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Further Study on Possible Violation of CP, T and CPT Symmetries in the K^0-$\overline{K^0}$ System — Remarks and Results —*

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Abstract

We demonstrate how one may identify or constrain possible violation of CP, T and CPT symmetries in the K^0-$\overline{K^0}$ system in a way as phenomenological and comprehensive as possibie. For this purpose, we first introduce parameters which represent violation of these symmetries in mixing parameters and decay amplitudes in a well-defined way. After discussing some characteristics of these parameters, we derive formulae which relate them to the experimentally measured quantities. We then carry out a numerical analysis with the help of the Bell-Steinberger relation to derive constraints to these violating parameters from available experimental data. Finally, we compare our parametrization and procedure of analysis with those employed in the recent literature.

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1 Introduction

Although, on the one hand, the standard field theory implies that CPT symmetry should hold exactly, and, on the other hand, all experimental observations up to now are perfectly consistent with this symmetry, continued experimental, phenomenological and theoretical studies of this and related symmetries are warrented.

In a series of papers[1-5], we have demonstrated how one may identify or constrain possible violation of CP, T and CPT symmetries in the K^0-$\overline{K^0}$ system in a way as phenomenological and comprehensive as possible. For this purpose, we have first introduced parameters which represent violation of these symmetries in mixing parameters and decay amplitudes in a well-defined way and related them to the experimentally measured quantities. We have then carried out a numerical analysis with little theoretical input to derive constraints to these violating parameters from available experimental data. It has been shown among other things that the most recent results on leptonic asymmetries obtained by the CPLEAR Collaboration[6] allow one for the first time to constrain to some extent possible CPT violation in leptonic decay modes.[a]

As discussed in [1-3], our parametrization is very unique in that it is manifestly invariant with respect to rephasing of the $|K^0\rangle$ and $|\overline{K^0}\rangle$ states,

$$|K^0\rangle \rightarrow |K^0\rangle' = |K^0\rangle e^{-i\xi_K} \ , \qquad |\overline{K^0}\rangle \rightarrow |\overline{K^0}\rangle' = |\overline{K^0}\rangle e^{i\xi_K} \ . \tag{1.1}$$

It has to be noted however that our parametrization is not invariant with respect to rephasing of final states $|f\rangle$, e.g.

$$\begin{aligned} |(2\pi)_I\rangle &\rightarrow |(2\pi)_I\rangle' = |(2\pi)_I\rangle e^{-i\xi_I} \ , \\ |\ell^-\rangle &\rightarrow |\ell^-\rangle' = |\ell^-\rangle e^{-i\xi_\ell} \ , \\ |\ell^+\rangle &\rightarrow |\ell^+\rangle' = |\ell^+\rangle e^{-i\overline{\xi}_\ell} \ . \end{aligned} \tag{1.2}$$

where $I = 0$ or 2 stands for the isospin of the 2π states, $|\ell^-\rangle = |\pi^+\ell^-\overline{\nu}_\ell\rangle$, $|\ell^+\rangle = |\pi^-\ell^+\nu_\ell\rangle$ and $\ell = e$ or μ. We have rather adopted a specific phase convention for the final states $|f\rangle$. Some of the constraints which we have claimed to follow from CP, T and/or CPT symmetries do depend on this phase convention and have to be distinguished from those constraints which are phase-convention-independent. We would like to clarify these points in Sec. 4 of the present work.

The main data we have used in our last analysis[5] are those reported by the CPLEAR Collaboration [6] and those compiled by the Particle Data Group in [8]. The latter article also contains a number of notes giving definition of parameters, formulae relevant for data processing and related remarks. In Sec. 7, we would like to compare our parametrization and procedure of analysis with those given or cited in [8].

[a] After our last paper[5] being sent to the major high energy physics centers, we became aware that the CPLEAR Collaboration themselves[7] had also, by an analysis more or less similar to ours, reached the similar conclusion.

To be self-contained, we need to recapitulate our parametrization(Sec. 2 and Sec. 3), formulae(Sec. 5) and main results(Sec. 6). These parts are essentially same with our previous works[1-5], except that the $\pi^+\pi^-\gamma$ state is taken into account as one of intermediate states in the Bell-Steinberger relation.

2 The K^0-$\overline{K^0}$ mixing and the Bell-Steinberger relation

Let $|K^0\rangle$ and $|\overline{K^0}\rangle$ be eigenstates of the strong interaction with strangeness $S = +1$ and -1, related to each other by (CP), (CPT) and T operations as[1, 2, 9]

$$\begin{aligned} (CP)|K^0\rangle &= e^{i\alpha_K}|\overline{K^0}\rangle \ , \quad (CPT)|K^0\rangle = e^{i\beta_K}|\overline{K^0}\rangle \ , \\ (CP)|\overline{K^0}\rangle &= e^{-i\alpha_K}|K^0\rangle \ , \quad (CPT)|\overline{K^0}\rangle = e^{i\beta_K}|K^0\rangle \ , \\ T|K^0\rangle &= e^{i(\beta_K-\alpha_K)}|K^0\rangle \ , \quad T|\overline{K^0}\rangle = e^{i(\beta_K+\alpha_K)}|\overline{K^0}\rangle \ . \end{aligned} \tag{2.1}$$

Note here that, given the first two where α_K and β_K are arbitrary real parameters, the rest follow from $(CP)T = T(CP) = (CPT)$ and anti-linearity of T and (CPT).

