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NEXT-TO-LEADING BFKL *

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Abstract

The representation of the total cross section at high energy $\sqrt{s}$ in the next-to-leading $\ln s$ approximation is given with definition of the impact factors and explicit expression for the BFKL kernel. The estimate of the Pomeron intercept and the next-to-leading contributions to anomalous dimensions near $j = 1$ are presented.

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At large c.m.s. energy $\sqrt{s}$ in the leading logarithmic approximation (LLA) the total cross-section $\sigma(s)$ for the high energy scattering of colourless particles A, B can be presented [1] in terms of their impact factors $\Phi_i(\vec{q_i})$ and the Green function $G(s; \vec{q}_1, \vec{q}_2)$ for the reggeized gluon scattering at zero momentum transfer. Let us consider the impact factors as (not normalized) wave functions of the t-channel two-particle states, denote these states $|A\rangle$ and $|B\rangle$ and use the complete set of the states $|\vec{q}\rangle$ in the transverse momentum space with the properties $\langle\vec{q}_1\,|\vec{q}_2\rangle = \delta(\vec{q}_1 - \vec{q}_2)\,,\ \hat{q^2}\,|\vec{q}_i\rangle = q_i^2\,|\vec{q}_i\rangle$, so that $\Phi_A(\vec{q}) \ = 2\pi q^2\,\langle\vec{q}\,|A\rangle$. In these denotations

$$\sigma(s) = \langle A|\ \hat{G}\ (s)\ \ |B\rangle\ ,\ \ \hat{G}\ (s) = \left(\frac{s}{s_0}\right)^{\hat{K}} \tag{1}$$

with the kernel

$$\langle\vec{q}_1|\ \hat{K}\ |\vec{q}_2\rangle = 2\,\omega(q_1)\,\delta(\vec{q}_1 - \vec{q}_2) + K_r(\vec{q}_1, \vec{q}_2) \tag{2}$$

which is expressed in terms of the gluon Regge trajectory $\omega(q)$ and the integral kernel $K_r(\vec{q}_1, \vec{q}_2)$, related with the real particle production. Taking separately $\omega(q)$ and $K_r(\vec{q}_1, \vec{q}_2)$ contain the infrared divergencies; after their cancellation the kernel averaged over the angle between the momenta $\vec{q}_1$ and $\vec{q}_2$ can be presented in LLA as

$$\overline{\langle\vec{q}_1|\ \hat{K}^B\ |\vec{q}_2\rangle} = \frac{\alpha_s(\mu^2)N_c}{\pi^2}\int\frac{dq^2}{|{q_1}^2 - q^{\ 2}|}\left(\delta(q^2 - q_2^2) - 2\frac{\min(q_1^2, q^2)}{(q_1^2 + q^2)}\delta(q_1^2 - q_2^2)\right). \tag{3}$$

Remind, that the dependence from the energy scale s_0 as well as from the argument μ^2 of the coupling constant is beyond the LLA accuracy.

The program of calculating next-to-leading corrections to the BFKL equation was formulated several years ago [2]. It was shown, that in the next-to-leading logarithmic approximation (NLLA), due to the gluon Reggeization, the form (1) of the cross section, as well as the representation (2) for the kernel remain unchanged, but the trajectory $\omega(q)$ should be taken with the two-loop correction (it was obtained in [3]) and the integral kernel $K_r(\vec{q_1}, \vec{q_2})$ - with one-loop accuracy. The one-loop correction to the integral kernel is obtained as a sum of two contributions. The first one is related with the one-loop virtual correction to the one-gluon production cross-section [4] and the second one is determined by the Born cross-sections for production of two gluons [5] and quark-antiquark pair [6].

For the total NLLA kernel it was obtained [7]:

$$\overline{\langle \vec{q}_1 |\ \hat{K}\ | \vec{q}_2 \rangle} = \frac{\alpha_s(\mu^2) N_c}{\pi^2} \int dq^2 \frac{1}{|{q_1}^2 - q^{\ 2}|} \left(\delta(q^2 - q_2^2) - 2 \frac{\min(q_1^2, q^2)}{(q_1^2 + q^2)} \delta(q_1^2 - q_2^2) \right)$$

$$\times \left[1 - \frac{\alpha_s(\mu^2) N_c}{4\pi} \left(\left(\frac{11}{3} - \frac{2n_f}{3N_c} \right) \ln \left(\frac{|{q_1}^2 - q^{\ 2}|^2}{\max(q_1^2, q^2)\mu^2} \right) - \left(\frac{67}{9} - \frac{\pi^2}{3} - \frac{10}{9} \frac{n_f}{N_c} \right) \right) \right]$$

$$- \frac{\alpha_s^2(\mu^2) N_c^2}{4\pi^3} \left[\frac{1}{32} \left(1 + \frac{n_f}{N_c^3} \right) \left(\frac{2}{{q_2}^2} + \frac{2}{{q_1}^2} + \left(\frac{1}{{q_2}^2} - \frac{1}{{q_1}^2} \right) \ln \frac{{q_1}^2}{{q_2}^2} \right) + \frac{\left(\ln(q_1^2/q_2^2) \right)^2}{|q_1^2 - q_2^2|} \right.$$

