Density Perturbations and the Cosmological Constant from Inflationary Landscapes

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Abstract

An anthropic understanding of the cosmological constant requires that the vacuum energy at late time scans from one patch of the universe to another. If the vacuum energy during inflation also scans, the various patches of the universe acquire exponentially differing volumes. In a generic landscape with slow-roll inflation, we find that this gives a steeply varying probability distribution for the normalization of the primordial density perturbations, resulting in an exponentially small fraction of observers measuring the COBE value of 10^{-5}. Inflationary landscapes should avoid this "σ problem", and we explore features that can allow them to do that. One possibility is that, prior to slow-roll inflation, the probability distribution for vacua is extremely sharply peaked, selecting essentially a single anthropically allowed vacuum. Such a selection could occur in theories of eternal inflation. A second possibility is that the inflationary landscape has a special property: although scanning leads to patches with volumes that differ exponentially, the value of the density perturbation does not vary under this scanning. This second case is preferred over the first, partly because a flat inflaton potential can result from anthropic selection, and partly because the anthropic selection of a small cosmological constant is more successful.

1   Introduction

The fundamental parameters of the standard model of particle physics and the standard Big-Bang cosmology are determined only from experiments and observations. One of the most important problems of physics is to provide a theoretical understanding for the values of these parameters. Such ideas as unification, symmetry and naturalness have had partial success, bringing radiative corrections under control and reducing the number of independent parameters. The small non-zero cosmological constant (CC), however, still seems to defy any explanation from these considerations [1].

The anthropic principle — that observed values of parameters must allow for the existence of observers — sets the stage for one of the rare successful explanations for why the CC is so small compared with its natural order of magnitude. It also predicts that the CC should be non-zero, and this may explain the observation that the expansion of the universe has recently begun to accelerate. Suppose that the fundamental theory of physics possesses many vacua with different values of the CC. The various vacua are realized cosmologically in different patches of the universe; ours survives anthropic selection only because the CC is sufficiently small to allow large scale structure and gravitationally bound systems to form [2]. This argument sets an upper bound on the CC

$$\Lambda^4 \lesssim \left[\rho_{\text{CDM}}\delta^3\right]_{rec}, \tag{1}$$

where ρ_{CDM}, the energy density of cold dark matter, and δ, the density perturbation for galactic sized modes, are evaluated at the epoch of recombination. The upper bound (1) is only about a factor of 10^2 higher than the observed value of the CC. This is an enormous improvement over the naturally expected value, which is 10^{120} times larger than the observed value.

While there is no direct experimental evidence that the CC is determined by this mechanism, stringent anthropic constraints on the values of the QED and QCD coupling constants [3] also suggest that there are plenty of vacua on which cosmological selection acts; otherwise our existence would be a remarkable coincidence. Cosmological selection may also explain why we live in a vast homogeneous universe created by inflation. Although fine-tuning of parameters is generically required to obtain a sufficiently flat potential for slow-roll inflation [4, 5], a vacuum with finely-tuned parameters that leads to successful inflation dominates the volume of the universe, giving an anthropic prediction for a flat potential [6]. The field space of the underlying theory, containing lots of vacua with different values of various parameters [7, 8], has recently been called the landscape [9] and has been studied extensively, mainly in the context of string theory.

Once we accept that there may be many vacua, realized in various patches of the universe, the notion of naturalness is replaced by probability. We assume that the probability, $\mathcal{P}$, of measuring a given value of a parameter is given by the fraction of observers in the universe who see that value. This probability takes into account not only the density of vacuum states in the landscape, but also appropriate weight factors arising from cosmological dynamics and selection, and can be decomposed as [6]:

$$d\mathcal{P}(\xi) \propto \mathcal{I}(\xi)\mathcal{V}(\xi)\mathcal{A}(\xi)d\xi, \tag{2}$$

where ξ denotes a collection of parameters of the low energy theory that vary from one vacuum to another. The factors $\mathcal{I}$, $\mathcal{V}$ and $\mathcal{A}$ stand for the initial volume distribution prior to slow-roll inflation, the cosmological volume increase due to slow-roll inflation, and the "anthropic factor", respectively. The first factor $\mathcal{I}(\xi)$ may come from a density of states, perhaps calculated from the underlying statistics of vacua in string theory [9], and weighted, for example, by some quantum creation process of the universe [10, 11, 12, 13, 14]. The number of observers is also proportional to the volume factor $\mathcal{V}(\xi)$, and it is the consequences of this very large factor that we explore in this paper. The last factor $\mathcal{A}(\xi)$ includes all other weightings associated with the existence of observers.

Rather than attempting to define the concept of an observer, we only consider a restricted set of patches of the universe where the low energy effective theories, cosmologies or environments are mildly perturbed about our own.[1] After inflation and reheating, observers are created at a certain rate per unit volume, and for a fixed period of time that ends when stars have burned up all of their available fuel. The factor $\mathcal{A}(\xi)$ is proportional to the number of observers produced per unit physical volume, and depends, for example, on the number density of acceptable galaxies formed.

As the CC approaches the upper bound (1), a smaller and smaller fraction of baryons form galaxies [16, 17], causing the anthropic factor $\mathcal{A}$ to shrink; fewer observers are expected to see the value of such a large CC. The authors of reference [17] assumed that the only relevant quantity that scans independently is the CC, i.e., $\xi = \Lambda^4$, and that $\mathcal{I}(\xi)\mathcal{V}(\xi)$ is Λ^4-independent. In this case, with every small value of Λ^4 represented equally in the density of states, they concluded that 5–10 % of observers in the universe, rather than a fraction 10^{-2}, would see a

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[1]Thus, we are not in a position to claim that certain collections of vacua, combined with anthropic selection, uniquely lead to the standard model and to the standard cosmology (*c.f.* [15]) with parameters that can be determined. Based on the restricted set, however, we can discuss necessary properties which must be satisfied by a landscape of vacua, along with the relevant cosmological dynamics, so that the cosmological constant and the density perturbations may be predicted correctly. The resulting conditions need not be sufficient, however.

CC smaller than the value observed by us. This is a remarkable success. It may be that the only parameter of nature that is significantly scanned in cosmology is the CC itself, so that this result justifies attempts to understand fundamental particle physics while ignoring the CC problem, and we have nothing to add.

However, if the CC scans, then why not other parameters? In this case one must study whether the scanning of multiple parameters can maintain a successful understanding of the CC. Suppose that the underlying theory possesses N parameters which scan. Some number n of the standard-model parameters, such as the gauge couplings, have allowed anthropic windows in $\mathcal{A}$ that are so narrow [3] that anthropic selection will determine n combinations of the scanning parameters, leaving $N_s = N - n$ freely scanning.

