## The  $m \to 0$  limit for massive graviton in  $dS_4$  and  $AdS_4$ How to circumvent the van Dam-Veltman-Zakharov discontinuity

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#### Abstract

We show that, by considering physics in  $dS_4$  or  $AdS_4$  spacetime, one can circumvent the van Dam - Veltman - Zakharov theorem which requires that the extra polarization states of a massive graviton do not decouple in the massless limit. It is shown that the smoothness of the  $m \to 0$  limit is ensured if the H ("Hubble") parameter, associated with the horizon of the  $dS_4$  or  $AdS_4$  space, tends to zero slower than the mass of the graviton m.

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## 1 Introduction

Recently, there has been a lot of activity on models that suggest that a part[[1](#page-10-0), [3](#page-10-0), [4](#page-10-0)] or all [\[2, 3](#page-10-0), [4](#page-10-0)] of gravitational interactions come from massive gravitons. The massive gravitons in these models occur as the result of the dimensional reduction of a theory of gravity in more that four dimensions, something well motivated from String Theory.

In the first kind of model[[1, 3](#page-10-0), [4](#page-10-0)] apart of the massless graviton, there exist ultralight Kaluza-Klein (KK) state(s) that lead to a "multigravity" scenario, in the sense that gravitational interactions are due to the net effect of the massless graviton and the ultralight state(s). In this case, the Cavendish bounds on gravitational interactions are satisfied, since it can be arranged that the rest of the KK tower is much heavier and contributes well below the submillimeter region. In this scenario, modifications to gravity at large scales will appear as we probe distances of the order of the Compton wavelength of the ultralight KK state(s). The phenomenological signature of this will be that gravitational interactions will be reduced or almost switched off (depending on the choice of parameters of the model) at ultralarge scales.

In the second kind of model  $[2, 3, 4]$ , there is no normalizable massless mode and  $4D$ gravity at intermediate scales is reproduced from a resonance-like behavior[[5](#page-10-0), [6\]](#page-10-0) of the wavefunctions of the KK states continuum. In other words 4D gravity in this case is effectively reproduced from a small band of KK states starting from zero mass. In this picture modifications of gravity will begin at scales that correspond to the width of the resonance that appears in the form of the coupling of these states to matter. The phenomenological signature of these modifications will be that the four dimensional Newton's Law  $(i.e.$  inverse square) will change to a five dimensional one *(i.e.* inverse cube) at ultralarge distances. In both kind of models, these modifications can be confronted with current observations of the CBM power spectrum[[7\]](#page-10-0) and are consistent with the data at present.

However, one should be careful when dealing with models with ultralight massive KK states because it is known that the extra polarizations of the massive gravitons do not decouple in the limit of vanishing mass, the famous van Dam - Veltman - Zakharov[[8\]](#page-10-0) discontinuity. This could make these models disagree[[6\]](#page-10-0) with standard tests of General Relativity, as for example the bending of the light by the sun. Furthermore, the moduli (radions) associated with the perturbations of the ′′−′′ branes are necessarily physical ghost fields[[9](#page-10-0)], therefore unacceptable. The latter problem is connected to the violation of the weaker energy condition [\[10](#page-11-0)] on  $''-''$  branes sandwiched between  $''+''$  branes. In the GRS model this radion cancels [\[11](#page-11-0)] the extra polarizations of the massive gravitons and gives the graviton propagator the correct tensorial structure at intermediate distances. However, the model has still an explicit ghost in the spectrum which reveals itself as scalar antigravity at cosmological scales. A mechanism of cancelling both the extra massive graviton polarizations and the radion field contribution was suggested in[[3, 4](#page-10-0)] and involves some bulk dynamics which are necessary to stabilize the system, based on a scenario described in[[12](#page-11-0)]. This mechanism is however non-local in the extra dimension and because of this may not be very attractive.