When the weak interaction H_{W} is switched on, the K^0 and $\overline{K^0}$ states decay into other states and become mixed. The time evolution of the arbitrary state

$$|\Psi(t)\rangle = c_1(t)|K^0\rangle + c_2(t)|\overline{K^0}\rangle$$

is described by a Schrödinger-like equation[10]

$$i\frac{d}{dt}\begin{pmatrix} c_1(t) \\ c_2(t) \end{pmatrix} = \Lambda \begin{pmatrix} c_1(t) \\ c_2(t) \end{pmatrix} \ . \tag{2.2}$$

$\Lambda = (\Lambda_{ij})$ is a 2×2 matrix related to H_{W}, e.g.

$$\Lambda_{12} = \sum_f \langle K^0|H_{\mathrm{W}}|f\rangle\langle f|H_{\mathrm{W}}|\overline{K^0}\rangle/(m_K - E_f + i\varepsilon) \ ,$$

and may be written as

$$\Lambda = M - i\frac{\Gamma}{2} \ , \tag{2.3}$$

$M(\Gamma)$ being an hermitian matrix called mass (decay) matrix. The two eigenstates of Λ and their respective eigenvalues may be written as

$$|K_S\rangle = \frac{1}{\sqrt{|p_S|^2 + |q_S|^2}}\left(p_S|K^0\rangle + q_S|\overline{K^0}\rangle\right) \ , \tag{2.4a}$$

$$|K_L\rangle = \frac{1}{\sqrt{|p_L|^2 + |q_L|^2}}\left(p_L|K^0\rangle - q_L|\overline{K^0}\rangle\right) \ ; \tag{2.4b}$$

$$\lambda_S = m_S - i\frac{\gamma_S}{2} \ , \tag{2.5a}$$

$$\lambda_L = m_L - i\frac{\gamma_L}{2} \ . \tag{2.5b}$$

λ_S, λ_L, q_S/p_S and q_L/p_L are related to Λ_{ij}, and $m_{S,L} = \mathrm{Re}(\lambda_{S,L})$ and $\gamma_{S,L} = -2\mathrm{Im}(\lambda_{S,L})$ are the mass and the total decay width of the $K_{S,L}$ state respectively. Parametrizing q_S/p_S and q_L/p_L as

$$\begin{aligned} \frac{q_S}{p_S} &= e^{i\alpha_K}\frac{1-\varepsilon_S}{1+\varepsilon_S} \ , \\ \frac{q_L}{p_L} &= e^{i\alpha_K}\frac{1-\varepsilon_L}{1+\varepsilon_L} \ , \end{aligned} \tag{2.6}$$

one may express $|K_S\rangle$ and $|K_L\rangle$ as

$$\begin{aligned} |K_S\rangle &= \frac{1}{\sqrt{2(1+|\varepsilon_S|^2)}}\left\{(1+\varepsilon_S)e^{-i\alpha_K/2}|K^0\rangle + (1-\varepsilon_S)e^{i\alpha_K/2}|\overline{K^0}\rangle\right\} \\ &= \frac{1}{\sqrt{1+|\varepsilon_S|^2}}\left(|K_1\rangle + \varepsilon_S|K_2\rangle\right) \ , \end{aligned} \tag{2.7a}$$

$$\begin{aligned} |K_L\rangle &= \frac{1}{\sqrt{2(1+|\varepsilon_L|^2)}}\left\{(1+\varepsilon_L)e^{-i\alpha_K/2}|K^0\rangle - (1-\varepsilon_L)e^{i\alpha_K/2}|\overline{K^0}\rangle\right\} \\ &= \frac{1}{\sqrt{1+|\varepsilon_L|^2}}\left(|K_2\rangle + \varepsilon_L|K_1\rangle\right) \ , \end{aligned} \tag{2.7b}$$

where

$$|K_{1,2}\rangle = \frac{1}{\sqrt{2}}\left(e^{-i\alpha_K/2}|K^0\rangle \pm e^{i\alpha_K/2}|\overline{K^0}\rangle\right) \tag{2.8}$$

are CP eigenstates with $CP = \pm 1$. Note that the overall phases of $|K_{1,2}\rangle$ and $|K_{S,L}\rangle$ are chosen in such a way as[1]

$$CPT|K_{1,2}\rangle = \pm e^{i\beta_K}|K_{1,2}\rangle \ ,$$

$$|K_{S,L}\rangle \to |K_{1,2}\rangle \quad \text{as} \quad \varepsilon_{S,L} \to 0 \ .$$

$\varepsilon_{S,L}$ will further be parametrized as

$$\varepsilon_{S,L} = \varepsilon \pm \delta \ . \tag{2.9}$$

From the eigenvalue equation of Λ, one may readily derive the well-known Bell-Steinberger relation[11]:

$$\left[\frac{\gamma_S+\gamma_L}{2} - i\Delta\right]\langle K_S|K_L\rangle = \langle K_S|\Gamma|K_L\rangle \ , \tag{2.10}$$

where

$$\langle K_S|\Gamma|K_L\rangle = 2\pi \sum_f \langle K_S|H_{\rm W}|f\rangle\langle f|H_{\rm W}|K_L\rangle \delta(m_K - E_f) \ , \tag{2.11}$$

$$\Delta = m_S - m_L \ . \tag{2.12}$$

One may further verify[3, 4]

$$\varepsilon_{\parallel} \equiv {\rm Re}[\varepsilon \exp(-i\phi_{SW})] \simeq \frac{-2{\rm Im}(M_{12}e^{i\alpha_K})}{\sqrt{(\gamma_S - \gamma_L)^2 + 4\Delta^2}} \ , \tag{2.13a}$$

$$\varepsilon_{\perp} \equiv {\rm Im}[\varepsilon \exp(-i\phi_{SW})] \simeq \frac{{\rm Im}(\Gamma_{12}e^{i\alpha_K})}{\sqrt{(\gamma_S - \gamma_L)^2 + 4\Delta^2}} \ , \tag{2.13b}$$

$$\delta_{\parallel} \equiv {\rm Re}[\delta \exp(-i\phi_{SW})] \simeq \frac{(\Gamma_{11} - \Gamma_{22})}{2\sqrt{(\gamma_S - \gamma_L)^2 + 4\Delta^2}} \ , \tag{2.14a}$$

$$\delta\perp \equiv {\rm Im}[\delta \exp(-i\phi_{SW})] \simeq \frac{(M_{11} - M_{22})}{\sqrt{(\gamma_S - \gamma_L)^2 + 4\Delta^2}} \ , \tag{2.14b}$$

where

$$\phi_{SW} = \tan^{-1}\left(\frac{-2\Delta}{\gamma_S - \gamma_L}\right) \tag{2.15}$$

is the so-called superweak phase.

