New Axial Interactions at a TeV

B. Holdom^{*} and T. Torma^{\dagger}

Department of Physics, University of Toronto Toronto, Ontario, M5S1A7, CANADA

Abstract

We consider a heavy fourth family with masses lying in the symmetry breaking channel of a new strong gauge interaction. This interaction generates a heavy quark axial-type operator, whose effects can be enhanced through multiple insertions. In terms of the strength of this operator we can express new negative contributions to the S and T parameters and the shifts of the Z couplings to the third family. In particular we find that the new contribution to T is strongly constrained by the experimental constraints on the Z coupling to the τ .

1 Introduction

A heavy fourth family could play a prominent role in dynamical electroweak symmetry breaking. However, the existence of additional families with masses greater than a few hundred GeV is thought to be strongly constrained by the value of the S and Telectroweak correction parameters. Each new weakly interacting degenerate fermion doublet contributes $1/6\pi$ to S. Also at least some mass splitting in new fermion doublets is difficult to avoid, which for weakly interacting doublets implies positive contributions to T. The current data, if anything, favors negative S and T.

If the fourth family fermions are strongly interacting, it is important to know how these results for weakly interacting fermions will change. When the new dynamics is QCD-like it is known that the new fermions will continue to give a positive contribution to S, since the sign of the corresponding parameter in low energy QCD is

^{*}bob.holdom@utoronto.ca

[†]kakukk@physics.utoronto.ca

known experimentally [1]. Here we will explore a situation which is distinctly non-QCD-like in two ways: 1) the strong interaction breaks down close to a TeV, 2) the fourth family quark masses are in a symmetry breaking channel with respect to this interaction.

A model based on this type of symmetry breaking pattern has been described in some detail in [2]. As far as providing some understanding of mass and flavor, the model has some attractive features in comparison with conventional extended technicolor models. In particular it was found that the dangerous isospin violating operators involving the fourth family quarks are naturally suppressed relative to the operator which feeds mass to the top. (More precisely, the latter operator is enhanced relative to the former operators.) In addition, the allowed set of effective 4-fermion operators have a rather different structure than usually considered, and provide a better chance of producing realistic quark and lepton masses and mixings. Both of these features relied on having fourth family quarks in a symmetry breaking channel. It is therefore worthwhile to understand the implications for precision electroweak observables, so that experimental data can be brought to bear on this and related models.

We can comment further on the plausibility of a nonorthodox symmetry breaking pattern. It is usually assumed that the dynamical mass will occur in the channel preferred by a single gauge boson exchange (the MAC hypothesis). But it was shown in [3] that the presence of a gauge boson mass has significant effects on the gap equation beyond the ladder approximation. As we briefly summarize here, the next order corrections can be large and of opposite sign to the leading order. Let the strong interaction be $SU(N)_X$ (we discuss the cause of its breakdown shortly). The 4-point kernel appearing in the gap equation for N_f fermions transforming as N or \overline{N} multiplets, to leading order in N or N_f , and after an angular integration, has the form

$$F(p,k) = \frac{2}{\pi} (2C_2(N) - C_2(R))(\alpha F_1(p,k) + \mathcal{R}\alpha^2 F_2(p,k)) + \dots$$
(1)

R is the $SU(N)_X$ representation of the condensing channel. \mathcal{R} characterizes the importance of the second order terms, given that F_2 is appropriately normalized relative to F_1 at the momentum scale which dominates the integrations. In the case of $N_f = 15$ (which is the case relevant for the model discussed below) it was

found [3] that $\mathcal{R} \approx -1.3$ for gauge boson masses corresponding to the breakdown $SU(N)_X \to SU(N-1)_X$, with the fermion mass in the symmetric tensor channel. Given that α may well be greater than unity, there is no reason to believe the MAC prediction of a strong repulsion in this channel.

