

Chiral Disorder and QCD Phase Transitions

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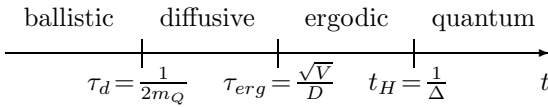
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If QCD is to undergo a second order phase transition, the light quark return probability is universal for large times at the critical point. We show that this behavior is distinct from the one expected at the mobility edge of a metal-insulator transition or a percolation transition in $d \leq 4$. Our results are accessible to current lattice QCD simulations.

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I. INTRODUCTION

Light quarks trapped in a finite Euclidean volume V behave like electrons in small metallic grains [1]. The spontaneous breaking of chiral symmetry amounts to diffusing quarks in proper time with a vacuum diffusion constant $D \approx 0.22$ fm [1]. For times larger than the ergodic time $\tau_{\text{erg}} = \sqrt{V}/D$ or small virtualities, the quarks undergo ergodic motion, while for times shorter than the elastic time $\tau_d = 1/2m_Q \approx 0.33$ fm [1] or large virtualities their motion is ballistic. Here m_Q is the constituent quark mass in the vacuum. Diffusion sets in at intermediate times and away from the quantum regime with the Heisenberg time $t_H = 1/\Delta = \Sigma V/\pi$ [2], and Σ the quark condensate.



The concept of the quark return probability in the QCD vacuum as a chirally disordered medium [1], borrows on the concept of the electron return probability in disordered metallic systems as first introduced by Anderson [3] in the context of localization. For long times and in the presence of sufficient disorder, Anderson noted that the electron return probability is finite. A similar observation can be made for the quark return probability in the QCD vacuum [1].

In this paper we would like to show that at the critical point of a second order phase transition, a quantitative change in the character of the disorder takes place, with consequences on the long time behavior of the quark return probability. In section 2 and 3 we analyze the quark return probability in the vacuum and at the critical point.

In section 4 and 5, we contrast our results to the expected ones from a metal-insulator and a classical percolation transition. In section 6, we use semi-classical arguments to show what this means for the light quark spectrum. Our conclusions are in section 7.

II. QUARK RETURN PROBABILITY

The eigenvalue equation of the massless Dirac operator for quarks in the fundamental representation and in the fixed gluon field A

$$i\nabla[A] q_k = \lambda_k[A] q_k \quad (1)$$

allows us to extend the theory into 4+1 dimensions with proper time t , and define the probability $p(t)$ for a light quark to start at $x(0)$ in V and return back to the same position $x(t)$ after a duration t , as

$$p(t) = \frac{V^2}{N} \left\langle \left| \langle x(0) | e^{i(\nabla[A] + im)|t} | x(0) \rangle \right|^2 \right\rangle_A. \quad (2)$$

The operator $i\nabla[A]$ in (1) acts as a four-dimensional Hamiltonian for the evolution in proper time t . Indeed, the expectation value in (2) is that of the proper time evolution operator, with the background gluon field A acting as a t -independent (static) random potential. The averaging in (2) is over all gluon configurations using the unquenched QCD measure, with light quark flavors of current mass m (flavor symmetric case). The normalization in (2) is per state, where N is the mean number of quark states in the four-volume V .

In terms of the quark-eigenfunctions (1) in V , the return probability reads

$$p(t) = \frac{V^2}{N} e^{-2m|t|} \sum_{j,k} \left\langle e^{i|t|(\lambda_j - \lambda_k)[A]} |q_j(x)|^2 |q_k(x)|^2 \right\rangle_A \quad (3)$$

where the exponent $e^{-2m|t|}$ is solely due to the current quark mass in (2). We note that (3) is gauge-invariant and amenable to lattice QCD simulation. This form is best suited for numerical estimates.

