# Superconducting cosmic string with propagating torsion

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### Abstract

We show that it is possible to construct a consistent model describing a currentcarrying cosmic string endowed with torsion. The torsion contribution to the gravitational force and geodesics of a test-particle moving around the SCCS are analyzed. In particular, we point out two interesting astrophysical phenomena in which the higher magnitude force we derived may play a critical role: the dynamics of compact objects orbiting the torsioned SCCS and accretion of matter onto it. The deficit angle associated to the SCCS can be obtained and compared with data from the Cosmic Background Explorer (COBE) satellite. We also derived a value for the torsion contribution to matter density fluctuations in the early Universe.

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### 1 Introduction

Cosmic strings have exact solutions [1] which represent topological defects that may have been formed during phase transitions in the realm of the Early Universe [2]. The GUT defects carry a large energy density and hence are of interest in Cosmology as potential sources for primordial density perturbations. These fluctuations would leave their imprint in the cosmic microwave background radiation (CMBR); a prediction not ruled out by COBE satellite observations yet[3], and hence would act as seeds for structure formation and thus as builders of the largestscale structures in the Universe[4], such as the very high redshift superclusters of galaxies as for instance the *great wall*. They may also help to explain the most energetic events in the Universe such as the cosmological gamma-ray bursts (GRBs)[5], ultra high energy cosmic rays (UHECRs) and very high energy neutrinos[4] and gravitational-wave bursts and backgrounds[6]. All these are issues deserving continuous investigation by many physicists nowadays [7].

Witten [8] has shown that the cosmic strings may possess superconducting properties and may behave like bosonic (see Ref.[[9]] and references therein) or fermionic strings[10]. In other communications it was supposed that the relevant superconductivity is generated during or very soon after the primary phase transition in which the string is formed. Cartan torsion has been connected previously with ordinary cosmic strings [11, 12] and also spinning cosmic strings [13] from quite distinct point of views.

In this work we consider the study case of bosonic SCCSs in Riemann-Cartan space-time with coupling terms in the potential. One should regard such an extension as a first step of a comprehensive study of cosmic string models in the context of theories including torsion [14]. We aim at dealing with most realistic models which demand supersymmetry, an essential ingredient of grand-unification theories, string theory, etc. Thus, we ought to combine both gravitational and spin degrees of freedom in the same formalism, and thus torsion is required.

The main-stream of this paper is as follows: we explored the physics of torsion coupling to cosmic strings in section II. An external solution for the SCCS metric in this scenario in presented in section III, while in section IV we derive the corresponding one for the internal structure of the SCCS by using the weak-field approximation. Two applications are provided. One focusing on the deviation of a particle moving near a torsion string. It is shown that such a high intensity of the gravitational force from the screwed SCCS (when compared with the one generated by a current-carrying string) may have important effects on the dynamics of compact objects orbiting around it, and also on matter being accreted by the string itself. The second one exploits the possibility that the temperature fluctuations in the cosmic microwave background radiation could have been, at least partially, generated after the SCCSs having interacted with it. We obtain a neat expression for the deficit angle in this context and a comparison is done with data from COBE satellite. We end this paper with a short summary of the picture here suggested.

### 2 Torsion coupling to cosmic strings

Here we construct a consistent framework for the torsion field pervading a cosmic string and define the vortex configuration for this problem. We choose here to analyse the simplest case where the torsion appears. In this line of reasoning, it is possible to describe torsion as a gradient-like field [16]

$$S_{\mu\nu}^{\ \lambda} = \frac{1}{2} [\delta^{\lambda}_{\mu} \partial_{\nu} \Lambda - \delta^{\lambda}_{\nu} \partial_{\mu} \Lambda], \qquad (2.1)$$

being the  $\Lambda$  field the source of torsion in the string that have only r-dependence.

The action representing the SCCS in a space-time with torsion can be written as:

$$S = \int d^4x \sqrt{g} \left[ \frac{1}{16\pi G} R(\{\}) + \alpha_1 \nabla_\mu S^\mu + \alpha_2 S_{\mu\nu k} S^{\mu\nu k} + \alpha_3 S_{\mu\nu k} S^{\mu k\nu} + \alpha_4 S_\mu S^\mu \right] + S_m, \quad (2.2)$$

where  $R(\{\})$  is the curvature scalar of the Riemannian theory and  $S_m$  is the matter action that describes the superconducting cosmic string (to be specified below). Here  $\alpha_1, \alpha_2, \alpha_3$  and  $\alpha_4$  are coupling arbitrary constants with  $\alpha_1$  connected with the torsion gradient term  $\nabla_{\mu}S^{\mu}$ . Where  $\nabla_{\mu}$  is a Riemannian covariant derivative which drops out from the action because it is a term involving a total derivative.  $S_{\mu\nu k}$  and  $S_{\mu}$  are SO(1,3) irreducible components of the torsion. For this extended Riemann-Cartan space (see Ref.[[17]] for a review), the affine connection can be written in terms of  $g_{\mu\nu}$  and  $S_{\alpha} = \partial_{\alpha}\Lambda$  as

$$\Gamma_{\lambda\nu}^{\ \alpha} = \{^{\alpha}_{\lambda\nu}\} + S^{\alpha}g_{\lambda\nu} - S_{\lambda}\delta^{\alpha}_{\nu} \tag{2.3}$$

so that  $S_{\mu}$  is the only piece that contributes to torsion, which here is the escalar derivative defined by Eq.(2.1).

