

One-Pion Charm Baryon Transitions in a Relativistic Three-Quark Model

M. A. Ivanov

*Bogoliubov Laboratory of Theoretical Physics, Joint Institute for Nuclear Research, 141980
Dubna, Russia*

J. G. Körner

*Johannes Gutenberg-Universität, Institut für Physik, Staudinger Weg 7, D-55099 Mainz,
Germany*

V. E. Lyubovitskij

*Bogoliubov Laboratory of Theoretical Physics, Joint Institute for Nuclear Research, 141980
Dubna, Russia and Department of Physics, Tomsk State University, 634050 Tomsk, Russia*

A. G. Rusetsky

*Bogoliubov Laboratory of Theoretical Physics, Joint Institute for Nuclear Research, 141980
Dubna, Russia and HEPI, Tbilisi State University, 380086 Tbilisi, Georgia
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Abstract

We study one-pion transitions between charm baryon states in the framework of a relativistic three-quark model. We calculate the charm baryon-pion coupling factors that govern the S-wave, P-wave and D-wave one-pion transitions from the s -wave and the lowest lying p -wave charm baryon states down to the s -wave charm baryon states. For these we obtain: $g_{\Sigma_c \Lambda_c \pi} = 8.88 \text{ GeV}^{-1}$, $f_{\Lambda_{c1} \Sigma_c \pi} = 0.52$ and $f_{\Lambda_{c1}^* \Sigma_c \pi} = 21.5 \text{ GeV}^{-2}$. We compare our rate predictions for the one-pion transitions with experimental results.

The last few years have seen significant progress in the study of the spectroscopy of ground state and excited state charm baryons and their strong one-pion decays [1]- [7]. The CLEO [2]- [5], ARGUS [6], and E687 [7] Collaborations reported on measurements of the mass difference between charm baryon states [2]- [7], upper limits for their total widths [1,3,4] and first estimations of excited Σ_c^* baryon decay rates [5]. Due to the small release of energy in these transitions the analysis of the one-pion decays of charmed baryons provide an excellent laboratory for tests of heavy quark symmetry predictions on the one hand and tests of the soft dynamics of the light-side one-pion diquark transitions on the

other hand. The one-pion transitions among charm baryons have been analyzed before in the framework of the Heavy Hadron Chiral Perturbation Theory (HHCHPT) [9,11–13], in the Constituent Quark Model [8,9] and in the Light-Front Quark Model [10]. In the HHCHPT analysis one makes no assumptions about the composition of the light-side states involved in the one-pion transitions of the heavy baryons. Using the measured rates of Σ_c^* baryons $\Gamma(\Sigma_c^{*0} \rightarrow \Lambda_c \pi^-) = 13.0_{-3.0}^{+3.7}$ MeV and $\Gamma(\Sigma_c^{*++} \rightarrow \Lambda_c \pi^+) = 17.9_{-3.2}^{+3.8}$ MeV one can give estimates of the unknown coupling parameters appearing in the effective chiral Lagrangian [9,11–13]. In the Constituent Quark Model and Light-Front Quark Model approaches one further assumes that the light-side state is composed of two constituent quarks. Using one more assumption still, namely that the one-pion transitions are governed by single quark transitions the authors of [8,9] were able to derive a number of relations between the various one-pion coupling constants of the charm baryons. The Constituent Quark Model calculations of [8,9] did not address the full dynamics issue in as far as no attempt was made to model and to calculate the complete wave function overlap of the states involved in the transition. A first true dynamical calculation of the one-pion couplings charm baryons characterizing the S-wave, P-wave and D-wave transitions $g_{\Sigma_c \Lambda_c \pi}$, $f_{\Lambda_{c1} \Sigma_c \pi}$ and $f_{\Lambda_{c1}^* \Sigma_c \pi}$ was done in [10] where use was made Light-Front quark model spin functions [10].

In this paper we report on the predictions of the Relativistic Three-Quark Model [14,15] for the one-pion transitions between charm baryon states. As in the Light-Front model [10] the Relativistic Three-Quark Model allows for a full dynamical evaluation of the one-pion transition strengths between charm baryons. We want to mention that the Relativistic Three-Quark Model approach was successfully applied before to a number of dynamical problems involving the properties of pions [14–16], light baryons [17] and heavy-light baryons [18–20].

The Lagrangian describing the couplings of a heavy baryon state to its constituent light and heavy quarks considerably simplifies in the heavy quark limit. One has

$$\begin{aligned} \mathcal{L}_{B_Q}^{\text{int}}(x) &= g_{B_Q} \bar{B}_Q(x) \Gamma_1 Q^a(x) \int d\xi_1 \int d\xi_2 F_B(\xi_1^2 + \xi_2^2) \\ &\times q^b(x + 3\xi_1 - \sqrt{3}\xi_2) C \Gamma_2 \lambda_{B_Q} q^c(x + 3\xi_1 + \sqrt{3}\xi_2) \varepsilon^{abc} + \text{h.c.} \end{aligned} \quad (1)$$

$$F_B(\xi_1^2 + \xi_2^2) = \int \frac{d^4 k_1}{(2\pi)^4} \int \frac{d^4 k_2}{(2\pi)^4} e^{ik_1 \xi_1 + ik_2 \xi_2} \tilde{F}_B \left\{ \frac{[k_1^2 + k_2^2]}{\Lambda_B^2} \right\}$$