For analytical considerations, it is best to rewrite (2) in terms of the standard Euclidean propagators for the quark field,

$$p(t) = \frac{V^2}{N} \lim_{y \rightarrow x} \int \frac{d\lambda_1 d\lambda_2}{(2\pi)^2} e^{-i(\lambda_1 - \lambda_2)|t|} \times \left\langle \text{Tr} (S(x, y; z_1) S^\dagger(x, y; z_2)) \right\rangle_A \quad (4)$$

with $z_{1,2} = m - i\lambda_{1,2}$, and

$$S(x, y; z) = \langle x | \frac{1}{i\nabla[A] + iz} | y \rangle. \quad (5)$$

Setting $\lambda_{1,2} = \Lambda \pm \lambda/2$ and neglecting the effects of Λ in the averaging in (4) (this is certainly true near the zero virtuality point), we find that in the flavor symmetric limit, the correlation function in (4) relates to the pion correlation function for a proper analytical continuation of the current quark mass [1]. Specifically,

$$p(t) = \frac{EV^2}{2\pi N} \lim_{y \rightarrow x} \int \frac{d\lambda}{2\pi} e^{-i\lambda|t|} \mathbf{C}_\pi(x, y; z) \quad (6)$$

where

$$\mathbf{1}^{ab} \mathbf{C}_\pi(x, y; z) = \left\langle \text{Tr} (S(x, y; z) i\gamma_5 \tau^a S(y, x; z) i\gamma_5 \tau^b) \right\rangle_A \quad (7)$$

with $z = m - i\lambda/2$ and $E = \int d\Lambda$. For $z = m$, pion-pole dominance (long paths) yields

$$\mathbf{C}_\pi(x, y; m) \approx \frac{1}{V} \sum_Q e^{iQ \cdot (x-y)} \frac{\Sigma^2}{F^2} \frac{1}{Q^2 + m_\pi^2} \quad (8)$$

with $Q_\mu = n_\mu 2\pi/L$ in $V = L^4$ and $\Sigma = |\langle \bar{q}q \rangle|$.

Using the GOR relation $F^2 m_\pi^2 = m\Sigma$, and the analytical continuation $m \rightarrow m - i\lambda/2$, we find [1]

$$\mathbf{C}_\pi(x, y; z) \approx \frac{1}{V} \sum_Q e^{iQ \cdot (x-y)} \frac{2\Sigma}{-i\lambda + 2m + DQ^2} \quad (9)$$

with the diffusion constant $D = 2F^2/\Sigma$. Inserting (9) into (6), and noting that $E/\Delta = N$ and $\rho = 1/\Delta V$, where Δ is the mean quantum level spacing with $\Sigma = \pi\rho$ (Banks-Casher relation), we conclude after a contour integration that

$$p(t) = e^{-2m|t|} \sum_Q e^{-DQ^2|t|}. \quad (10)$$

The validity of (10) is for $\tau_d < t < t_H$. For $m t_H \sim mV \ll 1$, (10) is dominated by the constant pionic mode, giving $p(t) \sim 1$. What happens to the present arguments if QCD is to undergo a phase transition?

III. SCALING AND UNIVERSALITY

During a second order transition, the quark condensate and the pion parameters undergo structural changes

which are governed by scaling arguments [4]. For $z = m$ (8) takes the general form

$$\mathbf{C}_\pi(x, y; m) \approx \frac{1}{V} \sum_Q e^{iQ \cdot (x-y)} \frac{Z_\pi}{Q^2 + Z_\pi/\chi_\pi} \quad (11)$$

where Z_π is the pion wavefunction renormalization, and χ_π the pion susceptibility. At the critical point T_C (generic of a second order transition), $V = L^3/T_C$. The representation (11) holds for long paths, or Euclidean momenta of order $\Lambda \leq (2\pi T_C)$. For larger Euclidean momenta or short paths, (11) is saturated by quarks in the dimensionally reduced theory [5].

At the critical point, we have [4]

$$\begin{aligned} Z_\pi &= \Sigma^{\nu\eta/\beta} \\ \chi_\pi &= \Sigma^{1-\delta} \\ \Sigma &= m^{1/\delta} \end{aligned} \quad (12)$$

where β, ν, δ, η are critical exponents. The prefactors in (12) are dimensionful constants that relate to the equation of state (the last equation). For convenience they were set to 1. Hence,

$$\mathbf{C}_\pi(x, y; m) \approx \frac{1}{V} \sum_Q e^{iQ \cdot (x-y)} \frac{m^A}{Q^2 + m^B} \quad (13)$$

where $A = B - 1 + 1/\delta$ and $B = 4/\delta(2 + \eta)$ after using scaling relations. For a mean-field transition there is no wave-function renormalization and $\eta = 0$. Therefore: $\delta = 3$, $A = 0$ and $B = 2/3$. After the analytical continuation $m \rightarrow m - i\lambda/2$, we obtain

$$\mathbf{C}_\pi(x, y; z) \approx \frac{1}{V} \sum_Q e^{iQ \cdot (x-y)} \frac{(m - i\lambda/2)^A}{Q^2 + (m - i\lambda/2)^B}. \quad (14)$$

