Probing the Nucleon's Transversity Via Two-Meson Production in Polarized Nucleon-Nucleon Collisions^{*}

Jian Tang

Center for Theoretical Physics Laboratory for Nuclear Science and Department of Physics Massachusetts Institute of Technology Cambridge, Massachusetts 02139

(MIT-CTP-2769

hep-ph/xxxxxxx

xx Submitted to Physical Review D

Abstract

We explore the possibility of probing the nucleon's transversity distribution $\delta q(x)$ through the final state interaction between two mesons $(\pi^+\pi^-, \pi K, \text{ or } K\overline{K})$ produced in transversely polarized nucleon-nucleon collisions. We present a single spin asymmetry and estimate its magnitude under some assumptions for the transversity distribution function and the unknown interference fragmentation function.

Typeset using REVT_EX

July 1998)

^{*}This work is supported in part by funds provided by the U.S. Department of Energy (D.O.E.) under cooperative research agreement #DF-FC02-94ER40818.

The quark transversity distribution in the nucleon $\delta q(x)$ measures the probability difference to find a quark polarized along versus opposite to the polarization of a nucleon polarized transversely to its direction of motion [1,2,3,4]. Along with unpolarized and longitudinally polarized quark distributions, it completely characterizes the state of quarks in the nucleon at leading twist in high-energy processes. While the other two have been studied extensively in the past through various high-energy experiments, very little is known about the transversity distribution $\delta q(x)$ since it decouples from hard QCD processes at the leading twist due to its chiral-odd property. For example, it is suppressed like $\mathcal{O}(m_q/Q)$ in totally-inclusive deep inelastic scattering (DIS).

In our recent works [5], we have studied the semi-inclusive production of two mesons $(e.g. \ \pi^+\pi^-, \pi K, \text{ or } K\overline{K})$ in the current fragmentation region in DIS on a transversly polarized nucleon. We have shown that the interference effect between the s- and p-wave of the two-meson system around the ρ (for pions), K^* (for πK), or ϕ (for kaons) provides a single spin asymmetry which may be sensitive to the quark transversity distribution in the nucleon. Such interference allows the quark's polarization information to be carried through the quantity $\vec{k}_+ \times \vec{k}_- \cdot \vec{S}_\perp$, where \vec{k}_+, \vec{k}_- , and \vec{S}_\perp are the three-momenta of π^+ (K), π^- (\overline{K}), and the nucleon's transverse spin, respectively. This effect appears at the leading twist level, and the production rates for pions and kaons are large in DIS. However, it would vanish by T-invariance in the absence of final state interactions, or by C-invariance if the two-meson mass dominated by a single resonance. However, both suppressions are evaded in the ρ ($\pi^+\pi^-$), K^* (πK), and ϕ ($K\overline{K}$) mass regions where both s- and p-wave production channels are active.

In this paper, we extend our study to discuss the possibility of probing the quark transversity distribution in the nucleon via two-meson semi-inclusive production in transversely polarized nucleon-nucleon collisions. Various processes in transversely polarized nucleonnucleon collisions have been suggested to measure the nucleon's transversity distribution since it was first introduced about two decades ago [1], among which are transversely polarized Drell-Yan [1] and two-jet production [6,7]. However, the Drell-Yan cross section is small and requires an antiquark transversity distribution, which is likely to be quite small. The asymmetry obtained in two-jet production is rather small due to the lack of a gluon contribution [6,7]. On the other hand, in the process described here, the gluon-quark scattering dominates and only one beam need be transversely polarized. Unless the novel interference fragmentation function is anomalously small, this will provide a feasible way to probe the nucleon's transversity distribution. The results of our analysis are summarized by Eq. (8) where we present the asymmetry for $\pi^+\pi^-(\pi K, K\overline{K})$ production in transversely polarized nucleon-nucleon collisions.

