

# Primordial Black Hole Formation in Supergravity

M. Kawasaki

*Institute for Cosmic Ray Research, University of Tokyo, Tanashi 188-8502, Japan*

T. Yanagida

*Department of Physics and RESCEU, University of Tokyo, Tokyo 113-0033, Japan*

(March 28, 2018)

## Abstract

We study a double inflation model (a preinflation + a new inflation) in supergravity and discuss the formation of primordial black holes which may be identified with massive compact halo objects (MACHOs) observed in the halo of our galaxy. The preinflation drives an inflaton for the new inflation close to the origin through supergravity effects and the new inflation naturally occurs. If the total  $e$ -fold number of the new inflation is smaller than  $\sim 60$ , both inflations produce cosmologically relevant density fluctuations. If the coherent inflaton oscillation after the preinflation continues until the beginning of the new inflation, density fluctuations on small cosmological scales can be set suitably large to produce black holes MACHOs of masses  $\sim 1M_{\odot}$  in a wide region of parameter space in the double inflation model.

arXiv:hep-ph/9807544v3 18 Dec 1998

## I. INTRODUCTION

Massive compact halo objects (MACHOs) observed by gravitational lensing effects [1] are one of the viable candidates for dark matter in the present universe. The analysis of the data taken by the MACHO collaboration [1] suggests that about half of our halo is composed of MACHOs whose masses are about  $(0.5 - 0.6)M_{\odot}$ . It is unlikely that MACHOs are low mass stars such as red dwarfs since the observed abundance of red dwarfs is small [2]. White dwarfs may account for the MACHOs if the initial mass function of the stellar population has a sharp peak at mass scale  $\sim 2M_{\odot}$  [3]. It is an unsolved problem whether or not such an initial mass function is consistent with observations. Thus, there arises another possibility, that MACHOs are primordial black holes of masses  $\sim 1M_{\odot}$ . In the early universe such black holes could be formed owing to the large density fluctuations at cosmic temperature  $T \sim 1$  GeV.

In a previous paper [4], Sugiyama and the present authors proposed<sup>1</sup> a model for the production of black hole MACHOs using a double inflation (a preinflation + a new inflation) model in supergravity.<sup>2</sup> This double inflation model was originally proposed to solve the initial value problem of the new inflation model [7]. In supergravity the reheating temperature of inflation should be low enough to avoid overproduction of gravitinos [8,9]. The new inflation model [10] generally predicts a very low reheating temperature and hence it is the most attractive among many inflation models. However, the new inflation suffers from a fine-tuning problem about the initial condition; i.e., for successful new inflation, the initial value of the inflaton should be very close to the local maximum of the potential in a large region whose size is much longer than the horizon of the universe. It was shown that this serious problem is solved by supergravity effects if there existed a preinflation (e.g., hybrid inflation) with a sufficiently large Hubble parameter before the new inflation [7].

In this double inflation model, if the total  $e$ -fold number of the new inflation is smaller than  $\sim 60$ , density fluctuations produced by both inflations are cosmologically relevant. In this case, the preinflation should account for the density fluctuations on large cosmological scales [including the Cosmic Background Explorer (COBE) scales] while the new inflation produces density fluctuations on small scales. Although the amplitude of the fluctuations on large scales should be normalized to the COBE data [11], fluctuations on small scales are free from the COBE normalization and can be large enough to produce primordial black holes which may be identified with MACHOs.

In the previous paper [4], however, it was found that the MACHO black holes are formed in a very restricted parameter space. This is because large quantum fluctuations of the inflaton for the new inflation are induced during the preinflation and they remain until the beginning of the new inflation, which results in a too inhomogeneous universe. The estimation of the quantum fluctuations was based on the assumption that the reheating takes place quickly after the preinflation. However, if the coherent oscillation of the inflaton continues for a long time, the amplitude of the quantum fluctuations of the new inflaton

---

<sup>1</sup>Different models for the primordial black hole formation have been studied in Ref. [5].

<sup>2</sup>For a review of inflation models in supergravity, e.g., see Ref. [6].

decreases during this oscillation phase and becomes negligible. Therefore, in this paper, we make a re-analysis of the double inflation model assuming a long period of the coherent oscillation of the (preinflation) inflaton field. Taking a hybrid inflation [12] as an example of the preinflation, we find that density fluctuations on small cosmological scales can have a suitable magnitude to produce black hole MACHOs of masses  $\sim 1M_\odot$  in a wide region of parameter space in the double inflation model if the coherent oscillation of an inflaton after the hybrid inflation continues until the beginning of the new inflation.

## II. BLACK HOLE FORMATION

In a radiation-dominated universe, primordial black holes are formed if the density fluctuations  $\delta$  at horizon crossing satisfy a condition  $1/3 \leq \delta \leq 1$  [13,14]. Masses of the black holes  $M_{BH}$  are roughly equal to the horizon mass:

$$M_{BH} \simeq 4\sqrt{3}\pi \frac{M^3}{\sqrt{\rho}} \simeq 0.066M_\odot \left( \frac{T}{\text{GeV}} \right)^{-2}, \quad (1)$$

where  $M (\simeq 2.4 \times 10^{18} \text{ GeV})$  is the gravitational scale and  $\rho$  and  $T$  are the total density and temperature of the universe, respectively. Thus, black holes of masses  $\sim 1M_\odot$  can be formed at temperature  $\sim 0.26 \text{ GeV}$ . Since we are interested in the black holes to be identified with the MACHOs, we assume hereafter that the temperature at the black hole formation epoch is  $T_* \simeq 0.26 \text{ GeV}$ .

