23]):

$$
\log\Gamma_1(x) = \int\limits_0^x dy\log\Gamma(y) + \frac{1}{2}x(x-1) - \frac{1}{2}x\log(2\pi). \quad (32)
$$

The finite temperature part can be calculated in a way, described in the previous section. In the dimensionless variables it looks as follows:

$$
\begin{aligned}
\omega_f &= -4\frac{\alpha}{\pi}\sum_f\Big\{\sum_{p=0}^{\infty}\sum_{n=1}^{\infty}(-1)^n\Big[\frac{(2ph+K_f\phi^2)^{1/2}h}{Bn}K_1((2ph+K_f\phi^2)^{1/2}Bn) \\
&+ \frac{(2p+2)h+K_f\phi^2)^{1/2}}{Bn}hK_1(((2p+2)h+K_f\phi^2)^{1/2}Bn)\Big]\Big\} \quad (33)
\end{aligned}
$$

Again, a number of terms from Eqs.(31) and (33) are cancelled being summed up, as in the bosonic sector.

These two expressions and the boson contributions obtained in Sect.2 will be used in numerical investigations of symmetry behaviour. More precise, we consider the difference $V'(H,T,\phi) = ReV(H,T,\phi) - ReV(H,T,\phi=0)$ giving possibility to determine the symmetry restoration. We will investigate the EP of two types - the one-loop contribution and the sum of that and the ring diagrams which are the next to leading order corrections at high temperature.

4. Contribution of ring diagrams

It was shown by Carrington [17] that at $T \neq 0$ a consistent calculation of the EP based on generalized propagators, which include the polarization operator insertions, requires that ring diagrams have to be added simultaneously with the one-loop contributions. These diagrams essentially affect the phase transition at high temperature and zero field [16],[17]. Their importance at T and $H \neq 0$ was also pointed out in literature [14],[15] but, as far as we know, this part of the EP has not been calculated, yet.

As is known [16], the sum of ring diagrams describes a dominant contribution of great distances. It differs from zero only in the case when massless states appear in a system. So, this type of diagrams has to be calculated when a symmetry restoration is investigated. To find the correction $V_{ring}(H,T)$ at high temperature and magnetic field the polarization operators of the Higgs particle, photon and Z-boson at the considered background have to be calculated. Just these calculations have been announced in Refs.[14], [15]. Then, $V_{ring}(H,T)$ is given by series depicted in figures 1,2.

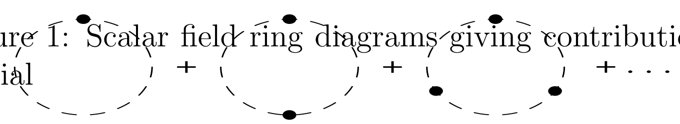

Figure 1: Scalar field ring diagrams giving contribution to the effective potential

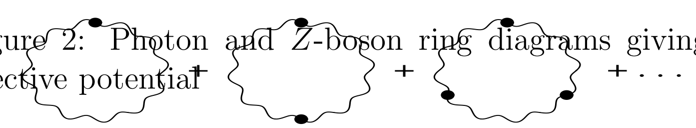

Figure 2: Photon and Z-boson ring diagrams giving contribution to the effective potential

Here, a dashed line describes the Higgs particles, the wavy lines represent photons and Z-bosons, the blobs represent the polarization operators in the limit of zero momenta. As also is known [17], in order to calculate the contribution of ring diagrams not the complete polarization operators $\Pi_{\mu\nu}(k,T,H)$ but only their limiting expressions at zero momenta, $\Pi_{00}(k=0,T,H)$, are sufficient. This limit, named the Debye mass, can be calculated from the EP of the special type. This fact considerably simplifies our task.

Now, let us turn to calculations of $V_{ring}(H,T)$. It is given by the standard expression [16],[17],[14]:

$$V_{ring} = -\frac{1}{12\pi\beta} Tr\{[M^2(\phi) + \Pi_{00}(0)]^{3/2} - M^3(\phi)\}, \tag{34}$$

where trace means the summation over all the contributing states, $M(\phi)$ is a tree mass of corresponding state and $\Pi_{00}(0) = \Pi(k=0,T,H)$ for the Higgs particle and $\Pi_{00}(0) = \Pi_{00}(k=0,T,H)$ are the zero-zero components of the polarization operators in the magnetic field taken at zero momenta. The above contribution has order $\sim g^3(\lambda^3)$ in coupling constant whereas the two-loop terms are to be of order $\sim g^4$. As $\Pi_{00}(0)$ the high temperature limits of polarization functions have to be substituted which have the order $\sim T^2$ for leading terms and $\sim g\phi_c T, (gH)^{1/2}T, \phi_c, (gH)^{1/2}/T << 1$ for subleading ones.