Then we may consider a theory of *gravitation* possessing torsion by writing that part of the action  $S_G$  stemming from the curvature scalar R as:

$$S_G = \int d^4x \sqrt{g} \left[ \frac{1}{16\pi G} R(\{\}) - \frac{\alpha}{2} \partial_\mu \Lambda \partial^\mu \Lambda \right], \qquad (2.4)$$

where the coupling constant  $\alpha$  is related with  $\alpha_2$ ,  $\alpha_3$  and  $\alpha_4$  and will be specified with the help of COBE data.

We can study the SCCS considering the Abelian Higgs model with two scalar fields,  $\phi$  and  $\tilde{\Sigma}$ . In this case, the action for all matter fields turns out to be:

$$S_m = \int d^4x \sqrt{g} \left[ -\frac{1}{2} D_\mu \phi (D^\mu \phi)^* - \frac{1}{2} D_\mu \Sigma (D^\mu \Sigma)^* - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{4} H_{\mu\nu} H^{\mu\nu} - V(|\phi|, |\Sigma|, \Lambda) \right], \quad (2.5)$$

where  $D_{\mu}\Sigma = (\partial_{\mu} + ieA_{\mu})\Sigma$  and  $D_{\mu}\phi = (\partial_{\mu} + iqC_{\mu})\phi$  are the covariant derivatives. The reason why the gauge fields do not minimally couple to torsion is well discussed in the works

of references [18, 19]. The field strengths are defined as usually as  $F_{\mu\nu} = \partial_{\mu}A_{\nu} - \partial_{\nu}A_{\mu}$  and  $H_{\mu\nu} = \partial_{\mu}C_{\nu} - \partial_{\nu}C_{\mu}$ , with  $A_{\mu}$  and  $C_{\nu}$  being the gauge fields.

The potential  $V(\varphi, \sigma, \Lambda)$  triggering the symmetry breaking can be fixed by:

$$V(\varphi,\sigma,\Lambda) = \frac{\lambda_{\varphi}}{4}(\varphi^2 - \eta^2)^2 + f_{\varphi\sigma}\varphi^2\sigma^2 + \frac{\lambda_{\sigma}}{4}\sigma^4 - \frac{m_{\sigma}^2}{2}\sigma^2 + l^2\sigma^2\Lambda^2, \qquad (2.6)$$

where  $\lambda_{\varphi}$ ,  $\lambda_{\sigma}$ ,  $f_{\varphi\sigma}$  and  $l^2$  are coupling constants, and the boson mass being defined by  $m_{\sigma}$ .

This action (Eq.2.2) has a  $U(1)' \times U(1)$  symmetry, where the U(1)' group, associated with the  $\phi$ -field, is broken by the vacuum and gives rise to vortices of the Nielsen-Olesen[20]

$$\phi = \varphi(r)e^{i\theta}, \qquad C_{\mu} = \frac{1}{q}[P(r) - 1]\delta^{\theta}_{\mu}, \qquad (2.7)$$

parametrized in cylindrical coordinates  $(t, r, \theta, z)$ , where  $r \ge 0$  and  $0 \le \theta < 2\pi$ . The boundary conditions for the fields  $\varphi(r)$  and P(r) are the same as those of ordinary cosmic strings[20]:

$$\varphi(r) = \eta \quad r \to \infty \quad \varphi(r) = 0 \quad r = 0 \qquad P(r) = 0 \quad r \to \infty \qquad P(r) = 1 \quad r = 0.$$
(2.8)

The other U(1)- symmetry, that we associate to electromagnetism, acts on the  $\Sigma$ -field. This symmetry is not broken by the vacuum; however, it is broken in the interior of the deffect. The  $\Sigma$ -field in the string core, where it acquires an expectation value, is responsible for a bosonic current being carried by the gauge field  $A_{\mu}$ . The only non-vanishing components of the gauge fields are  $A_z(r)$  and  $A_t(r)$  and the current-carrier phase may be expressed as  $\zeta(z,t) = \omega_1 t - \omega_2 z$ . Notwithstanding, we focus only on the magnetic case [9]. Their configurations are defined as:

$$\Sigma = \sigma(r)e^{i\zeta(z,t)}, \qquad A_{\mu} = \frac{1}{e}[A(r) - \frac{\partial\zeta(z,t)}{\partial z}]\delta_{\mu}^{z}, \qquad (2.9)$$

because of the rotational symmetry of the string itself. The fields responsible for the cosmic string superconductivity have the following boundary conditions:

$$\frac{d}{dr}\sigma(r) = 0 \quad r = 0 \quad \sigma(r) = 0 \quad r \to \infty \qquad A(r) \neq 0 \quad r \to \infty \qquad A(r) = 1 \quad r = 0.$$
(2.10)

