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FOUR-NEUTRINO MIXING, OSCILLATIONS AND BBN*

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ABSTRACT

We investigate the implications for neutrino mixing implied by the results of all neutrino oscillation experiments and by the standard Big-Bang Nucleosynthesis constraint on the number of light neutrinos.

Many experiments searching for neutrino oscillations have been done in the last 30 years using solar, atmospheric, reactor and accelerator neutrinos. The majority of these experiments reported a negative result, but there are three positive indications in favor of neutrino oscillations coming from the results of solar neutrino experiments[1], of atmospheric neutrino experiments[2] and of the LSND accelerator $\bar{\nu}_\mu \to \bar{\nu}_e$ experiment[3].

Neutrino oscillations can occur only if neutrinos are massive particles, if their masses are different and if neutrino mixing is realized in nature. In this case, the left-handed flavor neutrino fields $\nu_{\alpha L}$ ($\alpha = e, \mu, \tau$) are superpositions of the left-handed components ν_{kL} ($k = 1, \ldots, n$) of the fields of neutrinos with definite masses m_k: $\nu_{\alpha L} = \sum_{k=1}^{n} U_{\alpha k} \, \nu_{kL}$, where U is a unitary mixing matrix. The general expression for the probability of $\nu_\alpha \to \nu_\beta$ transitions in vacuum is

$$P_{\nu_\alpha \to \nu_\beta} = \left| \sum_{k=1}^{n} U_{\beta k} \, \exp\left(-i \, \frac{\Delta m^2_{k1} \, L}{2 \, p} \right) \, U_{\alpha k}^* \right|^2 , \qquad (1)$$

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where $\Delta m^2_{kj} \equiv m^2_k - m^2_j$, L is the distance between the neutrino source and detector and p is the neutrino momentum.

The three experimental indications in favor of neutrino oscillations require the existence of three different scales of neutrino mass-squared differences: $\Delta m^2_{\text{sun}} \sim 10^{-5}\,\text{eV}^2$ (MSW) or $\Delta m^2_{\text{sun}} \sim 10^{-10}\,\text{eV}^2$ (vacuum oscillations), $\Delta m^2_{\text{atm}} \sim 5 \times 10^{-3}\,\text{eV}^2$ and $\Delta m^2_{\text{SBL}} \sim 1\,\text{eV}^2$, where Δm^2_{SBL} is the neutrino mass-squared difference relevant for short-baseline (SBL) experiments, whose allowed range is determined by the positive result of the LSND experiment.

Three independent mass-squared differences require at least four massive neutrinos. The number of active light flavor neutrinos is known to be three from the measurement of the invisible width of the Z-boson, but there is no experimental upper bound for the number of massive neutrinos (the lower bound is three). In the following we consider the simplest possibility of existence of four massive neutrinos. In this case, in the flavor basis, besides the three light flavor neutrinos ν_e, ν_μ, ν_τ that contribute to the invisible width of the Z-boson, there is a light sterile neutrino ν_s that is a $\text{SU}(2)_L$ singlet and does not take part in standard weak interactions.

Two years ago we have shown[4] that among all the possible four-neutrino mass spectra only two are compatible with the results of all neutrino oscillation experiments:

$$\text{(A)} \quad \underbrace{\overbrace{m_1 < m_2}^{\text{atm}} \ll \overbrace{m_3 < m_4}^{\text{sun}}}_{\text{SBL}} \qquad \text{and} \qquad \text{(B)} \quad \underbrace{\overbrace{m_1 < m_2}^{\text{sun}} \ll \overbrace{m_3 < m_4}^{\text{atm}}}_{\text{SBL}} . \tag{2}$$

In these two schemes the four neutrino masses are divided in two pairs of close masses separated by a gap of about 1 eV, which provides the mass-squared difference $\Delta m^2_{\text{SBL}} = \Delta m^2_{41} \equiv m^2_4 - m^2_1$ that is relevant for the oscillations observed in the LSND experiment. In scheme A $\Delta m^2_{\text{atm}} = \Delta m^2_{21} \equiv m^2_2 - m^2_1$ is relevant for the explanation of the atmospheric neutrino anomaly and $\Delta m^2_{\text{sun}} = \Delta m^2_{43} \equiv m^2_4 - m^2_3$ is relevant for the suppression of solar ν_e's, whereas in scheme B $\Delta m^2_{\text{atm}} = \Delta m^2_{43}$ and $\Delta m^2_{\text{sun}} = \Delta m^2_{21}$.