For the next step of calculation, we remind that the effective potential is the generating functional of the one-particle irreducible Green functions at zero momenta transferred. So, to have $\Pi(0)$ we can just calculate the second derivative with respect to ϕ of the potential $V^{(1)}(H,T,\phi)$ in the limit of high temperature ,$T >> \phi, (eH)^{1/2}$ and then set $\phi = 0$. This limit can be calculated by means of the Mellin transformation technique (see for example [24]) and the result looks as follows:

$$\begin{aligned} V^{(1)}(H,\phi,T\text{ß}\infty) &= [\Big(\frac{C_f}{12}\phi_c^2 + \frac{\alpha\pi}{2cos^2\theta_w}\phi_c^2 + \frac{g^2}{16}\phi_c^2\Big)T^2] \\ &+ [\frac{\alpha\pi}{6}(3\lambda\phi_c^2 - \delta^2(0))T^2 - \frac{\alpha}{cos^3\theta}\phi^3 T \\ &- \frac{\alpha}{3}(\frac{3\lambda\phi_c^2 - \delta^2(0)}{2})^{3/2}T] \\ &- \frac{1}{2\pi}(\frac{1}{4}\phi_c^2 + gH)^{3/2}T + \frac{1}{4\pi}eHT(\frac{1}{4}\phi_c^2 + eH)^{1/2} \\ &+ \frac{1}{2\pi}eHT(\frac{1}{4}\phi_c^2 - eH)^{1/2}. \end{aligned} \tag{35}$$

The parameter $C_f = \sum_{i=1}^{3} G_{il}^2 + 3\sum_{i=1}^{3} G_{iq}^2$ determines the fermion contribution of the order $\sim T^2$ having relevance to our problem. We also omitted $\sim T^4$ contributions to the EP. The terms of the type $\sim log[T/f(\phi,H)]$ cancel the logarithmic terms in the temperature independent contributions (15),(30).

Considering the high temperature limit we restrict ourselves by linear and quadratic in T terms, only.

Now, differentiating this expression twice with respect to ϕ and setting $\phi = 0$, we obtain

$$\begin{aligned}\Pi_\phi(0) &= \frac{\partial^2 V^{(1)}(\phi, H, T)}{\partial \phi^2} |_{\phi=0} \\ &= \frac{1}{24\beta^2}\Big(6\lambda + \frac{6e^2}{\sin^2 2\theta_w} + \frac{3e^2}{\sin^2\theta_w}\Big) + \sum_f 2G_f^2/\beta^2 \\ &+ \frac{(eH)^{1/2}}{8\pi \sin^2\theta_w \beta} e^2 (3\sqrt{2}\zeta(-\frac{1}{2}, \frac{1}{2}) - 1). \end{aligned} \tag{36}$$

The terms $\sim T^2$ in Eq.(36) give standard contributions to temperature mass squared coming from the boson and fermion sectors. The H-dependent term is negative since the difference in the brackets is $3\sqrt{2}\zeta(-\frac{1}{2}, \frac{1}{2}) - 1 \simeq -0,39$. Formally, this may result in the negativeness of the $\Pi(0)_\phi$ for very strong fields $(eH)^{1/2} > T$. But this happens in the range of parameters where asymptotic expansion is not valid. Substituting expression (36) into Eq.(34) we obtain (in the dimensionless variables),

$$\mathcal{V}^\phi_{ring} = -\frac{1}{12B}\Big\{(\frac{3\phi^2 - 1}{2}K + \Pi_\phi(0)\Big\}^{3/2} + \frac{\alpha}{3B}K(\frac{3\phi^2-1}{2})^{3/2}. \tag{37}$$

As is seen, the last term of this expression cancels the fourth term in Eq.(35), which becomes imaginary at $3\phi^2 < 1$. This is the important cancellation preventing the infrared instability at high temperature.

Before to proceed, let us note that Eq.(35) contains other term (the last one) which becomes imaginary for strong magnetic fields or small ϕ^2. It reflects the known instability in the W-boson spectrum which is discussed for many years in literature (see papers [19],[14],[15], [24] and references therein). But it also will be cancelled out when the contribution of ring diagrams with the unstable mode is added.

To find the H-dependent Debye masses of photons and Z-bosons the following procedure will be used. We calculate the one-loop EP of the W-bosons and fermions in a magnetic field and some "chemical potential",μ, which plays the role of an auxiliary parameter. Then, by differentiating them twice with respect to μ and setting $\mu = 0$ the mass squared m_D^2 will be obtained. Let us demonstrate that in more detail for the case of fermion contributions where the result is known.

The temperature dependent part of the one-loop EP of constant magnetic field and a non-zero chemical potential induced by an electron-positron vacuum polarization is [22]:

$$V^{(1)}_{ferm.} = -\frac{1}{4\pi^2}\sum_{l-1}^{\infty}(-1)^{l+1}\int_0^{\infty}\frac{ds}{s^3}exp(\frac{-\beta^2 l^2}{4s} - m^2 s)eHscoth(eHs)cosh(\beta l\mu), \tag{38}$$

where m is the electron mass, $e = gsin\theta_w$ is electric charge and a proper-time representation is used. Its second derivative with respect to μ taken at $\mu = 0$ can be written in the form,

$$\frac{\partial^2 V^{(1)}_{ferm.}}{\partial\mu^2} = \frac{eH}{\pi^2}\beta^2\frac{\partial}{\beta^2}\sum_{l=1}^{\infty}(-1)^{l+1}\int_0^{\infty}\frac{ds}{s}exp(-m^2 s - \beta^2 l^2/4s)coth(eHs). \tag{39}$$

Expanding $coth(eHs)$ in series and integrating over s in accordance with formula (21) we obtain in the limit of $T >> m, (eH)^{1/2}$:

$$\sum_{l=1}^{\infty}(-1)^{l+1}[\frac{8m}{\beta l}K_1(m\beta l) + \frac{2}{3}\frac{(eH)^2 l\beta}{m}K_1(m\beta l) + \cdots] \tag{40}$$

