Escape of black holes from the brane

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TeV-scale gravity theories allow the possibility of producing small black holes at energies that soon will be explored at the LHC or at the Auger observatory. One of the expected signatures is the detection of Hawking radiation, that might eventually terminate if the black hole, once perturbed, leaves the brane. Here, we study how the 'black hole plus brane' system evolves once the black hole is given an initial velocity, that mimics, for instance, the recoil due to the emission of a graviton. The results of our dynamical analysis show that the brane bends around the black hole, suggesting that the black hole eventually escapes into the extra dimensions once two portions of the brane come in contact and reconnect. This gives a dynamical mechanism for the creation of baby branes.

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Introduction: The reason why gravity is so much weaker than all the other forces, aside from being still a mystery (the hierarchy problem), constitutes the major obstacle in performing experiments in the realm of quantum gravity, for the rather obvious reason that overcoming the fundamental Planck scale requires center of mass energies greater than 10^{18} GeV.

This common belief was radically questioned a few years ago. Although in four dimensions there is no hope to reach such an energy in a terrestrial experiment, the situation drastically changes when one assumes the existence of extra dimensions. In the latter case it might even be possible to reach a transplanckian regime. When geometry and scales of the extra (space-like) dimensions are appropriately chosen, the higher dimensional Planck scale may turn out to be much smaller than 10^{18} GeV, thus reformulating the hierarchy problem in geometrical terms.

The details depend upon the specific realisation of the model, two popular examples being scenarios with large [1] or warped extra dimensions [2], but the common features are a low fundamental Planck scale, $M \sim \text{TeV}$, the localisation of the standard model on branes, representing our directly observable universe, and the propagation of gravity throughout the higher dimensional space. This is, roughly, what is known as the brane world.

In the past few years many people started to use these simple ideas to build models of various type ranging from cosmology to particle physics and, more speculatively, investigate how the brane world can be used as a tool to solve long standing problems in physics. To date no complete solution to any problem has yet been found and many of the predictions of the brane world, relying on tunings of various sorts, do not provide definitive and clear-cut answers. However, it is widely believed that, if the fundamental scale of gravity truly lies in the TeV range, a very spectacular and relatively model independent prediction can be made, that is the creation of small black holes in the high energy collision of two particles [3, 4]. Thus the LHC or the Auger Observatory might be able to perform quantum gravity experiments and initiate the study of black hole microphysics.

A generic assumption is to consider the production of black holes whose mass exceeds the fundamental Planck scale M. In this regime a semiclassical treatment is possible and quantum gravity corrections can be ignored. The size of the black holes produced at colliders is typically assumed to be much smaller than the characteristic length of the extra dimensions. Then it seems reasonable to describe these objects by higher dimensional asymptotically flat solutions [5, 6]. In this approximation, the Schwarzschild radius of a d-dimensional black hole of mass m is given by

$$R_s = \frac{1}{\sqrt{\pi}} \left(\frac{8\Gamma((d-1)/2)}{d-2} \right)^{\frac{1}{(d-3)}} \left(\frac{m}{M} \right)^{\frac{1}{(d-3)}} \frac{1}{M} \,.$$

We use units in which $c = \hbar = 1$. Given these assumptions, one considers two particles with center of mass energy \sqrt{s} moving in opposite directions at impact parameter b. When b is smaller than R_s with m replaced by \sqrt{s} , semiclassical arguments show that a marginally trapped surface forms at the overlap between the two colliding shock waves describing the two scattering particles. This implies the formation of a common horizon unless naked singularities are formed. The black disk approximation, then, suggests the following formula for the cross section:

$$\sigma_{BH} \sim \pi R_s^2$$

Assuming a higher dimensional Planck scale $M \leq 3$ TeV, the cross section will range between 10^{-2} to 10^2 pb at the LHC energy, $\sqrt{s} \sim 14$ TeV. At the luminosity $L \sim 10^{34}$ cm⁻² s⁻¹, the LHC will be able to produce about 10^7 black holes per year.