In the present paper we will demonstrate that there is actually a way out of the first problem. The second problem can be avoided by considering models with only positive tension branes but this will be addressed in another publication[[13\]](#page-11-0). Here we demonstrate that due to an unusual property of the graviton propagators in  $dS_4$  or  $AdS_4$  spacetime, we are able to circumvent the van Dam - Veltman - Zakharov no go theorem (actually the $dS_4$  was considered in [[14](#page-11-0)] using a different method). In more detail, it is known that in flat spacetime the  $m \to 0$  limit of the massive graviton propagator does not give the massless one due to the non-decoupling of the additional longitudinal components. This generates the well known discontinuity between massive and massless states. Considering the massive graviton propagator in  $dS_4$  or  $AdS_4$  spacetime we can show that this result persists if  $m/H \to \infty$  where H is the "Hubble" parameter, *i.e.* the discontinuity is still present in the  $m \to 0$  limit if it happens that  $m/H \to \infty$ . However, in the case that  $m/H \to 0$ , we will explicitly show that the  $m \to 0$  limit is smooth. This is an important result since it gives us the possibility to circumvent the van Dam - Veltman - Zakharov no go theorem about the non-decoupling of the extra graviton polarizations. Thus, in the limit that  $m/H \to 0$  all the extra polarizations of the graviton decouple, giving an effective theory with massless graviton with just two polarization states.

Here we have to make an important comment about the relationship of the parameters  $m, H$ . From a four dimensional point of view these parameters are independent. However, the models that give the additional KK contributions of massive gravitons to gravity are higher dimensional models. After the dimensional reduction, they give us an effective four dimensional Lagrangian in which in general m and H depend on common parameters (e.g. the effective four dimensional cosmological constant). The behaviour of  $m/H$  is model dependent and explicit models that satisfy the smoothness requirement as  $m/H \to 0$  can be found. An interesting example will be given elsewhere[[13](#page-11-0)].

Furthermore, in such models if we keep  $m/H$  finite but small, the extra graviton po-

larization states couple more weakly to matter than the transverse states. This gives us the possibility that we can have a model with massive gravitons that do not violate the observational bounds of e.g. the bending of light by the sun. Moreover, such models predict modifications of gravity at all scales which could be measured by higher precision observations. However, in such a case even though the above can make "multigravity" models viable and interesting, the condition that the mass of the massive graviton must always scale faster that the "Hubble" parameter implies that the dramatic long distance effects of modifications of gravity (e.g. reduction of Newton's constant or transition to 5D law) will not reveal themselves until super-horizon scales. Thus, in this case the horizon acts as a curtain that prevents the long distance modifications of gravity due to the massive KK mode(s) to be observable.

### 2 Graviton propagator in flat spacetime

In order to understand how we can circumvent the van Dam-Veltman-Zakharov discontinuity, it is useful to review the forms of the massive and massless graviton propagators in flat spacetime and the resultant phenomenological differences.

The celebrated van Dam - Veltman - Zakharov discontinuity is evident from the different form of the propagators that correspond to the massive and massless graviton. In more detail, the form of the massless graviton propagator in flat spacetime (in momentum space) has the form:

$$
G^{\mu\nu;\alpha\beta} = \frac{1}{2} \frac{\left(\eta^{\mu\alpha}\eta^{\nu\beta} + \eta^{\nu\alpha}\eta^{\mu\beta}\right) - \eta^{\mu\nu}\eta^{\alpha\beta}}{p^2} + \cdots \tag{1}
$$

where we have omitted terms that do not contribute when contracted with a conserved  $T_{\mu\nu}$ .

On the other hand, the four-dimensional massive graviton propagator (in momentum space) has the form:

$$
G^{\mu\nu;\alpha\beta} = \frac{1}{2} \frac{\left(\eta^{\mu\alpha}\eta^{\nu\beta} + \eta^{\nu\alpha}\eta^{\mu\beta}\right) - \frac{2}{3} \eta^{\mu\nu}\eta^{\alpha\beta}}{p^2 - m^2} + \cdots \tag{2}
$$

This discontinuity has observable phenomenological implications in standard tests of Einsteinian gravity and particularly in the bending of light by the Sun. For example, if gravity is due to the exchange of a massive spin 2 particle, then the deflection angle of light would be 25% smaller than if it corresponds to the exchange of the massless graviton. The fact that the bending of the light by the sun agrees with the prediction of Einstein's theory to 1% accuracy, rules out the possibility that gravity is due to massive graviton exchange irrespective of how small the mass is<sup>4</sup>.