Let us consider a SCCS in a cylindrical coordinate system  $(t, r, \theta, z)$ , so that  $r \ge 0$  and  $0 \le \theta < 2\pi$  with the metric defined in these coordinates as:

$$ds^{2} = e^{2(\gamma - \psi)}(-dt^{2} + dr^{2}) + \beta^{2}e^{-2\psi}d\theta^{2} + e^{2\psi}dz^{2}, \qquad (2.11)$$

where  $\gamma, \psi$  and  $\beta$  depend only on r. We can write Einstein-Cartan equations in the quasi-Einsteinian form:

$$G^{\mu}_{\nu}(\{\}) = 8\pi G (2\alpha g^{\mu\alpha} \partial_{\alpha} \Lambda \partial_{\nu} \Lambda - \alpha \delta^{\mu}_{\nu} g^{\alpha\beta} \partial_{\alpha} \Lambda \partial_{\beta} \Lambda + T^{\mu}_{\nu}) = 8\pi G \tilde{T}^{\mu}_{\nu}$$
(2.12)

where ({}) stands for Riemannian geometric objects,  $\delta^{\mu}_{\nu}$  and  $T^{\mu}_{\nu}$  correspond to the identity and energy-momentum tensors, respectively.  $\tilde{T}^{\mu}_{\nu}$  tensor corresponds to an energy-momentum tensor containing the torsion field.

We have seen that the dependence upon torsion is represented, in the quasi-Einstenian form, by the  $\Lambda$ -field that has an equation of motion given by Eq.(2.18) below, whose solution shall be addressed subsequently.

The SCCS energy-momentum tensor is defined by  $T^{\mu}_{(scs)\nu} = \frac{2}{\sqrt{g}} \frac{\delta S_m}{\delta g_{\mu\nu}}$ , which yields:

$$T_{scs\ t}^{t} = -\frac{1}{2} \left\{ e^{2(\psi-\gamma)} [\varphi'^{2} + \sigma'^{2}] + \frac{e^{2\psi}}{\beta^{2}} \varphi^{2} P^{2} + e^{-2\psi} \sigma^{2} A^{2} + \frac{e^{2(2\psi-\gamma)}}{\beta^{2}} (\frac{P'}{q})^{2} + e^{-2\gamma} (\frac{A'}{e})^{2} + 2V(\varphi, \sigma, \Lambda) \right\}$$
(2.13)

$$T_{scs\ r}^{r} = \frac{1}{2} \left\{ e^{2(\psi-\gamma)} [\varphi'^{2} + \sigma'^{2}] - \frac{e^{2\psi}}{\beta^{2}} \varphi^{2} P^{2} - e^{-2\psi} \sigma^{2} A^{2} + \frac{e^{2(\psi-\gamma)}}{\beta^{2}} (\frac{P'}{q})^{2} + e^{-2\gamma} (\frac{A'}{e})^{2} - 2V(\varphi, \sigma, \Lambda) \right\} \right\}$$
(2.14)

$$T_{scs\ \theta}^{\theta} = -\frac{1}{2} \left\{ e^{2(\psi-\gamma)} [\varphi'^2 + \sigma'^2] - \frac{e^{2\psi}}{\beta^2} \varphi^2 P^2 + e^{-2\psi} \sigma^2 A^2 + -\frac{e^{2(\psi-\gamma)}}{\beta^2} (\frac{P'}{q})^2 + e^{-2\gamma} (\frac{A'}{e})^2 + 2V(\varphi,\sigma,\Lambda) \right\}$$
(2.15)

$$T_{scs\ z}^{z} = -\frac{1}{2} \left\{ e^{2(\psi-\gamma)} [\varphi'^{2} + \sigma'^{2}] + \frac{e^{2\psi}}{\beta^{2}} \varphi^{2} P^{2} - e^{-2\psi} \sigma^{2} A^{2} + \frac{e^{2(\psi-\gamma)}}{\beta^{2}} (\frac{P'}{q})^{2} - e^{-2\gamma} (\frac{A'}{e})^{2} + 2V(\varphi, \sigma, \Lambda) \right\}.$$
(2.16)

In these expressions Eqs.(2.13-2.16) only the usual fields of the string are present. The Euler-Lagrange equations result from the variation of the Eq.(2.2) together with the conditions for the Nielsen-Olesen [20] vortex Eqs.(2.7-2.9), and yield:

$$\begin{aligned} \varphi'' + \frac{1}{r}\varphi' + \frac{\varphi P^2}{r^2} - \varphi[\lambda_{\varphi}(\varphi^2 - \eta^2) + 2f_{\varphi\sigma}\sigma^2] &= 0 \\ \sigma'' + \frac{1}{r}\sigma' + \sigma[A^2 + (f_{\varphi\sigma}\varphi^2 + \lambda_{\sigma}\sigma^2 - m_{\sigma}^2 + l^2\Lambda^2)] &= 0 \\ P'' - \frac{1}{r}P' - q^2\varphi^2 P &= 0, \quad A'' + \frac{1}{r}A' + e^2\sigma^2 A &= 0, \end{aligned} \tag{2.17}$$

while the torsion wave equation is given by:

$$\Box_g \Lambda = \frac{l^2}{\alpha} \sigma^2 \Lambda. \tag{2.18}$$

Above, a prime denotes differentiation with respect to the radial coordinate r. The general solution for the SCCS will be found in the weak-field approximation together with junction conditions for the external metric.

