

Supersymmetric dark matter with a cosmological constant

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Abstract

Recent measurements of cosmological parameters from the microwave background radiation, type Ia supernovae, and the age of globular clusters help determine the relic matter density in the universe. It is first shown with mild cosmological assumptions that the relic matter density satisfies $\Omega_M h^2 < 0.6$ independent of the cosmological constant and independent of the SNIa data. Including the SNIa data, the constraint becomes $\Omega_M h^2 < 0.35$. This result is then applied to supersymmetric models motivated by generic features in supergravity mediated supersymmetry breaking. The result is an upper bound on gaugino masses within reach of the LHC and a 1.5 TeV lepton collider. Thus, cosmological considerations are beginning to limit the supersymmetric mass spectra in the experimentally verifiable range without recourse to finetuning arguments, and without assuming a zero cosmological constant.

The fields of particle physics and cosmology overlap and enlighten each other in many areas, including inflation, big bang nucleosynthesis, cosmic rays, and dark matter. The cosmology of dark matter, in particular, has been an effective slayer of otherwise reasonable particle physics models. Often times, the amount of cold dark matter left over today is calculable in a theory [1], and may even yield a relic density too high to be compatible with experiment. In supersymmetry, this “relic abundance constraint” has been a powerful one since it generally leads to upper bounds on the mass of supersymmetric particles [2, 3]. No other known physics argument limits the mass of superpartners. For this reason, dark matter relic abundance has a unique role in supersymmetry.

In this letter the recent cosmological measurements of the cosmic microwave background radiation (CMB), age of the universe, Hubble constant, and supernova type Ia data are combined to demonstrate that an upper bound exists on $\Omega_M \equiv \rho_M/\rho_c$ (matter relic abundance), independent of the cosmological constant. These cosmological measurements greatly restrict supersymmetric parameter space. One way to characterize the resulting allowed supersymmetry parameter space is to show mass limits in “generic supersymmetry”. Generic supersymmetry, as it will be defined in later paragraphs, merely implements the expected hierarchy in soft supersymmetry breaking masses: sleptons lighter than squarks, $|\mu| \gg M_1$, etc. In this case, the bino is the lightest supersymmetric partner (LSP), and its relic abundance depends mostly on the mass of right-handed sleptons.

One way to begin a discussion of relic matter density in the universe is to assume that the cosmological constant is zero. Historically, this has been an uncriticized assumption since one would naturally assume that the cosmological constant is greater than m_{weak}^4 or 0. Since m_{weak}^4 is grossly incompatible with experiment by many orders of magnitude, it appears that 0 is the most tenable option. However, recent experiments [4, 5] claim evidence for $\Omega_\Lambda \neq 0$, and recent theoretical work [6] has been entertaining once again the cosmological constant. This opens the mind to a non-zero, but small, cosmological constant.

From a particle physics point of view, limits on $\Omega_M h^2$ have traditionally been applied with the assumption that $\Lambda = 0$. The $\Lambda = 0$ assumption was so pervasive that it was not listed as a qualifier for the $\Omega_M h^2$ bounds presented in the 1996 Particle Data Group Book [7]. The most recent PDG book [8] rightly indicates that the listed bounds on $\Omega_M h^2$ are with $\Lambda = 0$. The most often cited limit on relic matter density is $\Omega_M h^2 < 1$, which is derivable from the assumptions that the universe is more than 10 billion years old, that $\Lambda = 0$, and

that the Hubble constant is greater than $40 \text{ km s}^{-1} \text{ Mpc}^{-1}$. This constraint has been widely applied in the particle physics community to place restrictions on particle parameter space.

If we relax the assumption that $\Lambda = 0$, the age of the universe can be calculated by integrating the Friedmann equation with appropriately scaled matter and vacuum energy densities,

$$t_{\text{age}} = H_0^{-1} \int_0^1 da \sqrt{\frac{a}{(1-a)\Omega_M + (a^3 - a)\Omega_\Lambda + a}}. \quad (1)$$

The lower limit on t_{age} and h alone can no longer put an upper limit on the relic matter density in the universe. It must be combined with another cosmological observable. One such observable is the anisotropy of the cosmic microwave background radiation. Measurements of the lower multipole moment power spectrum by COBE allow one to place bounds on the total energy density today [9, 10]:

$$0.3 < \Omega_M + \Omega_\Lambda < 1.5 \quad (\text{CMBR constraint}). \quad (2)$$

In the $\Omega_M - \Omega_\Lambda$ plane, this restriction is almost perpendicular to the age of the universe constraint. As we can see from Fig. 1, if we utilize both constraints a maximum value of Ω_M is derivable.