After the black hole is formed, it will decay by Hawking radiation at a temperature

$$T_H = \frac{d-3}{4\pi R_s} \; .$$

As it was argued in Ref. [7], Hawking evaporation must emit comparable amounts of energy into each low energy effective degree of freedom in the bulk and on the brane. Then, if we assume that only gravity propagates in the bulk, radiation on the brane will be the dominant component of the Hawking radiation.

However, this is not the whole story. As noticed in Ref. [8], the black hole is expected to be rotating and thus exhibits superradiance; this would enhance the emission of higher spin particles, possibly making the emission of gravitons a dominant effect, thus strongly perturbing the system and eventually resulting in the black hole leaving the brane with sudden termination of the Hawking radiation. Ref. [8] discusses this on the basis of a field theory model, where the black hole, treated as a point radiator, is described as a massive scalar field with internal degrees of freedom. Ref. [9] pushes this further and argues that it is possible to distinguish between Z_2 - and non Z_2 -symmetric scenarios due to the fact that a black hole cannot recoil if the spacetime is Z_2 -symmetric.

In this paper we would like to reconsider this problem. Although the interaction between defects and black holes is a relatively well studied subject [10], most of the investigations performed so far focus on the static case. Apart from this, it is assumed that a black hole may leave the brane somehow, but there is no direct example showing how such departure may occur. In the present work we will look at the motion of a brane in the gravitational field of a (small) black hole, and see explicitly how a black hole can escape from the brane.

As we will see, due to the presence of the black hole, even in the most crude approximation, any perturbation that will give the brane an initial velocity with respect to the black hole, will cause deformations in the brane itself. This is essential to understand whether or not a black hole can leave the brane. We will also see that these deformations induced in the brane can be indeed simulated in a precise way, allowing us to determine the time scale, τ_* , at which this separation eventually occurs and to compare it with the lifetime τ of the black hole. This will tell us whether or not the escape occurs before the black hole evaporation completes.

Membrane dynamics: We shall describe the set-up and the limitation of our approach. As we have mentioned, the question we would like to answer is whether or not a black hole on a brane leaves the brane once the system is perturbed and, if this is the case, work out the time scale at which this process occurs.

We will consider a general d-dimensional bulk spacetime and assume that our universe is a (p +1)-dimensional brane. We consider black holes whose gravitational radius is much smaller than the size of the extra dimensions, and this allows us to assume that our spacetime is adequately described by the asymptotically flat solution [5, 6]. This is of course a reasonable assumption only if the self-gravity of the brane is negligible, which is the case when the size of black holes is sufficiently small. As long as this approximation is reasonable, our results will give a model-independent prediction. We assume that the black hole is formed out of matter on the brane, and symmetries require that the brane initially lies on the equatorial plane of the black hole. From the point of view of accelerator generated black holes, the interesting possibility is to consider rotating black holes, however here we start with the case of Schwarzschild black holes as a first step, hoping to capture the essential features of the problem, and deferring the rotating case to our forthcoming work [11]. Thus the bulk space is described by the following line element:

$$ds^{2} = -f(r)dt^{2} + f^{-1}(r)dr^{2} + r^{2}d\Omega_{d-2}^{2}$$

where the function $f(r) = 1 - (1/r)^{d-3}$. We set the horizon radius to unity by adjusting the unit of the length. Then the area of the event horizon is equal to that of a unit (d-2)-sphere, Ω_{d-2} .

The brane in the leading approximation is described by a Dirac-Nambu-Goto action:

$$S = -\sigma \int d^{p+1} \zeta \sqrt{\gamma} \; ,$$

where σ is the tension of the brane and γ is the determinant of the induced metric γ_{ij} on the brane.