We view the breakdown of $SU(N)_X$ as part of the breakdown of a larger flavor interaction, which also couples to the lighter families. We suppose that this flavor dynamics produces hierarchies among the various flavor gauge boson masses, with the lightest being the broken $SU(N)_X$ gauge bosons. This hierarchy must be related to the contributions of different order parameters. Since we restrict ourselves to fermions which carry only standard model quantum numbers, no fermion bilinear condensates are allowed above the electroweak scale except for right-handed neutrino condensates. On the other hand a multitude of 4-fermion condensates are allowed, and we suppose that some of these are responsible for $SU(N)_X$ breaking [2]. For our purposes here we can take the effective theory describing the $SU(N)_X$ dynamics to have the gauge boson masses explicitly present.

We are therefore considering fourth family quarks q' with dynamical mass in the $N \times N$ symmetric tensor representation of a broken $SU(N)_X$ gauge interaction. That is, the q'_L is one component in an \overline{N} multiplet and the q'_R is one component in an N multiplet. (We discuss anomaly cancellation below.) The remaining quarks transforming under $SU(N-1)_X$ will not play an important role in our discussion if they are sufficiently lighter. Given our ignorance of strong interactions there is also the possibility that $SU(N)_X$ can be replaced by a $U(1)_X$, in which case there is no $SU(N-1)_X$ sector. This possibility will be kept implicit below.

Our approach will be to model the broken gauge interactions as effective 4-fermion interactions involving q'. This will be reasonable if, in the effective theory, the loops of interest are dominated by momenta smaller than the 4-fermion compositeness scale. This requires that there be a well defined separation between compositeness scale and the fourth family quark masses $m_{q'} \leq 1$ TeV (the fourth family leptons are taken to be somewhat lighter). We might add that our use of 4-fermion operators to model massive gauge boson dynamics is more justified than their frequent use in modeling QCD, where the gluons remain massless.

The 4-fermion operators represent the effects of integrating out the massive $SU(N)_X$

gauge bosons to all orders in the coupling, and not just the effects of one gauge boson exchange. These operators must respect the chiral flavor symmetries of the strong interactions. In fact there is a SU(12) flavor symmetry acting on the 12 fields q'_L and $(q'^c)_L$, accounting for the QCD colors and the up and down flavors. This is not an exact symmetry; it must be broken by the flavor physics at a higher scale, as well as by QCD and weak interactions.

To represent the massive gauge boson exchanges, we find that there is only one independent operator respecting the approximate flavor symmetry. We write it as a product of color singlet currents,

$$\frac{c}{2}(\overline{q}'\gamma_{\mu}\gamma_{5}q')(\overline{q}'\gamma^{\mu}\gamma_{5}q').$$
(2)

We take c > 0, which reflects our assumption that the underlying theory produces an attraction in the $\overline{q}'q'$ mass channel. This of course is opposite in sign to the result of the exchange of one massive $SU(N)_X$ gauge boson. Our interest in this operator originates in the fact that one operator insertion in the appropriate 2-loop diagrams will induce contributions to S and T proportional to -c.

Let $\hat{\Lambda}$ be the naive compositeness scale set by the mass of the $SU(N)_X$ gauge bosons. We now notice that in the effective theory below $\hat{\Lambda}$ we are able to sum up the effects of the above operator to leading order in the number of flavors. That is we can sum up multiple insertions of the operator in the form of bubble chains, illustrated in Fig. 1a. Each additional loop is an axial-vector 2-point function, which at zero momentum will come with a factor $4cf_{q'}^2$. By $f_{q'}^2$ we denote the heavy quark contribution, which is the dominant contribution, to $f^2 \approx (250 \text{ GeV})^2$. $f_{q'}^2$ includes a factor of $N_f = 6$, the number of Dirac fermion flavors in the loops. The important point is that these loops are dominated by momenta below $\hat{\Lambda}$, and thus are safely described by the effective theory.



Figure (1): a) The axial bubble chains modify the 4-fermion operator, at leading order in $1/N_f$. b) The axial bubble chains in the *T*-parameter. c) The axial bubble chains in the coupling of the Z to the third family. d) The "chain of chains" in the S-parameter.