Let $V(\phi)$ be the classical potential energy density of the universe, with ϕ representing all the scalar fields of the theory, including the inflaton. This potential contains many terms, each depending on a fundamental parameter and each typically much larger than the CC. Since $\Lambda^4 = V(\langle\phi\rangle)$, an anthropic understanding of the CC implies that some parameter(s) of $V(\phi)$ must scan, allowing cancellations between the various terms. The special case of $N_s = 1$ allows the CC to scan but nothing else. What happens in the more general case of $N_s > 1$? Since inflation is governed by $V(\phi)$, one now expects that the number of e-foldings of inflation, N_e, will also scan, leading to a crucial effect on the number of observers [6, 18].[2] When all else is held fixed, the number of observers is proportional to the total volume in which they live, so that

$$\mathcal{V}(\xi) \propto e^{3N_e(\xi)}, \tag{3}$$

where $3N_e(\xi) \gtrsim \mathcal{O}(100)$ varies as a function of parameters. If $N_s > 1$, allowing the parameters of inflation to scan, there is no doubt that the volume factor $\mathcal{V}$ is likely to be a decisive part of the probability calculation. If the universe undergoes eternal inflation [19, 20, 21], the corresponding volume factor may become even more important [6, 18].

The anthropic selection that results from maximizing $N_e(\xi)$ will have important consequences for the observed primordial density perturbations, assuming they are generated from the quantum fluctuations of a field during inflation. The amplitude for some specific mode will have a Gaussian probability distribution, proportional to $e^{-\delta^2/\sigma^2}$, where σ, the scale of the density perturbations, is computed in terms of the parameters of the inflaton potential. Unless

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[2]Even for the case of $N_s = 1$, with one continuous parameter controling both the CC and the inflaton potential, one might wonder whether the inflation volume factor $\mathcal{V}$ could be so important that the "a priori" distribution $\mathcal{IV}$ of [2, 17] is no longer flat in Λ^4, weakening the anthropic understanding of the CC. However, providing N_e is a mild function of the parameter and $N_e \lesssim 10^{120}$, the distribution $\mathcal{IV}$ is sufficiently flat for the small values of Λ^4 that are of interest.

specified otherwise, δ and σ will always refer to their values at the time the perturbations re-enter the horizon. The anthropically allowed window for σ is quite broad $10^{-6} \lesssim \sigma \lesssim 10^{-4}$ [22], with the number of observers, and hence the factor $\mathcal{A}$, falling off rapidly with σ outside the window.[3] If $N_s > 1$, allowing scanning in the inflaton sector, not only is each patch inflated by a different volume factor but the value of σ differs in each patch. The question immediately arises as to whether the patches with large N_e typically have σ close to the observed value of $\sim 10^{-5}$, or whether the volume factor (3) strongly favors other values.[4] In the generic case of $N_s > 1$, it is necessary to determine the combined probability distribution for both σ and Λ^4, and the success of the anthropic arguments of [2, 17] are far from guaranteed. In this paper we study whether such a probability distribution permits an anthropic understanding of the density perturbation as well as the CC, and, if so, in which landscapes.

In section 2 we study simple field-theory models of landscapes where the parameters of inflation are scanned cosmologically, calculating the volume factor as a function of σ. Its dependence is so steep that the fraction of observers measuring $\sigma \sim 10^{-5}$, in the center of the anthropic window, is exponentially small. We argue that this is a generic problem of landscapes where the parameters of inflation models are scanned. While such scanning offers the hope of understanding the flatness of the inflaton potential, it leads to a "σ problem" of proportions at least as overwhelming as that of the CC. Hence, we proceed to investigate whether landscapes with certain properties can overcome this σ problem. In section 3, we describe a class of landscapes where initial conditions prior to slow roll inflation solve the σ problem, and argue that eternal inflation may provide a mechanism to achieve this. We will show that such models can only give a probability between about 10^{-2} and 10^{-9} that we see a CC as small as we do, however. Section 4 is devoted to another class of landscapes, where a restricted scanning of parameters can avoid the volume factor from being exponentially sensitive to σ. In these cases, mild distributions for both σ and Λ^4 allow σ to naturally take a value 10^{-5} in the center of the anthropic window, and also allow an improved understanding of the observed CC, due to the anthropic factor $\mathcal{A}$ of [17]. Conclusions are drawn in section 5.

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[3]For larger σ, the density perturbations go non-linear when the average energy density of the universe was higher, so that the resulting structures are too crowded to guarantee a stable environment for life to evolve. Below the lower boundary of 10^{-6}, the majority of overdense regions are not able to cool quickly enough to form fragmented, structured galaxies. On the other hand, for $10^{-6} \lesssim \sigma \lesssim 10^{-4}$ the dependence of $\mathcal{A}$ on σ should be relatively mild.

[4]From (1) it appears that a higher value of σ is preferred since it allows a higher value for the CC, weakening the success of the anthropic argument for the CC [23]. However, the selection of σ is likely to be strongly dominated by the exponential appearing in the volume factor (3).

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2 Scanning in Models of Slow-Roll Inflation

2.1 One Parameter Model—Chaotic Inflation Ensemble

Let us first consider a simple field theory model of a landscape.[5] The scalar potential $V(\phi)$ on the landscape may contain many hills and valleys; some regions provide slow-roll inflation with sufficient e-folding numbers, while others do not. There will be many local minima; some contain the standard model as the low-energy effective theory, others do not. We are interested only in the inflationary regions leading to the standard-model minima. We expand the potential of these inflationary regions around the local minima, and approximate them by $V(\phi) = m^2\phi^2$, where m is a coefficient that has a different value for each region. We thus have an ensemble of chaotic inflation models. We assume that the quadratic approximation is valid even for ϕ significantly larger than M_{pl}; those local regions that do not satisfy this criterion are discarded from the ensemble since they do not give sufficient inflation. This model landscape will illustrate how we obtain the probability distribution on σ, and why it depends exponentially on σ.

Initially, the universe is assumed to have local patches scanning over the inflationary regions in different parts of the landscape. Prior to the period of slow-roll inflation that generates the density perturbations, the volume distribution of inflationary regions with mass parameters between m and $m + dm$ and field values between ϕ_i and $\phi_i + d\phi_i$ is $\mathcal{I}(m, \phi_i)dm\, d\phi_i$. Virtually nothing is known about the form of this distribution. Classical, slow-roll inflation occurs for field values in the range $M_{\text{pl}} < \phi < M_{\text{pl}}^{3/2}/m^{1/2}$. We do not consider the region with $\phi \gg M_{\text{pl}}^2/m$ where the vacuum energy density exceeds M_{pl}^4, nor even $\phi > M_{\text{pl}}^{3/2}/m^{1/2}$ where the field evolution is quantum rather than classical [20]. Any period of inflation governed by quantum evolution will have its effects included in the initial distribution $\mathcal{I}(m, \phi_i)$.

