A Color Mutation Hadronic Soft Interaction Model
– Eikonal Formalism and Branching Evolution

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ECOMB is established as a hadronic multiparticle production generator by soft interaction. It incorporates the eikonal formalism, parton model, color mutation, branching, resonance production and decay. A partonic cluster, being color-neutral initially, splits into smaller color-neutral clusters successively due to the color mutation of the quarks. The process stops at hadronic resonance, $q\bar{q}$ pair, formation. The model contains self-similar dynamics and exhibits scaling behavior in the factorial moments, e.g. the intermittency.

1   Introduction

The study of multiparticle production in low-p_T processes has been pursued for over twenty years. Since they involve soft interactions, they cannot be treated in perturbative QCD. In the absence of any reliable theoretical approach to the problem, many models have been proposed, most of which are represented in the review volumes published ten years ago [1]. Nearly all of those models have since been shown to be inadequate in light of the data on fluctuations and intermittency [2]. Indeed, there are very few models that have the appropriate dynamical content capable of reproducing the scaling behaviors observed in the experiments. In this paper we propose a model that incorporates some aspects of non-perturbative QCD and is capable of generating the features of the intermittency data, which are shown in the last figure of this paper. To our knowledge those data have not been fitted by any model that contains some features of the color dynamics. To reproduce those data has become the main motivator for this work.

Our approach [3] embraces many time-honored properties of hadronic collisions. Since hadrons are extended objects, the eikonal formalism is at the foundation of our model. Thus it is not difficult for our model to possess the virtues of geometrical scaling, approximate KNO scaling and unitarity. In order to build into the model features of chromodynamics, it is necessary to introduce quarks and gluons into the eikonal formalism, so the parton model is an essential gateway into the microscopic domain of color interactions. Once we enter that domain, we embark on an unconventional journey of studying color mutation of the constituents as a dynamical process by which the colors of the quarks evolve through the emission and absorption of gluons. The smallness of α_S is never assumed, so the evolution is not perturbative. With

all partons taken into consideration globally, we follow the evolution of the configuration in the color space. When the configuration exhibits color neutral subclusters, we allow branching to take place with a possible contraction of the cluster size in accordance to a reasonable rule consistent with confinement dynamics. Successive branching leads to smaller and smaller clusters, until they are finally identified as particles and resonances. The decay of resonances are also taken into account before the final state of an event is determined. Evidently, the model contains many features of soft interaction that are familiar and desirable at a qualitative level. Here we put them on a quantitative basis.

We shall call this model ECOMB[3], which stands for eikonal color mutation branching. Since the partons play a fundamental role in this model, it is significantly closer to QCD than any eikonal model on soft hadronic collisions has ever been.

There are a few parameters in the model. They are adjusted to fit a large body of experimental data on low-p_T processes for $\sqrt{s} \lesssim 100$ GeV. They include σ_{el}, σ_{inel}, $\langle n \rangle$, C_q, dn/dy, P_n, and F_q for all s in the range $10 < \sqrt{s} < 100$ GeV, and all rapidity intervals. F_q are the normalized factorial moments, whose power-law dependence on the rapidity bin-size δ has been referred to as the intermittency behavior [4]. Except for F_q, all the other pieces of data are global in nature; i.e., examined in or averaged over all rapidity space. They can be fitted by many models. F_q in small rapidity intervals exhibit local fluctuations, which are what invalidate most of those models.

2   Geometrical Effect on pp Soft Collision

To focus on processes in which hard subprocesses are unimportant, we confine our attention to the energy range $10 < \sqrt{s} < 100$ GeV, which covers the CERN ISR energies. From parton model point of view, this soft process with a relative smaller momentum transfer undergoes a relative longer time evolution of partons than hard subprocess. High energy hadronic collision provides a setting up of the initial condition for the partonic evolution. A proper description of the collision is the basis of any soft interaction model. Two of the well known experimental results, geometrical scaling and approximate Koba-Neilsen-Olesen (KNO) scaling, provide immediate constrains on the collision model building. The eikonal formalism of hadronic collisions gives us an ideal geometrical framework to fit all the constrains. Here is the highlight of it.