We consider the semi-inclusive nucleon-nucleon collision process with two-pion final states being detected: $N\vec{N}_{\perp} \rightarrow \pi^{+}\pi^{-}X$. (The analysis to follow applies as well to πK or $K\overline{K}$ production.) One of the nucleon beams is transversely polarized with polarization vector S_{μ} , and momentum P_{μ}^{A} . The other is unpolarized, with momentum denoted by P_{μ}^{B} . The experimentally observable invariant variables are defined as $s \equiv (P_{A} + P_{B})^{2}$, $t \equiv (P_{h} - P_{A})^{2}$, $u \equiv (P_{h} - P_{B})^{2}$, and the invariants for the underlying partonic processes are $\hat{s} \equiv$ $(p_{a} + p_{b})^{2}$, $\hat{t} \equiv (p_{c} - p_{a})^{2}$, $\hat{u} \equiv (p_{c} - p_{b})^{2}$, where P_{h} is the total momentum of the twopion system, p_{a} , p_{b} , p_{c} , p_{d} are the momenta for the underlying partonic scattering processes (see Fig. 1). The longitudinal momentum fractions x_a , x_b and z are given by $p_a = x_a P_A$, $p_b = x_b P_B$ and $P_h = z p_c$. The $\sigma[(\pi \pi)_{l=0}^{I=0}]$ and $\rho[(\pi \pi)_{l=1}^{I=1}]$ resonances are produced with momentum P_h . We recognize that the $\pi \pi$ s-wave is not resonant in the vicinity of the ρ and our analysis does not depend on a resonance approximation. For simplicity we refer to the non-resonant s-wave as the " σ ".

The invariant squared mass of the two-pion system is $m^2 = (k_+ + k_-)^2$, with k_+ and k_- the momentum of π^+ and π^- , respectively. The decay polar angle in the rest frame of the two-meson system is denoted by Θ , and the azimuthal angle ϕ is defined as the angle of the normal of two-pion plane with respect to the polarization vector \vec{S}_{\perp} of the nucleon, $\cos \phi = \vec{k}_+ \times \vec{k}_- \cdot \vec{S}_{\perp}/|\vec{k}_+ \times \vec{k}_-||\vec{S}_{\perp}|$. This is the analog of the "Collins angle" defined by the $\pi^+\pi^-$ system [8].

Since we are only interested in a result at the leading twist, we follow the helicity density matrix formalism developed in Refs. [9,10], in which all spin dependence is summarized in a *double* helicity density matrix. We factor the process at hand into basic ingredients (See Fig. 1): the $N \to q$ (or $N \to g$) distribution function, the hard partonic $q_a q_b \to q_c q_d$ cross section, the $q \to (\sigma, \rho)$ fragmentation, and the decay $(\sigma, \rho) \to \pi^+\pi^-$, all as density matrices in helicity basis:

$$\left[\frac{d^7\sigma(N\vec{N}_{\perp}\to\pi^+\pi^-X)}{dx_a\,dx_b\,d\hat{t}\,dz\,dm^2\,d\cos\Theta\,d\phi}\right]_{H'H}$$

$$= \left[\mathcal{F}(x_a) \otimes \frac{d^3 \sigma(q_a q_b \to q_c q_d)}{dx_a \, dx_b \, d\hat{t}} \otimes \frac{d^2 \hat{\mathcal{M}}}{dz \, dm^2} \otimes \frac{d^2 \mathcal{D}}{d \cos \Theta \, d\phi} \otimes \mathcal{F}(x_b) \right]_{H'H} \tag{1}$$

where H(H') are indices labeling the helicity states of the polarized nucleon. In order to incorporate the final state interaction, we have separated the $q \to \pi^+\pi^-$ fragmentation process into two steps. First, the quark fragments into the resonance (σ , ρ), then the resonance decays into two pions, as shown in the middle of the Fig. 1.

The s - p interference fragmentation functions describe the emission of a $\rho(\sigma)$ from a parton, followed by absorption of $\sigma(\rho)$ forming a parton. Imposing various symmetry (helicity, parity and time-reversal) restrictions, the interference fragmentation can be cast into a double density matrix notation [5]

$$\frac{d^2 \hat{\mathcal{M}}}{dz \, dm^2} = \Delta_0(m^2) \left\{ I \otimes \bar{\eta}_0 \, \hat{q}_I(z) + (\sigma_+ \otimes \bar{\eta}_- + \sigma_- \otimes \bar{\eta}_+) \, \delta \hat{q}_I(z) \right\} \Delta_1^*(m^2)
+ \Delta_1(m^2) \left\{ I \otimes \eta_0 \, \hat{q}_I(z) + (\sigma_- \otimes \eta_+ + \sigma_+ \otimes \eta_-) \, \delta \hat{q}_I(z) \right\} \Delta_0^*(m^2) ,$$
(2)