The horizon length at the black hole formation epoch ( $T = T_*$ ) corresponds to the scale  $L_*$  in the present universe given by

$$L_* \simeq \frac{a(T_0)}{a(T_*)} H^{-1}(T_*) \simeq 0.25 \text{ pc}, \quad (2)$$

where  $T_0$  is the temperature of the present universe.

The mass fraction  $\beta_*(= \rho_{BH}/\rho)$  of primordial black holes of mass  $M_*$  is given by [14]

$$\beta_*(M_*) = \int_{1/3}^1 \frac{d\delta}{\sqrt{2\pi}\bar{\delta}(M_*)} \exp\left(-\frac{\delta^2}{2\bar{\delta}^2(M_*)}\right) \simeq \bar{\delta}(M_*) \exp\left(-\frac{1}{18\bar{\delta}^2(M_*)}\right), \quad (3)$$

where  $\bar{\delta}(M_*)$  is the mass variance at the horizon crossing. Assuming that only black holes of mass  $M_*$  are formed (this assumption is justified later), the density of the black holes  $\rho_{BH}$  is given by

$$\frac{\rho_{BH}}{s} \simeq \frac{3}{4}\beta_*T_*, \quad (4)$$

where  $s$  is the entropy density. Since  $\rho_{BH}/s$  is constant at  $T < T_*$ , we can write the density parameter  $\Omega_{BH}$  of the black holes in the present universe as

$$\Omega_{BH}h^2 \simeq 5.6 \times 10^7 \beta_*, \quad (5)$$

where we have used the present entropy density  $2.9 \times 10^3 \text{ cm}^{-3}$  and  $T_* \simeq 0.26 \text{ GeV}$ , and  $h$  is the present Hubble constant in units of  $100 \text{ km/sec/Mpc}$ . Requiring that the black

holes (=MACHOs) be dark matter in the present universe, i.e.  $\Omega_{BH}h^2 \sim 0.25$ , we obtain  $\beta_* \sim 5 \times 10^{-9}$  which leads to

$$\bar{\delta}(M_*) \simeq 0.06. \quad (6)$$

This mass variance suggests that the amplitude of the density fluctuations at the mass scale  $M_*$  ( $\simeq 1M_\odot$ ) are given by

$$\frac{\delta\rho}{\rho} \simeq \frac{2}{3}\Phi \simeq 0.01, \quad (7)$$

where  $\Phi$  is the gauge-invariant fluctuations of the gravitational potential [15]. We will show later that such large density fluctuations are naturally produced during the new inflation.

### III. DOUBLE INFLATION MODEL

In this section we will consider a double inflation model in supergravity and see how MACHO black holes are formed. We adopt the double inflation model proposed in Refs. [7,4]. The model consists of two inflationary stages; the first one is called preinflation and we adopt a hybrid inflation [12] as the preinflation. We also assume that the second inflationary stage is realized by a new inflation model [16] and its  $e$ -fold number is smaller than  $\sim 60$ . Thus, the density fluctuations on large scales are produced during the preinflation and their amplitude should be normalized by the COBE data [11]. On the other hand, the new inflation produces fluctuations on small scales. Since the amplitude of the small scale fluctuations is free from the COBE normalization, we expect that the new inflation can produce density fluctuations large enough to form primordial black holes.

#### A. preinflation

First, let us discuss a hybrid inflation model which we adopt to cause the preinflation. The hybrid inflation model contains two kinds of superfields: one is  $S(x, \theta)$  and the others are  $\Psi(x, \theta)$  and  $\bar{\Psi}(x, \theta)$ . The model is also based on the  $U(1)_R$  symmetry. The superpotential is given by [17,12]

$$W(S, \Psi, \bar{\Psi}) = -\mu^2 S + \lambda S \bar{\Psi} \Psi. \quad (8)$$

The  $R$ -invariant Kähler potential is given by

$$K(S, \Psi, \bar{\Psi}) = |S|^2 + |\Psi|^2 + |\bar{\Psi}|^2 - \frac{\zeta}{4}|S|^4 + \dots, \quad (9)$$

where  $\zeta$  is a constant of order 1 and the ellipsis denotes higher-order terms, which we neglect in the present analysis. We gauge the  $U(1)$  phase rotation:  $\Psi \rightarrow e^{i\delta}\Psi$  and  $\bar{\Psi} \rightarrow e^{-i\delta}\bar{\Psi}$ . To satisfy the  $D$ -term flatness condition we take always  $\Psi = \bar{\Psi}$  in our analysis.

Here and hereafter, we set the gravitational scale  $M \simeq 2.4 \times 10^{18}$  GeV equal to unity and regard it as a plausible cutoff in supergravity. As is shown in Ref. [12] the real part of

$S(x)$  is identified with the inflaton field  $\sigma/\sqrt{2}$ . The potential is minimized at  $\Psi = \bar{\Psi} = 0$  for  $\sigma$  larger than  $\sigma_c = \sqrt{2}\mu/\sqrt{\lambda}$  and inflation occurs for  $0 < \zeta < 1$  and  $\sigma_c \lesssim \sigma \lesssim 1$ .