In terms of the eikonal function $\Omega(b)$, the geometrical scaling, i.e., σ_{el}/σ_{tot} is roughly constant, can be guaranteed, if Ω depends only on the scaled impact

parameter $R = b/b_0(s)$. The reduced inelasticity function

$$g(R) = 1 - e^{-2\Omega(R)} \tag{1}$$

satisfies the normalization condition $\int_0^\infty dR^2 g(R) = 1$ by setting $\sigma_{inel} = \pi b_0^2(s)$. The function $g(R)$ describes the probability of having an inelastic collision at R. For pp collisions the well-determined form for the eikonal function is [5]

$$\Omega(R) = -\ell n \left(1 - 0.71 e^{-1.17R^2} \right) \quad , \tag{2}$$

which has been used to give a good description of $d\sigma/dt$ [5].

The μth term of the inelasticity function expansion in a power series,

$$g(R) = \sum_{\mu=1}^\infty \pi_\mu(R) = \sum_{\mu=1}^\infty \frac{[2\Omega(R)]^\mu}{\mu!} e^{-2\Omega(R)}, \tag{3}$$

may be regarded as the μth-order rescattering contribution, and can be related to the μ-cut-Pomeron [6,7]. The average μ is estimated as 1.6 [6], according to (3).

To go further in multiparticle production, it is necessary to model the dynamics of soft interaction. This paper is devoted to develop such a model by incorporating the parton model into the formalism as far as we can.

To do so, we adopt the eikonal description to refer to parton number; let B_ν^μ be the probability of having ν partons in the μ-cut Pomeron when the two incident hadrons are separated by a scaled impact parameter R. If we use $\mathcal{E}_{\nu \to n}\{\cdots\}$ to denote the evolution process that takes the ν partons to the n hadrons, we may express symbolically the multiplicity distribution of hadron as

$$P_n = \mathcal{E}_{\nu \to n}\{\int dR^2 \sum_{\mu=1}^\infty \pi_\mu(R) B_\nu^\mu\} \quad . \tag{4}$$

The form invoked in the bracket essentially provides a parton number distribution which involved in the μ-cut Pomeron exchange soft interaction and the description of $\mathcal{E}_{\nu \to n}$ is the main task of this paper.

A way of thinking about B_ν^μ is to consider the $\mu = 1$ case, for which we have an one-Pomeron exchange diagram for the elastic scattering amplitude with the Pomeron being cut to expose the internal quark, antiquark and gluon lines on mass shell. We use B_ν^μ to represent the probability having ν such partons as μ Pomerons in the elastic amplitude are cut.

Since there exists no rigorous derivation of B_ν^μ, we shall assume that it is Poisson distributed around some mean number $\bar\nu$ of partons. It is reasonable,

since it is known that a cut ladder corresponds to a multiperipheral diagram for a ν-particle production amplitude, whose rapidity distribution is uniform, and multiplicity distribution Poissonian. The mean number can depend on both μ and s. Being guided by so-called $\log s$ physics, we adopt a parameterization for

$$\bar{\nu}(\mu, s) = \hat{\nu}(s)\mu^{a(s)} \quad , \tag{5}$$

where the parameters depend on the energy by $\hat{\nu}(s) = \nu_0 + \nu_1 \ln s$ and $a(s) = a_0 + a_1 \ln s + a_2 \ln^2 s$.

Once the number of parton, ν, is determined, one can move on the issue of evolution of the ν parton cluster. Before going further, I like to point out an important departure from the usual application of the parton model: We combine the property of hadrons being spatially extended objects with the property of hadrons being made up of smaller constituents. Clearly, for most collisions whose impact parameters are nonzero, only portions of the partons in the incident hadrons overlap and interact, so the number of hadrons produced depends on that overlap. The event-to-event fluctuation of the particle multiplicity therefore depends strongly on the impact-parameter fluctuation, and the event averaged parton distributions, as determined from the structure functions, are of no use in that respect.

As one of the consequences of the carefully consideration of event-to-event fluctuation, not only the average charged multiplicity of hadron $\langle n_{ch} \rangle$, but also the KNO scaling are reproduced. They are satisfactorily consistent with the data [8] in terms of the standard moments $C_q(s) = \langle n^q \rangle / \langle n \rangle^q$ over an energy range of $10 < \sqrt{s} < 70$ GeV as shown in Fig 1.

3 Color Mutation

We now consider the color dynamics of soft interaction. The initial state is that there are ν partons distributed in some fashion in a linear array in rapidity space. This is a consequence of the parton model in that the partons are in the incident hadrons to begin with, and the collision rearranges those partons and sets off the evolution process that takes those partons to the final state, where the produced hadrons are decoupled. The evolution process involves quarks and antiquarks emitting and absorbing gluons. Since the process is not perturbative, there is no analytical method to track the time development of the process. Thus we shall use Monte Carlo simulation to generate the configuration at each time step.