We use $\{\zeta^a\} \equiv \{t, r, \chi\}$ with a = 0, 1, ..., p as the coordinates on the brane, where χ represent coordinates of a (p-1)-dimensional sphere. Trajectory of a spherically symmetric brane is specified by the azimuthal inclination angle $\theta(t, r)$. The induced metric on the brane is then given by

$$\gamma_{ab}d\zeta^a d\zeta^b = -f(r)dt^2 + f^{-1}(r)dr^2 + r^2 \left[(\theta_t dt + \theta_r dr)^2 + \sin^2 \theta d\Omega_{p-1}^2 \right] ,$$

where the notation $h_x \equiv \partial h / \partial x$ is used, and we have

$$S_{\mathcal{B}} = -\sigma \Sigma_{p-1} \int dt dr \mathcal{L} ,$$

with

$$\mathcal{L} = (r\sin\theta)^{p-1} \sqrt{1 + r^2 f(r)\theta_r^2 - r^2 f^{-1}(r)\theta_t^2}$$

The case with d = 5 and p = 3 corresponds to a codimension one brane in a five dimensional bulk, whereas that with d = 4 and p = 1 corresponds to a string in



FIG. 1: Schematic picture of the system in the center of mass frame of the black hole. The intersection point is held fixed and the brane is given an initial velocity.

a four dimensional spacetime. From the above action the equation of motion can be obtained and it has to be supplemented by appropriate boundary conditions. As we have already mentioned we work in the center of mass frame of the black hole, and the fact that the intersection point between the black hole and the brane is not allowed to move fixes the boundary conditions at the horizon. The recoil of the black hole in this frame is described by giving some initial asymptotically uniform velocity to the brane. Pictorially the situation is described in fig. 1. The explicit form of the initial velocity we gave in our simulation is $u = v \left(1 - \frac{1}{r}\right)$, but the result does not depend much on our choice of the initial velocity profile.

The task at hand is to solve the dynamical evolution of the brane. This has been done first by writing the equation of motion in a new radial coordinate \tilde{r} , defined via the relation $r = 1/(1 - e^{-\tilde{r}})$, and then by applying finite difference methods along with the Newton-Raphson algorithm to solve the associated non-linear algebraic system. The grid spacing is chosen to be $\delta \tilde{r} = 0.025$ along the radial direction and $\delta t = 0.0025$ along the time direction. The number of points of the grid is fixed to be $N_{\tilde{r}} = 134$ along \tilde{r} and varied along t depending on the initial velocity. The chosen values are $N_t = 3400, 2420, 1970, 1700$ corresponding to v = 0.25, 0.50, 0.75, 1 respectively. The results of the simulation are reported in fig. 2. We find that in all cases the brane is bent and eventually the radius of the pinched part goes to zero. This result does not contradict with the stability analysis presented in Ref. [12], in which it was shown that there is no linear instability for the case with d = 4 and p = 2, since the imposed asymptotic boundary conditions are different. Roughly speaking, the time scale of the escape of a black hole is found to be given by $\tau_* \approx R_s/v$ where v is the initial recoil velocity. In the limit $v \ll 1$ back reaction due to the tension of the brane, which is neglected in our present calculation, may affect the evolution significantly. In this limit, however, the evolution will be well approximated by a sequence of static configurations. For the case with d = 4 and p = 2, static solutions are given in Ref. [10]. If we imagine the situation that there is an edge of the brane at a large radius $r = r_{edge}$, the force



FIG. 2: Unstable deformations of the brane: results of the simulation. v labels the initial velocity. All the simulations reported refer to a five dimensional bulk and 3+1 dimensional brane and the plots are drown in cylindrical coordinates $R = r \sin \theta$ and $Z = -r \cos \theta$.



FIG. 3: Creation of a baby brane wrapped around the black hole.

necessary to sustain the static configuration will be estimated by $F = \sigma \sum_{p-1} r^{p-1} d(r\theta)/dr|_{r=r_{edge}}$. For $p \ge 3$ this force becomes negative when θ is positive. This means that the force between a black hole and the brane is repulsive in this static limit. Therefore the effect of the brane tension will not prevent the black hole escaping from the brane.