The epoch of chaotic inflation multiplies the initial volume of each patch by an inflationary

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[5]In string-theory landscapes, space-time is not necessarily four-dimensional, and moreover, the Planck scale of the D=4 effective theory need not remain fixed relative to the string scale. All the compactified configurations that eventually (*c.f.* [24]) lead to decompactification in four dimensions, for example, through inflation involving D3-branes, are treated in our framework however. We do not consider a cosmological scan of the Planck scale in this article, because we think of it as the unit of all measurements: Any measurement is a comparison between two observables of the same dimensionality, and we take the local value of the Planck scale as the basis for comparison. We do this because i) the string scale is not directly observable for the moment, and ii) because it is the ratio of the Hubble parameter or the W-boson mass to the local value of the Planck scale, rather than to the string scale, that matters in physics within the sub-universes. Thus, when we say that an inflaton mass parameter m is scanned in a landscape, it can be interpretted in the application to string landscapes that a distribution of m/M_{pl} is obtained as a result of scanning of both/either m and/or M_{pl}.

factor $\mathcal{V} \propto e^{3N_e}$, with the e-fold number given by

$$N_e \sim \frac{\phi_i^2}{M_{\rm pl}^2}. \tag{4}$$

After reheating, local patches undergo power-law expansion until cold dark matter dominates the universe, structure begins to form, and hydrogen stars begin to shine. Such power-law expansion of the volume certainly has m dependence, through the reheating temperature for example, but this is a negligible effect compared with the exponential increase of volume during inflation. Thus, the final physical volume $dV_{\rm phys} \equiv \mathcal{I}(\xi)\mathcal{V}(\xi)d\xi$ of patches with initial inflationary parameters (m, ϕ_i) is roughly

$$dV_{\rm phys} \sim e^{3N_e(\phi_i)}\,\mathcal{I}(m,\phi_i)\,dm\,d\phi_i. \tag{5}$$

Density perturbations are generated by quantum fluctuations of the inflaton and have a magnitude

$$\sigma \sim \frac{m\phi_{eq}^2}{M_{\rm pl}^3}. \tag{6}$$

The density perturbations that we observe were created when the field value during inflation was ϕ_{eq}, given by $\phi_{eq}^2/M_{\rm pl}^2 \approx \ln(T_{RH}/T_{eq})$. We assume instant reheating after inflation to a temperature T_{RH}. In evaluating σ from (6) to leading order, we ignore the temperature logarithm and take $\phi_{eq} \approx M_{\rm pl}$, giving

$$\sigma \sim m/M_{\rm pl}. \tag{7}$$

Since σ depends only on m, and the parameter ϕ_i cannot be measured, we can obtain the volume distribution for the observable σ by integrating over ϕ_i:

$$\frac{dV_{\rm phys.}}{d\sigma} \propto \int_{\phi_i^{\rm min}}^{\phi_i^{\rm max}} e^{3\phi_i^2/M_{\rm pl}^2}\,\mathcal{I}(m,\phi_i)\,d\phi_i, \tag{8}$$

with $\phi_i^{\rm min} \sim \phi_{eq}$, and

$$\phi_i^{\rm max} \sim M_{\rm pl}^{3/2}/m^{1/2} \sim M_{\rm pl}/\sigma^{1/2}. \tag{9}$$

If the initial distribution $\mathcal{I}(m,\phi_i)$ has a milder dependence on parameters than the volume factor e^{3N_e} (we relax this assumption later), the integration over ϕ_i is dominated by $\phi_i^{\rm max}$. The probability distribution is then approximately given by:

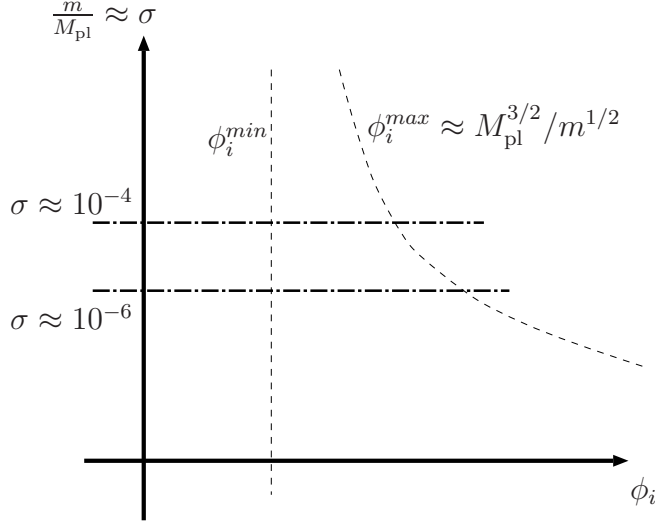$$\frac{d\mathcal{P}(\sigma)}{d\sigma} \propto e^{1/\sigma}\mathcal{A}(\sigma,\Lambda^4). \tag{10}$$

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Figure 1: Parameter space of the chaotic inflation landscape. The density perturbation σ depends on only one parameter of this model, m, while the volume increase due to slow-roll inflation is determined by ϕ_i. N_e and σ are related by an m-dependent upper bound on ϕ_i.

Since the anthropic factor $\mathcal{A}$ does not depend too strongly on σ for $10^{-6} \lesssim \sigma \lesssim 10^{-4}$, the volume factor $\mathcal{V}(\sigma) \propto e^{1/\sigma}$ will dominate the $\mathcal{P}(\sigma)$ distribution, making σ as small as possible. This implies that an exponentially small faction of observers in the universe see $\sigma \sim 10^{-5}$ in the middle of the anthropically allowed window; for example,

$$\mathcal{P}(10^{-5} < \sigma < 10^{-4}) \approx e^{-10^6} \times \mathcal{P}(10^{-6} < \sigma < 10^{-4}). \tag{11}$$

This clearly indicates that either some of the assumptions about the underlying landscape are wrong, or we are far from being generic observers in the universe.

It is interesting to note that the edges of the anthropic window for σ are not hard. If σ is less than 10^{-6}, the probability for a mode δ corresponding to typical large scale structures to fluctuate up to 10^{-6}, as required for acceptable structure formation, is roughly $e^{-(10^{-6}/\sigma)^2}$. Hence, with a flat initial distribution, $\mathcal{P} \propto \mathcal{V}\mathcal{A} \sim e^{1/\sigma}e^{-(10^{-6}/\sigma)^2}$. If more sub-structures are necessary in galaxies for anthropic reasons, and if more seeds of the density perturbations are necessary for that purpose, the anthropic factor may decrease faster than $\mathcal{A} \approx e^{-(10^{-6}/\sigma)^2}$ as σ becomes smaller than 10^{-6}. But as long as the anthropic conditions only require large enough density fluctuations for a moderate number of modes, the volume factor is so powerful that σ is pushed to smaller values, very far from the "anthropic window". This observation tells us that

$\mathcal{P}(10^{-6} < \sigma < 10^{-4})$ itself in (11) is much smaller than 1. The anthropic conditions require δ to arise from fluctuations further out on the exponential tail of the Gaussian distribution.

2.2   Multi-Parameter Model— Hybrid Inflation Ensemble

Above, we assumed the expansion $V = m^2\phi^2$ for the inflaton potential about each relevant local minimum of the landscape, with inflation occuring for field values $\phi > M_{\rm pl}$. However, it is much more reasonable to assume that the potential contains a constant term

$$V = M^4 + m^2\phi^2, \tag{12}$$

with inflation able to occur for field values less than $M_{\rm pl}$. Each patch now undergoes hybrid inflation.[6] We assume that cosmological scanning occurs for both parameters M and m, and for the intial and final values of the inflaton, ϕ_i and ϕ_f. This potential involves three free parameters, so that there is no longer a one-to-one correspondence between the density perturbation σ and the parameters in the inflaton potential. As we will see below, however, when the volume factor $\mathcal{V}$ is obtained as a function of σ by integrating over all unobservable parameters, it is exponentially sensitive to σ in this model as well.