One effect of these axial bubble chains is that the operator in (2) can have an effective compositeness scale smaller than $\hat{\Lambda}$. In other words our operator becomes effectively nonlocal, of the form

$$\overline{q}'\gamma_{\mu}\gamma_{5}q'\frac{C(k^{2})}{2}\overline{q}'\gamma^{\mu}\gamma_{5}q'$$
(3)

where k_{ν} is the momentum flowing from one $\overline{q}'\gamma^{\mu}\gamma_5 q'$ to the other. When $C(k^2)$ is calculated and expanded in k^2 we find

$$C(k^2) = \frac{3}{2} \frac{m_{q'}^2}{f_{q'}^2 \Lambda^2} \left(1 - \frac{k^2}{\Lambda^2} + \ldots\right)$$
(4)

where

$$\Lambda^2 = 6m_{q'}^2 (1/(4cf_{q'}^2) - 1).$$
(5)

 Λ becomes the new compositeness scale when $\Lambda < \hat{\Lambda}$. In succeeding sections we will let the smaller of Λ and $\hat{\Lambda}$ be denoted by Λ .

In any case, at momentum scales small compared to the true compositeness scale we have the original local 4-fermion operator with an enhanced coefficient.

$$C(0) = \frac{c}{1 - 4cf_{q'}^2} \tag{6}$$

We will restrict ourselves here to $4cf_{q'}^2 < 1$, and in particular to the case when Λ is greater than and not too close to $m_{q'}$. We stress the role played by a large N_f , which can imply a nonnegligible $4cf_{q'}^2$ even when there is a hierarchy between $\hat{\Lambda}$ and $m_{q'}$.

From the form of $C(k^2)$ one might wonder whether there is some sort of tachyonic pole at $k^2 \approx -\Lambda^2$. From the full form of $C(k^2)$ we find that a possible pole is pushed down to more negative values of k^2 , in which case it becomes physically meaningless if it occurs in the region of $-\hat{\Lambda}^2$. There remains the interesting question of what happens when $4cf_{q'}^2$ is fine-tuned to approach unity from below, so that a low 'mass' tachyonic axial meson apparently does appear. There is also the possibility of a real axial-vector meson resonance for $4cf_{q'}^2 > 1$. These issues will be considered elsewhere [4].

When Fierz transformed the operator in (2) contains a scalar-scalar $(\overline{q}'q')(\overline{q}'q')$ piece with the appropriate sign to induce a mass. But there is no large N_f justification (or large N justification, since we are discussing only one massive fermion in a $SU(N)_X$ multiplet) for the consideration of bubble chains in the scalar channel, in which case the existence of a light scalar particle with mass of order the fermion mass becomes questionable. In any case the physics of fermion mass generation is very sensitive to the physics at scale $\hat{\Lambda}$, in contrast to the physics in the effective theory which we claim is of interest for S and T. It is the latter which we focus on in this paper.

2 The Fourth and Third Families

Our first task is to produce an anomaly free set of fermions which can realize the previous description. Rather than introducing yet more fermions, we will instead assume that $SU(N)_X$ also couples to the third family in such a way that gauge anomalies are canceled between the third and fourth families. The consequence of this is another observable effect of the new physics, in the form of shifts in the Z couplings to the third family. This in turn will place strong constraints on how the new physics can affect S and T.

The third and fourth family quarks transform under $SU(3)_{\text{QCD}} \times SU(N)_X$ as follows.

$$q'_L = (3, \overline{N}), \quad q'_R = (3, N), \quad q_L = (3, N), \quad q_R = (3, \overline{N})$$
(7)

The broken $SU(N)_X$ dynamics is assumed to produce the fourth family quark masses, corresponding to $\overline{q}'_1 q'_1$, where the subscript denotes the first component of an N or \overline{N} multiplet. The third family quark masses corresponding to $\overline{q}_1 q_1$ are assumed not to be generated by the broken $SU(N)_X$ dynamics, even though their couplings are the same as the fourth family quarks. This is presumably due to a cross-channel coupling in some effective potential. (The reader may recall how a two Higgs potential, symmetric between the two Higgs and with a cross-coupling term $\phi_1^2 \phi_2^2$, will produce a vacuum expectation value for only one of the Higgs for a range of parameters.) The t and b will instead have masses fed down via isospin-violating 4-fermion operators generated by flavor physics at a higher scale. The quarks which transform under $SU(N-1)_X$ represent an additional sector in the theory, and we will return to it below.