Series in l can easily be calculated by means of the Mellin transformation (see Refs.[24],[15]). To have the Debye mass squared it is necessary to differentiate Eq.(39) with respect to β^2 and to take into account the relation of the parameter μ with the zero component of the electromagnetic potential : $\mu ß ieA_0$ [14]. In this way we obtain finally,

$$m_D^2 = g^2 sin^2\theta_w(\frac{T^2}{3} - \frac{1}{2\pi^2}m^2 + O((m\beta)^2, (eH\beta^2))). \tag{41}$$

This is the well known result calculated from the photon polarization operator [20]. As one can see, the dependence on H appears in the order $\sim T^{-2}$. To find the total fermion contribution to m_D^2 one should sum up expression (41) over all fermions and substitute their electric charges.

To calculate m_D^2 for Z-bosons it is sufficient to take into account the fermion couplings with Z-field. It can be done by substituting $\mu ß i(g/2cos\theta_w + gsin^2\theta_w)$ and the result differs from Eq. (41) by the coefficient at the bracket in the right-hand side which have to be replaced, $g^2 sin^2\theta ßg^2(\frac{1}{4cos^2\theta_w} + tang^2\theta_w)$. One also should add the terms coming due to the neutral currents and the part of fermion-Z-boson interaction which is not reproduced by the above substitution:

$$m_D^{2'} = \frac{g^2}{8cos^2\theta_w}(1 + 4sin^4\theta_w)T^2. \tag{42}$$

Now, let us apply this procedure for the case of the W-boson contribution. The corresponding EP (temperature dependent part) calculated at non-zero T, μ in the unitary gauge looks as follows,

$$V_w^{(1)} = -\frac{eH}{8\pi^2}\sum_{l=1}^{\infty}\int_0^{\infty}\frac{ds}{s^2}exp(-m^2 s - l^2\beta^2/4s)[\frac{3}{sinh(eHs)} + 4sinh(eHs)]cosh(\beta l\mu). \tag{43}$$

All the notations are obvious. The first term in the squared brackets gives the triple contribution of the charged scalar field and the second one is due to the interaction with a W-boson magnetic moment. Again, after differentiation twice with respect to μ and setting $\mu = 0$ it can be written as

$$\frac{\partial^2 V_w^{(1)}}{\partial\mu^2} = \frac{eH}{2\pi^2}\beta^2\frac{\partial}{\partial\beta^2}\sum_{l=1}^{\infty}\int_0^{\infty}\frac{ds}{s}exp(-\frac{m^2 s}{eH} - \frac{l^2\beta^2 eH}{4s})[\frac{3}{sinh(eHs)} + 4sinh(eHs)]. \tag{44}$$

Expanding $sinh^{-1}s$ in series over Bernoulli's polynomials,

$$\frac{1}{sinhs} = \frac{e^{-s}}{s}\sum_{k=0}^{\infty}\frac{B_k}{k!}(-2s)^k, \tag{45}$$

and carrying out all the calculations described above, we obtain for the W-boson contribution to m_D^2 of the electromagnetic field,

$$\begin{aligned} m_D^2 &= 3g^2 sin^2\theta_w[\frac{1}{3}T^2 - \frac{1}{2\pi}T(m^2 + gsin\theta_w H)^{1/2} - \frac{1}{8\pi^2}(gsin\theta_w H) \\ &\quad + O(m^2/T^2, (gsin\theta_w H/T^2)^2)]. \end{aligned} \tag{46}$$

Hence it follows that spin does not contribute to the Debye mass in the leading order. Other interesting point is that the next to leading terms are negative.

The contribution of the W-boson sector to the Z-boson mass m_D^2 is given by expression (46) with the replacement $g^2 sin^2\theta_w ß g^2 cos^2\theta_w$.

Summing up the expressions (41) and (46) and substituting them in Eq.(34), we obtain the photonic part V_{ring}^{γ} where it is necessary to express masses in terms of the vacuum value of the scalar condensate ϕ_c. In the same way the contribution of Z-bosons V_{ring}^{z} can be calculated. The only difference is an additional mass term of Z -field and the additional term in the Debye mass due to the neutral current $\sim \bar{\nu}\gamma_\mu \nu Z_{mu}$. These three fields - ϕ, γ, Z ,- which becomes massless in the restored phase, contribute into $V_{ring}(H, T)$ in the presence of the magnetic field. At zero field there is also a term due to the W-boson loops in Figs.1,2 . But when $H \neq 0$ the charged particles acquire $\sim eH$ masses. The corresponding fields remain short-range ones in the restored phase of the vacuum and therefore do not contribute.

A separate consideration should be spared to the tachyonic (unstable) mode in the W-boson spectrum: $p_0^2 = p_3^2 + M_w^2 - eH$. First of all we note that this mode is produced due to a spin interaction and it does not influence the $G_{00}(k)$ component of the W-boson propagator. Secondly, in the fields $eH \sim M_w^2$ the mode becomes a long range state. Therefore it should be included in $V_{ring}(H, T)$ side by side with other considered neutral fields. But in this case it is impossible to take advantage of formula (34) and one has to return to the initial EP with generalized propagators .