3 Decay amplitudes

The K^0 and $\overline{K^0}$ (or K_S and K_L) states have many decay channels, among which we concentrate on the following four relevant modes.

3.1 2π modes

We parametrize amplitudes for K^0 and $\overline{K^0}$ to decay into $(2\pi)_I$ as[1]

$$\begin{aligned} \langle(2\pi)_I|H_{\rm W}|K^0\rangle &= F_I(1+y_I)e^{i\alpha_K/2} \ , \\ \langle(2\pi)_I|H_{\rm W}|\overline{K^0}\rangle &= F_I^*(1-y_I^*)e^{-i\alpha_K/2} \ , \end{aligned} \tag{3.1}$$

and further introduce

$$z_I = \frac{{\rm Im}(F_I)}{{\rm Re}(F_I)} \ . \tag{3.2}$$

The experimentally measured quantities are η_{+-} and η_{00} defined by

$$\eta_{+-} = |\eta_{+-}|e^{i\phi_{+-}} = \frac{\langle\pi^+\pi^-, {\rm outgoing}|H_{\rm W}|K_L\rangle}{\langle\pi^+\pi^-, {\rm outgoing}|H_{\rm W}|K_S\rangle} \ , \tag{3.3a}$$

$$\eta_{00} = |\eta_{00}| e^{i\phi_{00}} = \frac{\langle \pi^0\pi^0, \text{outgoing}|H_{\rm W}|K_L\rangle}{\langle \pi^0\pi^0, \text{outgoing}|H_{\rm W}|K_S\rangle} \ . \tag{3.3b}$$

Defining

$$\eta_I = \frac{\langle (2\pi)_I|H_{\rm W}|K_L\rangle}{\langle (2\pi)_I|H_{\rm W}|K_S\rangle} \ , \tag{3.4}$$

$$\omega = \frac{\langle (2\pi)_2|H_{\rm W}|K_S\rangle}{\langle (2\pi)_0|H_{\rm W}|K_S\rangle} \ , \tag{3.5}$$

one gets

$$\eta_{+-} = \frac{\eta_0 + \eta_2\omega'}{1+\omega'} \ , \tag{3.6a}$$

$$\eta_{00} = \frac{\eta_0 - 2\eta_2\omega'}{1-2\omega'} \ , \tag{3.6b}$$

where

$$\omega' = \frac{1}{\sqrt{2}}\omega e^{i(\delta_2-\delta_0)} \ , \tag{3.7}$$

δ_I being the S-wave $\pi\pi$ scattering phase shift for the isospin I state at an energy of the rest mass of K^0. ω is a measure of deviation from the $\Delta I = 1/2$ rule, and may be inferred, for example, from

$$\begin{aligned} r &\equiv \frac{\gamma_S(\pi^+\pi^-) - 2\gamma_S(\pi^0\pi^0)}{\gamma_S(\pi^+\pi^-) + \gamma_S(\pi^0\pi^0)} \\ &= \frac{4\text{Re}(\omega') - 2|\omega'|^2}{1+2|\omega'|^2} \ . \end{aligned} \tag{3.8}$$

Here and in the following, $\gamma_{S,L}(f)$ denotes the partial width for $K_{S,L}$ to decay into the final state f.

3.2 3π and $\pi^+\pi^-\gamma$ modes

The experimentally measured quantities are

$$\eta_{+-0} = \frac{\langle \pi^+\pi^-\pi^0, \text{outgoing}|H_{\rm W}|K_S\rangle}{\langle \pi^+\pi^-\pi^0, \text{outgoing}|H_{\rm W}|K_L\rangle} \ , \tag{3.9a}$$

$$\eta_{000} = \frac{\langle \pi^0\pi^0\pi^0, \text{outgoing}|H_{\rm W}|K_S\rangle}{\langle \pi^0\pi^0\pi^0, \text{outgoing}|H_{\rm W}|K_L\rangle} \ , \tag{3.9b}$$

$$\eta_{+-\gamma} = \frac{\langle \pi^+\pi^-\gamma, \text{outgoing}|H_{\rm W}|K_L\rangle}{\langle \pi^+\pi^-\gamma, \text{outgoing}|H_{\rm W}|K_S\rangle} \ . \tag{3.10}$$

We shall treat the 3π ($\pi^+\pi^-\gamma$) states as purely CP-odd (CP-even).

3.3 Leptonic modes

We parametrize amplitudes for K^0 and $\overline{K^0}$ to decay into $|\ell^+\rangle = |\pi^-\ell^+\nu_\ell\rangle$ and $|\ell^-\rangle = |\pi^+\ell^-\overline{\nu}_\ell\rangle$, where $\ell = e$ or μ, as[1, 12, 13]

$$\begin{aligned}
\langle\ell^+|H_{\rm W}|K^0\rangle &= F_\ell(1+y_\ell)e^{i\alpha_K/2} \ , \\
\langle\ell^-|H_{\rm W}|\overline{K^0}\rangle &= F_\ell^*(1-y_\ell^*)e^{-i\alpha_K/2} \ , \\
\langle\ell^+|H_{\rm W}|\overline{K^0}\rangle &= x_{\ell+}F_\ell(1+y_\ell)e^{-i\alpha_K/2} \ , \\
\langle\ell^-|H_{\rm W}|K^0\rangle &= x_{\ell-}^*F_\ell^*(1-y_\ell^*)e^{i\alpha_K/2} \ .
\end{aligned} \tag{3.11}$$