We are in fact discussing a modified version of the model in [2], where the $U(1)_X$ described there is replaced by $SU(N)_X$. We refer the reader to that reference for more details on the generation of quark and lepton masses. A realistic mass spectrum requires that the charged leptons $(\tau'_L, \tau'_R, \tau_L, \tau_R)$ transform as $(\overline{N}, \overline{N}, N, N)$ respectively under $SU(N)_X$. There are no right-handed neutrinos in the theory at TeV scales, while the left-handed neutrinos (ν'_L, ν_L) transform as (\overline{N}, N) . Thus the fourth family charged lepton mass $\overline{\tau}'_1 \tau'_1$ is in the $N \times \overline{N}$ channel of $SU(N)_X$ while the fourth family Majorana neutrino mass $\nu'_{L1}\nu'_{L1}$ is in the symmetric tensor, like the quarks.

We are now able to derive the operators arising from integrating out the massive $SU(N)_X$ gauge bosons. We focus on the fourth family (fields with a prime) and the third family (fields without a prime) and omit the '1' subscripts. There is now an approximate $SU(15) \times SU(15)$ flavor symmetry acting on the 15-plets $(q'_L, (q'^c)_L, \tau'_L, (\tau^c)_L, \nu_L)$ and $(q_L, (q^c)_L, \tau_L, (\tau'^c)_L, \nu_L)$ respectively. In addition to this symmetry the effective 4-fermion operators should respect a discrete symmetry under interchange of these two multiplets. The allowed operators can then be written in the following form.

$$\frac{c^{-}}{2}J_{5\mu}^{-}J_{5}^{-\mu} + \frac{c^{+}}{2}J_{5\mu}^{+}J_{5}^{+\mu}$$
(8)

$$J_{\mu}^{-} = -\overline{q}'\gamma_{\mu}\gamma_{5}q' + \overline{q}\gamma_{\mu}\gamma_{5}q + \overline{\nu}_{L}'\gamma_{\mu}\nu_{L}' - \overline{\nu}_{L}\gamma_{\mu}\nu_{L} + \overline{\tau}'\gamma_{\mu}\tau' - \overline{\tau}\gamma_{\mu}\tau$$
(9)

$$J^{+}_{\mu} = \overline{q}'\gamma_{\mu}\gamma_{5}q' + \overline{q}\gamma_{\mu}\gamma_{5}q - \overline{\nu}'_{L}\gamma_{\mu}\nu'_{L} - \overline{\nu}_{L}\gamma_{\mu}\nu_{L} + \overline{\tau}'\gamma_{\mu}\gamma_{5}\tau' + \overline{\tau}\gamma_{\mu}\gamma_{5}\tau \qquad (10)$$

There is again a single operator involving the q' fields only. J^-_{μ} distinguishes between q and q' while J^+_{μ} does not. Thus it is only the c^- term which causes the $\overline{q}'q'$ mass channel to have a different interaction strength than the $\overline{q}'q$ mass channel. We expect that $c^- + c^+ > 0$ in order for the $\overline{q}'q'$ channel to be attractive, and $c^- > 0$ in order for the $\overline{q}'q'$ channel to be more attractive than the $\overline{q}'q$ channel.

There will also be many other operators arising from the flavor physics at a higher scale, which leave behind the approximate flavor symmetries at a TeV. Some of these operators may not be negligible at a TeV due to anomalous scaling [2]. One example is the operator which feeds mass to the t, and another is the operator $(\overline{\tau}'_L \tau'_R)(\overline{\tau}'_R \tau'_L)$ which can help to induce the τ' mass. There are also some operators (e.g. those labeled by \mathcal{C} and \mathcal{D} in [2]) which contribute to the breakdown of $SU(N)_X$. But none of these enhanced operators consist purely of q' fields. Thus the operators induced by flavor physics are not expected to compete with the $SU(N)_X$ induced operators in our discussion below.