For our purpose it will be convenient to use the expression for the generalized EP written as a sum over the modes in external magnetic field [14],[15]:

$$V_{gen}^{(1)} = \frac{eH}{2\pi\beta} \sum_{l=-\infty}^{+\infty} \int_{-\infty}^{+\infty} \frac{dp_3}{2\pi} \sum_{n=0,\sigma=0,\pm1}^{\infty} log[\beta^2(\omega_l^2 + \epsilon_{n,\sigma,p_3}^2 + \Pi(T, H))], \qquad (47)$$

where $\omega_l = \frac{2\pi l}{\beta}$, $\epsilon_n^2 = p_3^2 + M_w^2 + (2n + 1 - 2\sigma)eH$ and $\Pi(H, T)$ is the Debye mass of W-bosons in a magnetic field. Denoting as $D_0^{-1}(p_3, H.T)$ the sum $\omega_l^2 + \epsilon^2$, one can rewrite eq. (47) as follows:

$$\begin{aligned} V_{gen}^{(1)} &= \frac{eH}{2\pi\beta} \sum_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \frac{dp_3}{2\pi} \sum_{n,\sigma} log[\beta^2 D_0^{-1}(p_3, H, T)] \\ &+ \frac{eH}{2\pi\beta} \sum_{-\infty}^{+\infty} \int_{-infty}^{+\infty} \frac{dp_3}{2\pi} \{log[1 + (\omega_l^2 + p_3^2 + M_w^2 - eH)^{-1}\Pi(H, T)] \end{aligned}$$

$$+ \sum_{n\neq 0,\sigma\neq +1} log[1 + D_0(\epsilon_n^2, H, T)\Pi(H,T)]\}. \quad (48)$$

Here, the first term is just the one-loop contribution of W-bosons, the second one gives the sum of ring diagrams of the unstable mode (as it can easily be verified by expanding the logarithm into a series).The last term describes the sum of the short range modes and should be omitted.

Thus, to find $V_{ring}^{unstable}$ one has to calculate the second term in Eq. (48). In the high temperature limit we obtain:

$$V_{ring}^{unstable} = \frac{eH}{2\pi\beta}\{(M_w^2 - eH + \Pi(H,T))^{1/2} - (M_w^2 - eH)^{1/2}\}. \quad (49)$$

By summing up the one-loop EP and all the terms V_{ring} , we arrive at the total consistent in leading order effective potential.

Let us note the most important features of the above expression. It is seen that the last term in Eq.(49) exactly cancels the dangerous term in Eq.(35). So, no instabilities appear at sufficiently high temperatures when $\Pi(H,T) > M_w^2 - eH$ and the EP is real.To make a quantitative estimate of the range of validity of the total EP it is necessary to calculate the mass operator of W-boson in a magnetic field at finite temperature and hence to find $\Pi(H,T)$. This is a separate and enough complicated problem which will not be solved here completely. Instead that below we restrict ourselves by the contribution to $\Pi(T)$ of the neutral Higgs particles only which can easily be calculated to give $\Pi(T)_{Higgs} = \frac{1}{12}g^2T^2$. Since other particles have also to contribute into the temperature mass in leading T^2 order, the obtained mass to be lower value which can be substituted into $V_{ring}^{unstable}$. Just this value will be used in the following estimations.

5. Symmetry behaviour in a magnetic field at high temperature

Having obtained the one-loop EP described by formulae (16),(22), (28)-(33) and the ring diagram contributions V_{ring} we are going to investigate symmetry behaviour at high temperature and strong magnetic fields. We shall present the results in two steps. First, we consider the sum of the tree and one-loop effective potentials as the function of ϕ^2 at various fixed H, T and K . Then, we shall add the term V_{ring} and calculate symmetry behaviour for the total EP at the same fixed H, T, K. This will help to clarify the role of the plasmon contributions. Since constructed EP includes as an input all fundamental particles, we shall obtain new information about the electroweak phase transition. The one-loop EP contains the imaginary parts in some domains of ϕ^2. But these terms occur to be cancelled by the corresponding ones from V_{ring}. Thus, only the real part,$ReV^{(1)}$, is of interest.

As usually [23], to investigate symmetry behaviour let us consider the difference $\mathcal{V}' = Re[\mathcal{V}(h, \phi, K, B) - \mathcal{V}(h, \phi = 0, B)]$ which gives information about the symmetry restoration. Below, we consider the case when the mass m_H is equal to M_w. Typical curves for small fields h and different values of B are plotted in Fig.3.

It is seen that the well known symmetry restoration (for the heavy fermion case) takes place. There are two minima - local, produces due to the Higgs mechanism, and and global one, generated by heavy fermion contributions. At low temperatures (big B), the local metastable minimum is disposed near the value $\phi^2 = 1$ that corresponds to the spontaneously broken symmetry. With a temperature increasing the local minimum becomes shallower and at $B \sim 0.1$ removes to the value $\phi^2 = 0$ that signals the symmetry restoration. We see that typical second type phase transition takes place for $K = 1$. At the same time, the barrier separating two minima is increasing in height and width and so a tunneling to the global unbounded from below minimum is suppressed.

In Fig.4 we present the influence of the field on the symmetry behaviour at low temperature. As is seen, an increase in h leads to the getting deeper of the local minimum and to the growing up the barrier which separates the minima. In this way the magnetic field prevents tunneling to the global unbounded minimum.