# 3 Graviton propagator in  $dS_4$  and  $AdS_4$  space

In this section we will present the forms of the massless and massive graviton propagators in the case of  $dS_4$  and  $AdS_4$  spacetime with arbitrary "Hubble" parameter H and graviton mass  $m$ . Our purpose is to examine the behaviour of these propagators in the limit where  $H \to 0$  and the limit where the mass of the graviton tends to zero. For simplicity we will do our calculations in Euclidean  $dS_4$  or  $AdS_4$  space. We can use for metric the one of the stereographic projection of the sphere or the hyperboloid<sup>5</sup>:

$$
ds^{2} = \frac{\delta_{\mu\nu}}{\left(1 \mp \frac{H^{2}x^{2}}{4}\right)^{2}} dx^{\mu} dx^{\nu} \equiv g_{\mu\nu}^{0} dx^{\mu} dx^{\nu}
$$
 (3)

where  $x^2 = \delta_{\mu\nu} x^{\mu} x^{\nu}$  and the scalar curvature is  $R = \pm 12$   $H^2$ . From now on, the upper sign corresponds to  $AdS_4$  space while the lower to  $dS_4$  space. The fundamental invariant in these spaces is the geodesic distance  $\mu(x, y)$  between two points x and y. For convenience, we will introduce another invariant  $u$  which is related with the geodesic distance by the relation  $u = \cosh(H\mu) - 1$  for  $AdS_4$   $(u \in [0, \infty))$  and the relation  $u = \cos(H\mu) - 1$  for  $dS_4$  $(u \in [-2,0])$ . In the small distance limit  $u \sim \pm \frac{\mu^2 H^2}{2}$  $\frac{H^2}{2}$ .

This background metric is taken by the the Einstein-Hilbert action:

$$
S = \int d^4x \sqrt{g} \left( 2M^2 R - \Lambda \right) \tag{4}
$$

where the cosmological constant is  $\Lambda = \pm 12$   $H^2M^2$  and M the 4D fundamental scale. The spin-2 massless graviton field can be obtained by the linear metric fluctuations  $ds^2 =$  $(g_{\mu\nu}^0 + h_{\mu\nu}) dx^{\mu} dx^{\nu}$ . This procedure gives us the analog of the Pauli-Fierz graviton action

 $\overline{^{4}$ Of course there remains the possibility that a small fraction of the gravitational interactions are associated with a massive graviton component in the presence of a dominant massless graviton component. This can be realized by having an ultralight spin-2 particle with a very small coupling compared to graviton's one[[1\]](#page-10-0).

<sup>5</sup>Note that [\[17](#page-11-0)] and [\[21](#page-11-0)] whose results we use in the following have a different metric convention, but this makes no difference for our calculations.

in curved space:

$$
\frac{S_0}{2M^2} = \int d^4x \sqrt{g^0} \left\{ -\frac{1}{4}h\Box^0 h + \frac{1}{2}h^{\mu\nu}\nabla^0_\mu\nabla^0_\nu h + \frac{1}{4}h^{\mu\nu}\Box^0 h_{\mu\nu} - \frac{1}{2}h^{\mu\nu}\nabla^0_\nu\nabla^0_\kappa h^\kappa_\mu \right\} + \frac{1}{2}H^2\left(\frac{1}{2}h^2 + h_{\mu\nu}h^{\mu\nu}\right) \right\}
$$
(5)

The above action is invariant under the gauge transformation  $\delta h_{\mu\nu} = \nabla^0_\mu \xi_\nu + \nabla^0_\nu \xi_\mu$  which guarantees that the graviton has only two physical degrees of freedom. This is precisely the definitionof masslessness in  $dS_4$  or  $AdS_4$  space (for example see [[15\]](#page-11-0) and references therein) and it should be stressed that the gravitons do not have null cone propagation because the above action is not Weyl invariant[[16](#page-11-0)].