In a region of relatively small  $\sigma$  ( $\sigma_c \lesssim \sigma \lesssim \lambda/\sqrt{8\pi^2\zeta}$ ) radiative corrections are important for the inflation dynamics as shown by Dvali *et al.* [18]. Including one-loop corrections, the potential for the inflaton  $\sigma$  is given by

$$V \simeq \mu^4 \left[ 1 + \frac{\zeta}{2}\sigma^2 + \frac{\lambda^2}{8\pi^2} \ln \left( \frac{\sigma}{\sigma_c} \right) \right]. \quad (10)$$

The Hubble parameter  $H_{\text{pre}}$  and  $e$ -fold number  $N_{\text{pre}}$  are given by

$$H_{\text{pre}} \simeq \frac{\mu^2}{\sqrt{3}} \quad (11)$$

and

$$N_{\text{pre}} \simeq \begin{cases} \frac{1}{2\zeta} + \frac{1}{\zeta} \ln \frac{\sigma_{N_{\text{pre}}}}{\tilde{\sigma}} & (\sigma_{N_{\text{pre}}} > \tilde{\sigma}), \\ \frac{4\pi^2\sigma_{N_{\text{pre}}}^2}{\lambda^2} & (\sigma_{N_{\text{pre}}} < \tilde{\sigma}), \end{cases} \quad (12)$$

where

$$\tilde{\sigma} \simeq \frac{\lambda}{2\sqrt{2\zeta}\pi}. \quad (13)$$

Here  $\sigma_{N_{\text{pre}}}$  is the value of the inflaton field  $\sigma$  corresponding to an  $e$ -fold number  $N_{\text{pre}}$ .

If we define  $N_{\text{COBE}}$  as the  $e$ -fold number corresponding to the COBE scale, the COBE normalization leads to a condition for the inflaton potential:

$$\frac{V^{3/2}(\sigma_{\text{COBE}})}{|V'(\sigma_{\text{COBE}})|} \simeq 5.3 \times 10^{-4}, \quad (14)$$

where  $\sigma_{\text{COBE}} \equiv \sigma_{N_{\text{COBE}}}$ . Then, the scale  $\mu$  for the preinflation satisfies the following condition:

$$\frac{\mu^2}{\zeta\sigma_{\text{COBE}}} \simeq 5.3 \times 10^{-4} \quad (\sigma_{\text{COBE}} > \tilde{\sigma}), \quad (15)$$

$$\frac{8\pi^2\mu^2\sigma_{\text{COBE}}}{\lambda^2} \simeq 5.3 \times 10^{-4} \quad (\sigma_{\text{COBE}} < \tilde{\sigma}). \quad (16)$$

From Eqs.(12), (15), and (16), we obtain

$$\mu \simeq 6.5 \times 10^{-3} \lambda^{1/2} N_{\text{COBE}}^{-1/4}, \quad (17)$$

for  $\sigma_{\text{COBE}} < \tilde{\sigma}$ , and

$$\mu \simeq 6.0 \times 10^{-3} \zeta^{1/4} \lambda^{1/2} \exp(\zeta N_{\text{COBE}}/2), \quad (18)$$

for  $\sigma_{\text{COBE}} > \tilde{\sigma}$ .

## B. New inflation

Now, we consider a new inflation model. We adopt the new inflation model proposed in Ref. [7]. The inflaton superfield  $\phi(x, \theta)$  is assumed to have an  $R$  charge  $2/(n+1)$  and  $U(1)_R$  is dynamically broken down to a discrete  $Z_{2nR}$  at a scale  $v$ , which generates an effective superpotential [7]:

$$W(\phi) = v^2\phi - \frac{g}{n+1}\phi^{n+1}. \quad (19)$$

The  $R$ -invariant effective Kähler potential is given by

$$K(\phi, \chi) = |\phi|^2 + \frac{\kappa}{4}|\phi|^4 + \dots, \quad (20)$$

where  $\kappa$  is a constant of order 1.

The effective potential  $V(\phi)$  for a scalar component of the superfield  $\phi(x, \theta)$  in supergravity is obtained from the above superpotential (19) and the Kähler potential (20) as

$$V = e^{K(\phi)} \left\{ \left( \frac{\partial^2 K}{\partial \phi \partial \phi^*} \right)^{-1} |D_\phi W|^2 - 3|W|^2 \right\}, \quad (21)$$

with

$$D_\phi W = \frac{\partial W}{\partial \phi} + \frac{\partial K}{\partial \phi} W. \quad (22)$$

This potential yields a vacuum

$$\langle \phi \rangle \simeq \left( \frac{v^2}{g} \right)^{1/n}. \quad (23)$$

We have negative energy as

$$\langle V \rangle \simeq -3e^{\langle K \rangle} |\langle W \rangle|^2 \simeq -3 \left( \frac{n}{n+1} \right)^2 |v|^4 |\langle \phi \rangle|^2. \quad (24)$$

The negative vacuum energy (24) is assumed to be canceled out by a supersymmetry-(SUSY)-breaking effect [16] which gives a positive contribution  $\Lambda_{SUSY}^4$  to the vacuum energy. Thus, we have a relation between  $v$  and the gravitino mass  $m_{3/2}$ :

$$m_{3/2} \simeq \frac{\Lambda_{SUSY}^2}{\sqrt{3}} = \left( \frac{n}{n+1} \right) |v|^2 \left| \frac{v^2}{g} \right|^{\frac{1}{n}}. \quad (25)$$

The inflaton  $\phi$  has a mass  $m_\phi$  in the vacuum with ( for details, see Ref. [7])

$$m_\phi \simeq n |g|^{\frac{1}{n}} |v|^{2-\frac{2}{n}}. \quad (26)$$

The inflaton  $\phi$  may decay into ordinary particles through gravitationally suppressed interactions, which yields reheating temperature  $T_R$  given by

$$T_R \simeq 0.1 m_\phi^{3/2} \simeq 0.1 n^{\frac{3}{2}} |g|^{\frac{3}{2n}} |v|^{3-\frac{3}{n}}. \quad (27)$$

For example, the reheating temperature  $T_R$  is as low as  $2 - 6 \times 10^4$  GeV for  $v \simeq 10^{-8} - 10^{-6}$  ( $m_{3/2} \simeq 0.02$  GeV – 2 TeV),  $n = 4$  and  $g \simeq 1$ , which is low enough to solve the gravitino problem.<sup>3</sup>