$$\mathcal{L}_\pi^{\text{int}}(x) = \frac{i g_\pi}{\sqrt{2}} \vec{\pi}(x) \int d\xi F_\pi(\xi^2) \bar{q}(x + \xi/2) \gamma^5 \vec{\lambda}_\pi q(x - \xi/2) \quad (2)$$

$$F_\pi(\xi^2) = \int \frac{d^4 k}{(2\pi)^4} e^{ik\xi} \tilde{F}_\pi \left\{ \frac{k^2}{\Lambda_\pi^2} \right\}$$

where Γ_i and λ_{B_Q} are spin and flavor matrices, respectively; g_{B_Q} and g_π denote the couplings of the respective hadrons with the constituent quarks; Λ_B (Λ_π) are the cutoff parameters defining the distributions of light quarks in the heavy baryon (pion). The baryon cutoff parameter Λ_B is chosen to be the same for charm and bottom baryons such that one has the correct normalization of the baryonic Isgur-Wise function [18] in the heavy quark symmetry limit. The quantum numbers and matrices Γ_i and λ_{B_Q} define the structure of the relevant

three-quark charm baryon currents and are listed in TABLE I. The square brackets [...] and curly brackets {...} denote antisymmetric and symmetric flavour and spin combinations of the light degrees of freedom.

The vertex function which defines the matrix element of the process $B_Q^i(p) \rightarrow B_Q^f(p') + \pi(q)$ (see Fig. I) has the following form in the heavy quark limit

$$M_{inv}^\pi(B_Q^i \rightarrow B_Q^f \pi) = \frac{g_\pi}{\sqrt{2}} g_{\text{eff}}^i g_{\text{eff}}^f C_{\text{flavor}} \cdot \bar{u}(v) \Gamma_1^f \frac{(1 + \not{v})}{2} \Gamma_1^i u(v) \cdot I_{q_1 q_2}^{if}(v, q) \quad (3)$$

$$g_{\text{eff}}^2 = \frac{4C_{\text{color}}}{(4\pi)^4} \Lambda_B^4 g_{B_Q}^2, \quad C_{\text{flavor}} = \text{tr} \left(\lambda_\pi (\lambda_{B^i} + \lambda_{B^i}^\dagger) (\lambda_{B^f} - \lambda_{B^f}^\dagger) \right), \quad C_{\text{color}} = 6$$

$$\begin{aligned} I_{q_1 q_2}^{if}(v, q) &= \int \frac{d^4 k_1}{\pi^2 i} \int \frac{d^4 k_2}{\pi^2 i} \tilde{F}_B \left\{ -6 \left[k_1^2 + k_2^2 + (k_1 + k_2)^2 \right] \right\} \\ &\times \tilde{F}_B \left\{ -6 \left[(k_1 + q)^2 + (k_2 - q)^2 + (k_1 + k_2)^2 \right] \right\} \\ &\times \frac{\tilde{F}_\pi \left\{ -(k_2 - q/2)^2 \right\}}{[-k_1 v - \bar{\Lambda}_{q_1 q_2}]} \cdot \frac{1}{4} \text{tr} \left[\Gamma_2^i S_{q_2}(k_1 + k_2) \Gamma_2^f S_{q_1}(k_2 - q) \gamma^5 S_{q_1}(k_2) \right] \end{aligned} \quad (4)$$

where λ_π , λ_{B^i} and λ_{B^f} are the flavor matrices of the pion, the initial and the final baryons, respectively.

Here $S_q(k) = 1/(m_q - \not{k})$ is the light quark propagator ($q = u, d, s$). The parameter $\bar{\Lambda}_{q_1 q_2} = M_{Qq_1 q_2} - m_Q$ denotes the difference between the heavy baryon mass $M_{Qq_1 q_2}$ and the heavy quark mass m_Q . All dimensional parameters are expressed in units of Λ_B . The integrals are calculated in the Euclidean region both for internal and external momenta. Finally, the results for the physical region are obtained by analytic continuation of the external momenta after the internal momenta have been integrated out.

As an illustration of our calculational procedure we first evaluate the matrix element Eq. (4) in the simplified case where the pion has a local coupling to its constituent quarks, i.e. where the vertex $\pi q \bar{q}$ form factor $\tilde{F}_\pi(k^2) \equiv 1$. As it turns out this is already a good approximation. For example, we have calculated the integral Eq. (4) with the baryonic vertex function being chosen in the Gaussian form for two cases: (1) $\tilde{F}_\pi \equiv 1$ and (2) $\tilde{F}_\pi = \exp(-k^2/\Lambda_\pi^2)$, $\Lambda_\pi = 1$ GeV. The results differ from each other by $O(10\%)$.

In the calculation of (4) with $\tilde{F}_\pi(k^2) \equiv 1$ we use the α -parametrization for quark propagators and the Laplace transform for the vertex function

$$\frac{1}{A} = \int_0^\infty d\alpha e^{-\alpha A}, \quad \tilde{F}_B(6X) = \int_0^\infty ds \tilde{F}_B^L(6s) e^{-sX} \quad (5)$$

$$\begin{aligned} I^{if}(v, q) &= \int_0^\infty ds_1 \tilde{F}_B^L(6s_1) \int_0^\infty ds_2 \tilde{F}_B^L(6s_2) e^{2s_2 q^2} \int_0^\infty d^4 \alpha e^{\alpha_3 \bar{\Lambda} - (\alpha_1 + \alpha_4) m_{q_1}^2 - \alpha_2 m_{q_2}^2} \\ &\times \frac{1}{4} \text{tr} \left[\Gamma_2^i \left(m_{q_2} - \frac{\not{\partial}_1 + \not{\partial}_2}{2} \right) \Gamma_2^f \left(m_{q_1} - \frac{\not{\partial}_2 - \not{q}}{2} \right) \gamma^5 \left(m_{q_1} - \frac{\not{\partial}_2}{2} \right) \right] \int \frac{d^4 k_1}{\pi^2 i} \int \frac{d^4 k_2}{\pi^2 i} e^{k A k - 2k B} \end{aligned}$$