The propagator of the above spin-2 massless field can be written in the form:

$$
G^0_{\mu\nu;\mu'\nu'}(x,y) = (\partial_\mu \partial_{\mu'} u \partial_\nu \partial_{\nu'} u + \partial_\mu \partial_{\nu'} u \partial_\nu \partial_{\mu'} u) G^0(u) + g_{\mu\nu} g_{\mu'\nu'} E^0(u) + D[\cdots]
$$
 (6)

where  $\partial_{\mu} = \frac{\partial}{\partial x^{\mu}}$ ,  $\partial_{\mu'} = \frac{\partial}{\partial y^{\mu'}}$ . The last term, denoted  $D[\cdots]$ , is a total derivative and drops out of the calculation when integrated with a conserved energy momentum tensor. Thus, all physical information is encoded in the first two terms.

The process of finding the functions  $G^0$  and  $E^0$  is quite complicated and is the result of solving a system of six coupled differential equations[[17](#page-11-0)]. We will present here only the differential equation that  $G^0$  satisfies to show the difference between  $AdS_4$  and  $dS_4$  space. This equation results from various integrations and has the general form:

$$
u(u+2)G^{0}(u)'' + 4(u+1)G^{0}(u)' = C_{1} + C_{2}u
$$
\n(7)

where the constants  $C_1$  and  $C_2$  are to be fixed by the boundary conditions. For the case of the $AdS_4$  space [[17](#page-11-0)], these constants were set to zero so that the  $G^0$  function vanishes at the boundary at infinity  $(u \to \infty)$ . Using the same condition also for the  $E^0$  function, the exact form of them was found to be:

$$
G^{0}(u) = \frac{1}{8\pi^{2}H^{2}} \left[ \frac{2(u+1)}{u(u+2)} - \log \frac{u+2}{u} \right]
$$
  
\n
$$
E^{0}(u) = -\frac{H^{2}}{8\pi^{2}} \left[ \frac{2(u+1)}{u(u+2)} + 4(u+1) - 2(u+1)^{2} \log \frac{u+2}{u} \right]
$$
\n(8)

For the case of the  $dS_4$  space we iterated the procedure of Ref. [[17\]](#page-11-0) imposing the condition [\[18\]](#page-11-0) that the  $G^0$  and  $E^0$  functions should be non-singular at the antipodal point  $(u = -2)$ .

<span id="page-6-0"></span>The constants  $C_1$  and  $C_2$  were kept non-zero and played a crucial role in finding a consistent solution. It is straightforward to find the full expression of these functions, but we only need to know their short distance behaviour. Then with this accuracy the answer is:

$$
G^{0}(u) = -\frac{1}{8\pi^{2}H^{2}} \left[ \frac{1}{u} + \log(-u) \right] + \cdots
$$
  
\n
$$
E^{0}(u) = \frac{H^{2}}{8\pi^{2}} \left[ \frac{1}{u} + 2(u+1)^{2} \log(-u) \right] + \cdots
$$
\n(9)

If we define  $\Pi^0(u) = \frac{1}{H^4}$  $\frac{E^0(u)}{G^0(u)}$ , then for short distances  $(H^2x^2 \ll 1)$  where  $u \to 0$  we get:

$$
g_{\mu\nu}g_{\mu'\nu'} \rightarrow \delta_{\mu\nu}\delta_{\mu'\nu'}
$$
  
\n
$$
\partial_{\mu}\partial_{\nu'}u \rightarrow \mp H^2\delta_{\mu\nu'}
$$
  
\n
$$
G^0(u) \rightarrow \frac{1}{4\pi^2H^4\mu^2}
$$
  
\n
$$
\Pi^0(u) \rightarrow -1
$$
 (10)

and so we recover the short distance limit of the massless flat Euclidean space propagator:

$$
G^{0}_{\mu\nu;\mu'\nu'}(x,y) = \frac{1}{4\pi^2\mu^2} (\delta_{\mu\mu'}\delta_{\nu\nu'} + \delta_{\mu\nu'}\delta_{\nu\mu'} - \delta_{\mu\nu}\delta_{\mu'\nu'}) + \cdots
$$
 (11)

Of course this is just as expected.