$x_{\ell\pm}$, which measure deviation from the $\Delta S = \Delta Q$ rule, are further parametrized as

$$x_{\ell\pm} = x_\ell \pm x'_\ell \ . \tag{3.12}$$

Rather than the well measured time-independent asymmetry parameter

$$d_L^\ell = \frac{\gamma_L(\pi^-\ell^+\nu_\ell) - \gamma_L(\pi^+\ell^-\overline{\nu}_\ell)}{\gamma_L(\pi^-\ell^+\nu_\ell) + \gamma_L(\pi^+\ell^-\overline{\nu}_\ell)} \ , \tag{3.13a}$$

the CPLEAR Collaboration[6] have for the first time measured two kinds of time-dependent asymmetry parameters

$$d_1^\ell(t) = \frac{|\langle\ell^-|H_{\rm W}|K^0(t)\rangle|^2 - |\langle\ell^+|H_{\rm W}|\overline{K^0}(t)\rangle|^2}{|\langle\ell^-|H_{\rm W}|K^0(t)\rangle|^2 + |\langle\ell^+|H_{\rm W}|\overline{K^0}(t)\rangle|^2} \ , \tag{3.13b}$$

$$d_2^\ell(t) = \frac{|\langle\ell^+|H_{\rm W}|K^0(t)\rangle|^2 - |\langle\ell^-|H_{\rm W}|\overline{K^0}(t)\rangle|^2}{|\langle\ell^+|H_{\rm W}|K^0(t)\rangle|^2 + |\langle\ell^-|H_{\rm W}|\overline{K^0}(t)\rangle|^2} \ . \tag{3.13c}$$

4 Conditions imposed by CP, T and/or CPT symmetries

Although our amplitude parameters F_f and y_f as well as our mixing parameters ε and δ are all invariant with respect to rephasing of the $|K^0\rangle$ and $|\overline{K^0}\rangle$ states, Eq.(1.1), F_f and y_f are not invariant with respect to rephasing of the final state $|f\rangle$, Eq.(1.2). So are the relative CP phase α_ℓ between $|\ell^+\rangle$ and $|\ell^-\rangle$ and the relative CPT phase β_f between $|f\rangle$ and $|\overline{f}\rangle$ defined in such a way as

$$CP|\ell^+\rangle = e^{i\alpha_\ell}|\ell^-\rangle \ , \qquad CPT|f\rangle = e^{i\beta_f}|\overline{f}\rangle \ , \tag{4.1}$$

where α_ℓ and β_f are arbitrary real parameters and it is understood that $|\overline{f}\rangle = |f\rangle$ for $|f\rangle = |(2\pi)_I\rangle$ and $|\pi^+\pi^-\gamma\rangle$ and $|\overline{f}\rangle = -|f\rangle$ for $|f\rangle = |\pi^+\pi^-\pi^0\rangle$ and $|\pi^0\pi^0\pi^0\rangle$.

One may verify that CP, T and CPT symmetries impose such conditions as[b]

$$CP \text{ symmetry} \;:\; \mathrm{Im}(F_\ell)/\mathrm{Re}(F_\ell) = -\tan\alpha_\ell/2 \;; \tag{4.2a}$$

$$T \text{ symmetry} \;:\; 2\mathrm{Im}(y_f)/(1-|y_f|^2) = \tan(\beta_f - \beta_K) \;, \tag{4.2b}$$

$$\mathrm{Im}[(1+iz_I)^2(1-y_I^2)] = 0 \;, \tag{4.2c}$$

$$\mathrm{Im}[F_\ell^2(1-y_\ell^2)\exp(i\alpha_\ell)] = 0 \;; \tag{4.2d}$$

$$CPT \text{ symmetry} \;:\; \mathrm{Im}(y_f) = \tan(\beta_f - \beta_K)/2 \;, \tag{4.2e}$$

in addition to

$$\begin{aligned} CP \text{ symmetry} \;&:\; \varepsilon = 0,\; \delta = 0,\; z_I = 0,\; \mathrm{Re}(y_f) = 0,\; \mathrm{Im}(x_\ell) = 0,\; \mathrm{Re}(x'_\ell) = 0 \;; \\ T \text{ symmetry} \;&:\; \varepsilon = 0,\; \mathrm{Im}(x_\ell) = 0,\; \mathrm{Im}(x'_\ell) = 0 \;; \\ CPT \text{ symmetry} \;&:\; \delta = 0,\; \mathrm{Re}(y_f) = 0,\; \mathrm{Re}(x'_\ell) = 0,\; \mathrm{Im}(x'_\ell) = 0 \;. \end{aligned} \tag{4.3}$$

One sees from Eqs.(4.2a,b,d,e) that, since α_ℓ and β_f are completely arbitrary, $\mathrm{Im}(F_\ell)$ and $\mathrm{Im}(y_f)$ remain unconstrained even if one impose CP, T and/or CPT symmetries. It can be shown[14] however that it is possible by a choice of phase convention to set[c]

$$\mathrm{Im}(F_\ell) = 0 \;, \qquad \mathrm{Im}(y_f) = 0 \;. \tag{4.4}$$

Eq.(4.2c) then gives

$$T \text{ symmetry} : z_I = 0 \;. \tag{4.5}$$

5 Formulae relevant for numerical analysis

We shall adopt a phase convention which gives Eq.(4.4). Observed and expected smallness of violation of CP, T and CPT symmetries and of the $\Delta S = \Delta Q$ rule

[b] In our previous papers[2-4], having adopted from the outset the phase convention

$$\alpha_\ell = 0 \;, \qquad \beta_f = \beta_K \;, \tag{4.6}$$

we are led to claim that Eq.(4.4), too, would follow from CP, T and/or CPT symmetries and hence include $\mathrm{Im}(F_\ell)$ and $\mathrm{Im}(y_f)$ in our list of symmetry-violating parameters. Also, our classification of ε and δ as indirect parameters and of z_I, $\mathrm{Re}(y_f)$, $\mathrm{Im}(x_\ell)$ and x'_ℓ (and, erroneously, $\mathrm{Im}(F_\ell)$ and $\mathrm{Im}(y_f)$ as well) as direct parameters is not very consistent, since non-vanishing of the latter set of parameters will in general result in non-vanishing of the former set of parameters. More consistent is to refer to $\varepsilon_\parallel$ and $\delta_\perp$, which are related exclusively to the mass matrix, as indirect parameters and all the others, which are related to decay amplitudes or decay matrix, as direct parameters.