Let us discuss dynamics of the new inflation. Identifying the inflaton field  $\varphi(x)/\sqrt{2}$  with the real part of the field  $\phi(x)$ , we obtain a potential of the inflaton for  $\varphi < v$  from Eq. (21):

$$V(\varphi) \simeq v^4 - \frac{\kappa}{2} v^4 \varphi^2 - \frac{g}{2^{\frac{n}{2}-1}} v^2 \varphi^n + \frac{g^2}{2^n} \varphi^{2n}. \quad (28)$$

It has been shown in Ref. [16] that the slow-roll condition for the inflation is satisfied for  $0 < \kappa < 1$  and  $\varphi \lesssim \varphi_f$  where

$$\varphi_f \simeq \sqrt{2} \left( \frac{(1-\kappa)v^2}{gn(n-1)} \right)^{\frac{1}{n-2}}. \quad (29)$$

The new inflation ends when  $\varphi$  becomes larger than  $\varphi_f$ . The Hubble parameter of the new inflation is given by

$$H_{\text{new}} \simeq \frac{v^2}{\sqrt{3}}. \quad (30)$$

The  $e$ -fold number  $N_{\text{new}}$  is given by

$$N_{\text{new}} \simeq \frac{1}{\kappa} \ln \left( \frac{\tilde{\varphi}}{\varphi_{N_{\text{new}}}} \right) + \frac{1-n\kappa}{(n-2)\kappa(1-\kappa)}, \quad (31)$$

where

$$\tilde{\varphi} = \sqrt{2} \left( \frac{\kappa v^2}{gn} \right)^{\frac{1}{n-2}}. \quad (32)$$

Here, we have assumed that  $\kappa \leq 1/n$ .

The amplitude of primordial density fluctuations  $\delta\rho/\rho$  due to the new inflation is written as

$$\frac{\delta\rho}{\rho} \simeq \frac{1}{5\sqrt{3}\pi} \frac{V^{3/2}(\varphi_{N_{\text{new}}})}{|V'(\varphi_{N_{\text{new}}})|} = \frac{1}{5\sqrt{3}\pi} \frac{v^2}{\kappa\varphi_{N_{\text{new}}}}. \quad (33)$$

Notice here that we have larger density fluctuations for smaller  $\varphi_{N_{\text{new}}}$  and hence the largest amplitude of the fluctuations is given at the beginning of the new inflation. An interesting point on the above density fluctuations is that it results in a tilted spectrum with spectral index  $n_s$  given by (see Refs. [7,16])

---

<sup>3</sup>Since the reheating temperature is low, we assume that the baryon asymmetry is produced through the electroweak baryogenesis [19] or the Affleck-Dine mechanism [20].

$$n_s \simeq 1 - 2\kappa. \quad (34)$$

As shown later, we take  $\kappa \sim 0.2$  and  $n_s \sim 0.6$ .

Since only fluctuations produced during the new inflation have amplitudes large enough to form the primordial black holes, the maximum mass of the black holes is determined by fluctuations with wavelengths equal to the horizon at the beginning of the new inflation. We require that the maximum mass be  $\sim 1M_\odot$  to account for the black hole MACHOs. On the other hand, the formation of black holes with smaller masses is strongly suppressed since the spectrum of the density fluctuations predicted by the new inflation is tilted [see Eq. (34)]: the amplitude of the fluctuations with smaller wavelengths is smaller (Fig. 1). A tiny decrease of  $\bar{\delta}(M)$  results in a large suppression of the black hole formation rate as is seen from Eq. (3). Therefore, only black holes of masses in a narrow range are formed in the present model.

The  $e$ -fold number  $N_{\text{new}}$  is related to the present cosmological scale  $L$  by

$$N_{\text{new}} \simeq 60 + \ln \left( \frac{L}{3000 \text{ Mpc}} \right). \quad (35)$$

From Eq. (35), the density fluctuations corresponding to the MACHO scale  $L_*$  are produced when  $N_{\text{new}} = N_* \simeq 40$  during the new inflation. Since the fluctuations large enough to produce MACHOs are induced only at the beginning of the new inflation,  $N_*$  is also expressed as

$$N_* \simeq \frac{1}{\kappa} \ln \left( \frac{\tilde{\varphi}}{\varphi_b} \right) + \frac{1 - n\kappa}{(n - 2)\kappa(1 - \kappa)}, \quad (36)$$

where  $\varphi_b$  is the value of  $\varphi$  at the beginning of the new inflation.

### C. Initial value and fluctuations of $\varphi$

The crucial point observed in Ref. [7] is that the preinflation sets dynamically the initial condition for the new inflation. The inflaton field  $\varphi(x)$  for the new inflation gets an effective mass  $\sim \mu^2$  from the  $e^K[\dots]$  term in the potential [17,21] during the preinflation. Thus, we write the effective mass  $m_{\text{eff}}$  as

$$m_{\text{eff}} = c\mu^2 = \sqrt{3}cH, \quad (37)$$

where we introduce a free parameter  $c$  since the precise value of the effective mass depends on the details of the Kähler potential. For example, if the Kähler potential contains  $-f|\phi|^2|S|^2$ , the effective mass is equal to  $\sqrt{1+f}\mu^2$ .