$$\begin{aligned}
&= \int_0^\infty ds_1 \tilde{F}_B^L(6s_1) \int_0^\infty ds_2 \tilde{F}_B^L(6s_2) e^{2s_2 q^2} \int_0^\infty d^3 \alpha e^{\alpha_3 \bar{\Lambda} - (\alpha_1 + \alpha_4) m_{q_1}^2 - \alpha_2 m_{q_2}^2} \\
&\times \frac{1}{4} \text{tr} \left[\Gamma_2^i \left(m_{q_2} - \frac{\not{\partial}_1 + \not{\partial}_2}{2} \right) \Gamma_2^f \left(m_{q_1} - \frac{\not{\partial}_2}{2} - \not{q} \right) \gamma^5 \left(m_{q_1} - \frac{\not{\partial}_2}{2} \right) \right] \frac{e^{-BA^{-1}B}}{|A|^2}
\end{aligned}$$

Here

$$kAk - 2kB = \sum_{i,j=1}^2 k_i A_{ij} k_j - 2 \sum_{i=1}^2 k_i B_i, \quad \not{\partial}_i = \frac{\partial}{\partial \beta_i} \quad (6)$$

$$A_{ij} = \begin{pmatrix} 2(s_1 + s_2) + \alpha_2 & s_1 + s_2 + \alpha_2 \\ s_1 + s_2 + \alpha_2 & 2(s_1 + s_2) + \alpha_1 + \alpha_2 + \alpha_4 \end{pmatrix}$$

$$B_1 = -s_2 q - \alpha_3 v / 2 \quad B_2 = (s_2 + \alpha_1) q$$

The kinematics of the one-pion transitions allows one to make use of the approximations:

$$q^2 = m_\pi^2 \approx 0, \quad qv = \frac{1}{2m_i} (m_i^2 - m_f^2 + m_\pi^2) \approx 0 \quad (7)$$

where m_π , m_i and m_f are the masses of the pion, the initial and the final baryons, respectively, divided by Λ_B . Then, by making the variable replacement $\alpha_i \rightarrow (s_1 + s_2)\alpha_i$ and by using $\Gamma_2^i = \gamma_\mu$ and $\Gamma_2^f = \gamma_5$ the overlap integral can be seen to be proportional to q^μ such that

$$I^\mu(v, q) = q^\mu J \quad (8)$$

with

$$J = \int_0^\infty \frac{d^3 \alpha \alpha_1}{|A|^2} \tilde{F}_B^2(6z) \left\{ m_{q_1} m_{q_2} + \alpha_3 \frac{\partial z}{\partial \alpha_3} [A_{12}^{-1} + A_{22}^{-1}] \left[1 + \frac{(1 + \alpha_1) A_{22}^{-1} - A_{12}^{-1}}{2} \right] - \frac{\alpha_3^2}{4} A_{12}^{-1} [A_{11}^{-1} + A_{12}^{-1}] \right\}$$

$$z = \frac{\alpha_3^2}{4} A_{11}^{-1} - \alpha_3 \bar{\Lambda} + \alpha_1 m_{q_1}^2 + \alpha_2 m_{q_2}^2$$

$$A_{ij} = \begin{pmatrix} 2 + \alpha_2 & 1 + \alpha_2 \\ 1 + \alpha_2 & 2 + \alpha_1 + \alpha_2 \end{pmatrix}, \quad A_{ij}^{-1} = \frac{1}{|A|} \begin{pmatrix} 2 + \alpha_1 + \alpha_2 & -(1 + \alpha_2) \\ -(1 + \alpha_2) & 2 + \alpha_2 \end{pmatrix}$$

The last integral may be evaluated numerically for any given function \tilde{F}_B . Here we will use a Gaussian vertex functions both for baryons and the pion in Eq. (4).

In order to make contact with experimental numbers let us first define a set of coupling constants describing the one-pion transitions. For the transitions discussed in this paper the coupling constants are defined through the expansion of the the invariant one-pion transition matrix elements. One has [10]

$$\begin{aligned}
M_{inv}^\pi(\Sigma_c \rightarrow \Lambda_c \pi) &= \frac{1}{\sqrt{3}} g_{\Sigma_c \Lambda_c \pi} I_1 \bar{u}(v') \not{q}_\perp \gamma_5 u(v) \quad \underline{\text{p-wave transition}} \\
M_{inv}^\pi(\Sigma_c^* \rightarrow \Lambda_c \pi) &= g_{\Sigma_c^* \Lambda_c \pi} I_1 \bar{u}(v') q_{\perp \mu} u^\mu(v) \quad \underline{\text{p-wave transition}} \\
M_{inv}^\pi(\Lambda_{c1} \rightarrow \Sigma_c \pi) &= f_{\Lambda_{c1} \Sigma_c \pi} I_3 \bar{u}(v') u(v) \quad \underline{\text{s-wave transition}} \\
M_{inv}^\pi(\Lambda_{c1}^* \rightarrow \Sigma_c \pi) &= \frac{1}{\sqrt{3}} f_{\Lambda_{c1}^* \Sigma_c \pi} I_3 \bar{u}(v') \gamma_5 \not{q}_\perp u^\mu(v) q_{\perp \mu} \quad \underline{\text{d-wave transition}}
\end{aligned} \tag{9}$$