where $\sigma_{\pm} \equiv (\sigma_1 \pm i\sigma_2)/2$ with $\{\sigma_i\}$ the usual Pauli matrices. The η 's are 4×4 matrices in (σ, ρ) helicity space with nonzero elements only in the first column, and the $\bar{\eta}$'s are the transpose matrices $(\bar{\eta}_0 = \eta_0^T, \bar{\eta}_+ = \eta_-^T, \bar{\eta}_- = \eta_+^T)$, with the first rows (0, 0, 1, 0), (0, 0, 0, 1), and (0, 1, 0, 0) for $\bar{\eta}_0, \bar{\eta}_+$, and $\bar{\eta}_-$, respectively. The explicit definition of the fragmentation functions will be given in Ref. [11].

The final state interactions between the two pions are included explicitly in

$$\Delta_0(m^2) = -i\sin\delta_0 e^{i\delta_0} , \qquad \Delta_1(m^2) = -i\sin\delta_1 e^{i\delta_1} , \qquad (3)$$

where δ_0 and δ_1 are the strong interaction $\pi\pi$ phase shifts which can be determined by the $\pi\pi \mathcal{T}$ -matrix [12]. Here we have suppressed the m^2 dependence of the phase shifts for simplicity.

The decay process, $(\sigma, \rho) \to \pi^+ \pi^-$, can be easily calculated and encoded into the helicity matrix formalism. The result for the interference part is [5]

$$\frac{d^2 \mathcal{D}}{d\cos\Theta \, d\phi} = \frac{\sqrt{6}}{8\pi^2 m} \,\sin\Theta \left[i e^{-i\phi} \left(\eta_- - \bar{\eta}_- \right) + i e^{i\phi} \left(\eta_+ - \bar{\eta}_+ \right) - \sqrt{2} \cot\Theta \left(\bar{\eta}_0 + \eta_0 \right) \right] \,. \tag{4}$$

Here we have adopted the customary conventions for the ρ polarization vectors, $\vec{\epsilon}_{\pm} = \mp (\hat{e}_1 \pm i\hat{e}_2)/\sqrt{2}$ and $\vec{\epsilon}_0 = \hat{e}_3$ in its rest frame with \hat{e}_j 's the unit vectors.

In the double density matrix notation, the quark distribution function $\mathcal{F}_q(x)$ in the nucleon can be expressed as [10]

$$\mathcal{F}_q(x) = \frac{1}{2}q(x) \ I \otimes I + \frac{1}{2}\Delta q(x) \ \sigma_3 \otimes \sigma_3 + \frac{1}{2}\delta q(x) \ (\sigma_+ \otimes \sigma_- + \sigma_- \otimes \sigma_+) \ , \tag{5}$$

where the first matrix in the direct product is in the nucleon helicity space and the second in the quark helicity space. Here q(x), $\Delta q(x)$, and $\delta q(x)$ are the spin average, helicity difference, and transversity distribution functions, respectively, and their dependences on Q^2 have been suppressed.

The gluon distribution function $\mathcal{F}_{g}(x)$ in the nucleon can be written as

$$\mathcal{F}_g(x) = \frac{1}{3}G(x) \ I \otimes I_g + \frac{1}{3}\Delta G(x) \ \sigma_3 \otimes S_g^3 \ , \tag{6}$$

where I_g and S_g^3 are 3×3 matrices in gluon helicity space with nonzero elements only on the diagonal: diag $(I_g) = \{1, 1, 1\}$ and diag $(S_g^3) = \{1, 0, -1\}$. Here G(x) and $\Delta G(x)$ are the spin average and helicity difference gluon distributions in the nucleon, respectively, and, just like in Eq. (5), their dependences on Q^2 have been suppressed. Note that there is no gluon transversity distribution $\delta G(x)$ in the nucleon at the leading twist due to helicity conservation. This is one of the reasons why transverse asymmetries in two-jet production are typically small, as pointed out by Ji [6], Jaffe and Saito [7].