The evolution of the inflaton  $\varphi$  for the new inflation is described as

$$\ddot{\varphi} + 3H\dot{\varphi} + m_{\text{eff}}^2\varphi = 0. \quad (38)$$

Using  $\dot{H} \simeq 0$ , we get a solution to the above equation as

$$\varphi \propto a^{-3/2 + \sqrt{9/4 - 3c^2}}, \quad (39)$$



where  $a$  denotes the scale factor of the universe. Thus, for  $c \gtrsim \sqrt{3}/2$ ,  $\varphi$  oscillates during the preinflation and its amplitude decreases as  $a^{-3/2}$ . Thus, at the end of the preinflation the  $\varphi$  takes a value

$$\varphi \simeq \varphi_i \exp\left(-\frac{3}{2}N_{\text{pre,tot}}\right), \quad (40)$$

where  $\varphi_i$  is the value of  $\varphi$  at the beginning of the preinflation and  $N_{\text{pre,tot}}$  the total  $e$ -fold number of the preinflation.

The minimum of the potential for  $\varphi$  deviates from zero through the effect of the  $|D_S W|^2 + |D_\phi W|^2 - 3|W|^2$  term and this potential has a minimum as shown in Ref. [7]:

$$\varphi_{\text{min}} \simeq -\frac{\sqrt{2}}{c^2\sqrt{\lambda}}v\left(\frac{v}{\mu}\right). \quad (41)$$

Thus, at the end of the preinflation the  $\varphi$  settles down to this  $\varphi_{\text{min}}$ .

After the preinflation, the  $\sigma$  and  $\Psi(\bar{\Psi})$  start to oscillate and the universe becomes matter dominated.  $\Psi$  and  $\bar{\Psi}$  couple to the U(1) gauge multiplets and decay immediately to gauge fields if energetically allowed. We assume that masses for the gauge fields are larger than those of  $\Psi$  and  $\bar{\Psi}$ . We also assume that the SUSY standard model particles do not couple to the gauge multiplets. Thus,  $S$ ,  $\Psi$ , and  $\bar{\Psi}$  decay into light particles only through gravitationally suppressed interactions and the coherent oscillations of  $S$ ,  $\Psi$ , and  $\bar{\Psi}$  fields continue until the new inflation starts. In this period of the coherent oscillations the average potential energy of the scalar fields is the half of the total energy of the universe and hence the effective mass of  $\varphi$  is given by

$$m_{\text{eff}}^2 \simeq \frac{3}{2}H^2. \quad (42)$$

Here and hereafter, we take  $c = 1$ . The evolution of  $\varphi$  is described by Eq. (38). Taking into account  $\dot{H} = (3/2)H^2$ , one can find that the amplitude of  $\varphi$  decreases as  $a^{-3/4}$ . After the preinflation ends, the superpotential for the inflaton of the preinflation vanishes and hence the potential for  $\varphi$  has a minimum at  $\varphi \simeq 0$ . Since the scale factor increases by a factor  $(\mu/v)^{4/3}$  during the matter-dominated era between two inflations, the mean initial value  $\varphi_b$  of  $\varphi$  at the beginning of the new inflation is written as<sup>4</sup>

$$\varphi_b \simeq \frac{\sqrt{2}}{\sqrt{\lambda}}v\left(\frac{v}{\mu}\right)^2. \quad (43)$$

We now discuss quantum effects during the preinflation. It is known that in a de Sitter universe massless fields have quantum fluctuations whose amplitudes are given by  $H/(2\pi)$ . However, the quantum fluctuations for  $\varphi$  are strongly suppressed [22] in the present model since the mass of  $\varphi$  is larger than the Hubble parameter until the start of the new inflation.

---

<sup>4</sup>Since Eq.(43) represents the amplitude of the oscillating  $\varphi$ , the actual value of  $\varphi_b$  should be multiplied by a factor  $0 \leq \xi \leq 1$ . Here, we take  $\xi = 1$  for simplicity.

Let us consider the amplitude of fluctuations with comoving wavelength  $\ell_b$  corresponding to the horizon scale at the beginning of the new inflation. These fluctuations are induced during the preinflation and its amplitude at horizon crossing [ $\ell_b a(t_h = H_{\text{pre}})$  is given by  $H_{\text{pre}}/(2\pi)(H_{\text{pre}}/m_{\text{eff}})^{1/2}$ ]. Since those fluctuations reenter the horizon at the beginning of the new inflation ( $t = t_b$ ), the scale factor of the universe increases from  $t_h$  to  $t_b$  by a factor of  $(H_{\text{pre}}/H_{\text{new}}) = (\mu/v)^2$ . The amplitude of fluctuations decreases as  $a^{-3/2}$  during the preinflation and  $a^{-3/4}$  during the matter-dominated era between two inflations, and the amplitude of fluctuations with comoving wavelength  $\ell_b$  corresponding to the horizon scale at the beginning of the new inflation is now given by

$$\delta\varphi \simeq \frac{H}{2\pi} \left( \frac{H}{m_{\text{eff}}} \right)^{\frac{1}{2}} (\mu/v)^{(3/2)(2-4/3)} (\mu/v)^{(3/4)(4/3)} \simeq \frac{H}{3^{1/4} 2\pi} \left( \frac{v}{\mu} \right)^2. \quad (44)$$

Here, we have used the fact that the scale factor increases by  $(\mu/v)^{4/3}$  during the matter-dominated era. The fluctuations given by Eq. (44) are a little less than newly induced fluctuations at the beginning of the new inflation [ $\simeq v^2/(2\pi\sqrt{3})$ ]. Thus, we assume that the fluctuations of  $\varphi$  induced in the preinflation can be neglected when we estimate the fluctuations during the new inflation.

Finally, we make a comment on the domain wall problem in the double inflation model. Since the potential of the inflaton  $\phi$  has a discrete symmetry [see Eqs. (19) and (20)], domain walls are produced if the phases of  $\phi$  are spatially random. However, the preinflation make the phase of  $\phi$  homogeneous with the help of the interactions between two inflaton fields  $S$  and  $\phi$  [see Eq. (41)]. Therefore, the domain wall problem does not exist in the present model.