As a result, the quark return probability becomes

$$p(t) = \mathbf{C} \sum_Q \int \frac{d\lambda}{(2\pi)^2} e^{-i\lambda|t|} \frac{(2m - i\lambda)^A}{2^B Q^2 + (2m - i\lambda)^B} \quad (15)$$

where $\mathbf{C} = VE/N2^{A-B}$.

In contrast to the vacuum case (9), the result (14) at the critical point displays new singularities. For fixed Q^2 , the pole in the λ -plane in (9) characteristic of the vacuum phase is now changed to a branch point at $\lambda = -2im$, and a set of poles ($Q \neq 0$)

$$\lambda_n = -2im + 2ie^{i(2n+1)\pi/B} |Q^2|^{1/B} \quad (16)$$

with $n = 0, 1, \dots, n_{\max}$, and n_{\max} is the number of unit roots to $1 + z^B = 0$. The contribution of the cut is

$$p_c(t) = -\frac{VE}{N\pi^2 2^{1/\delta+1}} e^{-2m|t|} \times \sum_Q \int_0^\infty dx e^{-x|t|} \text{Im} \left(\frac{x^A e^{i\pi A}}{2^B Q^2 + x^B e^{i\pi B}} \right) \quad (17)$$

while the contribution of the poles is

$$p_p(t) = -\frac{VE}{\pi BN} \sum'_{Q,n} e^{-i\lambda_n|t|} (m - i\lambda_n/2)^{1/\delta}. \quad (18)$$

The primed sum in (18) retains only those poles in the lower half of the λ -plane. The result is still real as the poles occur in $(\lambda, -\lambda^*)$ pairs.

For large times, the dominant contribution to $p(t)$ results from the $Q = 0$ part (zero mode) of (17). Setting the Heisenberg time to be $t_H = 1/\Delta_* \sim V^{\delta/(1+\delta)}$ [6], and the mean number of levels to be $N = E/\Delta_* \gg 1$, the quark return probability simplifies

$$p(t) \approx \frac{1}{2\pi^2} \sin(\pi/\delta) \Gamma(1/\delta) e^{-\alpha_*|t/t_H|} \left(\frac{t_H}{2t}\right)^{1/\delta}. \quad (19)$$

For $\alpha_* = 2mV^{\delta/(\delta+1)} \ll 1$ (ergodic regime), the return probability for large times t is universal at the critical point with $p(t) \sim (t_H/t)^{1/\delta}$. This behavior is to be contrasted with the vacuum result $p(t) \sim 1$ in the ergodic regime and $(\tau_{\text{erg}}/t)^2$ in the diffusive regime [1].

We note that (14) obeys an ‘anomalous’ diffusion equation in $d=4$ at $m = 0$ (with the cutoff Λ) [7],

$$[-D(\lambda)\nabla^2 + (-i\lambda)] C_\pi(x, y; -i\lambda) = 0 \quad (20)$$

for $x \neq y$. At the critical point the diffusion constant is λ dependent and complex $D(\lambda) = 2^B(-i\lambda)^{1-B}$. For $\lambda \sim m \sim 0$ (zero virtuality) the diffusion constant vanishes as $|D(m)| \sim m^{1-B}$. In this case, the Heisenberg time is $t_H = 1/\Delta_* \sim V^{\delta/(\delta+1)}$, and the ergodic time is $\tau_{\text{erg}} = 1/E_c \sim \sqrt{V}/m^{1-B}$, where E_c is the Thouless energy [8]. Since the constituent quark mass (half the sigma mass) becomes comparable to the pion mass at the critical point, the elastic time is $\tau_d \sim 1/m^{B/2}$ (see (13)). To assess the hierarchy of scales near the critical point, we enhance artificially the contribution from the ergodic regime by setting $m/\Delta_* \sim 1$ in power counting, in generalization of a previous argument [9]. For a mean-field transition: $\tau_d \sim V^{1/4}$ and $\tau_{\text{erg}} \sim t_H \sim V^{3/4}$. Hence the Ohmic conductance $\sigma_L = t_H/\tau_{\text{erg}} \sim 1$ a situation reminiscent of the metal-insulator transition, except that in the present case the density of states vanishes as $\lambda^{1/\delta}$. The time scales are still ordered in the thermodynamical limit, with the diffusive regime stretching all the way to the quantum regime.