$$+ \left(3 + \left(1 + \frac{n_f}{N^3}\right) \left(\frac{3}{4} - \frac{(q_1^2 + q_2^2)^2}{32 q_1^2 q_2^2} \right) \right) \int_0^\infty \frac{dx}{q_1^2 + x^2 q_2^2} \ln \left| \frac{1+x}{1-x} \right| \Bigg] - \frac{1}{q_2^2 + q_1^2} \left(\frac{\pi^2}{3} \right.$$

$$\left. - 4L\left(\min\left(\frac{q_1^2}{q_2^2}, \frac{q_2^2}{q_1^2}\right)\right) \right) + \frac{\alpha_s^2(\mu^2) N_c^2}{4\pi^3} \left(6\zeta(3) - \frac{5\pi^2}{12} \left(\frac{11}{3} - \frac{2n_f}{3N_c} \right) \right) \delta(q_1^2 - q_2^2)\,, \qquad (4)$$

where

$$L(z) = \int_0^z \frac{dt}{t} \ln(1-t)\,,\ \zeta(n) = \sum_{k=1}^{\infty} k^{-n}. \qquad (5)$$

In NLLA the impact factors in (1) depend on the energy scale s_0:

$$\Phi_P(\vec{q}_R; s_0) = \frac{\sqrt{N_c^2 - 1}}{2\pi s} \int ds_{PR} I_{PR} \sigma_{PR}^{(s_0)}(s_{PR}) \theta(s_\Lambda - s_{PR})$$

$$- \int \frac{d\vec{q}}{(2\pi)^{D-1}} \Phi_P^{(B)}(\vec{q}) \frac{g^2 N_c \vec{q}_R^{\ 2}}{\vec{q}^{\ 2} (\vec{q}_R - \vec{q})^2} \ln \left(\frac{s_\Lambda^2}{(\vec{q}_R - \vec{q})^2 s_0} \right). \qquad (6)$$

Here D is the space-time dimension, $\sigma_{PR}^{(s_0)}(s_{PR})$ is the total cross section of the particle-Reggeon scattering at the c.m.s. energy $\sqrt{s_{PR}}$ averaged over colours of the Reggeon with the one-loop corrections calculated for case when the energy scale in the Regge factors is equal s_0; I_{PR} is the invariant flux, $I_{PR} = \sqrt{(s_{PR} - p_P^2 + \vec{q}_R^{\ 2})^2 + 4p_P^2 \vec{q}_R^{\ 2}}$, and $\Phi_P^{(B)}(\vec{q}_R)$ is the Born (LLA) value of the impact factor. The right hand part of the above equation is assumed to be taken in the limit $s_\Lambda \to \infty$, so that the dependence from s_Λ disappears due to the factorization properties of the Reggeon vertices in the regions of strongly ordered rapidities of the produced particles.

In Ref. [7] the energy scale s_0 was taken equal $q_1 q_2$ which is natural from the point of view of the Watson-Sommerfeld representation for high energy scattering amplitudes. It was pointed out in Ref. [7], that the change of the energy scale leads generally to the corresponding modification of the impact factors and the BFKL equation for the Green function G, but the physical results are not changed. Since with the NLLA accuracy

$$\left(\frac{s}{s_0}\right)^{\hat{K}} = s^{\hat{K}}(1 - \hat{K}^B \ln s_0) = \int d\vec{q}_1 \int d\vec{q}_2 \left(1 + \ln\left(\frac{f_1(\hat{q}^2)}{s_0}\right)\frac{\hat{K}^B}{2}\right)$$

$$\times \left|\vec{q}_1\right\rangle \left\langle \vec{q}_1\right| \left(\frac{s}{\sqrt{f_1(q_1^2) f_2(q_2^2)}}\right)^{\hat{K}} \left|\vec{q}_2\right\rangle \left\langle \vec{q}_2\right| \left(1 + \frac{\hat{K}^B}{2}\ln\left(\frac{f_1(\hat{q}^2)}{s_0}\right)\right) , \qquad (7)$$

where f_1 and f_2 are some functions, we obtain that at the transition from the scale s_0 to any factorizable scale $\sqrt{f_1(q_1^2) f_2(q_2^2)}$ we can keep the kernel unchanged, changing the impact factors to

$$\Phi_P(\vec{q}_i; f_i(q_i^2)) = \Phi_P(\vec{q}_i; s_0) + \frac{1}{2}\int d\vec{q}\, \Phi_P^B(\vec{q}) \ln\left(\frac{f_i(q^2)}{s_0}\right) K^B(\vec{q}, \vec{q}_i)\frac{q_i^2}{q^2}. \qquad (8)$$