#### IV. MACHO FORMATION

Since the density fluctuations corresponding to  $L_*$  are produced at the beginning of the new inflation ( $N_{\text{new}} = N_*$ ), from Eqs. (7), (33) and (36) we obtain

$$\frac{V^{3/2}}{V} \simeq \frac{v^2}{\kappa\varphi_b} \simeq 0.3, \quad (45)$$

where  $\varphi_b$  is given by Eq.(43) and we have used Eq.(7). Eqs.(32) and (36) lead to

$$v \simeq 0.3\kappa \exp(-\kappa N_*), \quad (46)$$

where we have taken  $n = 4$  and  $\sqrt{\kappa/(2g)} \simeq 1$ , and neglected the second term on the right-hand side of Eq. (36).

On the other hand, the density fluctuations produced in the preinflation should be normalized by the COBE data, which determine the scale of the preinflation as a function of  $N_{\text{COBE}}$  as Eqs.(14)–(18). In estimating  $N_{\text{COBE}}$  we must take into account the fact that fluctuations induced at  $e$ -fold numbers less than  $(2/3)\ln(\mu/v)$  reenter the horizon before the new inflation starts. Such fluctuations are cosmologically irrelevant since the new inflation produces much larger fluctuations. Thus,  $N_{\text{COBE}}$  is given by

$$N_{\text{COBE}} = 60 - N_* + \frac{2}{3} \ln \left( \frac{\mu}{v} \right). \quad (47)$$

From Eqs. (17), (18), and (45), the scale of the new inflation is written as

$$v \simeq 9.95 \times 10^{-5} \kappa^{-1} N_{\text{COBE}}^{-1/2} \lambda^{3/2}, \quad (48)$$

for small  $\zeta$  less than 0.021, which comes from the condition that the one-loop corrections govern the preinflation dynamics, i.e.,  $\sigma_{N_{\text{COBE}}} \equiv \sigma_{\text{COBE}} < \tilde{\sigma}$ . On the other hand, for  $\zeta > 0.021$ , we obtain

$$v \simeq 8.53 \times 10^{-5} \kappa^{-1} \lambda^{3/2} \zeta^{1/2} \exp(\zeta N_{\text{COBE}}). \quad (49)$$

From Eqs.(47), (17), (18), (48), and (49)  $N_{\text{COBE}}$  is approximately given by

$$N_{\text{COBE}} \simeq 23, \quad (50)$$

for both small and large  $\zeta$ . Here, we have neglected  $\mathcal{O}(\ln \lambda)$ ,  $\mathcal{O}(\ln \kappa)$ , and  $\mathcal{O}(\ln \zeta)$  corrections. Then, we obtain  $\mu$ ,  $v$ , and  $\sigma_{\text{COBE}}$  for  $\zeta < 0.021$  as

$$\mu \simeq 3.0 \times 10^{-3} \lambda^{1/2}, \quad (51)$$

$$v \simeq 2.1 \times 10^{-5} \kappa^{-1} \lambda^{3/2}, \quad (52)$$

$$\sigma_{\text{COBE}} \simeq 0.77 \lambda. \quad (53)$$

From Eq. (46),  $\kappa$  is given by

$$\kappa \simeq 0.24, \quad (54)$$

which results in

$$v \simeq 8.6 \times 10^{-5} \lambda^{3/2}. \quad (55)$$

The coupling  $\lambda$  cannot be taken too small because  $\sigma_{\text{COBE}}$  becomes less than  $\sigma_c$ , which leads to the constraint

$$\lambda \gtrsim \lambda_{\text{min}} = 5.5 \times 10^{-3}. \quad (56)$$

In the present model, the scale  $v$  of the new inflation is related to the gravitino mass as Eq. (25). Thus, the lower limit to  $\lambda$  predicts a lower bound of the gravitino mass:

$$m_{3/2} \gtrsim m_{3/2,\text{min}} = 0.42 \text{ GeV}, \quad (57)$$

which is consistent with both gauge-mediated [23] and gravity mediated [24] SUSY breaking models. If we require that the gravitino mass should be less than about 1 TeV, we obtain the upper limit to  $\lambda$  given by<sup>5</sup>

---

<sup>5</sup>If we disconnect the scale  $v$  from the gravitino mass  $m_{3/2}$  assuming some other contribution to cancel the energy in  $W$  in Eq. (24), this constraint may be relaxed.

$$\lambda \lesssim \lambda_{\max} = 4.3 \times 10^{-2}. \quad (58)$$

For the case of large  $\zeta$ , if we fix  $\zeta$ , we can obtain  $\mu$ ,  $v$ ,  $\kappa$ , and  $\sigma_{\text{COBE}}$  in the same way as for the case of small  $\zeta$ . The result is shown in Table I for  $\zeta = 0.1, 0.05$ , and  $0.025$ . In this case, lower limit  $\lambda_{\min}$  is obtained by requiring  $\tilde{\sigma} \gtrsim \sigma_c$ , and  $m_{3/2, \min}$  and  $\lambda_{\max}$  are determined in the same way as in the previous case. These limits are also shown in Table I. We require that  $\lambda_{\min} \leq \lambda_{\max}$  for consistency. From Table I one can see that the present model works for  $\zeta \lesssim 0.05$ .