3 Results

We first consider the contributions to T. We find that the basic loops can be expressed in terms of the contributions to $f^2 \approx (250 \text{ GeV})^2$ from the heavy q' quarks, the tquark, the τ' , and the ν'_L . We denote these contributions by $f^2_{q'} \equiv f^2_{t'} + f^2_{b'}$, f^2_{τ} , $f^2_{\tau'}$, and $2f^2_{\nu'}$ respectively. We can represent the various f's in terms of effective ultraviolet cutoffs in the corresponding loops [6].

$$f_{q'}^2 = \frac{3m_{q'}^2}{2\pi^2} \ln(\frac{\Lambda_{q'}}{m_{q'}}), \quad f_t^2 = \frac{3m_t^2}{4\pi^2} \ln(\frac{\Lambda_t}{m_t}), \quad f_{\tau'}^2 = \frac{m_{\tau'}^2}{4\pi^2} \ln(\frac{\Lambda_{\tau'}}{m_{\tau'}}), \quad f_{\nu'}^2 = \frac{m_{\nu'}^2}{4\pi^2} \ln(\frac{\Lambda_{\nu'}}{m_{\nu'}})$$
(11)

We allow the various cutoffs to be different, since they are provided either by the compositeness scale of the 4-fermion operators or by the momentum dependence of the mass in question.

In terms of these quantities we find

$$\alpha f^2 T = \frac{3\Delta m_{q'}^2 + \Delta m_{\ell'}^2}{12\pi^2} - f_{\nu'}^2 - 4(c^- D_1^2 + c^+ D_2^2) - 16f_{q'}^2(c^- D_1 + c^+ D_2)(\hat{c}^- D_1 + \hat{c}^+ D_2)$$
(12)

where

$$\hat{c}^{\pm} \equiv \frac{c^{\pm}}{1 - 4(c^{-} + c^{+})f_{q'}^{2}} \tag{13}$$

$$D_1 = \Delta f^2 - f_t^2 + f_{\nu'}^2 \tag{14}$$

$$D_2 = \Delta f^2 + f_t^2 + f_{\nu'}^2 - f_{\tau'}^2 \tag{15}$$

$$\Delta f^2 = f_{t'}^2 - f_{b'}^2 \approx \frac{\Delta m_{q'}}{m_{q'}} f_{q'}^2 \tag{16}$$

and $m_{q'} = (m_{t'} + m_{b'})/2$, $\Delta m_{q'} = m_{t'} - m_{b'}$ and $\Delta m_{\ell'} = m_{\nu'} - m_{\tau'}$. The first two terms in (12) arise at one loop where the first is the usual contribution and the $-f_{\nu'}^2$ term arises from the Majorana nature of the ν' mass, as described in [5]. It is clear that $f_{\nu'}^2$ must not be too large, if we wish to avoid fine-tuned cancellations.

The last two terms are the negative contributions from the 4-fermion operators. The third term is a two loop contribution and the fourth term sums up the axial bubble chains involving fourth family quarks, as illustrated in Fig. 1b. Bubble chains involving lighter fermions are safely neglected. From (11) we see that these bubble chains are characterized by a new logarithm for every operator insertion. As we have mentioned, these chains are leading in $1/N_f$, since each bubble sums over the three colors and two flavors of the q' quarks. This chain occurs in the isosinglet channel; isospin breaking masses occur in the two loops at the ends of the chain, allowing all the interior loops to be without τ_3 factors.