In order to describe a spin-2 massive field it is necessary to add to the above action a Pauli-Fierz mass term:

$$
\frac{S_m}{2M^2} = \frac{S_0}{2M^2} - \frac{m^2}{4} \int d^4x \sqrt{g^0} (h_{\mu\nu} h^{\mu\nu} - h^2)
$$
 (12)

By adding this term we immediately lose the gauge invariance associated with the  $dS_4$  or  $AdS_4$  symmetry group and the massive gravitons acquire five degrees of freedom.

The propagator of this massive spin-2 field can again be written in the form:

$$
G^{m}_{\mu\nu;\mu'\nu'}(x,y) = (\partial_{\mu}\partial_{\mu'}u\partial_{\nu}\partial_{\nu'}u + \partial_{\mu}\partial_{\nu'}u\partial_{\nu}\partial_{\mu'}u)G^{m}(u) + g_{\mu\nu}g_{\mu'\nu'}E^{m}(u) + D[\cdots]
$$
(13)

The last term of the propagator in eq. (13), denoted  $D[\cdots]$ , is again a total derivative and thus drops out of the calculation when integrated with a conserved  $T_{\mu\nu}$ .

At his point we should emphasize that in case of an arbitrary massive spin-2 field, the absence of gauge invariance means that there is no guarantee that the field will couple to a conserved current. However, in the context of a higher dimensional theory whose symmetry group is spontaneously broken by some choice of vacuum metric, the massive spin-2 graviton KK states couple to a conserved  $T_{\mu\nu}$ . One can understand this by the following example. Consider the case of the most simple KK theory, the one with one compact extra dimension. By the time we choose a vacuum metric e.g.  $g_{MN}^0 = \text{diag}(\eta_{\mu\nu}, 1)$ , the higher dimensional symmetry is broken. If we denote the graviton fluctuations around the background metric by  $h_{\mu\nu}$ ,  $h_{\mu 5}$  and  $h_{55}$ , there is still the gauge freedom:

$$
\delta h_{\mu\nu} = \partial_{\mu}\xi_{\nu} + \partial_{\nu}\xi_{\mu}
$$
  
\n
$$
\delta h_{\mu 5} = \partial_{\mu}\xi_{5} + \partial_{5}\xi_{\mu}
$$
  
\n
$$
\delta h_{55} = 2\partial_{5}\xi_{5}
$$
\n(14)

If we Fourier decompose these fields, their *n*-th Fourier mode acquires a mass  $m_n \propto n$ with  $n = 0, 1, 2, \ldots$ , but there is mixing between them. This means for example that  $h_{\mu\nu}^{(n)}$ is not a massive spin-2 eigenstate etc.. However, we can exploit the gauge transformations (14) to gauge away the massive  $h_{\mu 5}^{(n)}$  and  $h_{55}^{(n)}$  and construct a pure spin-2 field (see for example[[19](#page-11-0)] and references therein). For a comprehensive account of KK theories see[[20](#page-11-0)]. The new massive spin-2 field  $\rho_{\mu\nu}^{(n)}$  is invariant under (14) and so its Lagrangian does not exhibit a gauge invariance of the form  $\delta \rho_{\mu\nu} = \partial_\mu \chi_\nu + \partial_\nu \chi_\mu$ . However, since is originates from a Lagrangian that has the gauge invariance (14), it is bound to couple to a conserved  $T_{\mu\nu}$ . The argument goes on for more complicated choices of vacuum metric as for example warped metrics which are recently very popular in brane-world constructions.