Presently, the Hubble constant is known to be $H_0 = 100h \text{ km s}^{-1} \text{ Mpc}^{-1}$, where [8]

$$0.6 < h < 0.8 \quad (\text{Hubble constant range}). \quad (3)$$

New measurements and data analysis [11] indicate that the age of globular clusters is $t_{\text{age}} = 11.5 \pm 1.3 \text{ Gyr}$. This can be used to set a 95% C.L. lower limit on the age of the universe of $t_{\text{age}} > 9.5 \text{ Gyr}$. Therefore, the maximum area that the combined Hubble constant and age of the universe measurements fill in the $\Omega_M - \Omega_\Lambda$ plane is between the lines of $t_{\text{age}} = 9.5 \text{ Gyr}$ with $h > 0.6$ and approximately $t_{\text{age}} = 15 \text{ Gyr}$ with $h < 0.8$. In Fig. 1 these two lines are drawn and labelled, and the arrows indicate the area on the plot allowed.

The maximum acceptable value of Ω_M is therefore at the cross-point of the $t_{\text{age}} = 9.5 \text{ Gyr}$ with $h > 0.6$ line and the $\Omega_M + \Omega_\Lambda < 1.5$ line. Self-consistently using $h = 0.6$, one obtains the limit

$$\Omega_M < 1.7 \quad \text{and} \quad \Omega_M h^2 < 0.6 \quad (\text{limit on matter density}). \quad (4)$$

It is not completely obvious that choosing the constraints with $h = 0.6$ yields the maximum value of $\Omega_M h^2$. If we instead chose a value of h larger than 0.6 and self-consistently checked

where the age of the universe constraint line met the CMBR line, we would find a smaller value for the maximum $\Omega_M h^2$ allowed. Therefore, Eq. 4 is valid for all of allowed cosmological parameter space. The result does not take into account possible tensor fluctuations, etc., which if present, would alter the allowed range [12].

The recent data analysis of type Ia supernovae [4, 5, 13] show evidence for a cosmological constant. Fig. 1 plots in the $\Omega_M - \Omega_\Lambda$ parameter plane the published results of the Supernova Cosmology Project. Estimated *maximum* systematic errors have been included in the plot to make the allowed area conservatively large. The results of the high-Z Supernova Search Team are similar. In Fig. 1 we see that the supernova results are almost parallel in the plane to the age of the universe constraint. Therefore, the supernovae data can effectively replace the direct application of the universe's age measurement in relic matter density considerations. As it stands, the supernovae data combined with the CMBR implies

$$\Omega_M < 0.95 \quad \text{and} \quad \Omega_M h^2 < 0.35 \quad (\text{limit on } \Omega_M \text{ from SN1a and CMBR}), \quad (5)$$

which is presently much more restrictive than the age of universe constraint plus CMBR.

To apply these cosmological results to supersymmetry, a reasonable model of supersymmetry should be defined that has a tractable number of parameters. To this end, one can identify features of supersymmetry that are commonly manifested in different approaches to model building. One is that the soft-scalar masses are generation independent and that the masses fall in a hierarchy defined by the strength of their gauge interactions. This is true, for example, in minimal supergravity models and $SO(10)$ grand unified models with a common mass for all scalars. The hierarchy is generated by logarithms when the masses are renormalized from their high scale values to their low scale values from gauge interactions. It is therefore not unreasonable to assume a generic model of scalar superpartners arranged according to each scalar's gauge interactions. One should also note that gauge-mediated models also demonstrate a hierarchy in scalar masses according to the gauge interaction strength of each scalar. Although the gravitino is generally the LSP in gauge mediation models, it is nevertheless another example of how specific model building usually implies a hierarchy among the scalar states.

The important inference here from the generic supersymmetry model is that $m_{\tilde{t}_R}$ is the lightest scalar, and that the gauginos satisfy GUT relations ($M_i/\alpha_i = \text{const}$). One additional assumption is that the Higgsino mass term μ is sufficiently higher than M_1 so that the