where the I_1 and I_3 are the flavor factors which are directly connected with flavor coefficients C_{flavor} (see Eq. (3)) via relations $I_i = f_i \cdot C_{\text{flavor}}$, $i = 1$ or 3 . The sets of I_i and f_i are given in TABLE II. We have also indicated the orbital angular momentum of the pion in Eq. (9). The transversity in Eq. (9) is defined with regard of the velocity $v = p/m$ of the decaying baryon, i.e. $q_\perp^\perp = q_\mu - v_\mu(q \cdot v)$. In fact, to leading order in the HQET expansion one has $v = v'$. In general, however, $v \neq v'$ from momentum conservation. By keeping track of momentum conservation in Eq. (9) using $v \neq v'$ one is including a part of the nonleading effects. The structure of the covariants in Eq. (9) is patterned after the leading order HQET result which predicts $g_{\Sigma_c \Lambda_c \pi} = g_{\Sigma_c^* \Lambda_c \pi} = g$ [11]. It is an easy exercise to rewrite e.g. the p-wave $\not{q}_\perp \gamma_5$ -coupling in terms of the usual γ_5 -coupling. The expression for g is written as

$$\begin{aligned}
g &= \frac{1}{\Lambda_B} \cdot \frac{g_\pi}{\sqrt{2}} \cdot \frac{R_{\Sigma \Lambda \pi}}{\sqrt{R_\Lambda} \sqrt{R_\Sigma}} \tag{10} \\
R_{\Sigma \Lambda \pi} &= \int_0^\infty \frac{d\alpha \alpha^2}{1 + \alpha + t} \int_0^\infty d\beta \int_0^1 \frac{d\theta \theta \exp(\Delta_1)}{\frac{3}{4} + \alpha + \alpha^2 \theta(1 - \theta) + t(1 + \alpha(1 - \theta))} \\
&\quad \times \left\{ m_{q_1} m_{q_2} + \beta^2 \frac{\frac{1}{4} + \frac{\alpha}{2} + \alpha^2 \theta(1 - \theta)}{(1 + \alpha + t)^2} + \frac{1}{32[\frac{3}{4} + \alpha + \alpha^2 \theta(1 - \theta) + t(1 + \alpha(1 - \theta))]} \right\} \\
R_\Lambda &= \int_0^\infty \frac{d\alpha \alpha}{(1 + \alpha)^2} \int_0^\infty d\beta \beta \int_0^1 d\theta \exp(\Delta_2) \left\{ m_{q_1} m_{q_2} + \beta^2 \frac{\frac{5}{4} + \frac{3}{2}\alpha + \alpha^2 \theta(1 - \theta)}{(1 + \alpha)^2} - \frac{\bar{\Lambda} \beta}{1 + \alpha} \right\} \\
R_\Sigma &= \int_0^\infty \frac{d\alpha \alpha}{(1 + \alpha)^2} \int_0^\infty d\beta \beta \int_0^1 d\theta \exp(\Delta_2) \left\{ m_{q_1} m_{q_2} + \beta^2 \frac{\frac{3}{4} + \alpha + \alpha^2 \theta(1 - \theta)}{(1 + \alpha)^2} - \frac{\bar{\Lambda} \beta}{2(1 + \alpha)} \right\} \\
\Delta_1 &= -24 \left\{ \alpha [m_{q_1}^2 \theta + m_{q_2}^2 (1 - \theta)] + \beta(\beta - 2\bar{\Lambda}) \frac{\frac{3}{4} + \alpha + \alpha^2 \theta(1 - \theta) + t(\frac{1}{2} + \alpha(1 - \theta))}{1 + \alpha + t} \right\} \\
\Delta_2 &\equiv \Delta_1|_{t=0}, \quad t = (\Lambda_B / \Lambda_\pi \sqrt{24})^2
\end{aligned}$$

One can then go on and calculate the one-pion decay rates using the general formula

$$\Gamma = \frac{1}{2J + 1} \frac{|\vec{q}|}{8\pi M_{B_Q}^2} \sum_{\text{spins}} |M_{inv}^\pi|^2 \tag{11}$$

where $|\vec{q}|$ is the pion momentum in the rest frame of the decaying baryon. In terms of the above coupling constants one obtains

$$\Gamma(\Sigma_c \rightarrow \Lambda_c \pi) = g^2 I_1^2 \frac{|\vec{q}|^3}{6\pi} \frac{M_{\Lambda_c}}{M_{\Sigma_c}} \tag{12}$$

$$\Gamma(\Sigma_c^* \rightarrow \Lambda_c \pi) = g^2 I_1^2 \frac{|\vec{q}|^3}{6\pi} \frac{M_{\Lambda_c}}{M_{\Sigma_c^*}} \quad (13)$$

$$\Gamma(\Lambda_{c1} \rightarrow \Sigma_c \pi) = f_{\Lambda_{c1}\Sigma_c\pi}^2 I_3^2 \frac{|\vec{q}|}{2\pi} \frac{M_{\Sigma_c}}{M_{\Lambda_{c1}}} \quad (14)$$

$$\Gamma(\Lambda_{c1}^* \rightarrow \Sigma_c \pi) = f_{\Lambda_{c1}^*\Sigma_c\pi}^2 I_3^2 \frac{|\vec{q}|^5}{18\pi} \frac{M_{\Sigma_c}}{M_{\Lambda_{c1}^*}} \quad (15)$$