Now, let us investigate symmetry behaviour at high temperature and strong magnetic fields. The result of calculations is shown in Fig.5. From the plot it follows that the field tends to decrease the temperature and stimulates the symmetry breaking. First the metastable vacuum is generated with the field increasing. As is seen, this is a homogeneous transition. When the strength h is growing further the potential barrier separating the local and global vacua is diminishing and the depth of the former one is getting shallower. At $h \geq 2$ the electroweak minimum disappears at all. This picture is typical and realized at any high temperature. Therefore, it is possible to obtain an upper limit on the magnetic field strength requiring that the electroweak vacuum must be a long living state. From the above analysis one could conclude that the fields $H \geq 2H_0 = M^2/e \sim 2 \cdot 10^{24}G$ had not been generated in the early universe. In the opposite case our world would never been realized and the system from the very beginning suppose to be in the global unbounded minimum. Similar symmetry behaviour (with slightly different values of h, B) has also been determined for $K = 2$.

Fore completeness, let us describe the symmetry behaviour for the EP without the fermion contributions. In this case the symmetry restoration (for $K = 1, 2$) is also realized by the second type phase transition but typical temperatures to be of order $B \sim 0.5 - 0.6$. Naturally, no global minimum exists, so no limits on the magnetic field strength can be derived.

To summarize the above results we stress that fermions affect in a very essential way the symmetry behaviour in the field H. We also recall that our consideration was based on the one-loop EP only.

Now, let us include in our consideration the contribution of V_{ring} . In Fig. 6 we show the plot of V_{ring} . It represents a complicated dependence on h and this contribution for strong fields acts to remove the separation barrier and stimulate the transition to the global vacuum state.

In Figs.7 ,8 and 9, 10 the influence of the ring diagrams is represented for small B , weak fields $h \sim 0.01-0.1$ and $K = 0.5, 0.75$,respectively. To better clarify their role we show the plots in parallel for chosen K. As is seen, with V_{ring} included symmetry behaviour is considerably changed as compare with results presented in Fig. 5. Most important fact is that For $K = 1$ the local minimum is not realized at all even for weak field strengths. Actually, the value of $K = 0.75$ corresponding $m_H \sim 70$Gev is a bound value for the mass m_H and the field strengths $H \sim 0.01 - 0.1 M_w^2/e = 0.01 - 0.1 \cdot 10^{24}$ G give upper limits on the magnetic fields in the phase transition epoch. Stronger fields stimulates straight transition to the global minimum.Thus we see that due to V_{ring} term the upper limit on the magnetic field is decreased from $2 \cdot 10^{24}G$ derived with the EP $V^{(1)}$ to $0.1 \cdot 10^{24}G$. Moreover, they restrict the Higgs boson mass: $m_H < 70Gev$, otherwise the local electroweak minimum is not produced. We also observe that the phase transition is to be of the first type one.

It is very important that the described minima of the EP are stable at high temperatures even for strong fields h. Really, typical value of B when the symmetry restoration happens for $K \sim 1$ is $B \sim 0.1$. For the calculated lower temperature mass we have $\frac{1}{12}g^2/B^2 \sim 3$. Hence we find that the effective mass of the unstable mode $\phi^2 - h + \frac{1}{12}g^2/B^2$ is positive for all values of h considered. Thus, one has to conclude that classical constant magnetic field must be stable in the electroweak phase transition epoch. No W- and Z-boson condensates can be produced at high temperatures. These condensates would be realized at lower temperatures as the intermediate states of the vacuum.

6. Discussion

Let us discuss the vacuum stability in a magnetic field. This problem is of interest because of the presence in the W-boson spectrum the mode $p_0^2 = M_w^2 - eH$ which becomes unstable for $H > M_w^2/e$. The evolution of this state and its consequences have been investigated in various aspects by many authors [25],[23],[13]. At high temperatures, it was studied for the case when only the bosonic part of the electroweak theory has been included [12],[19],[13]. In Refs. [13] the classical equations of W, Z and ϕ fields were solved and the results that at high temperature the symmetry is restored and the magnetic field is screened by the W- boson and Z-boson condensates have been elaborated. The key point in this analysis is the assumption that symmetry restoration is the second type phase transition which can be taken into account by adding the term $\sim \phi^2 T^2$ in the field equations. For these results it was also important that the Higgs boson mass equals to m_Z. From the point of view of the present investigation the described approach is not trusty because it does not take into account the vacuum polarization which is very important at high temperature and strong fields. In fact, this is the only one term produced by the vacuum polarization and other relevant contributions must be included. Exactly the polarization effects produce the temperature masses which stabilize the vacuum. So, no conditions for generation of the W- and Z-boson condensates exist. These condensates could be realized in strong magnetic fields at intermediate temperatures when imaginary part of the EP is nonzero. But in any case, to have a consistent picture the vacuum polarization should be taken into account because of a complicated behaviour of the EP in an intermediate range of T, H [12],[19].

The present investigation carried out on the base of the complete EP with fermions included shown that at weak magnetic fields the symmetry restoration is to be the first type phase transition (for the values of K

considered ,$m_H < M_w$). For strong fields our metastable vacuum could never be realized. But at any fixed temperature T there exists a corresponding magnetic field strength at which a metastable vacuum with positive energy and non-zero scalar field is produced . This picture may be of interest for cosmology.