[c] See Ref.[12, 13] for related discussion on (non)observability of $\mathrm{Im}(y_\ell)$.

allows us to treat all our symmetry-violating parameters (i.e. those implicit in Eqs.(4.3) and (4.5)), and $\mathrm{Re}(x_\ell)$ as well, as small, [d] and, to the leading order, one finds

$$\omega \simeq \frac{\mathrm{Re}(F_2)}{\mathrm{Re}(F_0)} \ , \tag{5.1}$$

$$\eta_I \simeq \varepsilon - \delta + \mathrm{Re}(y_I) + iz_I \ , \tag{5.2a}$$

$$\eta_2 - \eta_0 \simeq \mathrm{Re}(y_2 - y_0) + i(z_2 - z_0) \ , \tag{5.2b}$$

$$d_1^\ell(t \gg 1/\gamma_S) \simeq -4\mathrm{Re}(\varepsilon) - 2\mathrm{Re}(y_\ell - x'_\ell) \ , \tag{5.3a}$$

$$d_2^\ell(t \gg 1/\gamma_S) \simeq -4\mathrm{Re}(\delta) + 2\mathrm{Re}(y_\ell - x'_\ell) \ . \tag{5.3b}$$

Furthermore, by taking 2π, 3π, $\pi^+\pi^-\gamma$ and $\pi\ell\nu_\ell$ intermediate states into account in the Bell-Steinberger relation, Eq.(2.10) with Eq.(2.11), one may, with the help of Eqs.(2.7a,b) and (2.9), express $\mathrm{Re}(\varepsilon)$ and $\mathrm{Im}(\delta)$ in terms of the measured quantities:

$$\begin{aligned}
\mathrm{Re}(\varepsilon) \ \simeq \ & \frac{1}{\sqrt{\gamma_S^2 + 4\Delta^2} + 4\cos\phi_{SW}\sum_\ell \gamma_L(\pi\ell\nu_\ell)} \times \\
& \Big[\ \gamma_S(\pi^+\pi^-)|\eta_{+-}|\cos(\phi_{+-} - \phi_{SW}) \\
& \quad + \gamma_S(\pi^0\pi^0)|\eta_{00}|\cos(\phi_{00} - \phi_{SW}) \\
& \quad + \gamma_S(\pi^+\pi^-\gamma)|\eta_{+-\gamma}|\cos(\phi_{+-\gamma} - \phi_{SW}) \\
& \quad + \gamma_L(\pi^+\pi^-\pi^0)\{\mathrm{Re}(\eta_{+-0})\cos\phi_{SW} - \mathrm{Im}(\eta_{+-0})\sin\phi_{SW}\} \\
& \quad + \gamma_L(\pi^0\pi^0\pi^0)\{\mathrm{Re}(\eta_{000})\cos\phi_{SW} - \mathrm{Im}(\eta_{000})\sin\phi_{SW}\} \\
& \quad + \sum_\ell \gamma_L(\pi\ell\nu_\ell)\{(2\mathrm{Re}(x'_\ell) - d_1^\ell(t \gg 1/\gamma_S))\cos\phi_{SW} \\
& \qquad\qquad\qquad\qquad\qquad\qquad -2\mathrm{Im}(x'_\ell)\sin\phi_{SW}\}\Big] \ ,
\end{aligned} \tag{5.4}$$

$$\begin{aligned}
\mathrm{Im}(\delta) \ \simeq \ & \frac{1}{\sqrt{\gamma_S^2 + 4\Delta^2}} \times \\
& \Big[\ -\gamma_S(\pi^+\pi^-)|\eta_{+-}|\sin(\phi_{+-} - \phi_{SW}) \\
& \quad - \gamma_S(\pi^0\pi^0)|\eta_{00}|\sin(\phi_{00} - \phi_{SW}) \\
& \quad - \gamma_S(\pi^+\pi^-\gamma)|\eta_{+-\gamma}|\sin(\phi_{+-\gamma} - \phi_{SW}) \\
& \quad + \gamma_L(\pi^+\pi^-\pi^0)\{\mathrm{Re}(\eta_{+-0})\sin\phi_{SW} + \mathrm{Im}(\eta_{+-0})\cos\phi_{SW}\} \\
& \quad + \gamma_L(\pi^0\pi^0\pi^0)\{\mathrm{Re}(\eta_{000})\sin\phi_{SW} + \mathrm{Im}(\eta_{000})\cos\phi_{SW}\} \\
& \quad + 2\sum_\ell \gamma_L(\pi\ell\nu_\ell)\{\mathrm{Re}(y_\ell)\sin\phi_{SW} + \mathrm{Im}(x'_\ell)\cos\phi_{SW}\}\Big] \ .
\end{aligned} \tag{5.5}$$

In deriving these equations, use has been made of the fact $\gamma_S \gg \gamma_L$.

[d] As a matter of fact, we have already assumed that CP, T and CPT violations are small in deriving Eqs.(2.13a) $\sim$ (2.14b).

6 Numerical results

As the first set of input data, we use the PDG-1996 data[8] as far as available (except those on η_{+-0} and η_{000}) and supplement them by Chell-Olsson's value[15] on $\delta_2 - \delta_0$ and CPLEAR's results[6] on η_{+-0}, η_{000}, $d_1^\ell(t \gg 1/\gamma_S)$ and $d_2^\ell(t \gg 1/\gamma_S)$. As the second set, we use the CPLEAR data[6] as far as available and supplement them by Gasser-Meissner's value[16] on $\delta_2 - \delta_0$ and the PDG-1996 data[8] for the rest. All the relevant data are recapitulated in Table 1.