We use different values for the parameter $\bar{\Lambda}_{q_1 q_2}$ for baryons containing only nonstrange light quarks and one or two strange quarks: $\bar{\Lambda}$, $\bar{\Lambda}_s$ and $\bar{\Lambda}_{ss}$, respectively. For the time being we shall avoid the appearance of unphysical imaginary parts in the Feynman diagrams by imposing the following condition: the baryon mass must be less than the sum of constituent quark masses. In the case of heavy-light baryons this restriction implies that the parameter $\bar{\Lambda}_{q_1 q_2}$ must be less than the sum of light quark masses. The last constraint serves as the upper limit for our choices of the parameter $\bar{\Lambda}_{q_1 q_2}$.

Let us now specify our model parameters. The coupling constants g_{B_Q} and g_π in Eqs. (1) and (2) are calculated from *the compositeness condition* (see, ref. [18]), which means that the renormalization constant of the hadron wave function is set equal to zero $Z_H = 1 - g_H^2 \Sigma'_H(M_H) = 0$ where Σ_H is the hadron (charm baryon and pion) mass operator. We thus remain with the cutoff parameters (Λ_B, Λ_π) and parameters $(\bar{\Lambda}, \bar{\Lambda}_s, \bar{\Lambda}_{ss})$ as the adjustable parameters in our approach. The masses of the u and the d quarks are set equal ($m_u = m_d = m_q$). The value of m_q is determined from an analysis of nucleon data: $m_q = 420$ MeV. The pion cutoff parameter $\Lambda_\pi = 1$ GeV is fixed from the description of low-energy pion observables (constants f_π and $g_{\pi\gamma\gamma}$, electromagnetic radii and form factors defining the transitions $\pi \rightarrow \pi\gamma$ and $\pi \rightarrow \gamma\gamma^*$) [14]- [16]. The parameters Λ_{B_Q} , m_s , $\bar{\Lambda}$ are taken from the analysis of the $\Lambda_c^+ \rightarrow \Lambda^0 + e^+ + \nu_e$ decay data. To reproduce the present average value of $B(\Lambda_c^+ \rightarrow \Lambda e^+ \nu_e) = 2.2\%$ we use the following values for our parameters: $\Lambda_{B_Q} = 1.8$ GeV, $m_s = 570$ MeV and $\bar{\Lambda} = 600$ MeV. The values of the unknown parameters $\bar{\Lambda}_s$ and $\bar{\Lambda}_{ss}$ are determined from the relations $\bar{\Lambda}_s = \bar{\Lambda} + (m_s - m)$ and $\bar{\Lambda}_{ss} = \bar{\Lambda} + 2(m_s - m)$, which give $\bar{\Lambda}_s = 750$ MeV and $\bar{\Lambda}_{ss} = 900$ MeV. Using the values of $\Lambda_{B_Q} = 1.8$ GeV and $\bar{\Lambda} = 600$ MeV one can describe the decay $\Lambda_b^0 \rightarrow \Lambda_c^+ e^- \bar{\nu}_e$ decay: the width $\Gamma(\Lambda_b^0 \rightarrow \Lambda_c^+ e^- \bar{\nu}_e) = 5.06 \times 10^{10} s^{-1}$ and the slope of the Λ_b Isgur-Wise function $\rho^2 = 1.44$. Finally, the mass values of the charm baryon states including current experimental uncertainties, are taken from TABLE I [1,13]. For $\Xi_c^{+'}$ and $\Xi_c^{0'}$ we use mass value with theoretical uncertainty $m_{\Xi_c} = 2600 \pm 30$ MeV. For the pion masses we take their experimental values $m_{\pi^\pm} = 139.6$ MeV and $m_{\pi^0} = 135$ MeV [1].

We now present our numerical results for the strong charm baryon-pion couplings and for the one-pion decay rates. In TABLE III we list our results for one-pion coupling constants. For comparison we give also the results of Light-Front (LF) Quark Model [10] which have been only available up to now. One can see that the values of the P -wave coupling constants are in qualitative agreement with the Light-Front quark model prediction. However, we disagree on the values of the S -wave and D -wave coupling constants $f_{\Lambda_{c1}\Sigma_c\pi}$ and $f_{\Lambda_{c1}^*\Sigma_c\pi}$. The disagreement can be traced to different choices of the momentum distribution of the light quarks in the charm baryon. The smaller values of $f_{\Lambda_{c1}\Sigma_c\pi}$ and $f_{\Lambda_{c1}^*\Sigma_c\pi}$ are welcome from comparison the results for exclusive one-pion rates of the Λ_{c1} and Λ_{c1}^* (see, TABLE IV) to the experimental data [1]. It is seen that our predictions are consistent with current