### 3 The external solution

Now, we proceed to solve the previous set of equations for an observer outside the SCCS stressed by torsion, focusing on the external metric which satisfies the constraint  $r_0 \leq r \leq \infty$ . The external contribution to the energy-momentum of the string reads

$$\mathcal{T}^{\mu}_{\nu} = \frac{1}{4} g^{\mu\alpha} g^{\beta\rho} F_{\alpha\beta} F_{\nu\rho} - \delta^{\mu}_{\nu} g^{\sigma\alpha} g^{\beta\rho} F_{\sigma\beta} F_{\alpha\rho}.$$
(3.1)

This tensor is the external energy-momentum tensor of a SCCS with no torsion. If we observe the asymptotic conditions, Eq.(2.8) and Eq.(2.10), we see that the only field that does not vanish is the  $A_{\mu}$ -field that is responsible for carrying off the string the effects of the current on it. The torsion contribution to the external energy-momentum tensor is given by

$$\mathcal{T}^{\mu}_{\nu_{tors}} = 2\alpha g^{\mu\alpha} \partial_{\alpha} \Lambda \partial_{\nu} \Lambda - \alpha \delta^{\mu}_{\nu} g^{\alpha\beta} \partial_{\alpha} \Lambda \partial_{\beta} \Lambda.$$
(3.2)

For this configuration, the energy-momentum tensor displays the following symmetry properties:

$$\mathcal{T}_t^t = -\mathcal{T}_r^r = \mathcal{T}_\theta^\theta = -\mathcal{T}_z^z.$$
(3.3)

Then, the only one component of  $\Lambda$  in Eq.(2.18) to be solved is the *r*-dependent function  $\Lambda(r)$ . The solution reads:

$$\Lambda(r) = \lambda \ln(r/r_0). \tag{3.4}$$

The vacuum solution of Eqs.(2.12) are found from the symmetries (3.3). Hence the solutions of  $\beta(r)$  and  $\gamma(r)$  are given by

$$\beta = Br, \qquad \gamma = m^2 \ln r/r_0. \tag{3.5}$$

To find the  $\psi$ -solution, we can use the condition:

$$R = 2\Lambda'^2 e^{2(\psi - \gamma)}.$$
(3.6)

This condition is different from the usual one[9] because the scalar curvature R does not vanish, and opposedly it is linked to the torsion-field  $\Lambda$ . Then, this condition has the same form as the one for a SCCS in a scalar-tensor theory [15]. By making use of solutions (3.4), (3.5), we find:

$$\psi = n \ln \left( r/r_0 \right) - \ln \frac{(r/r_0)^{2n} + k}{(1+k)}.$$
(3.7)

Thus we see that from the solutions of the SCCS Eqs.(3.5,3.7), there exists a relationship between the parameters  $n, \lambda$  and m given by  $n^2 = \lambda^2 + m^2$ .

With the above results, we find that the external metric for the SCCS takes the form:

$$ds^{2} = \left(\frac{r}{r_{0}}\right)^{-2n} W^{2}(r) \left[\left(\frac{r}{r_{0}}\right)^{2m^{2}} \left(-dt^{2} + dr^{2}\right) + B^{2}r^{2}d\theta^{2}\right] + \left(\frac{r}{r_{0}}\right)^{2n} \frac{1}{W^{2}(r)}dz^{2}, \qquad (3.8)$$

with  $W(r) = [(r/r_0)^{2n} + k]/[1+k].$ 

The external solution alone does not provide a complete description of the physical situation. We proceed hereafter to find the junction conditions to the internal metric in order to obtain an appropriate accounting for the nature of the source and its effects on the surrounding spacetime.

### 4 SCCS solution: The weak-field approximation

Nowlet us find the Einstein-Cartan solutions for a SCCS by considering the weak-field approximation. Thus, the space-time metric may be expanded in terms of a small parameter  $\varepsilon$  about the values  $g_{(0)\mu\nu} = diag(-1, 1, 1, 1)$ , then:

$$g_{\mu\nu} = g_{(0)\mu\nu} + \varepsilon h_{\mu\nu}, \qquad \tilde{T}_{\mu\nu} = \tilde{T}_{(0)\mu\nu} + \varepsilon \tilde{T}_{(1)\mu\nu}.$$
 (4.1)

The  $\tilde{T}_{(0)\mu\nu}$  tensor corresponds to the energy-momentum tensor in a space-time with no curvarture. However, torsion is embeeded.  $\tilde{T}_{(1)\mu\nu}$  represents the part of the energy-momentum tensor containing curvature and torsion. Next we proceed to define some important quantities useful for the analysis to come.