Let us define the quantities c_α, with $\alpha = e, \mu, \tau, s$, in the schemes A and B as

$$\text{(A)} \quad c_\alpha \equiv \sum_{k=1,2} |U_{\alpha k}|^2 \,, \qquad \text{(B)} \quad c_\alpha \equiv \sum_{k=3,4} |U_{\alpha k}|^2 \,. \tag{3}$$

Physically c_α quantify the mixing of the flavor neutrino ν_α with the two massive neutrinos whose Δm^2 is relevant for the oscillations of atmospheric neutrinos (ν_1, ν_2 in scheme A and ν_3, ν_4 in scheme B).

The probability of $\nu_\alpha \to \nu_\beta$ transitions ($\beta \neq \alpha$) and the survival probability of ν_α in SBL experiments are given by[5]

$$P_{\nu_\alpha \to \nu_\beta} = A_{\alpha;\beta} \, \sin^2 \frac{\Delta m^2_{\text{SBL}} L}{4p} \,, \qquad P_{\nu_\alpha \to \nu_\alpha} = 1 - B_{\alpha;\alpha} \, \sin^2 \frac{\Delta m^2_{\text{SBL}} L}{4p} \,, \tag{4}$$

with the oscillation amplitudes

$$A_{\alpha;\beta} = 4 \left| \sum_k U_{\beta k} \, U_{\alpha k}^* \right|^2 , \qquad B_{\alpha;\alpha} = 4 c_\alpha \, (1 - c_\alpha) \, , \tag{5}$$

where the index k runs over the values $1,2$ or $3,4$. The probabilities (4) have the same form as the corresponding probabilities in the case of two-neutrino mixing, $P_{\nu_\alpha \to \nu_\beta} = \sin^2(2\theta) \, \sin^2(\Delta m^2 L/4p)$ and $P_{\nu_\alpha \to \nu_\alpha} = 1 - \sin^2(2\theta) \, \sin^2(\Delta m^2 L/4p)$, which have been used by all experimental collaborations for the analysis of the data in order to get information on the parameters $\sin^2(2\theta)$ and Δm^2 (θ and Δm^2 are, respectively, the mixing angle and the mass-squared difference in the case of two-neutrino mixing). Therefore, we can use the results of their analyses in order to get information on the corresponding parameters $A_{\alpha;\beta}$, $B_{\alpha;\alpha}$ and Δm^2_{SBL}.

The results of all neutrino oscillation experiments are compatible with the schemes A and B only if[4]

$$c_e \leq a_e^0 \quad \text{and} \quad c_\mu \geq 1 - a_\mu^0 \, , \tag{6}$$

where

$$a_\alpha^0 \equiv \frac{1}{2} \left(1 - \sqrt{1 - B_{\alpha;\alpha}^0} \right) \qquad (\alpha = e, \mu) \tag{7}$$

and $B_{\alpha;\alpha}^0$ is the upper bound for the amplitude of $\overset{(-)}{\nu}_\alpha \to \overset{(-)}{\nu}_\alpha$ oscillations obtained from the exclusion plots of SBL reactor and accelerator disappearance experiments. Hence, the quantities a_e^0 and a_μ^0 depend on Δm^2. The exclusion curves obtained in the Bugey reactor experiment and in the CDHS and CCFR accelerator experiments[6] imply that both a_e^0 and a_μ^0 are small[7]: $a_e^0 \lesssim 4 \times 10^{-2}$ and $a_\mu^0 \lesssim 2 \times 10^{-1}$ for any value of Δm^2 in the range $0.3 \lesssim \Delta m^2 \lesssim 10^3 \, \text{eV}^2$.

The smallness of c_e in both schemes A and B is a consequence of the solar neutrino problem[4]. It implies that the electron neutrino has a small mixing with the neutrinos whose mass-squared difference is responsible for the oscillations of atmospheric neutrinos (ν_1, ν_2 in scheme A and ν_3, ν_4 in scheme B). Therefore, the transition probability of electron neutrinos and antineutrinos into other states in atmospheric and long-baseline (LBL) experiments is suppressed. Indeed, it can be shown[8] that the transition probabilities of electron neutrinos and antineutrinos into all other states and the probability of $\overset{(-)}{\nu}_\mu \to \overset{(-)}{\nu}_e$ transitions in vacuum are bounded by

$$1 - P^{(\text{LBL})}_{\overset{(-)}{\nu}_e \to \overset{(-)}{\nu}_e} \leq a_e^0 \left(2 - a_e^0 \right) \tag{8}$$

$$P^{(\text{LBL})}_{\overset{(-)}{\nu}_\mu \to \overset{(-)}{\nu}_e} \leq \min \left(a_e^0 \left(2 - a_e^0 \right) \, , \, a_e^0 + \frac{1}{4} \, A_{\mu;e}^0 \right) \tag{9}$$

where $A_{\mu;e}^0$ is the upper bound for the amplitude of $\overset{(-)}{\nu}_\mu \to \overset{(-)}{\nu}_e$ transitions measured in SBL experiments with accelerator neutrinos.