The number of e-foldings from this potential is:

$$N_e \sim \frac{M^4}{m^2 M_{\rm pl}^2} \ln\left(\frac{\phi_i}{\phi_f}\right). \tag{13}$$

As long as $N_e \gg 1$, $\phi_{eq} \approx \phi_f$ and the density perturbations at the epoch of matter-radiation equality are of order

$$\sigma \sim \frac{M^6}{M_{\rm pl}^3 m^2 \phi_f}. \tag{14}$$

Assuming that the parameters are scanned in the ranges $m \lesssim M_{\rm pl}$, $M_{min} \lesssim M \lesssim M_{\rm pl}$ with a phenomenological lower limit on M_{min} from reheating, and $\phi_f \lesssim \phi_i \lesssim M_{\rm pl}$, we find that the e-fold number becomes the largest for a given σ in a patch with ϕ_i as large as possible, M, m both as small as possible and $\phi_f \sim M_{\rm pl}/e$. Assuming again that the initial distribution factor $\mathcal{I}(M, m, \phi_i)$ is less important than the volume factor, the physical volume is exponentially

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[6]The potential in the waterfall direction is omitted here because it is irrelevant during the inflationary era. Various standard-model minima may be associated with different types of inflation models, such as new inflation, but, for simplicity, we consider only the ensemble of hybrid inflation models. The conclusion in this sub-section—that the volume factor tends to depend on σ exponentially—remains the same when a more generic ensemble of inflation models is considered.
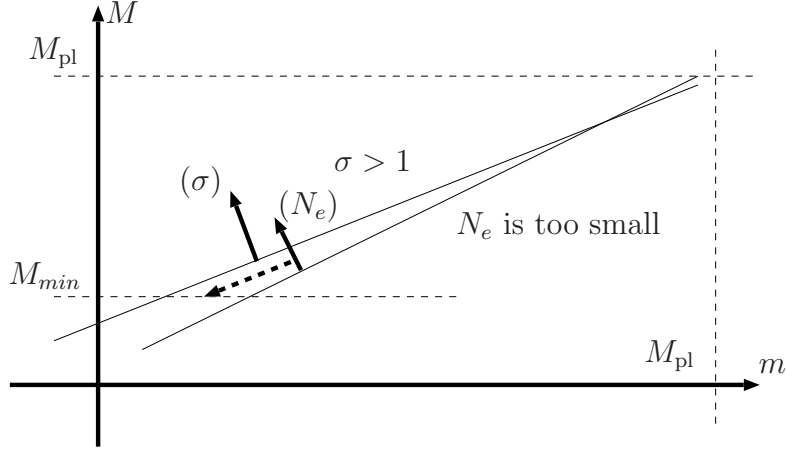
Figure 2: Schematic parameter space of hybrid inflation models for fixed ϕ_i and ϕ_f. Sufficient e-folding is not obtained in the lower-right region, while the density perturbation is too large in the upper-left region. Directions normal to the contours of σ and N_e are indicated by two arrows in the figure, and are slightly different. Thus, on a contour of σ, the e-folding number N_e increases in the direction shown by the broken arrow. For a given σ, $(M(\sigma), m(\sigma))$ on the line $M = M_{min}$ provides the largest e-folding number $N_e(M(\sigma), m(\sigma))$. With $\phi_i \sim M_{\rm pl}$ and $\phi_f \sim M_{\rm pl}/e$ the volume increase factor is roughly given by $e^{3N_e(M(\sigma),m(\sigma))}$.

dependent on σ

$$dV_{\rm phys.} \propto e^{\frac{3M_{\rm pl}^2}{M_{min}^2}\sigma} \, d\sigma, \tag{15}$$

and strongly favors larger values of σ, in contrast to the case of the chaotic inflationary regions.

As long as $\sigma \lesssim 10^{-4}$, the anthropic factor $\mathcal{A}$ has only a mild dependence on σ, and thus the distribution of the physical volume essentially determines the total probability distribution. As in the model in section 2.1, a negligibly small fraction of observers in the universe sees σ close to what we observe.[7] Although one might like to consider the possiblity of a landscape of vacua in order to solve problems such as that of the CC, the model landscape discussed here is clearly not an acceptable one.

As in the previous sub-section, the exponential volume factor is so powerful that the anthropic window is forced to open wider. In this hybrid inflation landscape a typical observer will measure $\sigma > 10^{-4}$. With such large density perturbations typical planets will have their orbits disrupted before observers can form [22], but a few planets will by chance avoid close

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[7]This result does not depend on the particular choice of the boundary of the parameter space $M_{min} \lesssim M$.

contact with foreign stars for a sufficent time for observers to form, and, given the huge increase in the volume of the patch from inflation, such observers will dominate.

2.3   Generalization

The above two examples demonstrate a rather generic feature of landscapes that can be approximated by an ensemble of slow-roll inflation models with scanned parameters: the probability distribution over σ contains a volume factor $\mathcal{V}(\xi)$ that depends exponentially on σ. Hence, barring an important effect from the initial volume distribution $\mathcal{I}(\xi)$, a negligibly small fraction of observers in the universe sees a scale-invariant density perturbation of order 10^{-5}. Whether the typical σ is larger or smaller than 10^{-5} depends on the ensemble of inflation models, but either way, the density perturbation is predicted incorrectly. Below we give a generalized argument for this "σ problem."

Consider any two sub-universes, i and j, in which large scale structure forms. We assume that all such sub-universes underwent a period of slow-roll inflation with a collection of parameters and fields, ξ, such as (m, ϕ_i) or (M, m, ϕ_i, ϕ_f), that scan from one patch to another. We assume that it is meaningful to discuss the relative probability for these two sub-universes[8]

$$\frac{\mathcal{P}(\xi_i)}{\mathcal{P}(\xi_j)} = \frac{\mathcal{I}(\xi_i)}{\mathcal{I}(\xi_j)} \frac{\mathcal{V}(\xi_i)}{\mathcal{V}(\xi_j)} \frac{\mathcal{A}(\xi_i)}{\mathcal{A}(\xi_j)}. \tag{16}$$

Again, $\mathcal{I}$ is an initial condition factor, giving the volume distribution prior to slow-roll inflation, $\mathcal{V}$ is the volume expansion factor from slow-roll inflation, generally having exponential dependence on ξ, and $\mathcal{A}$ contains all other anthropic factors including those that prefer σ to lie within the window $10^{-6} \lesssim \sigma \lesssim 10^{-4}$.

If the density perturbations arise from quantum fluctuations of the inflaton field, then their standard deviations are determined by ξ. The relative probability of finding two different values of the density perturbations, σ_1 and σ_2, will have the form

$$\frac{\mathcal{P}(\sigma_1)}{\mathcal{P}(\sigma_2)} = \frac{\sum_{\xi|\sigma=\sigma_1} \mathcal{I}(\xi)\mathcal{V}(\xi)A(\xi)}{\sum_{\xi|\sigma=\sigma_2} \mathcal{I}(\xi)\mathcal{V}(\xi)A(\xi)}, \tag{17}$$

where we sum over all sub-universes giving a particular value for σ.