Similar corrections are found for the Z couplings to the third family, as illustrated in Fig. 1c. We find¹

$$\Delta g_V^b = 0 \tag{17}$$

$$\Delta g_A^b = 2(c^- D_1 + c^+ D_2) + 8f_{q'}^2(c^- D_1 + c^+ D_2)(\hat{c}^- + \hat{c}^+)$$
(18)

$$\Delta g_V^{\tau} = 2c^- D_1 + 8f_{q'}^2 (c^- D_1 + c^+ D_2)\hat{c}^-$$
(19)

$$\Delta g_A^{\tau} = 2c^+ D_2 + 8f_{q'}^2 (c^- D_1 + c^+ D_2)\hat{c}^+ \tag{20}$$

$$\Delta g_L^{\nu_\tau} = c^- D_1 + c^+ D_2 + 4 f_{q'}^2 (c^- D_1 + c^+ D_2) (\hat{c}^- + \hat{c}^+)$$
(21)

These shifts involve the same two combinations of f_i^2 's which appeared in the *T* correction. The experimental constraints on the *Z* couplings are quite strong especially for the leptonic couplings, and in particular Δg_A^{τ} . This has the consequence that the

¹The Z couplings are normalized such that, for example, $g_A^b = -1/2$ and $g_L^{\nu_{\tau}} = 1/2$.

4-fermion contributions to T are constrained to be small compared to the first two terms in (12).

For S we find the following result.

$$S = \frac{15}{24\pi} - \frac{1}{3\pi} \ln(\frac{m_{\tau'}}{m_{\nu'}}) - \frac{2}{3\pi} f_{q'}^2(\hat{c}^+ + \hat{c}^-) \ln(\frac{\Lambda}{m_{q'}}) \frac{1 - \frac{1}{6} f_{q'}^2(\hat{c}^+ + \hat{c}^-)}{(1 - \frac{1}{3} f_{q'}^2(\hat{c}^+ + \hat{c}^-))^2}$$
(22)

The first two terms are the one-loop contributions to S from the massive fourth family (with no right-handed neutrino and with all masses sufficiently above the Z mass). The origin of the second term is described in [7].

The third term is the negative contribution from the 4-fermion operators. Here a loop integral emerges which is different from the $f_{q'}^2$ integral, and this produces a $\ln(\Lambda/m_{q'})$ dependence in addition to the one in $f_{q'}^2$. The \hat{c}^{\pm} 's appearing in this term again reflect the effect of the axial bubble chains. The last factor in the third term indicates that we have summed another bubble chain (in the isovector channel), where at each "vertex" the axial bubble chain is exchanged in the *t* channel. This is illustrated in Fig. 1d. This outer chain in the "chain of chains" is summing a particular subset of the subleading graphs (subleading in $1/N_f$). In the case that $\Lambda < \hat{\Lambda}$, there would be additional contributions, with loop momenta lying in the range between Λ and $\hat{\Lambda}$. These contributions are complicated by the momentum dependence of the axial bubble chain (as given by $C(k^2)$ in (3)); they could be of the same order as the terms we are keeping although they lack the $\ln(\Lambda/m_{q'})^2$ factor. We have also dropped terms proportional to any f_i^2 other than $f_{q'}^2$, since the $f_{q'}^2$ terms clearly dominate.

4 Another Sector?

We now consider the additional fermions transforming under the unbroken, and we assume confining, $SU(N-1)_X$. Here the discussion is much more model dependent. We will just comment on the possible significant changes to the S result, since any new contributions to T are just a reflection of the unknown up-down mass splittings in the new sector. There are four possibilities.

• The whole $SU(N-1)_X$ sector doesn't exist. This would assume that a $U(1)_X$ could replace and play the role of the $SU(N)_X$ [2].

— Our results remain as is.

• The $SU(N-1)_X$ fermion masses are small enough so that the one loop contributions to S are small, while the $SU(N-1)_X$ confining scale is large enough so that the bound states have so far escaped detection. We include here the possibility that the $SU(N-1)_X$ fermion masses are forbidden by certain discrete chiral symmetries [8].