	Quantity	PDG-1996	CPLEAR	Unit
	$1/\gamma_S$	0.8927 ± 0.0009		$10^{-10}s$
	$1/\gamma_L$	5.17 ± 0.04		$10^{-8}s$
	$-\Delta$	0.5304 ± 0.0014	0.5292 ± 0.0019	$10^{10}s^{-1}$
2π	$\gamma_S(\pi^+\pi^-)/\gamma_S$	68.61 ± 0.28		%
	$\gamma_S(\pi^0\pi^0)/\gamma_S$	31.39 ± 0.28		%
	$\|\eta_{+-}\|$	2.285 ± 0.019	2.316 ± 0.039	10^{-3}
	ϕ_{+-}	43.7 ± 0.6	43.5 ± 0.8	$^\circ$
	$\|\eta_{00}\|$	2.275 ± 0.019	2.49 ± 0.46	10^{-3}
	ϕ_{00}	43.5 ± 1.0	51.7 ± 7.3	$^\circ$
3π	$\gamma_L(\pi^+\pi^-\pi^0)/\gamma_L$	12.56 ± 0.20		%
	$\gamma_L(\pi^0\pi^0\pi^0)/\gamma_L$	21.12 ± 0.27		%
	$\mathrm{Re}(\eta_{+-0})$		-0.004 ± 0.008	
	$\mathrm{Im}(\eta_{+-0})$		-0.003 ± 0.010	
	$\mathrm{Re}(\eta_{000})$		0.15 ± 0.30	
	$\mathrm{Im}(\eta_{000})$		0.29 ± 0.40	
$\pi\ell\nu$	$\sum_\ell \gamma_L(\pi\ell\nu)/\gamma_L$	65.96 ± 0.30		%
	$\mathrm{Re}(x_\ell)$	0.006 ± 0.018	0.0085 ± 0.0102	
	$\mathrm{Im}(x_\ell)$	-0.003 ± 0.026	0.0005 ± 0.0025	
	d_L^ℓ	3.27 ± 0.12		10^{-3}
	$d_1^\ell(t \gg 1/\gamma_S)/4$		-1.57 ± 0.70	10^{-3}
	$d_2^\ell(t \gg 1/\gamma_S)/4$		-0.07 ± 0.70	10^{-3}
$\pi^+\pi^-\gamma$	$\gamma_S(\pi^+\pi^-\gamma)/\gamma_S$	0.178 ± 0.050		%
	$\|\eta_{+-\gamma}\|$	2.35 ± 0.07		10^{-3}
	$\phi_{+-\gamma}$	44 ± 4		$^\circ$
		Chell-Olsson	Gasser-Meißner	
2π	$\delta_2 - \delta_0$	-42 ± 4	-45 ± 6	$^\circ$

Table 1: Input data.

Our analysis goes as follows.

First, assuming ω being real (see Eq.(5.1)), we use Eqs.(3.8) to find ω. Equations (3.6a,b) are then used to estimate η_0 and $\eta_2 - \eta_0$. The value of ϕ_{SW} is obtained from Eq.(2.15). The results are shown in Table 2.

Quantity	PDG-1996	CPLEAR	Unit
ω	2.814 ± 0.357	2.961 ± 0.455	10^{-2}
ϕ_{SW}	43.44 ± 0.08	43.38 ± 0.11	$^\circ$
$\mathrm{Re}(\eta_0)$	1.652 ± 0.018	1.632 ± 0.124	10^{-3}
$\mathrm{Im}(\eta_0)$	1.575 ± 0.018	1.709 ± 0.136	10^{-3}
$\mathrm{Re}(\eta_2 - \eta_0)$	-0.121 ± 0.765	5.55 ± 5.13	10^{-3}
$\mathrm{Im}(\eta_2 - \eta_0)$	0.173 ± 0.446	-2.387 ± 7.238	10^{-3}

Table 2: Intermediate results.

Next, we use Eqs.(5.3a) and (5.4) to estimate $\mathrm{Re}(\varepsilon)$ and $\mathrm{Re}(y_\ell)$. For this purpose, in view of lack of accurate independent data on $x_{\ell+}$ and $x_{\ell-}$[6], we shall unwillingly neglect possible violation of CPT symmetry in the $\Delta S \neq \Delta Q$ amplitudes and assume

$$x'_\ell = 0 \ . \tag{6.1}$$

By combining with Eqs.(5.2a), (5.3b) and (5.5), one may further estimate $\mathrm{Re}(\delta)$, $\mathrm{Im}(\delta)$, $\mathrm{Re}(y_0)$ and $\mathrm{Im}(\varepsilon) + z_0$. Equation (5.2b) gives $\mathrm{Re}(y_2 - y_0)$ and $z_2 - z_0$ directly. All the results are compiled in Table 3, where the values of $\delta_\parallel$ and $\delta_\perp$ are also shown.

		PDG-1996	CPLEAR	Remark
CP/T Violating	$\mathrm{Re}(\varepsilon)$	1.639 ± 0.098	1.695 ± 0.141	$\times 10^{-3}$
	$\mathrm{Im}(\varepsilon) + z_0$	1.647 ± 0.102	1.707 ± 0.182	$\times 10^{-3}$
	$z_2 - z_0$	0.173 ± 0.446	-2.38 ± 7.24	$\times 10^{-3}$
	$\mathrm{Im}(x_\ell)$	-0.3 ± 2.6	0.05 ± 0.25	$\times 10^{-2}$; input
CP/CPT Violating	$\mathrm{Re}(\delta)$	0.01 ± 9.94	-0.55 ± 9.99	$\times 10^{-4}$
	$\mathrm{Im}(\delta)$	0.73 ± 1.01	-0.02 ± 1.21	$\times 10^{-4}$
	$\delta_\parallel$	0.51 ± 7.25	-0.42 ± 7.31	$\times 10^{-4}$
	$\delta_\perp$	0.52 ± 6.87	0.36 ± 6.92	$\times 10^{-4}$
	$\mathrm{Re}(y_0)$	0.01 ± 1.00	-0.12 ± 1.02	$\times 10^{-3}$
	$\mathrm{Re}(y_2 - y_0)$	-0.121 ± 0.765	5.55 ± 5.13	$\times 10^{-3}$
	$\mathrm{Re}(y_\ell)$	-0.13 ± 1.41	-0.25 ± 1.42	$\times 10^{-3}$
	$\mathrm{Re}(x'_\ell)$	0	0	assumed
T/CPT Violating	$\mathrm{Im}(x'_\ell)$	0	0	assumed

Table 3: Experimental constraints to the symmetry-violating parameters.