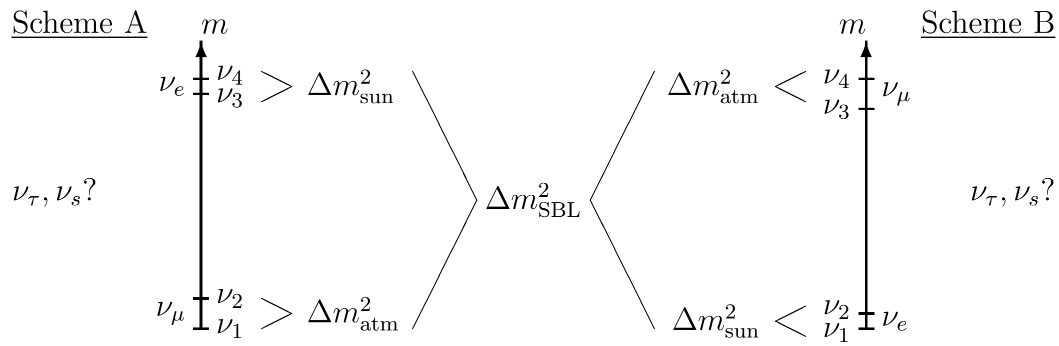

Figure 1

The two schemes A and B have identical implications for neutrino oscillation experiments, but very different implications for neutrinoless double-β decay experiments and for tritium β-decay experiments. Indeed, in scheme A

$$|\langle m \rangle| \leq m_4 \qquad \text{and} \qquad m(^3\mathrm{H}) \simeq m_4 \,, \tag{10}$$

whereas in scheme B

$$|\langle m \rangle| \leq a_e^0 \, m_4 \ll m_4 \qquad \text{and} \qquad m(^3\mathrm{H}) \ll m_4 \,, \tag{11}$$

where $\langle m \rangle = \sum_{i=1}^{4} U_{ei}^2 \, m_i$ is the effective Majorana mass that determines the matrix element of neutrinoless double-β decay and $m(^3\mathrm{H})$ is the neutrino mass measured in tritium β-decay experiments. Therefore, in scheme B $|\langle m \rangle|$ and $m(^3\mathrm{H})$ are smaller than the expected sensitivity of the next generation of neutrinoless double-β decay and tritium β-decay experiments. The observation of a positive signal in these experiments would be an indication in favor of scheme A.

Summarizing, the results of neutrino oscillation experiments indicate that only the two four-neutrino schemes (2) are allowed and the electron neutrino has a very small mixing with the two massive neutrinos that are responsible for the oscillations of atmospheric neutrinos (ν_1, ν_2 in scheme A and ν_3, ν_4 in scheme B). Hence, the two schemes have the form shown in Fig.1, where ν_e is associated with the two massive neutrinos neutrinos that are responsible for the oscillations of solar neutrinos (ν_3, ν_4 in scheme A and ν_1, ν_2 in scheme B), with which it has a large mixing, whereas ν_μ is associated with the two massive neutrinos neutrinos that are responsible for the oscillations of atmospheric neutrinos, with which the muon neutrino has a large mixing. The results of neutrino oscillation experiments do not provide yet an indication of where ν_τ and ν_s have to be placed in the two schemes represented in Fig.1. We will show in the following that the standard Big-Bang Nucleosynthesis constraint on the number of light neutrinos provide a stringent limit on the mixing of the sterile neutrino with the two massive neutrinos that are responsible for the oscillations of atmospheric neutrinos[9,10].

It is well known that the observed abundance of primordial light elements is predicted with an impressive degree of accuracy by the standard model of Big-Bang

Nucleosynthesis if the number N_ν of light neutrinos (with mass much smaller than 1 MeV) in equilibrium at the neutrino decoupling temperature ($T_{\rm dec} \simeq 2\,{\rm MeV}$ for ν_e and $T_{\rm dec} \simeq 4\,{\rm MeV}$ for ν_μ, ν_τ) is not far from three[11,12].