Again the functions  $G^m$  and  $E^m$  result from a complicated system of differential equa-tions[[21](#page-11-0)]. In that case, the differential equation that  $G<sup>m</sup>$  satisfies is:

$$
u(u+2)G^{m}(u)'' + 4(u+1)G^{m}(u)' \mp \left(\frac{m}{H}\right)^{2} G^{m}(u) = C_{1} + C_{2}u \tag{15}
$$

where the constants  $C_1$  and  $C_2$  are to be fixed by the boundary conditions. For the case ofthe  $AdS_4$  space [[21](#page-11-0)], these constants were set to zero so that the  $G^0$  function vanishes at the boundary at infinity. Imposing additionally the condition of fastest falloff at infinity  $(u \to \infty)$  [\[18\]](#page-11-0), the exact form of the  $G<sup>m</sup>$  and  $E<sup>0</sup>$  function was found to be:

$$
G^{m}(u) = \frac{\Gamma(\Delta)\Gamma(\Delta - 1)}{16\pi^{2}\Gamma(2\Delta - 2)H^{2}} \left(\frac{2}{u}\right)^{\Delta} F(\Delta, \Delta - 1, 2\Delta - 2, -\frac{2}{u})
$$

$$
E^{m}(u) = -\frac{2}{3} \frac{\Gamma(\Delta - 1)H^{2}}{16\pi^{2}\Gamma(2\Delta - 2)[2 + (m/H)^{2}]} \left(\frac{2}{u}\right)^{\Delta} \times
$$
  
 
$$
\times \left\{\begin{array}{l}\n3[2 + (m/H)^{2}]\Gamma(\Delta - 2)u^{2}F(\Delta - 1, \Delta - 2, 2\Delta - 2, -\frac{2}{u}) \\
-3(u + 1)uF(\Delta - 1, \Delta - 1, 2\Delta - 2, -\frac{2}{u}) \\
+ [3 + (m/H)^{2}]\Gamma(\Delta)F(\Delta, \Delta - 1, 2\Delta - 2, -\frac{2}{u})\n\end{array}\right\} (16)
$$

where  $\Delta = \frac{3}{2} + \frac{1}{2}$ 2  $\sqrt{9 + 4(m/H)^2}$ .

For the case of the  $dS_4$  space we iterated the procedure of Ref.[[21\]](#page-11-0) imposing the condition [\[18\]](#page-11-0) that the  $G^m$  and  $E^m$  functions should be non-singular at the antipodal point  $(u = -2)$ and also finite as  $m \to 0$ . Again we kept the constants  $C_1$  and  $C_2$  non-zero to obtain a consistent solution. It is straightforward to find the full expression of these functions, but we only need to know their short distance behaviour. Then with this accuracy the answer is:

$$
G^{m}(u) = \frac{\Gamma(\Delta)\Gamma(3-\Delta)}{16\pi^{2}H^{2}} \left[ F(\Delta, 3-\Delta, 2, \frac{u+2}{2}) - 1 \right] + \cdots
$$
  
\n
$$
E^{m}(u) = -\frac{2}{3} \frac{\Gamma(\Delta)\Gamma(3-\Delta)H^{2}}{4\pi^{2}[2-(m/H)^{2}]} \times
$$
  
\n
$$
\times \left\{ -3[2-(m/H)^{2}] \left[ \frac{2(u+2)}{(\Delta-1)(\Delta-2)} F(\Delta-1, 2-\Delta, 2, \frac{u+2}{2}) + \frac{u(u+2)}{2} \right] -3(u+1) \left[ \frac{2}{(\Delta-1)(\Delta-2)} F(\Delta-1, 2-\Delta, 1, \frac{u+2}{2}) + (u+1) \right] + [3-(m/H)^{2}] \left[ F(\Delta, 3-\Delta, 2, \frac{u+2}{2}) - 1 \right]
$$

where  $\Delta = \frac{3}{2} + \frac{1}{2}$ 2  $\sqrt{9-4(m/H)^2}.$ 

If we define  $\Pi^m(u) = \frac{1}{H^4}$  $E^m(u)$  $\frac{E^{m}(u)}{G^{m}(u)}$ , then for short distances  $(H^{2}x^{2} \ll 1)$  where  $u \to 0$  we get:

$$
G^{m}(u) \rightarrow \frac{1}{4\pi^{2}H^{4}\mu^{2}}
$$

$$
\Pi^{m}(u) \rightarrow -\frac{2}{3} \frac{3 \pm \left(\frac{m}{H}\right)^{2}}{2 \pm \left(\frac{m}{H}\right)^{2}}
$$
(18)