The energy-momentum density and tension of the thin SCCS are given by:

$$U = -2\pi \int_0^{r_0} \tilde{T}^t_{(0)t} r dr; \qquad T = -2\pi \int_0^{r_0} \tilde{T}^z_{(0)z} r dr \qquad (4.2)$$

The remaining components follows as

$$X = -2\pi \int_0^{r_0} \tilde{T}^r_{_{(0)}r} r dr; \qquad Y = -2\pi \int_0^{r_0} \tilde{T}^\theta_{_{(0)}\theta} r dr.$$
(4.3)

The energy conservation in the weak-field approximation, reduces to

$$r\frac{dT_{(0)}^{r}r}{dr} = (\tilde{T}_{(0)\theta}^{\theta} - \tilde{T}_{(0)r}^{r}), \qquad (4.4)$$

where  $\tilde{T}_{(0)\mu\nu}$  represents the trace of the energy-momentum tensor with tprsion.

For computing the overall metric, we use the Einstein-Cartan in the quasi-Einsteinian Eq.(2.12), where it gets the form  $G^{\mu\nu}(\{\}) = 8\pi G \tilde{T}^{\mu\nu}_{(0)}$  in the weak-field approximation, with the tensor  $\tilde{T}_{(0)\mu\nu}$  (being first order in G) containing torsion. After integration we have:

$$\int_{0}^{r_{0}} r dr (\tilde{T}^{\theta}_{(0)\theta} + \tilde{T}^{r}_{(0)r}) = r_{0}^{2} \tilde{T}^{r}_{(0)r}(r_{0}) = r_{0}^{2} \left[ \frac{A^{\prime 2}(r_{0})}{2e^{2}} + \frac{\alpha}{2} \Lambda^{\prime 2}(r_{0}) \right].$$
(4.5)

To find the internal energy-momentum tensor, it is more convenient to use Cartesian coordinates[9]. For this purpose we proceed to calculate the cross-section integrals of  $\tilde{T}^x_{_{(0)}x}$  and  $\tilde{T}^y_{_{(0)}y}$  that in cartesian coordinates reads

$$\tilde{T}_{(0)x}^{x} = c[\varphi'^{2} + \sigma'^{2} + \left(\frac{A'}{e}\right)^{2} + \alpha\Lambda'^{2}] + s\frac{\varphi^{2}P^{2}}{r^{2}} + \frac{1}{2}\left(\frac{P'}{qr}\right)^{2} - \frac{1}{2}\sigma^{2}A^{2} - 2V$$

$$\tilde{T}_{(0)y}^{y} = s[\varphi'^{2} + \sigma'^{2} + \left(\frac{A'}{e}\right)^{2} + \alpha\Lambda'^{2}] + c\frac{\varphi^{2}P^{2}}{r^{2}} + \frac{1}{2}\left(\frac{P'}{qr}\right)^{2} - \frac{1}{2}\sigma^{2}A^{2} - 2V,$$
(4.6)

where  $c = \cos^2 \theta - \frac{1}{2}$  and  $s = \sin^2 \theta - \frac{1}{2}$ . This way we found:

$$\int r dr d\theta \tilde{T}^x_{(0)x} = \int r dr d\theta \tilde{T}^y_{(0)y} = \pi \int r dr \left[\left(\frac{P'}{qr}\right)^2 - \sigma^2 A^2 - V\right] = -W.$$
(4.7)

Using the fact that  $\tilde{T}^r_{(0)r} + \tilde{T}^{\theta}_{(0)\theta} = \tilde{T}^x_{(0)x} + \tilde{T}^y_{(0)y}$ , then we have:

$$X + Y = 2W = -2\pi r_0^2 \left[ \frac{A^{\prime 2}(r_0)}{e^2} + \alpha \Lambda^{\prime 2}(r_0) \right], \qquad (4.8)$$

which can be computed by integration of Eq.(2.17)

$$A'(r) = \frac{eJ}{\sqrt{2\pi}r}, \qquad J = \sqrt{2\pi}e \int_0^{r_0} r dr \sigma^2 A,$$
 (4.9)

where J is the current density. Thus, the torsion density can be computed by integration of Eq.(2.18)

$$\Lambda' = \frac{S}{\sqrt{2\pi\alpha r}}, \qquad S = \sqrt{2\pi}l^2 \int_0^{r_0} r dr \sigma^2 \Lambda, \qquad (4.10)$$

where S is the torsion density. With these considerations we found the string structure. Then, we obtain

$$W = -\frac{1}{2\pi} \left( J^2 + \nu S^2 \right).$$
(4.11)

with  $\nu = 1/\alpha$ .