The value of N_ν is especially crucial for the primordial abundance of ^{4}He. This is due to the fact that N_ν determines the freeze-out temperature of the weak interaction processes $e^+ + n \leftrightarrows p + \bar{\nu}_e$, $\nu_e + n \leftrightarrows p + e^-$ and $n \leftrightarrows p + e^- + \bar{\nu}_e$ that maintain protons and neutrons in equilibrium, *i.e.* the temperature at which the rate $\Gamma_W(T) \simeq G_F T^5$ (G_F is the Fermi constant) of these weak interaction processes becomes smaller than the expansion rate of the universe

$$H(T) \equiv \frac{\dot{R}(T)}{R(T)} = \sqrt{\frac{8\pi^3}{90}\, g_*}\, \frac{T^2}{M_P} \tag{12}$$

(M_P is the Planck mass), where $R(T)$ is the cosmic scale factor and $g_* = 5.5 + 1.75 N_\nu$ for $m_e \lesssim T \lesssim m_\mu$. If $N_\nu = 3$ the primordial mass fraction of ^{4}He, $Y_P \equiv$ mass density of ^{4}He / total mass density, is $Y_P \simeq 0.24$, which agrees very well with the observed value[12] $Y_P^{\rm obs} = 0.238 \pm 0.002$. Since Y_P is very sensitive to variations of N_ν, it is clear that the observed value of Y_P puts stringent constraints on the possible deviation of N_ν from the Standard Model value $N_\nu = 3$.

In the four-neutrino schemes (2) standard BBN gives a constraint on neutrino mixing if the upper bound for N_ν is less than four. In this case the mixing of the sterile neutrino is severely constrained because otherwise neutrino oscillations would bring the sterile neutrino in equilibrium before neutrino decoupling, leading to $N_\nu = 4$. In particular, we will show that standard BBN with $N_\nu < 4$ implies that c_s is extremely small.

The amount of sterile neutrinos present at nucleosynthesis can be calculated using the differential equation[13]

$$\frac{{\rm d}n_{\nu_s}}{{\rm d}T} = -\frac{1}{2HT} \sum_{\alpha=e,\mu,\tau} \langle P_{\nu_\alpha\to\nu_s} \rangle_{\rm coll} \Gamma_{\nu_\alpha} \left(1 - n_{\nu_s}\right), \tag{13}$$

where n_{ν_s} is the number density of the sterile neutrino relative to the number density of an active neutrino in equilibrium and Γ_{ν_α} are the collision rates of the active neutrinos, including elastic and inelastic scattering[14], $\Gamma_{\nu_e} = 4.0\, G_F^2 T^5$ and $\Gamma_{\nu_\mu} = \Gamma_{\nu_\tau} = 0.7\, \Gamma_{\nu_e}$. The quantities $\langle P_{\nu_\alpha\to\nu_s} \rangle_{\rm coll}$ are the probabilities of $\nu_\alpha \to \nu_s$ transitions averaged over the collision time $t_{\rm coll} = 1/\Gamma_{\nu_\alpha}$. Hence, also n_{ν_s} has to be considered as a quantity averaged over the collision time.

Equation (13) describes non-resonant and adiabatic resonant neutrino transitions if $t_{\rm osc} \ll t_{\rm coll} \ll t_{\rm exp}$. The condition $t_{\rm osc} \ll t_{\rm coll}$ means that neutrino oscillations have to be fast relative to the collision time. The characteristic expansion time of the universe $t_{\rm exp}$ is given by $t_{\rm exp} = 1/H$ where H is the Hubble parameter $H \equiv \dot{R}/R$, which is related to the temperature T by $H = -\dot{T}/T \simeq 0.7\,(T/1\,{\rm MeV})^2\,{\rm s}^{-1}$ (this value can be obtained from Eq.(12) with $m_e \lesssim T \lesssim m_\mu$ and $N_\nu \simeq 3$). The relation