7 Comparison with other analyses

If one treats $|\omega'|$ as a small quantity, one gets from Eq.(3.6a,b)

$$\eta_{+-} \simeq \eta_0 + \varepsilon' \ , \tag{7.1a}$$

$$\eta_{00} \simeq \eta_0 - 2\varepsilon' \ , \tag{7.1b}$$

where

$$\varepsilon' = \frac{1}{\sqrt{2}} (\eta_2 - \eta_0)\, \omega \exp(i(\delta_2 - \delta_0)) \ . \tag{7.2}$$

If one further assumes CPT symmetry, one has, from Eqs.(5.2a,b),

$$\eta_0 = \varepsilon + iz_0 \ , \tag{7.3a}$$

$$\varepsilon' = \frac{1}{\sqrt{2}} i(z_2 - z_0)\omega \exp(i(\delta_2 - \delta_0)) \ . \tag{7.3b}$$

In a note [17] cited in [8], assuming CPT symmetry, the mixing parameter is parametrized as

$$\frac{q}{p} = \frac{1-\tilde{\varepsilon}}{1+\tilde{\varepsilon}} \ , \tag{7.4a}$$

and the 2π decay amplitude is simply denoted as

$$\langle (2\pi)_I | H_{\rm W} | K^0 \rangle = A_I \ . \tag{7.4b}$$

Comparing with Eqs.(2.6), (3.1), (3.2) and (5.1) and with the help of Eqs.(2.13b) and (2.15), one may verify[2] that

$$\begin{aligned} \tilde{\varepsilon} &= \frac{\varepsilon \cos \alpha_K/2 - i \sin \alpha_K/2}{\cos \alpha_K/2 - i\varepsilon \sin \alpha_K/2} \\ &= \frac{2{\rm Re}(\varepsilon) - i\{(1-|\varepsilon|^2)\sin \alpha_K - 2{\rm Im}(\varepsilon)\cos \alpha_K\}}{1+|\varepsilon|^2 + (1-|\varepsilon|^2)\cos \alpha_K + 2{\rm Im}(\varepsilon)\sin \alpha_K} \ , \end{aligned} \tag{7.5a}$$

$$\frac{{\rm Re}(A_2)}{{\rm Re}(A_0)} = \frac{\omega(\cos \alpha_K/2 - z_2 \sin \alpha_K/2)}{\cos \alpha_K/2 - z_0 \sin \alpha_K/2} \ , \tag{7.5b}$$

$$\frac{{\rm Im}(A_I)}{{\rm Re}(A_I)} = \frac{z_I \cos \alpha_K/2 + \sin \alpha_K/2}{\cos \alpha_K/2 - z_I \sin \alpha_K/2} \ . \tag{7.5c}$$

${\rm Im}(\tilde{\varepsilon})$ and ${\rm Im}(A_I)$ are *not* small in general. If, and only if, one restricts himself to the case in which α_K is allowed to be treated as small as our ε and z_I, one has

$$\tilde{\varepsilon} \simeq \varepsilon - i\alpha_K/2 \ , \tag{7.6a}$$

$$\frac{{\rm Re}(A_2)}{{\rm Re}(A_0)} \simeq \omega \ , \tag{7.6b}$$

$$\frac{{\rm Im}(A_I)}{{\rm Re}(A_I)} \simeq z_I + \alpha_K/2 \ , \tag{7.6c}$$

and η_0 and ε' may be expressed as

$$\eta_0 \simeq \tilde{\varepsilon} + i\frac{\mathrm{Im}(A_0)}{\mathrm{Re}(A_0)} \ , \tag{7.7a}$$

$$\varepsilon' \simeq \frac{i}{\sqrt{2}}\frac{\mathrm{Re}(A_2)}{\mathrm{Re}(A_0)}\left[\frac{\mathrm{Im}(A_2)}{\mathrm{Re}(A_2)} - \frac{\mathrm{Im}(A_0)}{\mathrm{Re}(A_0)}\right]\exp(i(\delta_2 - \delta_0)) \ . \tag{7.7b}$$

The parameters $\tilde{\varepsilon}$ and A_I are not invariant under the rephasing of the $|K^0\rangle$ and $|\overline{K^0}\rangle$ states, Eq.(1.1), and it is possible by a choice of phase convention to set α_K or $\mathrm{Im}(A_0)$ or $\mathrm{Im}(A_2)$ or $\mathrm{Im}(\tilde{\varepsilon})$ to 0. If one adopts the choice $\mathrm{Im}(A_0) = 0$ (i.e. the Wu-Yang phase convention[18]), one will have

$$\eta_0 = \tilde{\varepsilon} \ , \tag{7.8a}$$

$$\varepsilon' = \frac{i}{\sqrt{2}}\frac{\mathrm{Im}(A_2)}{\mathrm{Re}(A_0)}\exp(i(\delta_2 - \delta_0)) \ . \tag{7.8b}$$

Note however that the choice $\mathrm{Im}(A_0) = 0$ is, as seen from Eq.(7.5c) or (7.6c), equivalent to the choice $z_0 = -\tan\alpha_K/2$ or $\alpha_K \simeq -2z_0$ and hence Eqs.(7.6a,c) give

$$\tilde{\varepsilon} \simeq \varepsilon + iz_0 \ , \tag{7.9a}$$

$$\frac{\mathrm{Im}(A_2)}{\mathrm{Re}(A_2)} \simeq z_2 - z_0 \ . \tag{7.9b}$$

Inserting Eqs.(7.6b) and (7.9a,b) into Eqs.(7.8a,b), one goes back to Eqs.(7.3a,b).[e]