There are two spaces in which we must track the motion of the color charges. One is the two-dimensional color space; the other is the one-dimensional rapidity space. The latter can be extended to include the azimuthal angle ϕ

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and the transverse momentum $|\vec{p}_T|$, but in our first attempt here we integrate over those variables and examine the simpler problem of a 1D system. As for the color space, it is 2D, and spanned by the (I_3, Y) axes, as in $SU(3)$ flavor.

The configuration in the two spaces are coordinated in the following sense. Starting from the extreme left end of the rapidity (η) space, for which the system that contains no partons is by definition color neutral and therefore is represented by a point at the origin of the color space, we move in the positive direction in η space until we cross a color charge. At the point to the right of that color charge, the corresponding point in the color space jumps from the origin to a position that represents the color of the charge that has been crossed. Let us denote that jump by a vector. Then each time we cross a color charge in the η space, there is a corresponding vector added to the previous point in the color space. The succession of additions of those vectors, each one starting from the tip of the previous one, forms a path. Since the whole parton system is color neutral, by the time we have moved to the extreme right in the η space, the path in the color space returns to the origin, thus forming a closed loop. Such a loop may be self-intersecting at various points. We call such a closed path a configuration of the system in color space.

In order to initiate the time evolution, we need distribute the ν partons into both of those two spaces to form such an initial configuration.

(1) Initial Distribution in Rapidity Space

By rapidity we mean space-time rapidity η, where $\eta = \frac{1}{2} \ell n \frac{t+z}{t-z}$, because we are dealing with a variation of spatial separations between partons due to the color force during the time evolution.

The usual parton distribution in momentum fraction (at the lowest accessible Q^2) implies roughly a flat η distribution with rapid damping toward zero at large η. It is modeled as

$$\rho(\eta) = \rho^{(1)}(\eta) + \rho^{(2)}(\eta)\,, \tag{6}$$

$$\rho^{(1)}(\eta) = \rho_0 \ (\ \frac{\eta + \eta_0}{2\eta_0} \, \theta(\eta + \eta_0)\,\theta(\eta_0 - \eta) + \theta(\eta - \eta_0)\,\theta(\eta_c - \eta) +$$
$$\frac{\eta_{max} - \eta}{\eta_{max} - \eta_c} \theta(\eta - \eta_c)\,\theta(\eta_{max} - \eta))\,, \tag{7}$$

$$\rho^{(2)}(\eta) = \rho_0 \ (\ \frac{-\eta + \eta_0}{2\eta_0} \, \theta(\eta + \eta_0)\,\theta(\eta_0 - \eta) + \theta(\eta + \eta_c)\,\theta(-\eta_0 - \eta) +$$
$$\frac{\eta_{max} + \eta}{\eta_{max} - \eta_c} \theta(\eta + \eta_{max})\,\theta(-\eta_c - \eta))\,. \tag{8}$$

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in which, $\rho^{(1)}(\eta)$ and $\rho^{(2)}(\eta)$ indicate the forward and backward semi-spheres respectively. Only small portion of partons overlap in a limited central region $(-\eta_0, \eta_0)$ after collision follows from the known fact that the correlation between produced particles is short-ranged, because the soft interaction range is short in the conventional wisdom.

In the color space spanned by I_3 and Y, a quark is represented by a vector, which has the coordinates of one of the triplets: $(1/2, 1/3), (-1/2, 1/3)$, and $(0, -2/3)$. An antiquark is represented by a vector directed opposite to one of the above. As a distribution of partons in the η space is generated, say, $\rho^{(2)}(\eta)$ between $-\eta_{\max} \leq \eta \leq \eta_0$, we assign to each parton a color vector consistent with the requirement that the quarks and antiquarks come in pairs, but their orderings in the η and the color spaces are totally random. The partons in $\rho^{(1)}(\eta)$ are distributed similarly, but completely independently, between $-\eta_0 \leq \eta \leq \eta_{\max}$. Due to the merge in the central region $(-\eta_0, \eta_0)$, ν partons are no longer divided into two separate color neutral groups. Usually, they mix up into a whole neutral system according to the rule to count the color charge as described in the beginning of this section.