## V. CONCLUSION

In this paper we have studied the formation of primordial black holes of masses about  $1M_{\odot}$  by taking a double inflation model in supergravity. We have shown that in a wide range of parameter space primordial black holes are produced of masses  $\sim 1M_{\odot}$  which may be identified with MACHOs in the halo of our galaxy. It should be noticed that the present model allows natural values for the coefficient  $c$  of the effective mass of  $\varphi$  during the preinflation [i.e.,  $c = \mathcal{O}(1)$ ]. In contrast, we must assume unnaturally large  $c$  [4] if the inflaton decays quickly after the preinflation.

Our double inflation model consists of a preinflation (e.g., hybrid inflation) and a new inflation. The preinflation provides the density fluctuations observed by COBE and it also dynamically sets the initial condition of the new inflation through supergravity effects. The predicted power spectrum has almost a scale invariant form ( $n_s \simeq 1$ ) on large cosmological scales which is favored for the structure formation of the universe [25]. On the other hand, the new inflation gives the power spectrum which has large amplitude and shallow slope ( $n_s < 1$ ) on small scales. Thus, this power spectrum has a large and sharp peak on the scale corresponding to the turning epoch from preinflation to new inflation. With taking the MACHO scale ( $\sim 0.25$  pc) as the turning scale, our model leads to the formation of the black hole MACHOs in a narrow mass range, which is quite consistent with the observation by the MACHO collaboration [1]. We should stress that although we have adopted a specific model for the preinflation and the new inflation, our result is quite general in double inflation models which consist of a generic preinflation and a new inflation. We may construct a larger class of double inflation models which explain naturally primordial black hole MACHOs.

In the present paper we set the turning epoch from one inflation to another so that the masses of black holes are  $\sim 1M_{\odot}$  to account for the MACHOs. However, the double inflation model can produce black holes of different masses by changing the  $e$ -fold number of the new inflation. One of interesting cases is that produced black holes have masses of  $\sim 10^{-19}M_{\odot}$  and are just evaporating now. Such black holes are one of the interesting candidates for the sources of antiproton fluxes recently observed in the BESS detector [26].

The primordial black holes play the role of the usual cold dark matter on the large scale structure formation. The scales of the fluctuations for primordial black hole formation themselves are much smaller than the galactic scale and thus we cannot see any signals for such fluctuations in  $\delta T/T$  measurements. The primordial black holes are also attractive as a source of gravitational waves. If the primordial black holes dominate dark matter of the present universe, some of them likely form binaries. Such binary black holes coalesce and produce significant gravitational waves [27] which may be detectable in future detectors.

## **ACKNOWLEDGMENT**

M.K. thanks S. Kasuya and H. Suzuki for useful discussions. We also thank S. Olito for information on the BESS results.

TABLES

TABLE I.  $\mu$ ,  $v$ ,  $\kappa$ ,  $\sigma_{\text{COBE}}$ ,  $m_{3/2,\text{min}}$ ,  $\lambda_{\text{max}}$  and  $\lambda_{\text{min}}$  for  $\zeta = 0.1, 0.05, 0.025$

$\zeta$	0.1	0.05	0.025
$\mu$	$1.1 \times 10^{-2} \lambda^{1/2}$	$5.1 \times 10^{-3} \lambda^{1/2}$	$3.2 \times 10^{-3} \lambda^{1/2}$
$v$	$1.5 \times 10^{-3} \lambda^{3/2}$	$2.8 \times 10^{-4} \lambda^{3/2}$	$1.0 \times 10^{-4} \lambda^{3/2}$
$\kappa$	0.18	0.21	0.24
$\sigma_{\text{COBE}}$	$2.2\lambda$	$0.96\lambda$	$0.77\lambda$
$m_{3/2,\text{min}}$	$1.3 \times 10^6 \text{ GeV}$	$3.0 \times 10^2 \text{ GeV}$	1.1 GeV
$\lambda_{\text{max}}$	$6.3 \times 10^{-3}$	$2.0 \times 10^{-2}$	$3.9 \times 10^{-2}$
$\lambda_{\text{min}}$	$4.2 \times 10^{-2}$	$1.4 \times 10^{-2}$	$6.3 \times 10^{-3}$