experimental estimations whereas the Light-Front model results lay above the experimental rates. Also our predictions for S -wave and D -wave transitions are preferable if one sums up the three exclusive one-pion rates of the Λ_{c1} and Λ_{c1}^* and compares the sums to the total experimental rates $\Gamma(\Lambda_{c1}) = 3.6_{-1.3}^{+2.0}$ MeV and $\Gamma(\Lambda_{c1}^*) < 1.9$ MeV. From the results of our model calculation we obtain $\Gamma(\Lambda_{c1}) > 2.6 \pm 0.3$ MeV and $\Gamma(\Lambda_{c1}^*) > 0.25 \pm 0.03$ MeV consistent with the experimental total rate of Λ_{c1} and lower limit for rate of Λ_{c1}^* whereas the Light-Front model has $\Gamma(\Lambda_{c1}) > 6.49$ MeV and $\Gamma(\Lambda_{c1}^*) > 2.19$ MeV which lay above the experimental rates. One can hope that more precise experimental study of strong decays of excited Λ_{c1} baryons in the near future can test the predictions of our approach and the LF quark model. All our results for the one-pion decay rates of charm baryons are collected in TABLE IV. The uncertainties for the calculated rates reflect the experimental errors in the charm baryon masses (see, TABLE I). For comparison we have also listed the predictions of the Light-Front quark model [10] and experimental results, where available. One can only hope that there will be more precise data on the one-pion transitions of the excited Λ_{c1} baryon states to the ground states in the near future such that one can perform a more detailed comparison with the model predictions of dynamical models such as described in our approach and in the LF quark model.

Let us add a comment of the relation of our approach to the chiral invariant coupling method used for example in [11]. The chiral formalism implies that all one-pion coupling factors are proportional to the factor $1/f_\pi$ associated with the pion field. In our approach the corresponding factor emerges in the following way. The pion-quark-antiquark coupling g_π can be seen to obey the Goldberger-Treiman relation $g_\pi \cdot f_\pi = 2m_q$ with an accuracy of a few percent. Hence our approach agrees with the chiral approach [11] in that the pion leptonic constant f_π effectively appears as a dimensional parameter in the coupling factors.

In conclusion, we have calculated strong one-pion decays of charm baryons. We have obtained predictions for the values of couplings of charm baryons with pions and for the rates of the two-body transitions $B_c^i(p) \rightarrow B_c^j(p') + \pi(q)$. We have compared our results with data obtained with the use Light-Front Quark Model [10]. As a next step we plan to study one-photon transitions between charm baryons. Also we intend to extend our results to the bottom baryon sector.

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REFERENCES

- [1] C. Caso et.al. (Particle Data Group), Eur. Phys. J. C **3**, 1 (1998).
- [2] P. Avery et.al., CLEO Coll., Phys. Rev. Lett. **75**, 4364 (1995).
- [3] K. W. Edwards et.al., CLEO Coll., Phys. Rev. Lett. **74**, 3331 (1995).
- [4] L. Gibbons et.al., CLEO Coll., Phys. Rev. Lett **77**, 810 (1996).
- [5] G. Brandenburg et.al., CLEO Coll., Phys. Rev. Lett **78**, 2304 (1997).
- [6] H. Albrecht et.al., ARGUS Coll., Phys. Lett. B **317**, 227 (1993).
- [7] P.L. Frabetti et.al, E687 Coll., Phys. Lett B **365**, 461 (1996);
Phys. Rev. Lett. **72**, 961 (1994).
- [8] F. Hussain, J.G. Körner and S. Tawfiq, Preprints MZ-TH/96-10, IC/96/35, 1996.
- [9] D. Pirjol and T. M. Yan, Phys. Rev. D **56**, (1997) 5483.
- [10] S. Tawfiq, P. J. O'Donnell and J.G. Körner, Preprint MZ-TH/98-08, UTPT-98-03, 1998;
Preprint UTPT-98-08, 1998.
- [11] T. M. Yan et. al., Phys. Rev. D **46**, 1148 (1992).
- [12] H.-Y. Cheng, Phys. Lett. B **399**, 281 (1997).
- [13] G. Chiladze and A. Falk, Phys. Rev. D **56**, 6738 (1997).
- [14] I.V. Anikin, M.A. Ivanov, N.B. Kulimanova and V.E. Lyubovitskij,
Physics of Atomic Nuclei **57**, 1021 (1994).
- [15] I.V. Anikin, M.A. Ivanov, N.B. Kulimanova and V.E. Lyubovitskij,
Z. Phys. C **65**, 681 (1995).
- [16] M.A. Ivanov and V.E. Lyubovitskij, Phys. Lett. **B408**, 435 (1997).
- [17] M.A. Ivanov, M.P. Locher, V.E. Lyubovitskij, Few-Body Syst. **21**, 131 (1996).
- [18] M.A. Ivanov, V.E. Lyubovitskij, J.G. Körner and P. Kroll, Phys. Rev. D **56**, 348 (1997).
- [19] M.A. Ivanov, J.G. Körner, V.E. Lyubovitskij and A.G. Rusetsky,
Phys. Rev. D **57**, 5632 (1998).
- [20] M.A. Ivanov, J.G. Körner, V.E. Lyubovitskij and A.G. Rusetsky,
Mod. Phys. Lett. A **13**, 181 (1998).
- [21] F. Hussain, J.G. Körner, J. Landgraf and Salam Tawfiq, Z. Phys. C **69**, (1996) 655;
J.G. Körner, M. Krämer and D. Pirjol, Progr. Part. Nucl. Phys., **33** , 787 (1994).

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TABLE I Quantum numbers of charm baryons ($\lambda_u = \text{diag}\{1,0,0\}$, $\lambda_d = \text{diag}\{0,1,0\}$)

TABLE II Flavor coefficients I_1, I_3 and f_1, f_3 .

TABLE III Charm baryon-pion couplings.

TABLE IV Decay rates Γ (in MeV) for charm baryon states.

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FIG. I One-pion charm baryon transition: $\Sigma_c \rightarrow \Lambda_c \pi$ decay.