The action of the kernel (4) on the eigenfunctions $q_2^{2(\gamma-1)}$ of the Born kernel gives us [7]

$$\int d\vec{q}_2\;\; K(\vec{q}_1, \vec{q}_2)\left(\frac{q_2^2}{q_1^2}\right)^{\gamma-1} = \frac{\alpha_s(q_1^2)\, N_c}{\pi}\left(\chi^{(B)}(\gamma) + \frac{\alpha_s(q_1^2) N_c}{\pi}\chi^{(1)}(\gamma)\right) , \qquad (9)$$

were $\chi^{(B)}(\gamma) = 2\psi(1) - \psi(\gamma) - \psi(1-\gamma)$ is proportional to the eigenvalue of the Born kernel, $\psi(\gamma) = \Gamma'(\gamma)/\Gamma(\gamma)$, and the correction $\chi^{(1)}(\gamma)$ is:

$$\chi^{(1)}(\gamma) = -\frac{1}{4}\left[\left(\frac{11}{3} - \frac{2n_f}{3N_c}\right)\frac{1}{2}\left(\chi^2(\gamma) - \psi'(\gamma) + \psi'(1-\gamma)\right)\right.$$

$$-6\zeta(3)+\frac{\pi^2\cos(\pi\gamma)}{\sin^2(\pi\gamma)(1-2\gamma)}\left(3+\left(1+\frac{n_f}{N_c^3}\right)\frac{2+3\gamma(1-\gamma)}{(3-2\gamma)(1+2\gamma)}\right)$$

$$-\left(\frac{67}{9}-\frac{\pi^2}{3}-\frac{10}{9}\frac{n_f}{N_c}\right)\chi(\gamma)-\psi''(\gamma)-\psi''(1-\gamma)-\frac{\pi^3}{\sin(\pi\gamma)}+4\phi(\gamma)\Bigg] \qquad (10)$$

with

$$\phi(\gamma)=-\int_0^1\frac{dx}{1+x}\left(x^{\gamma-1}+x^{-\gamma}\right)\int_x^1\frac{dt}{t}\ln(1-t)$$

$$=\sum_{n=0}^{\infty}(-1)^n\left[\frac{\psi(n+1+\gamma)-\psi(1)}{(n+\gamma)^2}+\frac{\psi(n+2-\gamma)-\psi(1)}{(n+1-\gamma)^2}\right]. \qquad (11)$$

For the relative correction $r(\gamma)$ defined as $\chi^{(1)}(\gamma)=-r(\gamma)\chi^{(B)}(\gamma)$ in the symmetrical point $\gamma=1/2$, corresponding to the Pomeron singularity we have $r(1/2)\simeq 6,46+0.05\frac{n_f}{N_c}+2.66\frac{n_f}{N_c^3}$, i.e., the correction is large. In some sense, the large value of the correction is natural and is a consequence of the large value of the Born intercept $\omega_P^B=4N_c\ln 2\alpha_s(q^2)/\pi$. If we express the corrected intercept ω_P in terms of the Born one, we obtain

$$\omega_P=\omega_P^B(1-\frac{r\left(\frac{1}{2}\right)}{4\ln 2}\omega_P^B)\simeq\omega_P^B(1-2.4\omega_P^B). \qquad (12)$$

The coefficient 2.4 does not look very large. Moreover, it corresponds to the rapidity interval where correlations become important in the hadron production processes. Nevertheless, if we take (12) for $\alpha_s(q^2)=0.15$ we obtain $\omega_P\simeq 0.07$ that is too small. But it is necessary to realize that, firstly, the estimate (12) is quite straightforward and does not take into account neither the influence of the running coupling on the eigenfunctions nor the nonperturbative effects [9]; secondly, the value of the correction strongly depends on its representation. For example, if one takes into account the next-to-leading correction by the corresponding increase of the argument of the running QCD coupling constant, ω_P at $\alpha_s(q^2)=0.15$ turns out to be only two times smaller, than its Born value.

The results obtained for the BFKL kernel can be applied for the calculation of anomalous dimensions of the local twist-2 operators near point $\omega = J - 1 = 0$, which are determined [7] from the solution of the equation

$$\omega = \frac{\alpha_s N_c}{\pi} \left(\chi^{(B)}(\gamma) + \frac{\alpha_s N_c}{\pi} (\chi^{(1)}(\gamma) - 2\chi^{(B)}(\gamma)(\chi^{(B)}(\gamma))') \right). \tag{13}$$

For the low orders of the perturbation theory the solution of (13) reproduces the known results and gives the higher loop correction [7] for $\omega \to 0$:

$$\gamma \simeq \frac{\alpha_s\, N_c}{\pi} (\frac{1}{\omega} - \frac{11}{12} - \frac{n_f}{6N_c^3}) - \left(\frac{\alpha_s}{\pi}\right)^2 \frac{n_f\, N_c}{6\,\omega} (\frac{5}{3} + \frac{13}{6N_c^2})$$
$$- \frac{1}{4\omega^2} \left(\frac{\alpha_s N_c}{\pi}\right)^3 \left(\frac{395}{27} - 2\zeta(3) - \frac{11}{3}\frac{\pi^2}{6} + \frac{n_f}{N_c^3}(\frac{71}{27} - \frac{\pi^2}{9}) \right) . \tag{14}$$

The results obtained in [7] for the BFKL kernel and the anomalous dimensions were confirmed in [8].

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