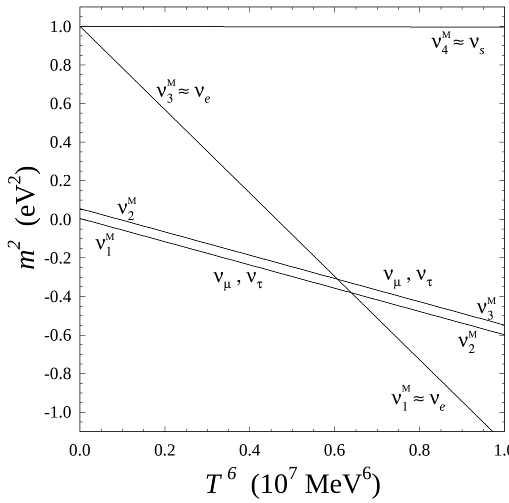

Figure 2

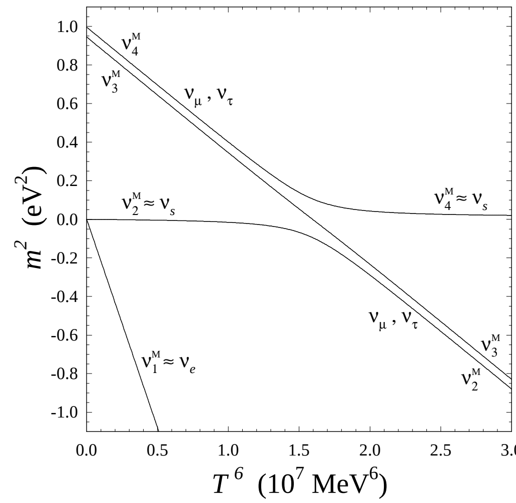

Figure 3

$\Gamma_{\nu_e}/H \simeq 1.2\,(T/1\,\mathrm{MeV})^3$ shows that for temperatures larger than 2 MeV the collision time is always much smaller than the expansion time[13].

Since by definition N_ν is the effective number of massless neutrino species at $T_{\rm dec}$, in order to get a constraint on the mixing of sterile neutrinos we need to calculate the value of n_{ν_s} at $T_{\rm dec}$ produced by neutrino oscillations. With the initial condition $n_{\nu_s}(T_i) = 0$ ($T_i \sim 100$ MeV), the integration of Eq.(13) gives[16] $n_{\nu_s}(T_{\rm dec}) = 1 - e^{-F}$ with

$$F = \int_{T_{\rm dec}}^{T_i} \frac{1}{2HT} \sum_{\alpha=e,\mu,\tau} \langle P_{\nu_\alpha \to \nu_s} \rangle_{\rm coll} \Gamma_{\nu_\alpha} \mathrm{d}T \,. \tag{14}$$

Imposing the upper bound $n_{\nu_s}(T_{\rm dec}) \leq \delta N \equiv N_\nu - 3$ one obtains the condition $F \leq |\ln(1-\delta N)|$.

For the calculation of F the averaged transition probabilities $\langle P_{\nu_\alpha \to \nu_s} \rangle_{\rm coll}$ must be evaluated and the effective potentials of neutrinos and antineutrinos due to coherent forward scattering in the primordial plasma[15],

$$V_e = -6.02\, G_F\, p\, \frac{T^4}{M_W^2} \equiv V \,, \quad V_{\mu,\tau} = \xi V \quad \text{and} \quad V_s = 0 \,, \tag{15}$$

must be taken into account (in the absence of a lepton asymmetry the effective potentials of neutrinos and antineutrinos are equal). Here p is the neutrino momentum, which we approximate with its temperature average $\langle p \rangle \simeq 3.15\, T$, G_F is the Fermi constant, M_W is the mass of the W boson and $\xi = \cos^2\theta_W/(2+\cos^2\theta_W) \simeq 0.28$, where θ_W is the weak mixing angle. The propagation of neutrinos and antineutrinos is governed by the effective hamiltonian in the weak basis

$$H_W = p + \frac{1}{2p}\, U\, \mathrm{diag}\Big[m_1^2, m_2^2, m_3^2, m_4^2\Big]\, U^\dagger + \mathrm{diag}\Big[V, \xi V, \xi V, 0\Big] \,. \tag{16}$$

It is convenient to subtract from H_W the constant term $p + m_1/2p + \xi V$, which does not affect the relative evolution of the neutrino flavor states, in order to get

$$H'_W = \frac{1}{2p} \, U \, \mathrm{diag}\Big[0, \Delta m^2_{21}, \Delta m^2_{31}, \Delta m^2_{41}\Big] \, U^\dagger + \mathrm{diag}\Big[(1-\xi)V, 0, 0, -\xi V\Big] \,. \qquad (17)$$