It is interesting to consider two massless flat limits. In the first one  $m \to 0$  and  $H \to 0$ while  $m/H \to \infty$ . In this case, from eq. (18) we see that we recover the Euclidean propagator for a massive graviton in flat space:

$$
G^{m}_{\mu\nu;\mu'\nu'}(x,y) = \frac{1}{4\pi^2\mu^2} (\delta_{\mu\mu'}\delta_{\nu\nu'} + \delta_{\mu\nu'}\delta_{\nu\mu'} - \frac{2}{3}\delta_{\mu\nu}\delta_{\mu'\nu'}) + \cdots
$$
 (19)

This is in agreement with the van Dam - Veltman - Zakharov theorem. The second limit has  $m \to 0$  and  $H \to 0$  but  $m/H \to 0$ . In this case the propagator passes smoothly to the one of the flat massless case [\(11](#page-6-0)):

$$
G^{0}_{\mu\nu;\mu'\nu'}(x,y) = \frac{1}{4\pi^2\mu^2} (\delta_{\mu\mu'}\delta_{\nu\nu'} + \delta_{\mu\nu'}\delta_{\nu\mu'} - \delta_{\mu\nu}\delta_{\mu'\nu'}) + \cdots
$$
 (20)

This is in contrary to the van Dam - Veltman - Zakharov discontinuity in flat space.

In general, we may consider the limit with  $m/H$  finite. Then for small  $m/H$  the contribution to the  $\delta_{\mu\nu}\delta_{\mu'\nu'}$  structure is  $-1 \pm m^2/6H^2$ . Since observations agree to 1% accuracy with the prediction of Einstein gravitational theory for the bending of light by the sun, we obtain the limit  $\frac{m}{H} \lesssim 0.1$ .

### 4 Conclusions

In summary, in this paper we showed that, by considering physics in  $dS_4$  or  $AdS_4$  spacetime, one can circumvent the van Dam - Veltman - Zakharov theorem about non-decoupling of the extra polarization states of a massive graviton. It is shown that the smoothness of the  $m \rightarrow$ 0 limit is ensured if the H ("Hubble") parameter, associated with the horizon of  $dS_4$  or  $AdS_4$ space, tends to zero slower than  $m$ . The above requirement can be realized in various models and an interesting example will be given elsewhere [\[13\]](#page-11-0). Furthermore, if we keep  $m/H$  finite, we can obtain models where massive gravitons contribute to gravity and still have acceptable and interesting phenomenology. Gravity will then be modified at all scales with testable differences from the Einstein theory in future higher precision observations. However, the dramatic modifications of gravity at large distances that "multigravity" models suggest, will be hidden by the existence of the horizon which will always be well before the scales that modifications would become relevant. It will be interesting to investigate if the above smooth limit can also be obtained in the case of non-maximally symmetric spacetimes (*e.g.*) considering FRW or Schwarzschild backgrounds) when the inverse of the graviton mass is much bigger than the characteristic curvature radius. This issue is important for the phenomenology of "multigravity" and will be addressed in other publication [\[22\]](#page-11-0).

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Addendum: One day after this work had appeared in the hep-archives, ref.[[23\]](#page-11-0) appeared, reaching the same result as this paper (for the  $AdS<sub>4</sub>$ ) with a different approach. From this paper we became aware of ref. [\[14\]](#page-11-0) where the smoothness of the limit of the graviton propagator in  $dS_4$  had been shown.

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