As follows from the results of Sect.5 , the role of fermions is very essential in the symmetry dynamics. Actually, their contribution determines the properties of the phase transition due to magnetic field. We have seen that at low temperature the field acts to prevent the phase transition from the metastable to stable minimum of the EP. Since the charged fermions and gauge bosons oppositely influence the symmetry behaviour in a magnetic field, for the actual values of particle masses these two contributions compensate each other and the metastable minimum position remains near the initial point $\phi^2 = 1$ for any values of h. The influence of the field is expressed in the change of the potential barrier separating two minima. As is also occurred, at high temperatures the role of the ring diagrams is important (as also takes place at $h = 0$ [16],[17]). Thus, we conclude that the EW phase transition in a magnetic field acquires substantial changes. For its detailed investigation it is necessary to calculate the bubble nucleation parameters, the metastability bound on m_H, etc.

We would like to complete our discussion with a few remarks concerning the magnetic field in the restored phase. As was mentioned in the Introduction, in current literature a hypercharge magnetic field is discussed as a relevant one there [9]. There is an important difference between this field and ordinary magnetic field investigated in the present paper. The former field is an abelian one and requires an external source to maintain it in space. On the contrary, ordinary magnetic field may be considered as the projection of a nonabelian field produced spontaneously at high temperatures via Savvidy's mechanism. Investigation of the state was given in Refs.[7],[14],[24]. A final conclusion about this phenomenon can be done when the complete temperature mass of the unstable mode $\Pi(H, T)$ will be calculated. This work is in progress now.

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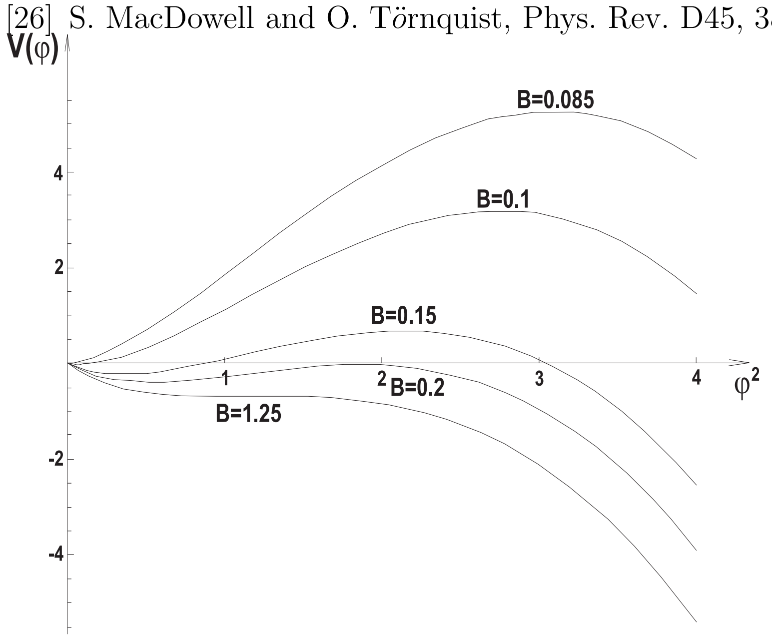

Figure 3: Symmetry restoration at high temperatures and small magnetic fields determined by the one-loop effective potential $\mathcal{V}(\phi^2, h, B, K)$.

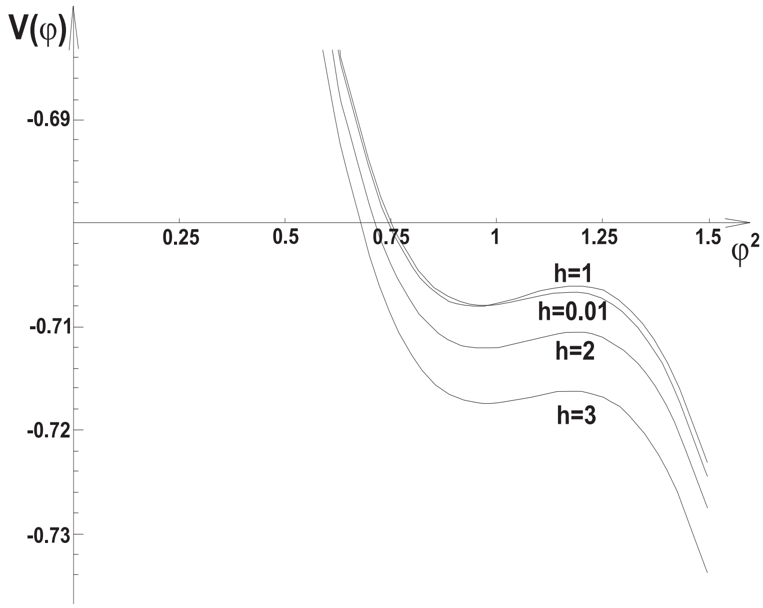

Figure 4: Symmetry behaviour at zero temperature and strong magnetic fields determined by the one-loop effective potential $\mathcal{V}(\phi^2, h, K)$.