We note that the same scaling arguments we have used at $T = T_c$ also indicates that near but above the critical temperature $T > T_c$, the pion mass vanishes in the symmetric phase in general with $m_\pi^2 \sim \Sigma^{\nu/\beta}$ as $\Sigma \sim m$ [4], owing to wave-function renormalization [10]. Some of our previous arguments may apply provided that due care is paid to the pion dispersion relation with $Q^2 \rightarrow Q^{2-\eta}$ [4]. Similar behavior has been noted at the mobility edge of a metal-insulator transition in metallic grains [11].

IV. METAL-INSULATOR TRANSITION

How does the present chiral phase transition compare to the metal-insulator (MI) transition in QCD in vacuum we discussed recently in $d=4$ [1]? For comparison, we note that in the latter and for long times the quark return probability scales as [1,12]

$$p(t) \approx \frac{e^{-2m|t|}}{|t|^{1-\eta/4}} \quad (21)$$

with a multifractal exponent [1]

$$\frac{\eta}{4} = 2\chi = 4 \frac{1 - 0.577 + \ln 4}{\beta(2\pi)^4} \frac{1}{\sigma_*}. \quad (22)$$

In QCD $\beta = 2$ and the critical conductance σ_* is equal to the microscopic conductance $\sigma_l = 2F^2 l^2/\pi$ for one mean-free path l , that is [1,13]

$$\sigma_l = \frac{8}{\pi} \frac{F^4}{\Sigma m_Q} \approx 0.041 \quad (23)$$

where $F \approx 93$ MeV is the pion decay constant, $\Sigma \approx (250)$ (MeV)³ is the vacuum quark condensate and $m_Q \approx 300$ MeV a typical constituent quark mass. Hence $\eta/4 \approx 0.057$, which yields a larger critical exponent than in (19). We note that our prediction for the critical exponent in QCD is in qualitative agreement with the numerical estimates $\eta/4 \approx 0.08 - 0.16$ obtained subsequently using the instanton liquid model [15]. Remarkably, the smallness of $\eta/4$ makes the result (21) close to a 2-dimensional diffusion process.

In the MI transition, (23) follows from the ballistic region and is related to a careful consideration of the null spectral sum rule [1,11,16]. At the critical point this is necessary because of the multifractal character of the corresponding wavefunctions at the mobility edge [12]. In the metallic regime, the quark wavefunctions are spread through the metal, while in the insulator regime they are localized. At the edge, the wavefunctions are ‘filamentary’ with self-similar structure [17]. In a second order transition we expect the quark wavefunctions to be persistently ‘metallic’.

V. PERCOLATION TRANSITION

Multifractal wavefunctions have been deemed as ‘quantum’ percolation by Aharony [18]. Could a ‘classical’ percolation transition take place at finite temperature in QCD? Recently, some arguments have been put forward by Satz [19]. In the present framework this can be addressed by noting that (11) valid at T_C by standard scaling arguments [4] does not incorporate the fact that for temperatures $0 < T < T_C$ the Goldstone modes disperse asymmetrically in matter [20,21]. Indeed, in these range

of temperatures and for space-like momenta we have instead of (9),

$$\begin{aligned} \mathbf{C}_\pi(x, y, z) &\approx \frac{1}{V} \sum_Q e^{iQ \cdot (x-y)} \\ &\times \frac{2\Sigma_T}{D_4 Q_4^2 + D_S \vec{Q}^2 + 2m - i\lambda} \end{aligned} \quad (24)$$

with $\Sigma_T = |\langle \bar{q}q \rangle_T|$ the temperature dependent quark condensate. Here, the ‘temporal’ D_4 and ‘spatial’ D_S diffusion constants are related to the ‘temporal’ F_4 and ‘spatial’ F_S weak decay constants of the pion [20]. In the space-like regime they are both real, and we have generically

$$\begin{aligned} D_4 &= 2F_4^2/\Sigma_T, \\ D_S &= 2F_4 F_S/\Sigma_T. \end{aligned} \quad (25)$$

If we were to denote by m_4^2 the squared pion ‘time-like’ mass, and by m_S^2 the squared pion ‘space-like’ mass, then $D_4 = 2m/m_4^2$ and $D_S = 2m/m_S^2$ where $m_S \geq m_4$ by causality [20]. Hence $D_4 \geq D_S$.