TABLE I

Baryon	J^P	Quark Content	$\Gamma_1 \otimes C\Gamma_2$	λ_{B_Q}	Mass (MeV) [1,13]
Λ_c^+	$\frac{1}{2}^+$	c[ud]	$I \otimes C\gamma^5$	$i\lambda_2/2$	2284.9 ± 0.6
Ξ_c^+	$\frac{1}{2}^+$	c[us]	$I \otimes C\gamma^5$	$i\lambda_5/2$	2465.6 ± 1.4
Ξ_c^0	$\frac{1}{2}^+$	c[ds]	$I \otimes C\gamma^5$	$i\lambda_7/2$	2470.3 ± 1.8
$\Xi_c^{+'}$	$\frac{1}{2}^+$	c{us}	$\gamma^\mu \gamma^5 \otimes C\gamma_\mu$	$\lambda_4/(2\sqrt{3})$	2600 ± 30
$\Xi_c^{0'}$	$\frac{1}{2}^+$	c{ds}	$\gamma^\mu \gamma^5 \otimes C\gamma_\mu$	$\lambda_6/(2\sqrt{3})$	2600 ± 30
Σ_c^{++}	$\frac{1}{2}^+$	c{uu}	$\gamma^\mu \gamma^5 \otimes C\gamma_\mu$	$\lambda_u/\sqrt{6}$	2452.8 ± 0.6
Σ_c^+	$\frac{1}{2}^+$	c{ud}	$\gamma^\mu \gamma^5 \otimes C\gamma_\mu$	$\lambda_1/(2\sqrt{3})$	2453.6 ± 0.9
Σ_c^0	$\frac{1}{2}^+$	c{dd}	$\gamma^\mu \gamma^5 \otimes C\gamma_\mu$	$\lambda_d/\sqrt{6}$	2452.2 ± 0.6
Ξ_c^{*+}	$\frac{3}{2}^+$	c{us}	$I \otimes C\gamma_\mu$	$\lambda_4/2$	2644.6 ± 2.1
Ξ_c^{*0}	$\frac{3}{2}^+$	c{ds}	$I \otimes C\gamma_\mu$	$\lambda_6/2$	2643.8 ± 1.8
Σ_c^{*++}	$\frac{3}{2}^+$	c{uu}	$I \otimes C\gamma_\mu$	$\lambda_u/\sqrt{2}$	2519.4 ± 1.5
Σ_c^{*0}	$\frac{3}{2}^+$	c{dd}	$I \otimes C\gamma_\mu$	$\lambda_d/\sqrt{2}$	2517.5 ± 1.4
Λ_{c1}	$\frac{1}{2}^-$	c[ud]	$\not{\partial}_{\xi_1} \gamma_5 \otimes C\gamma^5$	$i\lambda_2/(2\sqrt{3})$	2593.9 ± 0.8
Λ_{c1}^*	$\frac{3}{2}^-$	c[ud]	$\partial_{\xi_1}^\mu \otimes C\gamma^5$	$i\lambda_2/2$	2626.6 ± 0.8
Ξ_{c1}^*	$\frac{3}{2}^-$	c[us]	$\partial_{\xi_1}^\mu \otimes C\gamma^5$	$i\lambda_5/2$	2815

TABLE II

Decay mode	I_1	f_1	Decay mode	I_3	f_3
$\Sigma_c^+ \rightarrow \Lambda_c \pi^0$	1	$\sqrt{3}/2$	$\Lambda_{c1}(2593) \rightarrow \Sigma_c^0 \pi^+$	1	3/2
$\Sigma_c^0 \rightarrow \Lambda_c \pi^-$	1	$\sqrt{3}/2$	$\Lambda_{c1}(2593) \rightarrow \Sigma_c^+ \pi^0$	1	3/2
$\Sigma_c^{++} \rightarrow \Lambda_c \pi^+$	1	$\sqrt{3}/2$	$\Lambda_{c1}(2593) \rightarrow \Sigma_c^{++} \pi^-$	1	3/2
$\Sigma_c^{*0} \rightarrow \Lambda_c \pi^-$	1	1/2	$\Xi_{c1}^*(2815) \rightarrow \Xi_c^{*0} \pi^+$	$1/\sqrt{2}$	1/2
$\Sigma_c^{*++} \rightarrow \Lambda_c \pi^+$	1	1/2	$\Xi_{c1}^*(2815) \rightarrow \Xi_c^{*+} \pi^0$	1/2	1/2
$\Xi_c^{*0} \rightarrow \Xi_c^0 \pi^0$	1/2	1/2	$\Lambda_{c1}^*(2625) \rightarrow \Sigma_c^0 \pi^+$	1	$\sqrt{3}/2$
$\Xi_c^{*0} \rightarrow \Xi_c^+ \pi^-$	$1/\sqrt{2}$	1/2	$\Lambda_{c1}^*(2625) \rightarrow \Sigma_c^+ \pi^0$	1	$\sqrt{3}/2$
$\Xi_c^{*+} \rightarrow \Xi_c^0 \pi^+$	$1/\sqrt{2}$	1/2	$\Lambda_{c1}^*(2625) \rightarrow \Sigma_c^{++} \pi^-$	1	$\sqrt{3}/2$
$\Xi_c^{*+} \rightarrow \Xi_c^+ \pi^0$	1/2	1/2	$\Xi_{c1}^*(2815) \rightarrow \Xi_c^{*0} \pi^+$	$1/\sqrt{2}$	$\sqrt{3}/2$
			$\Xi_{c1}^*(2815) \rightarrow \Xi_c^{*+} \pi^0$	1/2	$\sqrt{3}/2$