Several hard partonic processes contribute here, as shown in the middle of Fig. 1. The cross sections can be written as follows (here we list only the relevant parts, i.e. spin-average and transversity-dependent ones),

$$\frac{d^3\sigma(q_a q_b \to q_c q_d)}{dx_a dx_b d\hat{t}} = \frac{\pi \alpha_s^2}{2\hat{s}^2} I_b \otimes \left[\hat{\bar{\sigma}}_{ab}^{cd} I_a \otimes I_c + 4\delta\hat{\sigma}_{ab}^{cd} \left(\sigma_a^+ \otimes \sigma_c^- + \sigma_c^- \otimes \sigma_a^+\right)\right] , \qquad (7)$$

where subscripts a, b, c means that the helicity matrices above are in a, b, c parton helicity spaces, respectively (See Fig. 1). $\hat{\sigma}_{ab}^{cd}$ and $\delta \hat{\sigma}_{ab}^{cd}$ are the spin-average and transversitydependent cross sections for the underlying partonic processes $q_a q_b \rightarrow q_c q_d$, respectively, which are shown in the Table I.

Combining all the above ingredients together, and integrating over Θ to eliminate the \hat{q}_I dependence, we obtain a single spin asymmetry as follows,

$$\mathcal{A}_{\perp\top} \equiv \frac{d\sigma_{\perp} - d\sigma_{\top}}{d\sigma_{\perp} + d\sigma_{\top}} = -\frac{\sqrt{6}\pi}{4} \sin \delta_0 \sin \delta_1 \sin(\delta_0 - \delta_1) \cos \phi$$
$$\otimes \frac{[\delta q(x_a) \otimes G(x_b) \otimes \delta \hat{q}_I(z)] \delta \hat{\sigma}_{qg}^{qg} + [\delta \bar{q}(x_a) \otimes G(x_b) \otimes \delta \hat{\bar{q}}_I(z)] \delta \hat{\sigma}_{\bar{q}g}^{\bar{q}g} + \dots}{\{[G(x_a) \otimes G(x_b)] \hat{\sigma}_{gg}^{q\bar{q}} + [q(x_a) \otimes G(x_b)] \hat{\sigma}_{qg}^{qg} + \dots\} \otimes [\sin^2 \delta_0 \hat{q}_0(z) + \sin^2 \delta_1 \hat{q}_1(z)]}$$
(8)

where $\hat{q}_0(z)$ and $\hat{q}_1(z)$ are spin-average fragmentation functions for the σ and ρ resonances, respectively, and the summation over flavor is suppressed for simplicity. The terms denoted by ... include quark quark and quark antiquark scattering contributions. This asymmetry can be measured either by flipping the target transverse spin or by binning events according to the sign of the crucial azimuthal angle ϕ (See Fig. 2). The "figure of merit" for this asymmetry, $\sin \delta_0 \sin \delta_1 \sin(\delta_0 - \delta_1)$, is shown in Fig. 3.

The flavor content of the asymmetry $\mathcal{A}_{\perp \top}$ can be revealed by using isospin symmetry and charge conjugation restrictions. For $\pi^+\pi^-$ production, isospin symmetry gives $\delta \hat{u}_I = -\delta \hat{d}_I$ and $\delta \hat{s}_I = 0$. Charge conjugation implies $\delta \hat{q}_I^a = -\delta \hat{q}_I^a$. Thus there is only one independent interference fragmentation function for $\pi^+\pi^-$ production, and it may be factored out of the asymmetry, e.g. $\sum_a \delta q_a \delta \hat{q}_I^a = [(\delta u - \delta \bar{u}) - (\delta d - \delta \bar{d})]\delta \hat{u}_I$. Similar application of isospin symmetry and charge conjugation to the ρ and σ fragmentation functions that appear in the denominator of Eq. (8) leads to a reduction in the number of independent functions: $\hat{u}_i = \hat{d}_i = \hat{\bar{u}}_i = \hat{\bar{d}}_i$ and $\hat{s}_i = \hat{\bar{s}}_i$ for $i = \{0, 1\}$. For other systems the situation is more complicated due to the relaxation of the Bose symmetry restriction. For example, for the $K\overline{K}$ system, $\delta \hat{q}_I^a = -\delta \hat{\bar{q}}_I^a$ still holds, but $\delta \hat{u}_I$, $\delta \hat{d}_I$, and $\delta \hat{s}_I$, are in general independent. We also note that application of the Schwartz inequality puts an upper bound on the interference fragmentation function, $\delta \hat{q}_I^2 \leq 4\hat{q}_0\hat{q}_1/3$ for each flavor.