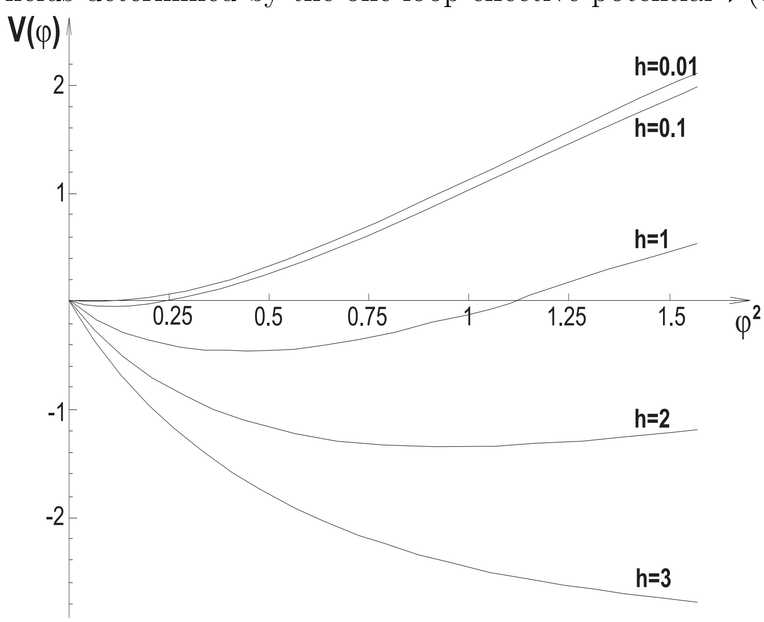

Figure 5: Symmetry behaviour at fixed high temperature and a number of values h described by the one-loop efective potential $\mathcal{V}(\phi^2, h, B, K)$.

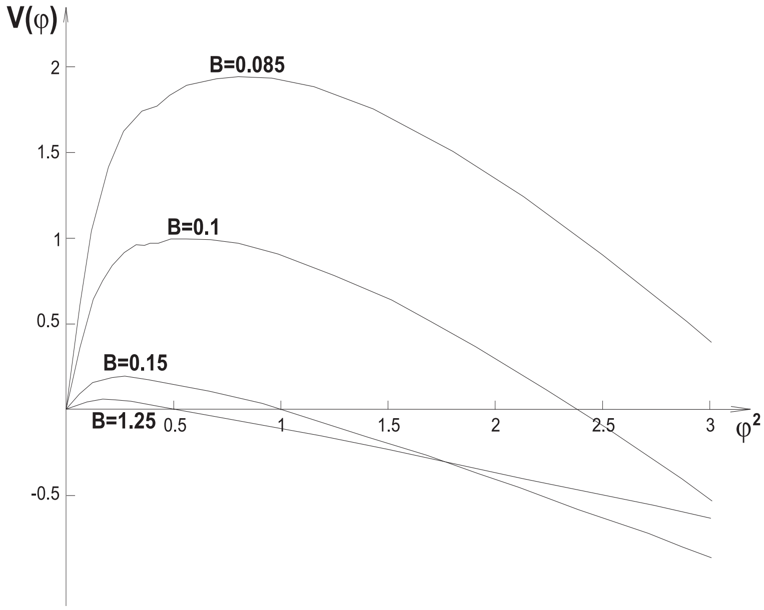

Figure 6: The V_{ring} curves as the functions of ϕ^2 for fixed temperature and a number of h.

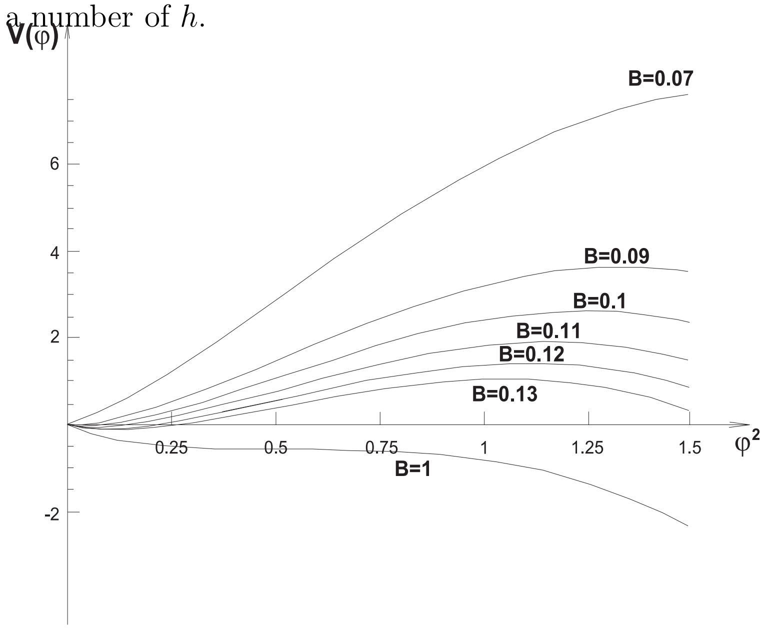

Figure 7: Symmetry behaviour at high temperature and 'weak' magnetic fields determined within the one-loop effective potential $\mathcal{V} = \mathcal{V}^{(0)} + \mathcal{V}^{(1)}$ for $K = 0.5$.