Such an approximate relation as

$$\frac{\varepsilon'}{\tilde{\varepsilon}} \simeq \mathrm{Re}\left(\frac{\varepsilon'}{\tilde{\varepsilon}}\right) \simeq \frac{1}{3}\left(1 - \left|\frac{\eta_{00}}{\eta_{+-}}\right|\right) \ , \tag{7.10}$$

is used to estimate $\varepsilon'/\tilde{\varepsilon}$ (or ε'/η_0 in our notation) in [8, 17]. On the other hand, from Eqs.(7.1a,b) and the first set of the data in Table 1, 2 and 3, we find

$$\begin{aligned} \mathrm{Re}\left(\tfrac{\varepsilon'}{\eta_0}\right) &= (1.5 \pm 5.5) \times 10^{-3} \ , \\ \mathrm{Im}\left(\tfrac{\varepsilon'}{\eta_0}\right) &= (1.2 \pm 5.5) \times 10^{-3} \ . \end{aligned} \tag{7.11}$$

It appears therefore that the second (first) near equality in Eq.(7.10) is (may not be) justifiable.[f]

[e] We expect that Eqs.(7.1a,b) with η_0 and ε' given either by Eqs.(7.7a,b) or Eqs.(7.8a,b) should coincide exactly with Eqs.(4a,b) and (5) derived in [17]. It seems to us that a term propotional to $\mathrm{Im}(A_0)$ is missing in their Eqs.(4a,b) or has to be omitted from their Eq.(5). See also Eqs.(13.4a,b,c) of [19] in this respect.

[f] It is argued that the first near equality in Eq.(7.10) follows, since

$$\text{The phase of } \ \tilde{\varepsilon} \ \text{ in the Wu-Yang phase convention} \simeq \phi_{SW} \ , \tag{7.12}$$

and since ϕ_{SW} is accidentally close to the phase of ε', $\delta_2 - \delta_0 + \pi/2$. Note however that Eq.(7.12) holds only when direct CP violation (i.e. CP violation in decay amplitudes and decay matrix) is negligible (see Eqs.(2.13b), (2.15) and (7.9a)) and that ε' *is* a quantity related to direct CP violation.

In [8],

$$\zeta = \frac{m_{K^0} - m_{\overline{K^0}}}{m_{K^0}} , \tag{7.13a}$$

is quoted as a typical quantity which signals CPT violation. ζ is related to our parameters defined in Eq.(2.14b) as

$$\zeta = \frac{\delta_\perp \sqrt{\gamma_S^2 + 4\Delta^2}}{m_{K^0}} , \tag{7.13b}$$

and it is true that $\zeta \neq 0$ would imply violation of both CP and CPT symmetries. It seems however that $\delta_\perp$ itself (rather than ζ) is better to be regarded as a parameter which characterizes symmetry violation[13, 20]. In [21] cited in [8], a couple of assumptions and approximations are made to relate $\delta_\perp$ directly to the measured quantities such as η_{+-}, η_{00} and ϕ_{SW}. Among the asumptions is direct CPT violation being negligible, which would however at the same time lead to $\delta_\parallel = 0$. It appears that, in view of our numerical results shown in Table 3, such an assumption may not be justifiable. [g]

8 Concluding remarks

We have introduced a set of parameters to describe possible violation of CP, T and CPT symmetries and of the $\Delta S = \Delta Q$ rule in the K^0-$\overline{K^0}$ system in a well-defined way and attempted to derive constraints to these parameters from the presently available experimental data in a way as phenomenological and comprehensive as possible.

From our numerical results shown in Table 3, it is seen that, in contrast to $\mathrm{Re}(\varepsilon)$ and $\mathrm{Im}(\varepsilon) + z_0$, which are definitely non-vanishing and are of the order of 10^{-3}, all the other symmetry-violating parameters are consistent with being vanishing and are at most of the order of 10^{-3}. This implies, on the one hand, that all the present observations are consistent with no CPT violation and no direct CP and T violations, and, on the other hand, that CPT violation and direct CP and T violations up to a level comparable to that of indirect CP and T violations are at present not excluded. It is therefore not advisable to neglect direct symmetry violation in phenomenological analyses.

We have to admit that our analysis is not totally free from theoretical prejudices and is subject heavily to experimental uncertainties, among which we mention:

(1) We have unwillingly accepted Eq.(6.1). In this respect, we would like to stress that measurements on various leptonic asymmetries without this or that theoretical inputs are highly desirable and that $\mathrm{Im}(x'_\ell)$ is the only parameter which characterizes T and CPT violation but has nothing to do with CP violation.

(2) We have treated the 3π ($\pi^+\pi^-\gamma$) state as purely CP-odd (CP-even) and taken these states into account when using the Bell-Steinberger relation to estimate

[g] A similar remark was also raised in [22].

Re(ε) and Im(δ). As a result, our final numerical results are subject to uncertainties which come largely from experimental errors on η_{+-0}, η_{000} and $\eta_{+-\gamma}$. It is hoped that, in the near future, more abundant and accurate data on these and other relevant quantities will become available and enable one to identify and/or constrain CP, T and/or CPT violations in a more precise way.

(3) The Bell-Steinberger relation has played a very important role in our analysis. It is to be noted in this respect that fully time-dependent measurements on leptonic asymmetries of various types will allow one to identify or constrain Re(ε) and Im(δ) and thereby test this relation itself[12, 14].

Finally, as mentioned earlier, our parametrization is not fully rephasing invariant. A more thorough discussion of phase ambiguities associated with final state as well as the K^0 and $\overline{K^0}$ states will be given elsewhere[14].

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