— There will be additional negative contributions to S from 4-fermion operators, for example operators formed as the product of $SU(N)_X$ currents, which involve both the $SU(N-1)_X$ fermions and the fourth family quarks. The light $SU(N-1)_X$ fermions then appear in a loop which only depends logarithmically on the light fermion mass, giving contributions like the third term in (22) except with a different log factor. These new negative terms can easily be larger than the one appearing in (22).

• The $SU(N-1)_X$ fermion masses are large enough so that the usual one loop contributions to S apply, but are still significantly lower than the fourth family quark masses.

— We would have to include the positive one loop contributions from $SU(N-1)_X$ fermions. This could be offset to some extent by terms like the second term in (22), coming from isospin splittings in the quarks and leptons of that sector. The one loop results are also modified by the $SU(N-1)_X$ strong interactions. 4-fermion induced effects beyond those in previous case would not be substantial as long as the decay constant of the $SU(N-1)_X$ fermions is sufficiently less than $f_{q'}$.

• The $SU(2)_X$ fermion masses are of the same order as the fourth family quark masses.

— There would be many additional contributions from the 4-fermion effects, but the individual fermion masses are reduced because f^2 is fixed. The basic bubble loop contributing to S would be enhanced relative to those appearing in T and the Z couplings by a $SU(3)_X$ color factor.

5 Discussion

For illustration we provide some numbers for the case when we ignore all contributions from the model-dependent $SU(N-1)_X$ sector. We note that the strongest constraints come from $\Delta g_V^{\tau} = .00083 \pm .00158$ and $\Delta g_A^{\tau} = .00015 \pm .00063$ [9].

- The masses can be fine-tuned, m_{q'} = 670, Δm_{q'} = 44, m_{τ'} = 735, m_{ν'} = 304 GeV with cutoffs Λ_{q'} = Λ_t = 2m_{q'}, Λ_{τ'} = 2m_{τ'}, Λ_{ν'} = 2m_{ν'}, to give T = D₁ = D₂ = 0 (and f = 250 GeV). Then c⁻ and c⁺ are free to vary to produce a negative S. But this is not a serious possibility in the absence of a dynamical mechanism to produce these particular masses.
- If we insist on no significant fine tuning among the terms in D_1 and D_2 then we expect that $|D_1|/f^2 \gtrsim 0.03$ and $|D_2|/f^2 \gtrsim 0.03$, which in turn would require that $c^-f^2 \lesssim 0.03$ and $c^+f^2 \lesssim 0.02$. This would imply that the contributions of the 4-fermion operators to both S and T are negligible. The one loop contributions are such that T is small as long as there is a suitable hierarchy in the masses $m_{q'} > m_{\tau'} > m_{\nu'}$ [5], while $S \approx 0.13(0.08)$ for $m_{\tau'}/m_{\nu'} = 2(3)$.
- It has been suggested recently that the τ-polarization data shows internal inconsistencies [10]. If we exclude the τ-polarization data then we have Δg_V^τ = -.0059 ± .0042 and Δg_A^τ = .0007 ± .0007 [9]. For example the masses m_{q'} ≈ 700, Δm_{q'} ≈ 25, m_{τ'} ≈ 550, m_{ν'} ≈ 245 GeV yield D₁/f² ≈ -0.03 and D₂/f² ≈ 0.03 which along with c⁻f² ≈ 0.12 and c⁺f² ≈ 0.08 would produce values of Δg_V^τ and Δg_A^τ consistent with the remaining data. In this case the 4-fermion contribution to S could easily be comparable to the other contributions, resulting in a much reduced or even negative S. T is still completely dominated by the first two terms in (12), which need to cancel at the 10% level (for these masses and when T ≈ -0.2).

In summary we have considered fourth family quarks in a symmetry breaking channel of a new strong interaction. Below the symmetry breaking scale we find a single effective 4-fermion operator involving the fourth family quarks. A single insertion of this operator generates a negative contribution to both S and T. We summed multiple insertions in a complete model in which gauge anomalies are cancelled through the couplings of the new interaction to the third family. The constraints on the shifts of Z couplings to the third family then strongly constrain the new contributions to T. Significant negative contributions to S are still possible.

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