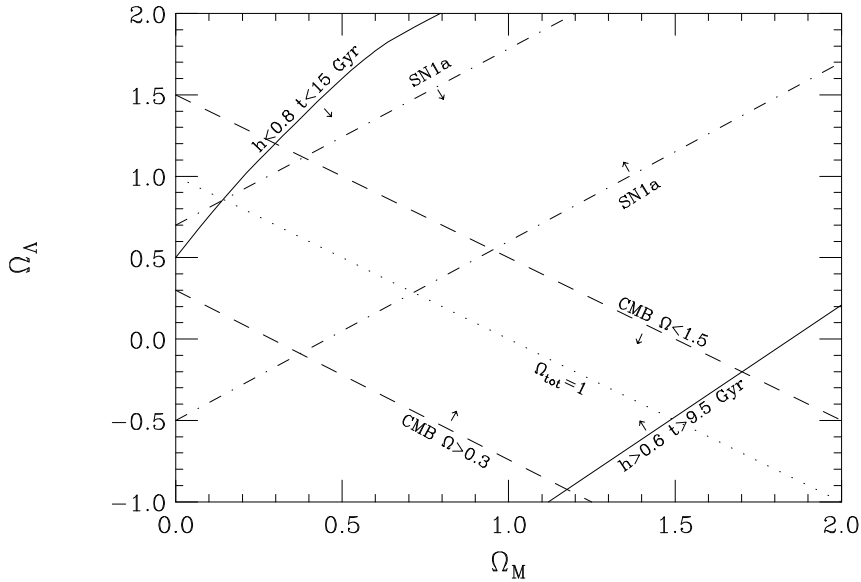


Figure 1: The allowed area from the combination of the Hubble constant measurement and the age of the universe measurement are between the two solid lines. The allowed region from SN1a data is between the two dot-dashed lines. And the allowed region from the cosmic microwave background measurements are between the two dashed lines.

lightest gaugino is almost pure bino [14]. This is a reasonable supposition, and is realized, for example, over most of parameter space in minimal supergravity [3].

If nature is described by a supersymmetric spectrum roughly along the pattern described above, then the relic abundance will be calculable [15, 2] from only two parameters: the masses of $m_{\tilde{l}_R}$ and $m_{\tilde{B}}$. The formula is

$$\Omega_\chi h^2 = \frac{(m_{\tilde{l}_R}^2 + m_\chi^2)^4}{M^2 \sqrt{N_F} m_\chi^2 (m_{\tilde{l}_R}^4 + m_\chi^4)} \quad (6)$$

where $M \simeq 460 \text{ GeV}$ and N_F is the number of degrees of freedom at χ -decoupling.

In Fig. 2 the contours of $\Omega_\chi h^2$ are plotted in the $m_{\tilde{l}_R} - m_{\tilde{B}}$ plane. The old constraint, $\Omega_\chi h^2 < 1$, based on the age of the universe and $\Lambda = 0$ limits [16] $m_{\tilde{l}_R} \lesssim 500 \text{ GeV}$. The corresponding requirement of $m_{\tilde{B}} \lesssim 500 \text{ GeV}$ is similar to the limit of $m_{1/2} \lesssim 1 \text{ TeV}$ found in ref. [3] (with $m_t = 175 \text{ GeV}$). This is another indication that the model discussed above yields very similar results to minimal supergravity, even though generic supersymmetry does not specify the precise values of the squark masses, etc.

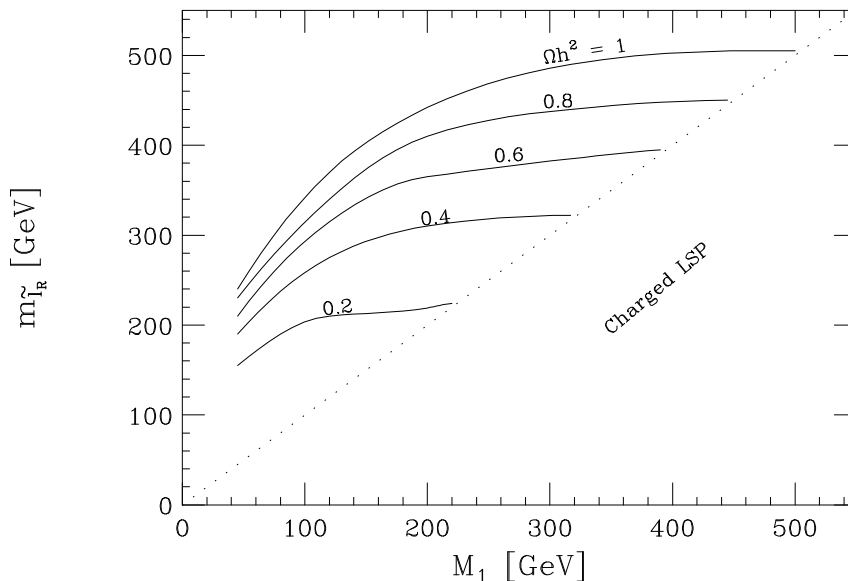


Figure 2: Contours of constant $\Omega_\chi h^2$ in the $m_{\tilde{l}_R} - m_\chi$ plane. The old constraint $\Omega_\chi h^2 < 1$ allowed bino (gluino) masses above 500 GeV (3 TeV), whereas the new constraint $\Omega_\chi h^2 < 0.4$ allows bino (gluino) masses only up to 300 GeV (2 TeV) at most, and therefore is detectable at the LHC. The contours terminate at the left due to chargino mass bounds from LEP2.