In addition, we can assume that the string is infinitely thin so that its stress-energy tensor is given by

$$\tilde{T}_{string}^{\mu\nu} = diag[U, -W, -W, -T]\delta(x)\delta(y).$$
(4.12)

It worths to note that definitions for both string energy U and tension T, as in equations (4.2), already incorporate information on the torsion.

By virtue of the presence of the external current we use the form Eq.(4.12) for the string energy-momentum tensor as well as Eq.(3.1) and Eq.(3.2) for the external energy-momentum tensor in linearized solution to zeroth order in G. In the sense of distributions we have,  $\nabla^2 ln(r/r_0) = 2\pi\delta(x)\delta(y), \nabla^2(\ln(r/r_0))^2 = 2/r^2$  and  $\nabla^2(r^2\partial_i\partial_j ln(r/r_0)) = 4\partial_i\partial_j ln(r/r_0)$ .

The energy-momentum tensor of the string source  $\tilde{T}_{(0)\mu\nu}$ , in Cartesian coordinates, possesses no curvature, which is the well-known result [9, 15], but does have torsion which produces the following energy-momentum tensor

$$\tilde{T}_{(0)tt} = U\delta(x)\delta(y) + \frac{(J^2 + \nu S^2)}{4\pi} \nabla^2 \left( ln\frac{r}{r_0} \right)^2, 
\tilde{T}_{(0)zz} = -T\delta(x)\delta(y) + \frac{(J^2 - \nu S^2)}{4\pi} \nabla^2 \left( ln\frac{r}{r_0} \right)^2, 
\tilde{T}_{(0)ij} = (J^2 + \rho S^2)\delta_{ij}\delta(x)\delta(y) - \frac{(J^2 + \nu S^2)}{2\pi} \partial_i \partial_j ln(r/r_0),$$
(4.13)

where the trace is given by  $\tilde{T}_{(0)} = -(U + T - J^2 - \nu S^2)\delta(x)\delta(y) - \frac{\nu S^2}{2\pi}\nabla^2 \left(\ln \frac{r}{r_0}\right)^2$ . Now let us find the matching conditions to the external solution. For this purpose, we shall

use the linearized Einstein-Cartan equation in the form

$$\nabla h_{\mu\nu} = -16\pi G(\tilde{T}_{(0)\mu\nu} - \frac{1}{2}g_{(0)\mu\nu}\tilde{T}_{(0)}).$$
(4.14)

The internal solution to equation (4.14) with source yields:

$$h_{tt} = -4G[J^{2}(\ln(r/r_{0}))^{2} + (U - T + J^{2} + \nu S^{2})\ln(r/r_{0})]$$
  

$$h_{zz} = -4G[J^{2}\ln(r/r_{0}))^{2} + (U - T - J^{2} - \nu S^{2})\ln(r/r_{0})]$$
  

$$h_{ij} = -2G(J^{2} + \nu S^{2})r^{2}\partial_{i}\partial_{j}\ln(r/r_{0}) - 4G\delta_{ij}\left[(U + T + J^{2} + \nu S^{2})\ln(r/r_{0})) + S^{2}\left(\ln\frac{r}{r_{0}}\right)^{2}\right].$$
(4.15)

This corresponds to the solution in Cartesian coordinates. We note that the torsion appears explicitly in the transverse components of the metric. To analyse the solution for the junction condition to the external metric let us transform it back into cylindrical coordinates.

#### 5 Matching Conditions

It is possible to find the matching conditions [22] to the external solution. In the case of a spacetime with torsion we can find the junction conditions using the fact that  $[\{\alpha_{\mu\nu}\}]_{r=r_0}^{(+)} = [\{\alpha_{\mu\nu}\}]_{r=r_0}^{(-)}$ , and the metricity condition  $[\nabla_{\rho}g_{\mu\nu}]_{r=r_0}^{+} = [\nabla_{\rho}g_{\mu\nu}]_{r=r_0}^{-} = 0$ , to find the continuity conditions

$$[g_{\mu\nu}]_{r=r_0}^{(-)} = [g_{\mu\nu}]_{r=r_0}^{(+)},$$
  
$$[\frac{\partial g_{\mu\nu}}{\partial x^{\alpha}}]_{r=r_0}^{(+)} + 2[g_{\alpha\rho}K_{(\mu\nu)}^{\ \rho}]_{r=r_0}^{(+)} = [\frac{\partial g_{\mu\nu}}{\partial x^{\alpha}}]_{r=r_0}^{(-)} + 2[g_{\alpha\rho}K_{(\mu\nu)}^{\ \rho}]_{r=r_0}^{(-)}$$
(5.1)

where (-) represents the internal region and (+) corresponds the external region around  $r = r_0$ . In analysing the junction conditions we notice that the contortion contributions do not appear neither in the internal nor in the external regions [22, 23].

To match our solution with the external metric we used the metric in cylindrical coordinates, which is obtained from the coordinate transformations:

$$r^2 \partial_i \partial_j ln(r/r_0) dx^i dx^j = r^2 d\theta^2 - dr^2, \tag{5.2}$$

Unfortunately, for this goal we cannot use the metric the way it stands. Therefore, we have to change the radial coordinate to  $\rho$ , using the constraint (symmetry)  $g_{\rho\rho} = -g_{tt}$ , to have, to first order in G,  $\rho = r[1 + a_1 - a_2 \ln(r/r_0) - a_3(\ln(r/r_0)^2)]$ .