Concluding remarks: We have considered a system consisting of a brane plus a black hole. We assumed that the size of the black hole is small compared to the extra dimensions and that the tension of the brane is negligible. In this approximation, the spacetime with a black hole is well described by asymptotically flat solution as given in Refs. [5, 6]. If \mathcal{Z}_2 -symmetry is not imposed, it seems natural to consider the situation in which the black hole acquires some initial velocity with respect to the brane, say, due to anisotropic emission of particles. We simulated this process by studying the dynamical evolution of a brane in a fixed black hole spacetime. The main result of our study is that, irrespectively of the initial velocity v, the brane tends to wrap around the black hole, suggesting that the black hole might escape in the extra dimensions after two portions of the brane come in contact and reconnect (see fig. 3). Such a set-up is of relevance in a number of physical situations.

The first application we have in mind is that of accelerator generated black holes: will it be possible to observe the black hole in its final state and the process of evaporation or will the black hole disappear in the extra dimensions? The results of our analysis strongly suggests that the black hole will indeed escape from the brane. In fact, this process is likely to occur before the black hole evaporation completes if the initial mass of the produced black hole is sufficiently large. The time scale of Hawking evaporation will be estimated as $\tau \approx (dM/Mdt)^{-1} \approx M^{-1}(m/M)^{n+3/n+1}$. On the other hand, the time scale of the escape of a black hole was found to be given by $\tau_* \approx R_s/v$. If the origin of the recoil is the inhomogeneous emission of particles due to Hawking radiation, we will have a rough estimate $v \approx \alpha/\sqrt{N}$, where α is the fraction of the evaporated mass and $N \approx \alpha M/T_H$ is the number of emitted particles. Then we have $\tau_* \approx M^{-1} (m/M)^{\frac{n+4}{2(n+1)}} / \sqrt{\alpha}$. For $\alpha = O(1)$ and $m \gtrsim M$, the time scale of escape is shorter than the time scale of evaporation. Furthermore, α at τ_* is estimated as $\alpha \approx \tau_*/\tau$. Using this relation, we find $\tau_* \approx M^{-1} (m/M)^{\frac{2n+7}{3(n+1)}}.$

Aside from more phenomenological application mentioned above, it is amusing to speculate about other possible situations where the process of baby brane nucleation may be relevant. An interesting application is the possibility to generate global charge non-conserving processes on the defect. This has already been noticed in Ref. [13], where the quantum fluctuations of the geometry induce nucleation of baby branes and this is used as a mechanism to generate baryon asymmetry in the early universe. Our example, though, is different in that it is entirely classical and provides an explicit example of how this nucleation takes place. All this is also reminiscent of the baryogenesis mechanism proposed in Refs. [14, 15], where the evaporation of primordial black hole is used as a means to generate the asymmetry. Of course a more detailed study is necessary to quantify whether or not such a possibility is feasible [11].

One can push things even further by relaxing the condition that the fundamental Planck scale is in the TeV range, but still assume a higher dimensional spacetime. This will not change the process of baby brane creation but will reduce the size of such objects from 10^{-16} cm to the traditional Planck length $l_P \sim 10^{-33}$ cm. It is, then, tempting to imagine the spacetime at the Planck scale as a foam of baby branes and the extra dimensions filled with such bubbles [11].

In the present paper, we completely neglected the selfgravity of the brane. When \mathcal{Z}_2 -symmetry across the brane is imposed, the brane bending is not allowed in our current treatment. Once the self-gravity of the brane is turned on, however, it is not completely clear whether the escape of black holes from the brane is forbidden or not. In fact, such a process is thought to be one possible mechanism to explain the proposed conjecture of classical black hole evaporation in the Randall-Sundrum II model [16, 17]. This is a challenging problem which requires development of numerical relativity with a self-

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gravitating singular hypersurface and a black hole.

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