The size of the asymmetry $\mathcal{A}_{\perp \top}$ critically depends upon the ratio of the s - p interference fragmentation function and ρ and σ fragmentation functions, which is unknown at present. In order to estimate the magnitude of $\mathcal{A}_{\perp \top}$, we saturate the Schwartz inequality and replace the interference fragmentation with its upper bound, i.e. $\delta \hat{q}_I^2 = 4\hat{q}_0\hat{q}_1/3$ for each flavor. Meanwhile, we assume the σ and ρ fragmentation functions are equal to each other. Thus, the fragmentation function dependences cancel out in $\mathcal{A}_{\perp \top}$. We also assume that the transversity $\delta q(x, Q^2)$ saturates the Soffer inequality [13]: $2|\delta q(x, Q^2)| =$ $q(x, Q^2) + \Delta q(x, Q^2)$. We use the polarized structure functions obtained by Gehrmann and Stirling through next-to-leading order analysis of experimental data [14]. We also go to the region m = 0.83GeV, around which the phase factor $|\sin \delta_0 \sin \delta_1 \sin(\delta_0 - \delta_1)|$ is large (See Fig. 3), and let $\cos \phi = 1$. The asymmetry as function of p_T^{jet} at $\sqrt{s} = 500$ GeV and $\sqrt{s} = 200$ GeV for pseudo-rapidity $\eta = 0.0$ and $\eta = 0.35$ is shown in Fig. 4, where p_T^{jet} is the transverse momentum of the jet. The size of asymmetry is about 12 - 15% at $p_T^{\text{jet}} = 120$ GeV for $\sqrt{s} = 500$ GeV and about 17 - 20% at $p_T^{\text{jet}} = 90$ GeV for $\sqrt{s} = 200$ GeV, which would be measurable at RHIC.

A few comments can be made about our numerical results. Firstly, under the above approximations, the asymmetry is independent of z. Of course, the experiment may not be able to determine p_T^{jet} — the transverse momentum of the jet, so direct comparison between our asymmetry and experimental data will require event simulation. Secondly, because we don't know the sign of the unknown quantities yet, we can not determine the sign of the asymmetry shown in Fig. 4 should only be taken as its magnitude. Finally, in order to estimate the asymmetry, we have made very optimistic assumptions about the novel interference fragmentation functions and transversity distribution functions, so our estimates here should be regarded as "the high side".

To summarize, we have studied the possibility of probing the quark transversity distribution in the nucleon via two-meson semi-inclusive production of (only one beam) transversely polarized nucleon-nucleon collisions. We obtained a single spin asymmetry that is sensitive to the quark transversity and estimated its magnitude.

I would like to thank Bob Jaffe for encouraging and enlightening conversations relating to this subject. I would also like to thank Xuemin Jin for helpful discussions. I am also grateful to N. Saito and M. Baker for informations related to RHIC.

REFERENCES

- [1] J. Ralston and D. E. Soper, Nucl. Phys. **B152**, 109 (1979).
- [2] R. L. Jaffe and X. Ji, Phys. Rev. Lett, 67, 552 (1991).
- [3] X. Artru and M. Mekhfi, Z. Phys. C 45, 669 (1990).
- [4] J. L. Cortes, B. Pire, J. P. Ralston, Z. Phys. C 55, 409 (1992).
- [5] R.L. Jaffe, Xuemin Jin, and Jian Tang, Phys. Rev. Lett. 80, 1166(1998); R.L. Jaffe, Xuemin Jin, and Jian Tang, Phys. Rev. D57, 5920(1998).
- [6] X. Ji, Phys. Lett. **B284**, 137(1992).
- [7] R. L. Jaffe and N. Saito, Phys. Lett. **B382**, 165(1996).
- [8] J. C. Collins, Nucl. Phys. B396, 161 (1993); J. C. Collins, S. F. Heppelmann, and G. A. Ladinsky, Nucl. Phys. B420, 565 (1994).
- [9] K. Chen, G. R. Goldstein, R.L. Jaffe, X. Ji, Nucl. Phys. **B445**, 380 (1995).
- [10] R. L. Jaffe, Phys. Rev. D 54, 6581 (1996).
- [11] R. L. Jaffe, X. Jin, and J. Tang, in preparation.
- [12] M. L. Goldberger and K. M. Watson, *Collision Theory* (J. Wiley&Sons, New York, 1964).
- [13] J. Soffer, Phys. Rev. Lett. **74**, 1292(1995).
- [14] T. Gehrmann, W.J. Stirling, Phys.Rev.**D53**, 6100(1996).
- [15] P. Estabrooks and A. D. Martin, Nucl. Phys. **B79**, 301 (1974).