(3) Color Interaction

We now consider the evolution of a configuration due to QCD dynamics. A nonperturbative treatment of ν simultaneously interacting color charges is, of course, too difficult to contemplate here. We reduce the problem by considering pairwise near-neighbor interaction via the exchange of a gluon in any of the s-, t-, or u-channel, whichever is applicable. Starting from the extreme left in the η space, we regard the ordered chain of ν partons as having $\nu - 1$ links (with varying link lengths). Pairwise near-neighbor interaction means that we consider the $\nu - 1$ links in η space one at a time, according to a rule to be specified below. After the interactions at all $\nu - 1$ links are considered, the evolution of the whole configuration is regarded as having taken one time step, and the process is then repeated.

For every global configuration α of the ν partons, there is an associated energy E_α. E_α is determined by

$$E_\alpha = \sum_{i=1}^{\nu-1} \left| \vec{C}_i \right|^2 = \sum_{i=1}^{\nu-1} \left| \sum_{j=1}^{i} \vec{c}_j \right|^2 \quad , \tag{9}$$

where $\vec{c}_j$ is the color vector for jth parton, $\vec{C}_i$ is the color charge the i-link seeing on its left side and sums obviously depends on the particular path in color space. Whenever a local pair of partons interact, the outcome may

or may not affect the global configuration. Our statistical approach to the determination of the global configuration α, consistent with favoring the lowest energy state, requires that the probability for configuration α to occur is

$$P_\alpha = e^{-\beta E_\alpha}/Z, \qquad Z = \sum_{\alpha=1}^{c} e^{-\beta E_\alpha}, \tag{10}$$

where β is a free parameter. Using the Metropolis algorithm, we use P_α to determine the outcome of a local interaction at every link.

(4) Spatial Fluctuations

In the meanwhile the system undergoing color mutation due to the exchanging of gluon, the spatial position of quarks also fluctuates due to the same reason. We track the change in the length of the ith link in the η space, $d_i = \eta_{i+1} - \eta_i$, to quantify the spatial evolution of ith link. Whether the link length contracts or expands depends on the attractiveness or repulsiveness of the net color forces that act on the two ends of the link. To determine the nature of that force is beyond the scope of this treatment. We shall model the change in link length by a stochastic approach, consistent with how we have handled the color mutation part of the dynamics of the complex system. We allow d_i to change by m_i, i.e., $d_i \rightarrow d_i + m_i$, where the probability for m_i is specified by a distribution

$$\mathcal{P}_i(m) = \gamma_1 \, \theta(m + d_i) \, \theta(\gamma_2 - m) + [1 - \gamma_1(\gamma_2 + d_i)] \, \delta_{m,-d_i} \quad, \tag{11}$$

which satisfies $\sum_{m=-d_i}^{\gamma_2} \mathcal{P}_i(m) = 1$. This distribution allows m to be positive (expansion) uniformly up to γ_2, and negative (contraction) down to $-d_i$. γ_1 and γ_2 are two free parameters.

4 Branching and Hadronization

A fission of the color-singlet cluster occurs, when it contains two color-singlet subclusters, since no confining force exists between them. Thereafter, the color mutation process is applied to the two subsystems separately and independently. This is repeated again and again until all subclusters consist of only $q\bar{q}$ pairs. At every stage of the evolution process, a cluster shrinks due to overall spatial contraction. We always keep the center of the cluster invariant to conserve momentum. When a branching occurs, the two daughter clusters will have their own respective centers, which will remain invariant during contraction, until they themselves branch. At the end when a hadron is formed

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from a $q\bar{q}$ pair, the hadron momentum rapidity y will be identified with a value of η taken randomly between the η values of the quark and antiquark. The reason for doing this is that without knowing the mass and the transverse momentum p_T of the hadron. A rough identification of y with η is justified for free particles at high energy.

When branching process terminates, the $q\bar{q}$ pairs are identified as hadrons, which may be pions, kaons, or resonances. Those resonances must be allowed to decay before the total number and distribution of particles are counted for the final state of the event. The probabilities of producing various resonances and stable (in strong interaction) particles have been studied experimentally in pp collisions in [9]. We use that reference as a generic guide for the proportions of all particles produced in any general hadronic collision. We give each hadron a transverse momentum according to a "standard" exponential distribution of transverse momentum squared. The average transverse momentum is a parameter to be fixed actually as 400MeV. If the $q\bar{q}$ pair forms a resonance, then we assume an isotropic decay distribution in the rest frame of the resonance. The azimuthal angle is assigned randomly. After boosting back to the cm system, the 3-momenta $\vec{p}$ of the decay particles are then determined.