In this case we have  $a_1 = G(4U + J^2 + \nu S^2)$ ,  $a_2 = 4GU$  and  $a_3 = -2G(J^2 + \nu S^2)$ , which corresponds to the magnetic configuration of the string fields[9]. The transformed metric yields:

$$g_{tt} = -\{1 + 4G[J^{2}(\ln(\rho/r_{0}))^{2} + (U - T + J^{2} + \nu S^{2})\ln(\rho/r_{0})]\} = -g_{\rho\rho}$$
  

$$g_{zz} = \{1 - 4G[(J^{2} + \nu S^{2})(\ln(\rho/r_{0}))^{2} + (U - T + J^{2} - \nu S^{2})\ln(\rho/r_{0})]\}$$
  

$$g_{\theta\theta} = \rho^{2}\{1 - 8G(U + \frac{(J^{2} + \rho S^{2})}{2}) + 4G(U - T - J^{2} - \rho S^{2})\ln(\rho/r_{0})] + 4GJ^{2}(\ln(\rho/r_{0}))^{2}\}.$$
(5.3)

Now we can find the external parameters B, n and m as functions of the source structure. If we consider the junction of the equation (5.1), after the linearization, and using the limit  $|n \ln(\rho/r_0)| \ll 1$ , we have:

$$n\left(\frac{1-k}{1+k}\right) = 2G\left(U - T - J^2 - \nu S^2\right)$$
  

$$B^2 = 1 - 8G\left(U + \frac{(J^2 + \rho S^2)}{2}\right)$$
  

$$m^2 = 4G(J^2 + \nu S^2).$$
(5.4)

and using the derivative of the expression Eq.(3.4) and the Eq.(4.10), we find

$$\lambda = \frac{\nu}{\sqrt{2\pi}}S.$$
(5.5)

This expression completes the derivation of the full metric components. In analysing the metric of the SCCS with torsion we note that the contribution of torsion appears in the  $\theta\theta$ -metric component, which is important in astrophysical applications such as gravitational lensing studies because this component is linked to the deficit angle. Next we present two preliminary applications of the formalism here introduced assuming that such a kind of torsioned SCCSs really exist. Firstly, we focus on the issue of the deviation of a particle moving near the string, and later on we attempt to perform a comparative analysis of the observations performed by the COBE satellite, with the effects this sort of string may produce on the CMBR, supposed to interact with it as discussed in this paper.

### 6 Particle deflection near a torsion SCCS

We know that when the string possesses current there appear gravitational forces. We shall consider the effect that torsion plays on the gravitational force generated by SCCS on a particle moving around the defect, initially with no charge. We consider the particle speed  $|\mathbf{v}| \leq 1$ , condition under which the geodesic equation becomes:

$$\frac{d^2x^i}{d\tau^2} + \Gamma^i_{tt} = 0, \tag{6.1}$$

where i is the spatial coordinate and the connection can be written as Eq.(2.3)), in this manner the gravitational force of the string (per unit length) gets the form

$$F_G = \frac{1}{2} (\nabla h_{tt} - \frac{S}{\rho}), \tag{6.2}$$

with  $g_{tt} = -1 - h_{tt}$  in Eq.(5.3). We also note that the gravitational force is related to the  $h_{tt}$  component that has no explicit dependence on the torsion. From the last equation, the force the SCCS exerts on a test particle can be explicitly written as

$$F_G = -\frac{1}{\rho} \left[ 2GJ^2 \left( 1 + \frac{(U - T + \nu S^2)}{J^2} + 2\ln(\rho/r_0) \right) + S \right].$$
(6.3)

A quick glance at the last equation allows us to understand the essential role torsion may in the context of the present formalism. As we show below, this extra-term yields an amplification of the total force a particle close to the SCCS will undergo.

If torsion is present, even in the case the string has no current, an attractive gravitational force appears. In the context of the SCCS torsion acts as a enhancer of the force a test particle feels outside the string. In our summary, we discuss a bit further potential applications of this new result to astrophysics and cosmology.

## 7 Angular deficit and COBE map

Recently many works [24, 25] have shown that the COBE data are compatible with Einstein-Cartan gravity. In this section, we analyse the effects of a screwed superconducting cosmic string on the primordial microwave background radiation using the COBE data. To this end, we need to compute the angular deficit introduced by torsion. The hidden idea here is that the cosmic large-scale density fluctuations could have had origin during the appearance of Cosmic String defects or due to interaction with them during the late stages of the Universe's evolution. The torsion would modify properties of light and radiation interacting with a cosmic string pervaded by screw dislocations in such a way that the density fluctuations induced might match those ones measured by COBE[3]. The DMR (Differential Microwave Radiometer) instrument of COBE has provided temperature sky maps leading to the rms sky variation where the beam separation in the COBE experiment is  $\theta_1 - \theta_2 = 60^{\circ}$ .