In terms of (24) the quark return probability in the range $0 < T < T_C$ reads

$$p(t) = e^{-2m|t|} \sum_Q e^{-D_4 Q_4^2 |t| - D_S \vec{Q}^2 |t|} \quad (26)$$

where we have used the facts that $\Sigma_T = \pi\rho_T$, $\Delta_T = 1/\rho_T V$ and $N = E/\Delta_T$. At finite temperature both the density of states ρ_T and the level spacing Δ_T change in a 4-volume $V = L^3/T$. The momenta in (26) are $Q_4 = n_4 2\pi T$ and $\vec{Q} = \vec{n} 2\pi/L$. (26) implies the existence of two Thouless energies $E_4 = D_4 T^2$ and $E_S = D_S/L^2$, hence two ergodic times $\tau_{4,\text{erg}} = 1/E_4$ and $\tau_{S,\text{erg}} = 1/E_S$, with $\tau_{4,\text{erg}} < \tau_{S,\text{erg}}$. The corresponding Ohmic conductances are $\sigma_4 = E_4/\Delta_T$ and $\sigma_S = E_S/\Delta_T$. These scales allow a simple organization of the disorder in an asymmetric (finite temperature) Euclidean box. In particular, (26) becomes universal for $mV \ll 1$ and times $t \gg \tau_{S,\text{erg}} > \tau_{4,\text{erg}}$. Most of the vacuum arguments presented in [1] can be extended to the present finite temperature phase.

If we were to denote the ‘temporal’ ϱ_4 and ‘spatial’ ϱ_S quark conductivities, then by the Kubo formulae: $\varrho = D\rho_T$, we have $\varrho_4 \geq \varrho_S$. For temperatures $0 < T < T_C$ the quark conductivity in the ‘spatial’ directions is weaker than the quark conductivity in the ‘temporal’ direction. This is easily understood by the fact that ‘spatially’ the system ‘screens’ all charges including the singlet ones. Indeed, the pion screening length at high temperature asymptotes $m_S^2 \sim 2\pi T$ by standard arguments [5], so that the spatial conductivity $D_S \sim 2m/(2\pi T)^2$ is parametrically small in the high temperature phase.

The asymmetry in the conduction properties may cause an ‘asymmetric’ percolation from d=4 to d=1 as D_S becomes much smaller than D_4 , a situation reminiscent of finite density [22]. From (26), it follows that

$$p(t, T) \approx \frac{e^{-2m|t|}}{\sqrt{4\pi E_4 |t|}} \quad (27)$$

in the limit $D_4 \gg D_S$. We note with Aharony [18] that in a ‘classical’ percolation transition in either d=3 or d=4, the quark return probability on an infinite critical cluster is expected to scale as

$$p(t) \approx \frac{e^{-2m|t|}}{|t|^{\tilde{D}/(2+\theta)}} \quad (28)$$

where $\tilde{D} = 2.5, 3.2$ is the fractal dimension and $\theta = 1.77, 2.70$ in d=3 and d=4 respectively. Hence, $\tilde{D}/(2+\theta) = 0.66, 0.68$ for d=3,4 respectively, which are similar to 0.5 in (27). Both exponents are markedly different from the MI behavior (21).