From this expression it is clear that the relative evolution of the flavor neutrino states depends on the three mass-squared differences and not on the absolute scale of the neutrino masses. The effective hamiltonian in the mass basis is given by $H'_M = U^\dagger \, H'_W \, U$:

$$H'_M = \frac{1}{2p} \, \mathrm{diag}\Big[0, \Delta m^2_{21}, \Delta m^2_{31}, \Delta m^2_{41}\Big] + U^\dagger \, \mathrm{diag}\Big[(1-\xi)V, 0, 0, -\xi V\Big] \, U \,. \qquad (18)$$

In the mass basis the mixing has been transferred from the mass term to the potential term. In order to calculate the evolution of the neutrino flavors it is necessary to parameterize the 4×4 neutrino mixing matrix U. However, since the second and third rows and the second and third columns of the diagonal potential matrix in Eq.(18) are equal to zero, it is clear that the values of the second and third rows of U, corresponding to ν_μ and ν_τ, are irrelevant and do not need to be parameterized. Furthermore, since c_e is small in both schemes A and B, it does not have any effect on neutrino oscillations before BBN and the approximation $c_e = 0$ is allowed. Hence, the 4×4 neutrino mixing matrix in scheme A can be partially parameterized as

$$U = \begin{pmatrix} 0 & 0 & \cos\theta & \sin\theta \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \sin\varphi\sin\chi & -\sin\varphi\cos\chi & -\cos\varphi\sin\theta & \cos\varphi\cos\theta \end{pmatrix} , \qquad (19)$$

with $0 \leq \varphi \leq \pi/2$. The partial parameterization of the mixing matrix in scheme B can be obtained from Eq.(19) with the exchanges $1 \leftrightarrows 3$ and $2 \leftrightarrows 4$ of the columns of U. In this way $c_s = \sin^2\varphi$ in both schemes A and B. The dots in Eq.(19) indicate the elements of the mixing matrix belonging to the ν_μ and ν_τ rows ($U_{\mu i}$ and $U_{\tau i}$ with $i = 1, \ldots, 4$), which do not need to be parameterized. In Eq.(19) we have parameterized only the elements of the mixing matrix belonging to the ν_e and ν_s lines (U_{ei} and U_{si} with $i = 1, \ldots, 4$) in terms of the three mixing angles θ, χ, φ. It is clear that this partial parameterization of the mixing matrix (with the approximation $U_{e1} = U_{e2} = 0$) is much easier to manipulate than a complete parameterization, which would require the introduction of 6 mixing angles and 3 complex phases.

Notice that no complex phase is needed for the partial parameterization of the mixing matrix in Eq.(19), because the elements U_{ei} and U_{si} with $i = 1, \ldots, 4$ can be chosen real. Indeed, the line U_{si} with $i = 1, \ldots, 4$ and the element U_{e4} can be chosen real because all observable transition probabilities are invariant under the phase transformation $U_{\alpha j} \to e^{i x_\alpha} \, U_{\alpha j} \, e^{i y_j}$, where x_α and y_j are arbitrary parameters.

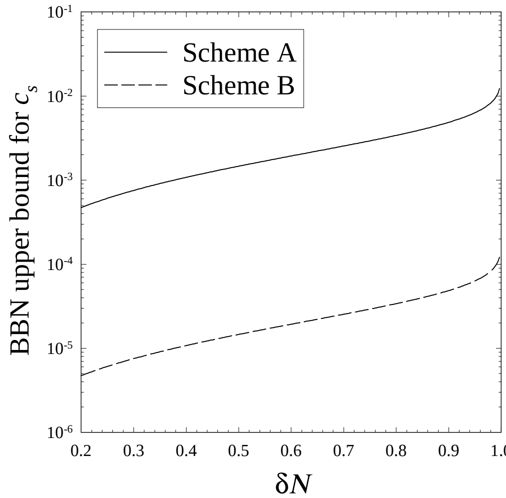

Figure 4

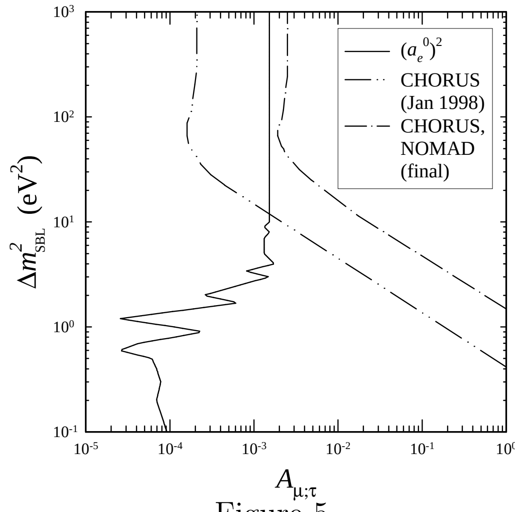

Figure 5

In order to show that also U_{e3} can be chosen real, we multiply the unitarity relation $\sum_{k=1}^{4} U_{ek}\, U_{sk}^{*} = 0$ with $U_{e3}^{*}\, U_{s3}$. The imaginary part of the resulting relation gives