Each string that affects the photon beam induces a temperature variation [26, 27], of the order of magnitude as  $\frac{\delta T}{T} \sim \delta \leq 10^{-6}$  (COBE), where  $\delta$  is the angular deficit. If we consider the metric Eqs.(5.3), projected into the space-time perpendicular to the string, i. e., dz = 0, then we have:

$$ds_{\perp}^{2} = (1 - h_{tt})[-dt^{2} + dr^{2} + (1 - b)r^{2}d\theta^{2}], \qquad (7.1)$$

with  $h_{tt}$  given by Eq.(5.3), and b calculated from junction conditions using  $\beta^2 = (1-b)\rho^2$ . Then, in first order in G, the deficit angle gets:

$$\delta = b\pi = 8\pi G \{ U + \frac{J^2 (1 + \ln(\rho/r_0))}{2} + \frac{1}{2}\nu S^2 (1 - 2\ln(\rho/r_0)) \}.$$
(7.2)

We can interpret this angular deficit  $\delta$  as being due to three different contributions:  $\delta_s$  to an ordinary cosmic string,  $\delta_J$  to the current and  $\delta_{tors}$  to the torsion field, respectively. In the case of the ordinary cosmic string the angular deficit is given by  $\delta_s = 8\pi GU$ , which in this work corresponds to the case where both current J and torsion S vanish. In this situation[28], it is demonstrated that cosmic string models are more consistent with the COBE data [29] for a wider range of cosmological parameters than the standard CDM models, and the numerical simulations have confirmed these predictions [30].

When the cosmic string carries current, we have used results of Ref.[31] for the current, that is, a configuration with the maximum current  $J \sim \eta$ , for  $\eta = 10^{16} GeV$  as well-known for grand unification theories. In such a case, we found  $\delta_J = 8\pi G J^2 \sim 10^{-6}$  or less what is compatible with COBE data. As it is easy to see, we neglected the logarithmic term because we consider the experiment is being performed close to the string surroundings.

However, in the situation where the cosmic string is stressed by torsion, the issue is more difficult because we have no idea about the energy density torsion puts in the Universe via cosmic strings. Therefore, if a cosmic string actually formed (and it is a good mechanism to generate density fluctuations that can be measured by COBE), then we can estimate the density of torsion the string induces in the cosmic background. To this purpose we choose the value  $\nu \sim 10^{38} GeV^2 \sim 1/G$ ; in this case, we have the torsion energy density  $S \sim 10^{-3}$  with  $\delta_{tors} = 8\pi G\nu S^2$ . As one can check by substituting in the previous section, the inferred value for S enlarges the intensity of the net force undergone by a test particle encircling the SCCS.

### 8 Summary

It is possible for torsion to have had a physically relevant role during the early stages of the Universe's evolution. Along these lines, torsion fields may be potential sources of dynamical stresses which, when coupled to other fundamental fields (i. e., the gravitational field), might have performed an important action during the phase transitions leading to formation of topological defects such as the SCCSs here we focused on. It therefore seems a crucial issue to investigate basic models and scenarios involving cosmic defects within the torsion context. We showed that in this picture there exists the possibility for SCCSs to effect the spectrum of primordial density perturbations, whose imprints could be seen in the relic cosmic microwave background radiation as observed by COBE.

We also showed that torsion has a non-negligible contribution to the geodesic equation obtained from the contortion term. From a physical point of view, this contribution is responsible for the appearance of a stronger attractive force acting on a test-particle. Using the COBE data, we found that the torsion density contribution S is the order of  $\sim 10^{-3}$ . If we compute the force strength, Eq.(6.3), in association with the above estimative and data coming from COBE observations, we can show that the torsion contribution to this force is  $10^3$  times bigger than the corresponding to a current-carrying string compared to the one induced by the gravitational interaction itself.

This peculiar fact may have meaningful astrophysical and cosmological effects. Let us imagine for a while a compact object (CO): a black hole or an exotic cosmic relic such as a boson, strange mirror star, for instance, orbiting around the SCCS. Because the acceleration induced on the radial component of its orbital motion is about one thousand stronger than in ordinary cases, then, we can expect the changes it provokes in the quadrupole moment of the system (SCCS + CO) to be enhanced by a large factor so that the gravitational wave (GW) signal expected from the CO inspiraling onto the SCCS could be above the lower strain sensitivity threshold of planned LIGO, VIRGO, GEO-600, etc. interferometric GW observatories, for distances even as the Hubble radius. Moreover, this very strong force may also turn the SCCS a potential source of hard X-ray and  $\gamma$ -ray transient emissions. These radiations can be emitted by matter (primordial gas and/or dust clouds, or something else) accreting onto the SCCS as the material gets closer and becomes heated due to the powerful tidal stripping. All these issues, we plan to address to in a forthcoming work including the Sachs-Wolfe effect in space-time with torsion [14].

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