The new constraint, $\Omega_M h^2 < 0.35$, based on the supernovae data and the CMB radiation limits [16] $m_{\tilde{l}_R} \lesssim 300$ GeV. This is, of course, a quantitative improvement over the old constraint. However, this is not just to say that a portion of parameter space has been hacked off by this new constraint. Rather, there is a qualitative difference between a 500 GeV and a 300 GeV mass limit on $m_{\tilde{B}}$. In the first case, a 500 GeV bino mass corresponds to a gluino mass over 3 TeV. Gluinos over about 2 – 2.5 TeV are not likely to be detected at the LHC [17]. However, the 300 GeV mass limit on $m_{\tilde{B}}$ corresponds to a mass limit of the gluino of about 2 TeV. This is detectable at the LHC [17], and also a 1.5 TeV center-of-mass energy lepton collider. Of course, these are upper bounds on the mass within the generic supersymmetry model, and the actual mass may be much lower. The conclusion here is that *general expectations of supersymmetry along with cosmology requirements produce a spectrum of supersymmetric states visible at the LHC and a 1.5 TeV lepton collider*. Finetuning arguments are not needed in this framework to make the prediction that these colliders can find superpartners.

The correlation made above between relic abundance of the LSP and detectability at the

LHC and a 1.5 TeV lepton collider is straightforward. This is because the LSP mass correlates with all gaugino masses, and detectability at the colliders is closely linked to gaugino production processes. In other words, a minimum cross-section of non-SM signatures is guaranteed just from gauginos whose properties are known from relic abundance considerations and a few assumptions about the supersymmetry mass spectrum enumerated above. Correlations between dark matter and lower limits from LEP2 collider searches also yield interesting restrictions on parameters space [18].

Correlating dark matter relic abundance in generic supersymmetry with direct searches for dark matter is not so straightforward. For example, a small higgsino component to the LSP will not change the relic abundance calculation in a noteworthy way; however, a small higgsino component can change the cryogenic direct detection rates of LSP-nucleon scattering substantially [19, 20]. Therefore, the direct detection method depends very sensitively on the M_1/μ ratio, whereas the relic abundance does not as long as $|\mu| \gtrsim 2M_1$.

Searches for supersymmetric dark matter from annihilations of LSPs in the galactic halo are also not easily correlated with the relic abundance in generic supersymmetry. The reason is because LSP virial velocity in the galactic halo is only a few hundred kilometers per second, and thus non-relativistic. All annihilations proceed through a helicity suppressed S -wave and want to terminate in a heavy quark, such as the b quark, rather than a lepton. Annihilations in the early universe which dictate the relic abundance are done in a sufficiently relativistic regime such that P -wave annihilations, which are not helicity suppressed, can dominate. Final state leptons from $\chi\chi$ annihilations through a t -channel slepton are of primary importance. Therefore, since a small higgsino content or heavy squarks can mediate significant S -wave annihilations in the galactic halo (non-relativistic limit), they are in principle totally uncorrelated with the relic abundance. For this reason, no firm predictions can be made about these processes, even if one had full knowledge of the density profile of dark matter in the galactic halo.

Although colliders cannot easily tell if a particle lives for more than a few meters of $c\tau$, they are effective probes of dark matter [20, 21]. As was discussed above, collider physics observables are cleanly correlated with relic abundance in generic supersymmetry, whereas all dark matter specific experiments are not necessarily correlated.

In conclusion, $\Omega_M h^2 < 0.35$ is perhaps a more appropriate constraint to apply to particle physics models than the old $\Omega_M h^2 < 1$ constraint. This new constraint relies on the lower

bound of the age of the universe and Hubble constant, SNIa data, and the upper bound of total energy density of the universe from microwave background measurements. It does not depend on $\Lambda = 0$. Applying this constraint to generic supersymmetry – a model of supersymmetry based on the gauge coupling hierarchy – one finds that the allowed parameter space is within reach of the LHC, and a 1.5 TeV lepton collider. Other approaches to supersymmetry breaking which have family dependent masses, for example, also can be correlated with relic abundance constraints [22, 23, 24] and limits can be obtained in these frameworks. In short, recent and forthcoming cosmological parameter measurements are significantly restricting the parameter space of any model of supersymmetry breaking which has a heavy, stable LSP.

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