$$\mathrm{Im}[U_{e3}\, U_{s3}^{*}\, U_{e4}^{*}\, U_{s4}] = \mathrm{Im}[U_{e1}\, U_{s1}^{*}\, U_{e3}^{*}\, U_{s3}] + \mathrm{Im}[U_{e2}\, U_{s2}^{*}\, U_{e3}^{*}\, U_{s3}]\,. \qquad (20)$$

In the approximation $U_{e1} = U_{e2} = 0$ the right-hand part of Eq.(20) vanishes. Therefore, since U_{s3}, U_{e4}, U_{s4} have been chosen to be real, also U_{e3} must be real.

Since the mass-squared differences have a hierarchical structure, $\Delta m^2_{43} \ll \Delta m^2_{21} \ll \Delta m^2_{41}$ in scheme A and $\Delta m^2_{21} \ll \Delta m^2_{43} \ll \Delta m^2_{41}$ in scheme B, the effective hamiltonian H'_M can be diagonalized approximately taking into account only one of the three Δm^2's for different ranges of the temperature T. Then, it can be shown that[9,10] the condition $F \leq |\ln(1 - \delta N)|$ gives the bound

$$920 \left(\frac{\Delta m^2_{\mathrm{SBL}}}{1\,\mathrm{eV}^2} \right)^{1/2} d_s \sqrt{1 - d_s} + 33 \left(\frac{\Delta m^2_{\mathrm{atm}}}{10^{-2}\,\mathrm{eV}^2} \right)^{1/2} \frac{\sin^2 2\chi}{\sqrt{1 + \cos 2\chi}}\, c_s^{3/2} \leq |\ln(1 - \delta N)|\,, \qquad (21)$$

with $d_s \equiv c_s$ in scheme A and $d_s \equiv 1 - c_s$ in scheme B.

Both terms in the left-hand side of Eq.(21) are positive and must be small if $\delta N < 1$. The SBL term, depending on $\Delta m^2_{\mathrm{SBL}}$, is small if either c_s is small or large, but the atmospheric term, which depends on $\Delta m^2_{\mathrm{atm}}$, is small only if c_s is small. Indeed, if c_s is close to one we have $(U_{\mu 1}, U_{\mu 2}) \sim (\cos\chi, \sin\chi)$ in scheme A and $(U_{\mu 3}, U_{\mu 4}) \sim (\cos\chi, \sin\chi)$ in scheme B. This means that, in order to accommodate the atmospheric neutrino anomaly, $\sin^2 2\chi$ cannot be small. This is in contradiction with the inequality (21) and we conclude that the bound (21) implies that c_s is small.

Since c_s is small only non-resonant transitions of active into sterile neutrinos due to $\Delta m^2_{\mathrm{SBL}}$ are possible in scheme A, as illustrated in Fig.2 where we have plotted the effective squared masses (obtained from a numerical diagonalization of the hamiltonian (16)) as functions of T^6 (ν_e does not have resonant transitions into ν_μ or ν_τ

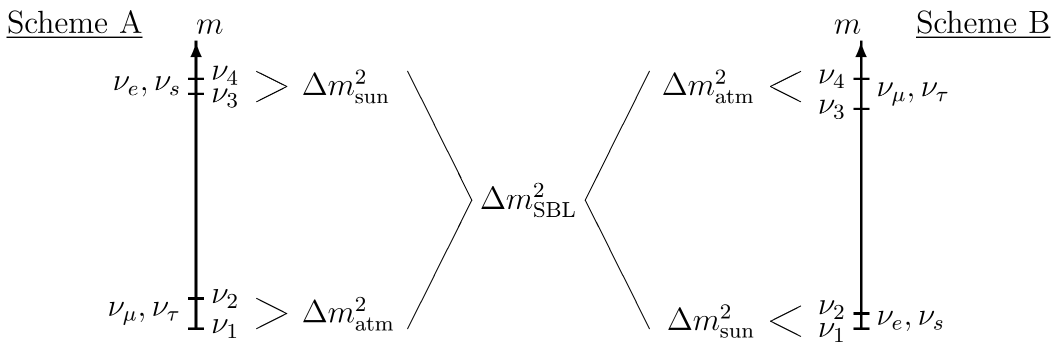

Figure 6

because we have chosen $c_e = 0$). Hence, the conditions for the validity of Eq.(13) are satisfied and the SBL term in Eq.(21) gives the bound

$$c_s \leq 1.1 \times 10^{-3} \left(\frac{\Delta m^2_{\rm SBL}}{1\,{\rm eV}^2} \right)^{-1/2} |\ln(1-\delta N)| \,. \tag{22}$$