TABLE III

Coupling	Our	Ref. [10]
$g_{\Sigma_c \Lambda_c \pi}$	8.88 GeV ⁻¹	6.81 GeV ⁻¹
$g_{\Xi_c^* \Xi_c \pi}$	8.34 GeV ⁻¹	
$f_{\Lambda_{c1} \Sigma_c \pi}$	0.52	1.16
$f_{\Xi_{c1}^* \Xi_c^* \pi}$	0.32	
$f_{\Lambda_{c1}^* \Sigma_c \pi}$	21.5 GeV ⁻²	96.0 GeV ⁻²
$f_{\Xi_{c1}^* \Xi_c^* \pi}$	20 GeV ⁻²	

TABLE IV

$B_Q \rightarrow B'_Q \pi$	Our	Ref. [10]	Experiment
P-wave transitions			
$\Sigma_c^+ \rightarrow \Lambda_c \pi^0$	3.63 ± 0.27	1.70	
$\Sigma_c^0 \rightarrow \Lambda_c \pi^-$	2.65 ± 0.19	1.57	
$\Sigma_c^{++} \rightarrow \Lambda_c \pi^+$	2.85 ± 0.19	1.64	
$\Sigma_c^{*0} \rightarrow \Lambda_c \pi^-$	21.21 ± 0.81	12.40	$13.0^{+3.7}_{-3.0}$
$\Sigma_c^{*++} \rightarrow \Lambda_c \pi^+$	21.99 ± 0.87	12.84	$17.9^{+3.8}_{-3.2}$
$\Xi_c^{*0} \rightarrow \Xi_c^0 \pi^0$	1.01 ± 0.15	0.72	
$\Xi_c^{*0} \rightarrow \Xi_c^+ \pi^-$	2.11 ± 0.29	1.16	$\Gamma(\Xi_c^{*0}) < 5.5$
$\Xi_c^{*+} \rightarrow \Xi_c^0 \pi^+$	1.78 ± 0.33	1.12	
$\Xi_c^{*+} \rightarrow \Xi_c^+ \pi^0$	1.26 ± 0.17	0.69	$\Gamma(\Xi_c^{*+}) < 3.1$
S-wave transitions			
$\Lambda_{c1}(2593) \rightarrow \Sigma_c^0 \pi^+$	0.83 ± 0.09	2.61	$0.86^{+0.73}_{-0.56}$
$\Lambda_{c1}(2593) \rightarrow \Sigma_c^+ \pi^0$	0.98 ± 0.12	1.73	$\Gamma(\Lambda_{c1}) = 3.6^{+2.0}_{-1.3}$
$\Lambda_{c1}(2593) \rightarrow \Sigma_c^{++} \pi^-$	0.79 ± 0.09	2.15	$0.86^{+0.73}_{-0.56}$
$\Xi_{c1}^*(2815) \rightarrow \Xi_c^{*0} \pi^+$	0.91 ± 0.03	4.84	
$\Xi_{c1}^*(2815) \rightarrow \Xi_c^{*+} \pi^0$	0.48 ± 0.02	2.38	$\Gamma(\Xi_{c1}^*) < 2.4$
D-wave transitions			
$\Lambda_{c1}^*(2625) \rightarrow \Sigma_c^0 \pi^+$	0.080 ± 0.009	0.77	< 0.13
$\Lambda_{c1}^*(2625) \rightarrow \Sigma_c^+ \pi^0$	0.095 ± 0.012	0.69	$\Gamma(\Lambda_{c1}^*) < 1.9$
$\Lambda_{c1}^*(2625) \rightarrow \Sigma_c^{++} \pi^-$	0.076 ± 0.009	0.73	< 0.15
$\Xi_{c1}^*(2815) \rightarrow \Xi_c^{*0} \pi^+$	0.46 ± 0.39	0.30	
$\Xi_{c1}^*(2815) \rightarrow \Xi_c^{*+} \pi^0$	0.25 ± 0.21	0.15	$\Gamma(\Xi_{c1}^*) < 2.4$

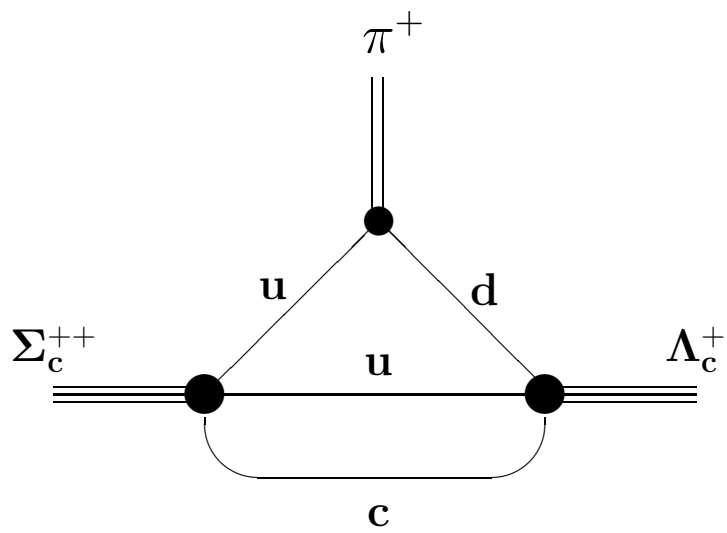


FIG. I