## REFERENCES

- [1] MACHO Collaboration, C. Alcock *et al.*, *Astrophys. J.* **486**, 697 (1997); *Nature* **365**, 621 (1993).
- [2] D.S. Graff and K. Freese, *Astrophys. J.* **456**, L49 (1996);  
J. Bahcall, C. Flynn, A. Gould and S. Kirhakos, *Astrophys. J.* **435**, L51 (1994).
- [3] F.C. Adams and G. Laughlin, *Astrophys. J.* **468**, 586 (1996).
- [4] M. Kawasaki, N. Sugiyama and T. Yanagida, *Phys. Rev.* **D57**, 6050 (1998).
- [5] J. Yokoyama, *Astron. Astrophys.* **318**, 673 (1997), *atrp-ph/9802357*;  
J. Garcia-Bellido, A. Linde and D. Wands, *ibid* **54**, 6040 (1996).
- [6] D.H Lyth and A. Riotto, *hep-ph/9807278*.
- [7] K.I. Izawa, M. Kawasaki and T. Yanagida, *Phys. Lett.* **B411**, 249 (1997).
- [8] M. Yu. Khlopov and A.D. Linde, *Phys. Lett.* **B138**, 265 (1984) ;  
J. Ellis, E. Kim and D.V. Nanopoulos, *Phys. Lett.* **B145**, 181 (1984);  
J. Ellis, G.B. Gelmini, J.L. Lopez, D.V. Nanopoulos and S. Sarker, *Nucl. Phys.* **B373**, 399 (1992);  
M. Kawasaki and T. Moroi, *Prog. Theor. Phys.* **93**, 879 (1995).
- [9] T. Moroi, H. Murayama and M. Yamaguchi, *Phys. Lett.* **B303**, 289 (1993).
- [10] A. Albrecht and P.J. Steinhardt, *Phys. Rev. Lett.* **48**, 1220 (1982);  
A.D. Linde, *Phys. Lett.* **B108**, 389 (1982).
- [11] C.L. Bennett *et al.*, *Astrophys. J.* **464**, L1 (1996).
- [12] C. Panagiotakopoulos, *Phys. Rev.* **D55**, 7335 (1997);  
A. Linde and A. Riotto, *Phys. Rev. D* **56**, R1841 (1997).
- [13] B.J. Carr, *Astrophys. J.* **201**, 1 (1975);  
B.J. Carr, J.H. Gilbert and J.E. Lidsey, *Phys. Rev.* **50**, 4853 (1994).
- [14] A.M. Green and A.R. Liddle, *Phys. Rev.* **D56**, 6166 (1997);  
A.M. Green and A.R. Liddle and A. Riotto, *ibid* **D56**, 7554 (1997).
- [15] For example, A.D. Linde, *Particle Physics and Inflationary Cosmology*, (Harwood, Chur, Switzerland, 1990).
- [16] K.I. Izawa and T. Yanagida, *Phys. Lett.* **B393**, 331 (1997).
- [17] E.J. Copeland, A.R. Liddle, D.H. Lyth, E.D. Stewart and D. Wands, *Phys. Rev.* **D49**, 6410 (1994).
- [18] G. Dvali, Q. Shafi and R.K. Shaefer, *Phys. Rev. Lett.* **73**, 1886 (1994).
- [19] For a review, see V.A. Rubakov and M.E. Shaposhnikov, *Usp. Fiz. Nauk.* **166**, 493 (1996) [*Phys. Usp.* **39**, 461 (1996)].
- [20] I. Affleck and M. Dine, *Nucl. Phys.*, **B249**, 361(1985).
- [21] K. Kumekawa, T. Moroi and T. Yanagida, *Prog. Theor. Phys.* **92**, 437 (1994);  
M. Dine, L. Randall and S. Thomas, *Phys. Rev. Lett.* **75**, 398 (1995).
- [22] K. Enqvist, K.W. Ng and K.A. Olive, *Nucl. Phys.* **B303**, 713 (1988).
- [23] For a review, see G.F. Giudice and R. Rattazzi, *hep-ph/9801271*.
- [24] For a review, see H.P. Nilles, *Phys. Rep.* **110**, 1 (1984).
- [25] M. White, D. Scott, J. Silk and M. Davis, *Mon. Not. R. Astron. Soc.* **276**, L69 (1995).
- [26] K. Yoshimura *et al.* *Phys. Rev. Lett.* **75**, 3792 (1995).
- [27] T. Nakamura, M. Sasaki, T. Tanaka and K.S. Thorne, *Astrophys. J.* **487**, L139 (1997).

FIGURES

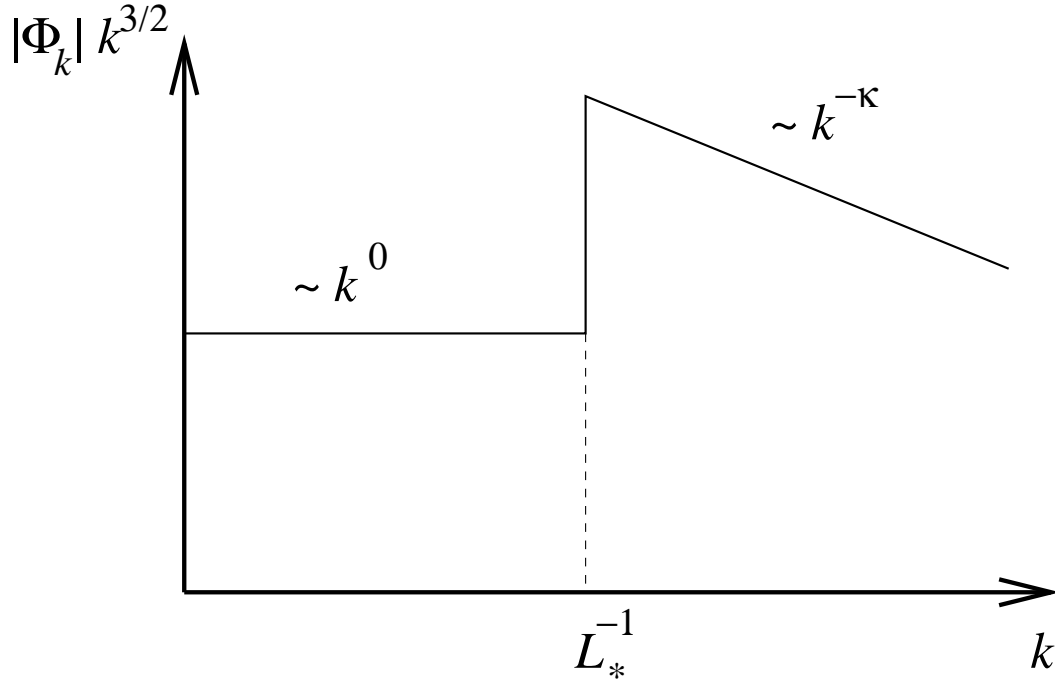


FIG. 1. Spectrum of the fluctuations of the gravitational potential  $\Phi_k$  [ see Eq. (7)].  $k$  is the comoving wave number of the fluctuations and  $L_*$  is the MACHO scale. The fluctuations on large scales ( $k < L_*^{-1}$ ) are almost scale invariant ( $\sim k^0$ ) and the spectrum is tilted ( $\sim k^{-\kappa}$ ) on small scales ( $k > L_*^{-1}$ ).