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[8]There is a subtlety when infinite numbers of observers have to be accounted for in the probabilites (see e.g., [18, 25, 26]). One can try to deal with this problem by regularizing the infinites and taking a limit. Here, we just assume that there is a meaningful definition of relative probability, and we do not specify what it is. Our conclusion should not be affected by the definition, unless a really specific choice is made.

As a result of the exponential dependence on ξ, such sums will generally be dominated by a particular value of ξ, i.e., $\sum_{\xi|\sigma=\sigma_1} \mathcal{I}(\xi)\mathcal{V}(\xi)\mathcal{A}(\xi) \sim \mathcal{I}(\xi^*(\sigma_1))\mathcal{V}(\xi^*(\sigma_1))\mathcal{A}(\xi^*(\sigma_1))$, where $\xi^*(\sigma_1)$ is the exponentially most probable value of ξ with $\sigma = \sigma_1$. That is to say, even though many different sub-universes will generally have a given value of σ, we expect one of them to be exponentially more probable than all the others. Then the probability of finding a given σ is roughly the same as the probability of finding that specific sub-universe, i.e.,

$$\frac{\mathcal{P}(\sigma_1)}{\mathcal{P}(\sigma_2)} \approx \frac{\mathcal{I}(\xi^*(\sigma_1))\mathcal{V}(\xi^*(\sigma_1))A(\xi^*(\sigma_1))}{\mathcal{I}(\xi^*(\sigma_2))\mathcal{V}(\xi^*(\sigma_2))A(\xi^*(\sigma_2))}. \tag{18}$$

Within the window $10^{-6} \lesssim \sigma \lesssim 10^{-4}$, where $\mathcal{A}$ has mild dependence on σ, we therefore expect exponential sensitivity in the probability distribution, arising from the volume factor $\mathcal{V}$. This exponential sensitivity persists even if $\mathcal{I}(\xi)$ has a strong dependence on ξ, either steeply increasing or falling with σ across the anthropic window. In these cases, to demonstrate a σ problem we do not need to calculate the form of $\mathcal{I}$, we need only assume that it does not have an exponential dependence on σ that precisely cancels that of $\mathcal{V}$. Since $\mathcal{I}$ and $\mathcal{V}$ have entirely different physical origins, such cancelling exponentials could only be accidental.

Although we have discussed so far only the case in which the density perturbations are generated by inflaton fluctuations, it is straightforward to extend the argument to the scenario in which they originate from fluctuations of another light field, s [27, 28, 29]. Analogous arguments lead generically to a distribution $dV_{\text{phys}}(H) \propto e^{f(H)}dH$, with exponential dependence on the Hubble parameter H. The density perturbation is given by $\sigma \sim H/\langle s \rangle$, and the probability distribution for σ is given by convoluting those of H and $\langle s \rangle$, and is generically exponentially sensitive to σ.

3   Dominant Selection from $\mathcal{I}$

In this section we consider a special form for the initial volume distribution that avoids the σ problem. We take $\mathcal{I}$ to have a sharp peak within the anthropic window for σ — a peak that is so sharp that $\mathcal{IV}$ is also sharply peaked, so that the exponential behavior of $\mathcal{V}$ is sub-dominant. Indeed it may be that the discreteness of the vacua is relevant, so that a single anthropically acceptable vacuum has an initial volume very much larger than all the others.[9] Virtually all

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[9]As before, we do not consider vacua that are anthropically unacceptable, say, because the QED fine-structure constant is outside its anthropic window, or because the CC has already dominated the universe by the epoch of recombination. These are excluded from consideration because there are so few observers in such vacua, and the associated anthropic factors $\mathcal{A}$ are essentially zero.

the observers in the universe, including us, will then see the physics of this vacuum, hopefully with $\sigma \sim 10^{-5}$ and $\Lambda^4 \sim (3 \times 10^{-3}\,\text{eV})^4$.

What kind of physics can prepare such an initial condition, and how probable is it that such a uniquely chosen vacuum happens to be ours? In sub-sections 3.1 and 3.2, we argue that eternal inflation can lead to such a strong selection of vacua; if the conditions for eternal inflation are met in some patch of the universe, the enormous volume factor that results will play an important role in determining the distribution of observers in that patch. Although we present eternal inflation as a possible example mechanism, we stress that our essential conclusions in sub-section 3.3 on probabilities depend only on the assumptions made for $\mathcal{I}$, and not on any particular mechanism for obtaining $\mathcal{I}$.

3.1   False-Vacuum Eternal Inflation

We consider a landscape of vacua with differing energies, as required for an anthropic solution to the CC problem. Regions of the universe in local minima with positive vacuum energy expand exponentially, while regions in local minima with large negative vacuum energy shrink to cosmological singularities. We are not interested in the latter regions, because no observers live there. A positive vacuum energy region, on the other hand, continually nucleates bubbles of vacua with lower energy [30], while inflating with its associated Hubble parameter, H. This process of inflation and bubble nucleation continues forever if $H_{\text{eff.}} \equiv H - \Gamma_{\text{tot}}/H^3$ is positive, where Γ_{tot} is the sum of all the bubble nucleation rates. This sort of eternal inflation may be a generic feature of landscapes [21, 31, 32].

There may be more than one eternally inflating local minimum, each with its own effective expansion rate. As you look infinitely far into the future, however, the false vacuum with the largest $H_{\text{eff.}}$ will be arbitrarily larger in physical volume than all the others, and will dominate the universe [6, 18].[10] This will be true even if its expansion rate is only very slightly larger than all the others. This feature is convenient because it causes any prior initial condition of the universe, such as those found in [10, 11, 12, 13, 14], to be erased [20]. The universe converges to a fixed asymptotic state, dominated by a single local minimum and its associated bubbles.

How does this asymptotic-state universe[11] prepare an environment in which observers can

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[10]The physical volume of each false vacuum depends on the choice of equal-time surface. If inflation ends within a finite time, this subtlety is not a problem. But, since the false vacua are inflating forever, it is quite subtle to compare the two infinite numbers of observers produced in the bubbles nucleated from two different false vacua. For more about this issue, see [18, 25, 26].

[11]The dominant percentage of observers live in bubbles that nucleated at later times. This is why any local minima other than the asymptotic-state local minimum are irrelevant.

live? Some of the nucleated bubbles will go to standard-model vacua[12] with moderate values of the CC, and also with anthropically acceptable values for other parameters. Note, however, that simple bubble nucleation to a standard-model vacuum is not sufficient to create a habitable universe. The space inside the bubble must be reheated, and furthermore, must become a flat universe, rather than an open universe.[13] These conditions are most readily satisfied if a nucleated bubble goes not directly to a standard-model minimum, but rather to a slow-roll inflationary region that reheats to a standard-model minimum. For this reason we neglect bubbles that do not nucleate to slow-roll inflationary regions. The result is that false-vacuum eternal inflation sets the initial volume distribution $\mathcal{I}(\xi)$ for the slow-roll inflation ensemble.