On the other hand, since c_s is small, a resonance occurs in scheme B at the temperature $T_{\rm res} = 16 (\Delta m^2_{\rm SBL}/1\,{\rm eV}^2)^{1/6} |1-2c_s|^{1/6}\,{\rm MeV}$, as illustrated in Fig.3. The condition $\delta N < 1$ implies that this resonance must not be passed adiabatically. In this case the conditions for the validity of Eq.(13) are not fulfilled and the SBL term of Eq.(21) does not apply. Using an appropriate formula[17] for the calculation of the amount of sterile neutrinos produced at the resonance through non-adiabatic transitions one can show[10] that the BBN bound on c_s in scheme B is given by

$$c_s \leq 1.1 \times 10^{-5} \left(\frac{\Delta m^2_{\rm SBL}}{1\,{\rm eV}^2} \right)^{-1/2} |\ln(1-\delta N)| \,. \tag{23}$$

Figure 4 shows the values of the bounds (22) and (23) obtained from the LSND[3] lower bound $\Delta m^2_{\rm SBL} \gtrsim 0.27\,{\rm eV}^2$ for $0.2 \leq \delta N < 1$. One can see that standard BBN implies that c_s is extremely small. Therefore, ν_s is mainly mixed with the two massive neutrinos that contribute to solar neutrino oscillations (ν_3 and ν_4 in scheme A and ν_1 and ν_2 in scheme B) and the unitarity of the mixing matrix implies that ν_τ is mainly mixed with the two massive neutrinos that contribute to the oscillations of atmospheric neutrinos. Adding this information to the two schemes depicted in Fig.1 we obtain the schemes shown in Fig.6. These schemes have the following testable implications for solar, atmospheric, long-baseline and short-baseline neutrino oscillation experiments:

- The solar neutrino problem is due to $\nu_e \to \nu_s$ oscillations. This prediction will be checked by future solar neutrino experiments that can measure the ratio of neutral-current and charged-current events[18].

- The atmospheric neutrino anomaly is due to $\nu_\mu \to \nu_\tau$ oscillations. This prediction will be checked by LBL experiments.

- $\nu_\mu \to \nu_\tau$ and $\nu_e \to \nu_s$ oscillations are strongly suppressed in SBL experiments. With the approximation $c_s \simeq 0$, for the amplitude of $\nu_\mu \to \nu_\tau$ oscillations we have the upper bound $A_{\mu;\tau} \leq (a_e^0)^2$, that is shown in Fig.5 (solid curve) together with a recent exclusion curve obtained in the CHORUS experiment (dash-dotted curve) and the final sensitivity of the CHORUS and NOMAD experiments (dash-dot-dotted curve)[19].

If these prediction will be falsified by future experiments it could mean that some of the indications given by the results of neutrino oscillations experiments are wrong and neither of the two four neutrino schemes A and B is realized in nature, or that Big-Bang Nucleosynthesis occurs with a non-standard mechanism[20].

In conclusion, we would like to emphasize that if the analysis presented here is correct and one of the two four neutrino schemes depicted in Fig.6 is realized in nature, at the zeroth-order in the expansion over the small quantities c_e and c_s the 4×4 neutrino mixing matrix has an extremely simple structure in which the ν_e, ν_s and ν_μ, ν_τ sectors are decoupled. For example, in scheme A

$$U \simeq \begin{pmatrix} 0 & 0 & \cos\theta & \sin\theta \\ \cos\gamma & \sin\gamma & 0 & 0 \\ -\sin\gamma & \cos\gamma & 0 & 0 \\ 0 & 0 & -\sin\theta & \cos\theta \end{pmatrix}, \tag{24}$$

where θ and γ are, respectively, the two-generation mixing angles relevant in solar and atmospheric neutrino oscillations. Therefore, the oscillations of solar and atmospheric neutrinos are independent and the two-generation analyses of solar and atmospheric neutrino oscillations yield correct information on the mixing of four-neutrinos.

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