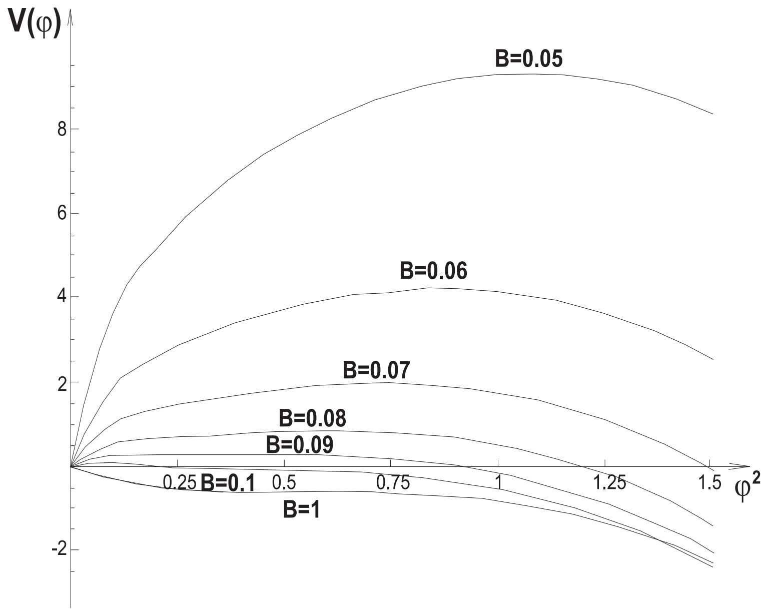

Figure 8: Symmetry behaviour at high temperature and 'weak' magnetic fields determined within the total effective potential $\mathcal{V} = \mathcal{V}^{(0)} + \mathcal{V}^{(1)} + \mathcal{V}_{ring}$ for $K = 0.5$.

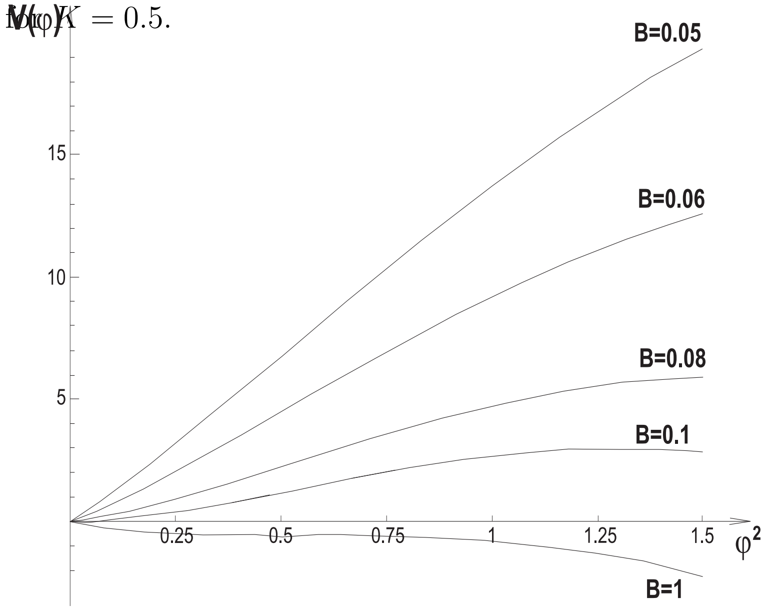

Figure 9: Symmetry behaviour at high temperature and 'weak' magnetic fields determined within the total effective potential $\mathcal{V} = \mathcal{V}^{(0)} + \mathcal{V}^{(1)} + \mathcal{V}_{ring}$ for $K = 0.75$.

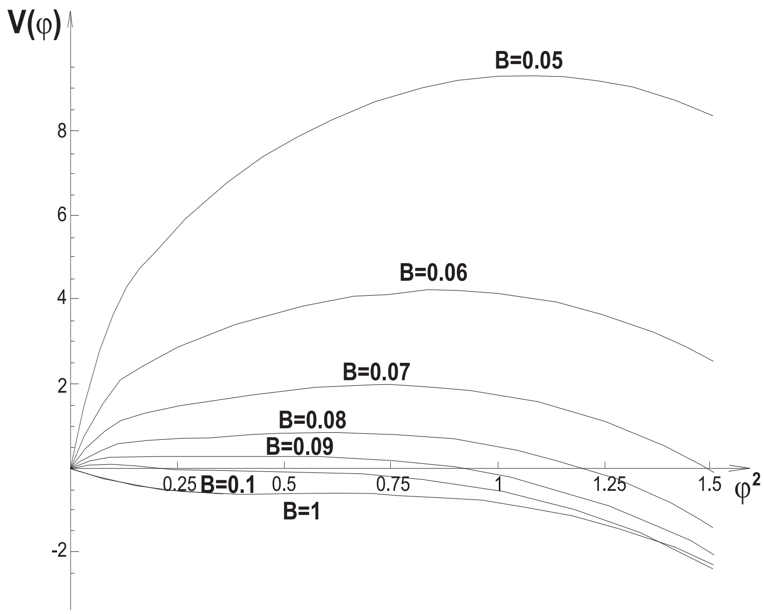

Figure 10: Symmetry behaviour at high temperature and 'weak' magnetic fields determined within the total effective potential $\mathcal{V} = \mathcal{V}^{(0)} + \mathcal{V}^{(1)} + \mathcal{V}_{ring}$ for $K = 0.75$.