In a linear sigma model the diffusion constants D_4 and D_S can be estimated in a weak-coupling (1/n) expansion [20]. The results in our case and to 1-loop are

$$\begin{aligned} D_4 &\sim D \left(1 - \frac{T^2}{24F^2} + \frac{6\pi^2}{45} \frac{T^4}{F^2 m_\sigma^2} \right), \\ D_S &\sim D \left(1 - \frac{T^2}{24F^2} - \frac{2\pi^2}{45} \frac{T^4}{F^2 m_\sigma^2} \right) \end{aligned} \quad (29)$$

where $m_\sigma \sim 500$ MeV is a typical sigma mass. To the same-order we have $\Sigma_T \sim \Sigma(1 - T^2/8F^2)$. Similar calculations can be performed in the context of the non-linear sigma model as well using instead 2-loops. A fine tuning of the running logarithms in this case [21] should bring both estimations into agreement. From (29) it follows that an order of magnitude drop in the relative conductivities $\varrho_S/\varrho_4 \sim 0.1$ or equivalently

$$\frac{D_S}{D_4} \sim 1 - \frac{8\pi^2}{45} \frac{T^4}{F^2 m_\sigma^2} \sim 0.1 \quad (30)$$

takes place for $T \sim 180$ MeV, which is where we expect the percolation transition to possibly set in. At this temperature only 50% of the chiral condensate or the density of quark states at zero virtuality has depleted by the 1-loop argument. It would be interesting to investigate the effects of strangeness on the present arguments.

VI. SEMI-CLASSICAL APPROXIMATION

What does the result (19) mean for the light quark spectrum in QCD? A qualitative answer to this question can be obtained using semi-classical arguments [23,24]. Since in our case the density of states $\rho(\lambda) \sim \lambda^{1/\delta}$ changes in the energy band of interest, the standard semi-classical arguments have to be modified. Indeed, a rerun of the standard arguments [23,24] yields for the quark return probability

$$p(t) = \frac{V^2}{N} \left\langle \sum_j |A_j(x)|^2 \delta(t - T_j) \right\rangle_A \quad (31)$$

and the spectral form-factor

$$K(t) = \frac{1}{2\pi^2} \left\langle \sum_j \left| \int dx A_j(x) \right|^2 \delta(t - T_j) \right\rangle_A \quad (32)$$

where the sum is over closed classical paths of reduced action $S_j(\Lambda)$, virtuality Λ , and period $T_j = dS_j/d\Lambda$. $A_j(x)$ is the Gaussian contribution around the classical paths to the propagator $S(x, x; m - i(\Lambda \pm \lambda/2))$. The averaging in (32) is over the gauge-configurations (static or t-independent disorder).

The contributions to (31) result from closed classical paths labeled by j , starting at x and returning back to x with probability $P_j = |A_j(x)|^2$. The contributions to (32) differ from those of (31) by the overlap integration over the ‘4-volume’ of a given classical path [23,24]. For a virtuality $\lambda_j \sim 1/T_j$ about the mean $\Lambda \sim 0$, the ‘4-volume’ is of order $T_j/\rho(1/T_j)$, as $\rho(\lambda)$ counts the number of states per ‘4-volume’ per virtuality. Inserting this result into (32) and using (31), we conclude that

$$K(t) \approx \mathbf{K} \frac{\Delta_*^2 |t|^{1+1/\delta}}{(2\pi)^2 \beta} p(t) \quad (33)$$

The dimensionless coefficient \mathbf{K} is not fixed by these estimates. For QCD $\beta = 2$ [1], (33) in combination with (19) yields $K(t) \sim |t|$ for large times ($m = 0$). This result at $T = T_c$ is reminiscent of the result at $T = 0$ and suggests once more that it is universal. Hence, the corresponding spectral rigidity is still logarithmic but with a coefficient that is not fixed by the present semi-classical arguments. It can be fixed using our recent arguments on critical scaling in 0-dimension [6,25].

VII. CONCLUSIONS

We have shown that the long time behavior of the quark return probability in a QCD phase transition carries a quantitative information on the character of a phase change, that could be used to discriminate between a second order, a metal-insulator or a percolation transition. For times comparable to the Heisenberg time $t_H \sim 1/\Delta_*$ and in the narrowing ergodic regime $m \ll \Delta_*$, we have found that the quark return probability at the critical point scales as $|t|^{-1/\delta}$ for a second-order transition. In contrast, in a metal-insulator transition it is of order $|t|^{-0.943}$ while for an asymmetric percolation transition it is of order $|t|^{-1/2}$ from $d=4$ to $d=1$. Our results can be tested by numerically assessing (3) using current lattice QCD algorithms.

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