What sort of initial volume distribution is obtained? Let Γ_i be the bubble nucleation rate from the dominant local minimum to a given inflationary region, labeled by i; the nucleation rates to two different such regions of the landscape will generally have different values. Now, the total volume of all sub-universes produced in the region i will simply be proportional to Γ_i, with the physical volume of the dominant eternally inflating local minimum factoring out; $\mathcal{I}(\xi_i) \propto \Gamma_i$. An individual decay rate Γ_i takes the form $M_i^4 e^{-S_i}$, where M_i is a characteristic energy scale of the potential barrier and the distance of the tunneling, and S_i is the classical action of a bounce solution interpolating between the two vacua. Now note that $\mathcal{I}(\xi_i)$ cannot be expected to vary mildly as a function of the low-energy parameters ξ_i; while two slow-roll inflationary regions neighboring each other in a landscape may have much the same tunneling rate Γ_i's, their low energy parameters, such as the inflaton mass and the CC, may be totally different, as in the case where the CC is given by the mechanism found in [34]. Two inflationary regions with almost the same low-energy parameters may generally be far away from each other in the landscape of vacua, with e^{-S_i} factors differing by a huge amount.

If the landscape does not have large numbers of anthropically acceptable standard-model vacua, then it will not be possible to treat the initial volume distribution $\mathcal{I}(\xi)$ as a continuous mild distribution over the low-energy parameters, even after binning and averaging. It will rather become an essentially isolated distribution with perhaps a few exponentially high peaks. We assume that this exponential dependence from e^{-S_i} is more important than the volume

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[12]The decay to standard-model vacua includes cascade decays through various other vacua in intermediate steps. What we call the asymptotic-state local minimum is assumed to have a non-zero decay rate to a standard-model vacuum. If it does not, it is replaced by the one with largest $H_{\text{eff.}}$ among those that have non-zero decay rates to standard-model vacua.

[13]This may be an anthropic condition because density perturbations do not grow in curvature dominated backgrounds [33]. If it is not, however, we just assume that the bubble with the largest Γ_i (see what follows in the text) happens to go to a slow-roll region. More discussion on this issue is found in [32] and references therein.

increase from slow-roll inflation. This is how eternal inflation might be able to prepare the sort of initial volume distribution proposed at the beginning of this section. Perhaps the standard-model vacuum in closest proximity to the dominant eternally inflating vacuum will be almost uniquely selected.

Essentially unique values are thus chosen for various parameters in this type of scenario, including σ and Λ^4, in the sense that the same values are observed by virtually all observers in the universe. They therefore should clearly be the values that *we* observe. We cannot presently test this idea, since we do not at this moment have a guess as to the details of the underlying landscape. One could in principle work out which is the dominant eternally inflating local minimum in a given landscape, which inflationary region has the largest bubble nucleation rate from the minimum, and what is the value of the CC for all the relevant standard-model vacua. This might be doable once a concrete landscape, such as the Type IIB string landscape, is adopted.

3.2   Large-Field Eternal Inflation

Eternal inflation can also take place by another mechanism [20]. When a slow-roll inflaton potential is so flat that the condition

$$H \gtrsim \frac{\dot{\phi}}{H} \approx \frac{V'}{H^2} \tag{19}$$

is satisfied, the evolution of the inflaton is mostly governed by quantum fluctuations, and not by the classical equations of motion. If this is the case, the average value of the inflaton field, weighted by the physical volume, does not descend the potential, but goes uphill, because the expansion rate of the volume is higher for a larger energy density [20, 35]. One such eternally inflating region eventually dominates the volume of the universe: the one with the highest expansion rate [18], just as in the false-vacuum eternal inflation case.

The quantum fluctuations of the inflaton occasionally bring its value outside of the range satisfying (19), converting some part of the eternally inflating spacetime into a classical slow-roll inflation "bubble". Once such a bubble enters a stage of slow-roll inflation, it is eventually reheated and leads to the standard cosmology. The bubble nucleation process in the false-vacuum type eternal inflation is replaced by the creation of quantum fluctuation bubbles in this scenario. Note that eternal inflation must be followed by a period of standard slow-roll inflation in this scenario as well; during the eternal inflation density fluctuations are generated which are of order $\sigma \approx H^2/\dot{\phi}$; this is larger than 1 because of (19) [20]. When density fluctuations of order

one enter the horizon, primordial black holes are produced, leading to a black-hole dominated universe [36, 22]. Thus, the period of slow-roll inflation cannot be skipped. Eternal inflation of large-field type thus also sets an initial condition $\mathcal{I}(\xi)$ for the slow-roll inflation ensemble.

Since the exiting process from the eternal inflation epoch is through quantum fluctuations, the history after the exit is not determined completely. There may be several paths from the dominant eternally inflating region to standard-model vacua, passing through slow-roll inflationary regions. The initial volume distribution $\mathcal{I}(\xi)$ is non-zero for such inflationary regions, and the relative ratio is calculated from the rates for quantum fluctuations to exit along these various routes. Rates for quantum fluctuations thus replace the bubble nucleation rates Γ_i from the false vacuum eternal inflation case. If only one path is favored significantly over the others, then one set of parameters is observed by almost everyone in the universe. All of the arguments based on false-vacuum eternal inflation thus hold true in the large-field eternal inflation case.

3.3   The Probability for Choosing Our Vacuum

We now turn to the question of how likely it would be for the chosen value of σ to be 10^{-5} and the chosen value of the CC to be $(3 \times 10^{-3}\text{eV})^4$. We assume here only that the initial distribution factor selects a particular anthropically acceptable vacuum as being the most probable one. Specifically, we do not need to assume anything about eternal inflation in this sub-section. The analysis will be a bit subtle, but the basic idea is simple: the larger the range of allowed values for a parameter, the less likely it is that the particular value we see would be chosen. Let us define a value to be "choosable" if any suppression in observers from the anthropic factor $\mathcal{A}$ is less important than the increase in the number of observers from the initial distribution $\mathcal{I}$. Since the initial distribution factor is expected to be quite strong, the range of "choosable" values for a parameter tends to be larger than one might expect from anthropic considerations alone. The result is to make it *less* likely that we observe the values for σ and the CC that we do. This may be a problem for this scenario.

For the moment, let us take it for granted that σ is chosen correctly, and consider only the selection of the CC. We feel it to be a reasonable assumption that the initial distribution factor $\mathcal{I}$ doesn't have much dependence on the actual value of the very small CC that emerges after reheating and the various phase transitions of late-time cosmology. The location of the vacuum within the landscape will be relevant to $\mathcal{I}$, but whether its energy is $10^{-120}M_{\text{pl}}^4$ or $10^{-121}M_{\text{pl}}^4$ probably will not be. We will then say that any anthropically acceptable vacuum has an "equal chance" to be the one with the largest $\mathcal{I}$. This is simply a statement of our ignorance about

the precise details of the underlying landscape. Since the volume factor $\mathcal{V}$ is irrelevant in this picture, as explained above, the result is that for anthropically acceptable CC's, the probability for a specific value to be chosen is given primarily by the fundamental density of states in the landscape. This logic may apply to other parameters as well. We will then assume that the density of states gives a flat distribution for small values of the CC, as in [2, 17]. Since standard-model vacua with CC's satisfying

$$\Lambda^4 \lesssim \left[\rho_{\text{CDM}} \sigma^3\right]_{rec} \tag{20}$$

are certainly satisfactory, at most only one part in a hundred anthropically acceptable standard-model vacua have a CC as small as ours: $P[\Lambda^4 < (3 \times 10^{-3}\text{eV})^4] < 10^{-2}$. Here we use P, rather than the $\mathcal{P}$ used earlier, to emphasize that this new probability is a statement of our ignorance about which vacuum happens to be selected, rather than a measure of a fraction of observers.