FIG. 1. Hard scattering diagram for two-meson semi-inclusive production in nucleon-nucleon collision.



FIG. 2. Illustration of the pp collision at the center-of-mass frame and the so-called "Collins angle" $\phi.$



FIG. 3. The factor, $\sin \delta_0 \sin \delta_1 \sin(\delta_0 - \delta_1)$, as a function of the invariant mass m of two-pion system. The data on $\pi\pi$ phase shifts are taken from Ref. [15].



FIG. 4. The single spin symmetry as function of p_T^{jet} for two-pion production in pp collision at $\sqrt{s} = 500 \text{ GeV}(\text{solid})$ and $\sqrt{s} = 200 \text{ GeV}(\text{dashes})$ (pseudo-rapidity $\eta = 0.0$ and $\eta = 0.35$).

Partonic process	Spin Average	Transversity Dependent
$ab \rightarrow cd$	Cross Section $-\hat{\sigma}^{cd}_{ab}$	Cross Section $-\delta \hat{\sigma}^{cd}_{ab}$
qg ightarrow qg	$rac{\hat{s}^2+\hat{u}^2}{\hat{t}^2}-rac{4}{9}rac{\hat{s}^2+\hat{u}^2}{\hat{s}\hat{u}}$	$rac{\hat{s}\hat{u}}{\hat{t}^2}-rac{4}{9}$
$\bar{q}g ightarrow \bar{q}g$	$rac{\hat{s}^2 + \hat{u}^2}{\hat{t}^2} - rac{4}{9}rac{\hat{s}^2 + \hat{u}^2}{\hat{s}\hat{u}}$	$rac{\hat{s}\hat{u}}{\hat{t}^2}-rac{4}{9}$
qq ightarrow qq	$\frac{4}{9} \left(\frac{\hat{s}^2 + \hat{u}^2}{\hat{t}^2} + \frac{\hat{s}^2 + \hat{t}^2}{\hat{u}^2} \right) - \frac{8}{27} \frac{\hat{s}^2}{\hat{u}\hat{t}}$	$rac{4}{27}rac{\hat{s}}{\hat{t}} - rac{4}{9}rac{\hat{s}\hat{u}}{\hat{t}^2}$
$qq' \rightarrow qq'$	$rac{4}{9}rac{\hat{s}^2+\hat{u}^2}{\hat{t}^2}$	$-rac{4}{9}rac{\hat{s}\hat{u}}{\hat{t}^2}$
$q\bar{q} \to q\bar{q}$	$\frac{4}{9} \left(\frac{\hat{s}^2 + \hat{u}^2}{\hat{t}^2} + \frac{\hat{u}^2 + \hat{t}^2}{\hat{s}^2} \right) - \frac{8}{27} \frac{\hat{u}^2}{\hat{s}\hat{t}}$	$rac{8}{27}rac{\hat{u}}{\hat{t}} - rac{4}{9}rac{\hat{s}\hat{u}}{\hat{t}^2}$
$q\bar{q}' ightarrow q\bar{q}'$	$rac{4}{9}rac{\hat{s}^2+\hat{u}^2}{\hat{t}^2}$	$-rac{4}{9}rac{\hat{s}\hat{u}}{\hat{t}^2}$
gg ightarrow q ar q	$\frac{1}{6}\frac{\hat{t}^2 + \hat{u}^2}{\hat{t}\hat{u}} - \frac{3}{8}\frac{\hat{t}^2 + \hat{u}^2}{\hat{s}^2}$	

TABLES

TABLE I. Partonic cross sections for $q_a q_b \rightarrow q_c q_d$ (only spin-average and transversity-dependent parts are shown here).