5 Results

In the previous section we have introduced six parameters: η_c, $\eta_{\max}$ and η_0 in (7) and (8), β in (10), γ_1 and γ_2 in (11). They are to be varied to fit the data on inclusive distributions and on fluctuations of the exclusive distributions.

The parameters η_c, $\eta_{\max}$ and $\left\langle k_T^2 \right\rangle = 400$MeV are essentially kinematical; they set the boundaries of the phase space in which the partons are placed. They do not affect the dynamics of color mutation and the spatial fluctuation of the clusters. The data used to determine them are the rapidity distribution $dn/d\eta$ [10] and the transverse momentum distribution. The energy range of the data in [10] is $22 < \sqrt{s} < 63$ GeV, for which hard scattering is negligible. The values of η_c and $\eta_{\max}$ depend on s. In Fig. 2 we show the pseudorapidity distributions to compare with ECOMB as an example. The agreement is clearly satisfactory.

The parameters that influence the evolution process of mutation and branching are η_0 , β, γ_1 and γ_2. η_0 specifies the overlap region of the initial color-neutral clusters, β pertains to the probability of color mutation, and γ_1 and γ_2 characterize the fluctuations of the link lengths. The data used to determine them are on factorial moments F_q for $q = 1, 2, ...$ and their variants [2]. It is here that we can underline the importance of those data on fluctuations, without which we have no guidance on how to restrict the detail dynamics of particle

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production. Putting that in another way, in the absence of a procedure to calculate from first principles, any model that fails to fit the fluctuation data is missing some aspect of the basic dynamics. To the extent of our awareness, very few models on soft interaction have been put to the test of confronting those data on the factorial moments for varying bin sizes.

The parameters that we have varied to give the best fit of the intermittency data at $\sqrt{s} = 22$ GeV are: $\hat{\nu}(s) = 9.1, a(s) = 0.63, \eta_{\max} = 5, \eta_c = 3.5, \eta_0 = 1.9, \beta = 0.0015, \gamma_1 = 0.077, \gamma_2 = 5$. In Fig. 3 we show the intermittency results calculated with ECOMB and data of [11] which to our knowledge have not been reproduced by any model, except ECCO[6,2]. It should be noted that to achieve the fits attained is highly nontrivial. If any part of the dynamical process in generating the hadrons is altered, one would not be able to obtain the rising factorial moments, no matter how many parameters are used. By working with the many parts of our model, all of which affect the determination of F_q, we have gained confidence in regarding the dynamics of color mutation and branching as having captured the essential properties of soft interaction.

6   Conclusion

It is rather satisfying that we have been able to reproduce the intermittency data in Fig. 3. Scaling behavior of that type implies self-similarity in the dynamics of particle production.

We have amalgamated many concepts that form various elements of the conventional wisdom about soft interaction. They include: (a) hadrons having sizes, (b) eikonalism, (c) parton model, (d) interaction of quarks via gluons, (e) statistical properties of a many-body system, (f) spatial contraction and expansion of a color system, and (g) resonance production. They are interlaced by intricate connections described in this paper.

The most important omission in this paper is Bose-Einstein correlation, without which our model is not completely realistic. That defect must be corrected in an improved version.

Acknowledgments

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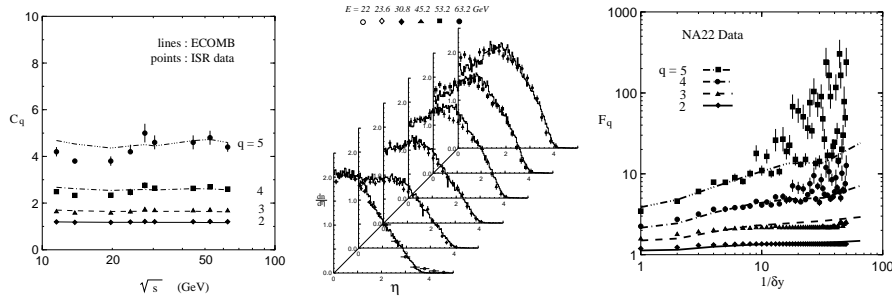
Figure 1: Standard moments of the multiplicity distribution. The data are from Ref. [8].

Figure 2: Rapidity distributions at various cm energies. Symbols are the data from Ref. [10], while the histograms are the result of ECOMB.

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Figure 3: Intermittency data of normalized factorial moments for $q = 2 - 5$. The data are from Ref. [11]. The lines are determined from ECOMB.