Those vacua satisfying (20), however, are not all the "choosable" ones, in the sense defined at the beginning of this sub-section. Since each slow-roll inflationary region is associated only with the standard deviation σ of the density perturbations, there is a chance that the actual density fluctuations δ could be larger than σ, so that the true anthropic condition (1) is satisfied even for a CC larger than (20). If we adopt the estimate for the anthropic factor in [17]

$$\mathcal{A}(\Lambda^4) \approx e^{-[(\Lambda^4/\rho_{\text{CDM}})^{2/3}/\sigma^2]_{rec}} \qquad \text{for} \quad \Lambda^4 \lesssim 10^5 \times (3 \times 10^{-3}\text{eV})^4, \tag{21}$$

with $\sigma_{rec}(\text{Mpc})$ of order a few times 10^{-3} corresponding to the COBE normalization [17], we have $\mathcal{A} \sim e^{-100}$ for a CC 10^5 times larger than ours. Thus, the suppression of gaussian fluctuations "slightly" disfavors such a large CC, reducing the number of observers by a factor of e^{-100}. But this effect is not as significant as that of the volume factor, which is expected to vary from one inflationary region to another by at least of order $e^{3N_e} > e^{100}$. Since we have assumed in this section that the hierarchy among $\mathcal{I}$ values is more than that among the volume factors, the anthropic factor is relatively negligible for CC's 10^5 times larger than ours, and perhaps larger. If we then consider vacua with $\Lambda^4 < 10^5 \times (3 \times 10^{-3}\text{eV})^4$ to be "choosable", the probability $P[\Lambda^4 < (3 \times 10^{-3}\text{eV})^4]$ is less than 10^{-5}. The anthropic factor $\mathcal{A}$ above, however, comes from the assumption that the density perturbation of a *single* wavenumber is required to go non-linear and form a massive clump before the CC dominates the energy density of the universe. This will certainly be a necessary condition for observers to exist, but may not be a sufficient condition [17]. Thus the anthropic factor may decrease much faster than (21), and it is not a certainty that the upper bound on the probability is less than 10^{-2}.

The lower bound for the probability is clearer, however; there is no chance for a reasonable scenario of structure formation when the CC is almost as large as the energy density at the

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epoch of recombination. Thus, vacua with $\Lambda^4 > [\rho_{CDM}]_{rec}$ are not regarded as "choosable" [22], even if there are $\mathcal{I}$ values for such vacua that are very large. We thus have

$$10^{-9} \lesssim P\left[\Lambda^4 < (3 \times 10^{-3}\text{eV})^4\right] \lesssim 10^{-2}, \tag{22}$$

with 10^{-9} coming from the ratio $(3 \times 10^{-3}\text{eV})^4/[\rho_{CDM}]_{rec}$.

One could in principle ask a similar question about the probability that σ would be chosen to be $\approx 10^{-5}$ in this scenario. The nature of the density of states distribution on σ, however, is hard to estimate, as σ is a model dependent function of fundamental parameters of the landscape, and therefore we do not attempt to calculate it. Even if it turned out that $\sigma \approx 10^{-5}$ was chosen, however, we would not have an explanation for why this chosen value is within the anthropically preferred window of $10^{-6} \lesssim \sigma \lesssim 10^{-4}$; the window is not a hard cut-off, just as the upper bound on the CC (20) is not. The strength of the initial distribution factor can overcome suppression in the number of observers when σ lies outside the window.

Now, the e-fold number N_e of the last slow-roll inflation epoch is also an observable for values of about 60. Thus, the density of states can also be represented as a function of both σ and N_e. Reference [32] discusses the density of states as a function of N_e, based on a simple model. While that paper ignored volume factors in probabilities, the scenario outlined here could provide a justification for this approach. One need only keep in mind that the resulting probabilities are statements of ignorance rather than distributions of observers. Discussions in [32] and references therein concerning the lower multipoles of the CMB, for example, would then be applicable.

In summary, the σ problem of section 2 could be solved by an extremely sharp peak in the $\mathcal{I}(\sigma)$ distribution. This could possibly be achieved from eternal inflation. By selecting roughly a single standard-model vacuum, and thus single values for both σ and Λ^4, this mechanism could circumvent the exponential dependence on σ coming from slow-roll inflation. The biggest problems with this scenario however are

- The probability $P(\Lambda^4 < (3 \times 10^{-3}\text{eV})^4)|_{\sigma=2\times10^{-5}}$ is at least as small as 10^{-2} and may even be as small as 10^{-9}. The broad range comes from uncertainty in anthropic conditions as well as model dependence. In any case this probability is worse than the 5–10% of [17], but better than 10^{-120}.

- There is no reason for the observed spectrum of density perturbations to fall within the middle of the anthropically preferred window $10^{-6} \lesssim \sigma \lesssim 10^{-4}$.

4 Preferred Landscapes Without a σ Problem

In the previous section we have shown that the σ problem may be solved if the dominant vacuum selection is determined by the initial condition factor $\mathcal{I}$, rather than by the volume factor $\mathcal{V}$, but at the cost of the two problems listed above. In this section we seek alternative solutions to the σ problem; in particular ones in which the physical volume distribution $\mathcal{IV}$, named the "a priori factor" in [17], has a flat distribution in the CC, leading to the success $\mathcal{P}(\Lambda \leq 3 \times 10^{-3}\text{eV})|_{\sigma=2\times10^{-5}} \sim 5\text{--}10\%$ for the CC problem [17]. If $\mathcal{IV}$ is moderately peaked near $\sigma \sim 10^{-5}$, we have essentially the assumption made in [17]. If $\mathcal{IV}$ is flat (or at most power-law) in σ across the anthropic window as well (c.f. [23]), the anthropic factor naturally accounts for why σ_{COBE} happens to lie within the anthropic window. In both cases the key is to avoid an exponential behavior for $\mathcal{V}(\sigma)$, the σ problem in section 2. In fact it seems that the scanning in the inflaton sector must be restricted in some way.

The most obvious solution to the σ problem is that none of the parameters of slow-roll inflation scan significantly. The parameters may be uniquely determined, or the density of states as a function of the parameters may have a sharply peaked behavior [37, 38]. Since σ is not scanned, the σ problem does not exist. Scanning of the CC can still occur, for example from the scanning of the parameters of the hidden sector that leads to supersymmetry breaking. As long as $\mathcal{I}$ is roughly flat in the CC, the successful result of [17] follows. This solution to both the σ and CC problems, however, may not leave behind an anthropic explanation for the flatness of the inflaton potential.

Another solution to the σ problem results if the scanning of the inflaton sector is restricted in such a way that while N_e and σ scan they do not depend on a common scanning parameter of the theory. In this case N_e is scanned cosmologically and the flatness problem of the inflaton potential is solved. Since the scanning of N_e is independent of the scanning of σ, $\mathcal{V}$ has no exponential sensitivity to σ. Thus, the σ problem is avoided. As long as $\mathcal{I}$ is approximated by a mild function of Λ^4 and scanning parameters of slow-roll inflation, $\mathcal{IV}$ varies mildly across the anthropic window of σ, and most of the observers in the universe are likely to see σ in the middle of the anthropic window $10^{-6} \lesssim \sigma \lesssim 10^{-4}$. The physical volume distribution $\mathcal{IV}$ may be flat in Λ^4 as above, and then it follows that $\mathcal{P}(\Lambda < 3 \times 10^{-3}\text{eV})|_{\sigma=2\times10^{-5}} \approx 0.05\text{--}0.10$ [17] or $\mathcal{P}(\Lambda < 3 \times 10^{-3}\text{eV}) \sim 10^{-4}$ as in [23]. Such restricted scanning is possible in both chaotic and hybrid inflation models, as shown below.

For the chaotic inflation ensemble, in contrast to sub-section 2.1 we now assume that the quadratic potential extends only to field values somewhat larger than the Planck scale, say

$10 \times M_{\rm pl}$, rather than to the very large m dependent $\phi_i^{\rm max}$ of (9). In this case (10) is replaced by

$$\frac{d\mathcal{P}}{d\sigma} \propto e^{100} \mathcal{I}(m, \phi_i^{max})|_{m \sim \sigma M_{\rm pl}} \mathcal{A}. \tag{23}$$

Since the upper bound on the initial field value is no longer tied to the mass parameter or to the observed density perturbation, the volume increase factor from slow-roll inflation, e^{3N_e}, is no longer an exponential function of σ. Actually, ϕ_i^{max} could have a moderate m dependence, so long as such dependence did not lead to a dominant exponential distribution in σ.

For the hybrid inflation ensemble of 2.2, we now scan m^2 and the initial and final field values, but fix the parameter M. This restriction on the scanning might result if the model is extended to include grand unification [39], so that M is the unified symmetry breaking scale. Different values of M lead to different QED and QCD coupling constants, so that the anthropic factor $\mathcal{A}$ very strongly selects a narrow range for M. Since m^2 is scanned and so is N_e, the fine-tuning problem of the inflaton mass is solved anthropically [6]. However, since both N_e and σ depend on m^2, which scans, we have not yet achieved our objective, which was to solve the σ problem. The solution is for density perturbations to arise from a mechanism such as those found in [27, 28, 29], rather than from the inflaton fluctuation itself. The initial field value of another light field $\langle s \rangle$ gives rise to the density perturbation $\sigma \sim H/\langle s \rangle$, which is independent of m^2, as the Hubble parameter during inflation depends only on M and does not scan.[14] Thus the volume of a patch is not correlated with its density perturbation σ. The behavior of $\mathcal{I}$ across the anthropic window depends on the distribution of the initial value for $\langle s \rangle$, and could be flat or power-law in $\langle s \rangle$. Then the likely value of σ would be determined mainly by the anthropic factor.

A crucial assumption made in these solutions is that $\mathcal{I}$ can be treated as a mild distribution in Λ^4 and the scanning parameters of slow-roll inflation. Could this assumption still be reasonable if eternal inflation occurs? For false-vacuum eternal inflation, the previous section used the picture that the exponential dependence coming from $\Gamma_i \propto e^{-S_i}$ was so large that $\mathcal{I}$ was very sharply peaked at particular vacua. This would not necessarily be the case, however. If there were large numbers of anthropically acceptable standard-model vacua in the landscape, or if there were clumps of such vacua in close proximity to each other resulting in similar Γ_i

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[14]We should clarify how it is possible to scan the CC but not the vacuum energy of slow-roll inflation. The vacuum energy of inflation has two contributions: one from the CC and the other from the "waterfall energy" released at the end of inflation. Scanning of the CC piece certainly does affect the Hubble parameter during inflation, but it is anthropically constrained to be so small that its effect on the inflation Hubble parameter is negligible.

values, then it might be possible for $\mathcal{I}$ to be a mild distribution even in the presence of eternal inflation.

5   Conclusions and Discussion

We have considered models of landscapes motivated by the CC problem and by the severe anthropic constraints on various other parameters of the standard model. If the parameters of a slow-roll inflation model are scanned cosmologically, this may further explain the existence of a flat inflaton potential; although very flat potentials may be rare within the landscape, the exponential increase in volume that results would more than make up for this rarity. A large volume factor leads to a large number of observers to see it.

On the other hand, the volume factor discussed here is likely to be so powerful in determining the observed density perturbations, that its effect should be carefully studied. When a landscape of vacua is approximated by an ensemble of slow-roll inflationary regions, we find that the volume factor is generically exponentially sensitive to the density perturbation σ, so that an *exponentially* small fraction of observers in the universe see σ of order 10^{-5}. Hence, such landscapes do not provide a viable setting for understanding the value of the small but non-zero CC.

Two ideas to avoid the σ problem have been presented in this article. There may well be others. In one of them, we assume an initial volume distribution $\mathcal{I}$ that gives an exceedingly large weight to one of the anthropically acceptable standard-model vacua. In particular, this weight was assumed to be much larger than the other factors, so that the volume factor was relatively unimportant and the σ problem was absent. Virtually all observers in the universe, including ourselves, see the same values for the low-energy parameters in this scenario. The values for parameters that would actually be chosen cannot be identified, however, unless a particular landscape of vacua is specified, and the resulting $\mathcal{I}$ determined. Replacing our ignorance of the landscape with a density of states that is independent of the CC, the probability of the CC being chosen to be smaller than what we see is in the range $10^{-9} \lesssim P[\Lambda^4 < (3 \times 10^{-3}\text{eV})^4] \lesssim 10^{-2}$, with uncertainties from anthropic conditions and model dependence. Here we have just assumed that the density perturbation was chosen to be 10^{-5}, and we stress that there is no understanding of why σ lies in the middle of its "anthropic window" in this scenario. The key point is that the strength of $\mathcal{I}$ widens the anthropically allowed ranges for parameters in this picture, reducing the probabilities that we measure *our* values. Eternal inflation may have occured prior to slow-roll inflation, and could provide a dynamical

mechanism for obtaining the sharply peaked $\mathcal{I}$ assumed here.

In our second idea, we consider landscapes where the physical volume distribution $\mathcal{IV}$ does not have a σ problem to begin with and in which the distribution on the CC is roughly flat. If the e-folding number N_e and σ do not depend on a common scanning parameter of the inflation model, then the volume factor $\mathcal{V}$ does not necessarily depend exponentially on σ, solving the σ problem. The naturalness problem for the inflaton mass is also solved through the scanning of N_e. If $\mathcal{I}$ then has sufficiently mild dependence on parameters, most of the observers in the universe will then see σ to be within the "anthropic window" $10^{-6} \lesssim \sigma \lesssim 10^{-4}$, and furthermore, the successful anthropic explanation for the CC will be in full form, with $\mathcal{P}[\Lambda^4 < (3 \times 10^{-3}\text{eV})^4] \sim 5 - 10\%$.

Acknowledgments

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