

LTH-661
HEP-TH/0509054
AUGUST 2005

FICTITIOUS EXTRA DIMENSIONS

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String theory requires additional degrees of freedom to maintain world-sheet reparameterisation invariance at the quantum level. These are often interpreted as extra dimensions, beyond the 4 space-time. I discuss a class of quasi-realistic string models in which all the untwisted geometrical moduli are projected out by GSO projections. In these models the extra dimensions are fictitious, and do not correspond to physical dimensions in a low energy effective field theory. This raises the possibility that extra dimensions are fictitious in phenomenologically viable string vacua. I propose that self-duality in the gravitational quantum phase-space provides the criteria for the string vacuum selection.

1. Introduction

String theory, and its various modern incarnations, provides a consistent and most developed framework to study the unification of all the observed fundamental forces and interactions. This quest for unification is an everlasting theme in modern physics. Early proponents included Newton who unified celestial and terrestrial gravity; Maxwell who unified the electric and magnetic forces; and Einstein who unified electromagnetism and mechanics. In more recent times all the observed fundamental processes in nature are described in terms of the electromagnetic, weak and strong, gauge interactions; and in terms of gravitational general relativity. String theory affords the inclusion of all of those in a consistent framework, and is the reason for its continued appeal and interest. This, however, is not a speedy enterprise. Adjudicating whether it succeeds or fails will likely require the

*to appear in the proceedings of the fourth international conference on quantum theory and symmetries, 15-21/8/ 2005, varna bulgaria

efforts of more than one generation. One should consider, however, that it took more than two millenia to reach a decisive conclusion on heliocentrism versus geocentrism. The reason being not merely the dogma of well fashioned clergy, but rather the mundane interpretation of the available data.

In the classical string we can always gauge fix the two dimensional world-sheet metric to the flat metric. Preserving this property in the quantised string requires that we embed it in 26 space-time dimensions in the case of the bosonic string; and 10 in the case of the fermionic string. The closed string allows for independent treatment of the left- and right-moving modes on the string world-sheet. Hence, it gives rise to the heterotic-string in which the left-movers are fermionic and the right-movers are bosonic.

In the real world, we only observe four space-time dimensions, and internal symmetries of the particle spectrum. The standard lore to rectify this apparent discrepancy is to compactify the quantized string on an internal compactified manifold. In the case of the heterotic string 16 of the right-moving dimensions are compactified on an even self-dual lattice with fixed radii. Six right-moving coordinates, combined with six left-moving dimensions, are compactified on a six dimensional real manifold, or on a three dimensional complex manifold. The size and shape of this internal compact manifold are parametrized by the moduli. At present there is no known mechanism that selects and fix these moduli. Unravelling it is one of the major hurdles facing string theory.

On the other hand, over the past two decades, phenomenological studies of string theory have continued in earnest, and numerous quasi-realistic string models have been constructed. A natural question to ask therefore is whether these phenomenological string vacua can offer a guide to the issue of moduli selection and fixation. In this note I propose that the answer is affirmative. The quasi-realistic heterotic string models in the free fermionic formulation ¹, which are associated with $Z_2 \times Z_2$ orbifold compactifications at special points in the moduli space, points in the direction of the self-dual point under T-duality as playing a special role in the vacuum selection, and to the independence of the left-right moving modes as allowing for asymmetric conditions, that result in fixation of all of the geometrical moduli, as well as all of the twisted sector moduli ²

2. Moduli fixing in realistic string models

The general structure of the quasi-realistic free fermionic models and their phenomenological characteristics have been amply discussed and reviewed in the past ¹. Here I focus on the question of moduli fixing in these mod-

els. The relation of these models to $Z_2 \times Z_2$ orbifold compactifications is elaborated in ³. The untwisted sector of the $Z_2 \times Z_2$ orbifold gives rise to an $SO(10)$ GUT gauge group, which is broken down further, by the string boundary conditions, to one of its sub-group. The three twisted sectors produce three spinorial 16 representations of $SO(10)$ decomposed under the unbroken $SO(10)$ subgroup. In this manner the models give rise to three generations, which possess the canonical $SO(10)$ GUT embedding. These models were primarily studied using the free fermionic formalism ⁴, in which all the string boundary conditions are given in terms of the free fermion transformation properties on the string world-sheet. These fermionic models correspond to bosonic compactifications, in which the moduli are a priori fixed at a special point in the moduli space.

The geometrical moduli are the untwisted Kähler and complex structure moduli of the six dimensional compactified manifold. Additionally, the string vacua contain the dilaton moduli whose VEV governs the strength of the four dimensional interactions. The VEV of the dilaton moduli is a continuous parameter from the point of view of the perturbative heterotic string, and its stabilization requires some nonperturbative dynamics, or some input from the underlying quantum M-theory, which is not presently available. The problem of dilaton stabilization is therefore not addressed in this work, as the discussion here is confined to perturbative heterotic string vacua. Additionally, the models contain twisted sector moduli. Since the moduli fields correspond to scalar fields in the massless string spectrum, the moduli space is determined by the set of boundary condition basis vectors that define the string vacuum and encodes its properties. The first step therefore is to identify the fields in the fermionic models that correspond to the untwisted moduli. The subsequent steps entail examining which moduli fields survive successive GSO projections and consequently the residual moduli space.

The four dimensional fermionic heterotic string models are described in terms of two dimensional conformal and superconformal field theories of central charges $C_R = 22$ and $C_L = 9$, respectively. In the fermionic formulation these are represented in terms of world-sheet fermions. A convenient starting point to formulate such a fermionic vacuum is a model in which all the fermions are free. The free fermionic formalism facilitates the solution of the conformal and modular invariance constraints in terms of simple rules ⁴. Such a free fermionic model corresponds to a string vacuum at a fixed point in the moduli space. Deformations from this fixed point are then incorporated by including world-sheet Thirring interactions among the world-sheet fermions, that are compatible with the conformal

and modular invariance constraints. The coefficients of the allowed world-sheet Thirring interactions correspond to the untwisted moduli fields. For symmetric orbifold models, the exactly marginal operators associated with the untwisted moduli fields take the general form $\partial X^I \bar{\partial} X^J$, where X^I , $I = 1, \dots, 6$, are the coordinates of the six-torus T^6 . Therefore, the untwisted moduli fields in such models admit the geometrical interpretation of background fields, which appear as couplings of the exactly marginal operators in the non-linear sigma model action. The untwisted moduli scalar fields are the background fields that are compatible with the orbifold point group symmetry.

It is noted that in the Frenkel-Kac-Segal construction of the Kac-Moody current algebra from chiral bosons, the operator $i\partial X^I$ is a $U(1)$ generator of the Cartan sub-algebra. Therefore, in the fermionic formalism the exactly marginal operators are given by Abelian Thirring operators of the form $J_L^i(z)\bar{J}_R^j(\bar{z})$, where $J_L^i(z)$, $\bar{J}_R^j(\bar{z})$ are some left- and right-moving $U(1)$ chiral currents described by world-sheet fermions. Abelian Thirring interactions preserve conformal invariance, and are therefore marginal operators. One can therefore use the Abelian Thirring interactions to identify the untwisted moduli in the free fermionic models. The untwisted moduli correspond to the Abelian Thirring interactions that are compatible with the GSO projections induced by the boundary condition basis vectors, which define the string models.

I now turn to examine the moduli space in concrete free fermionic constructions. The models are constructed recursively by adding additional boundary condition basis vectors, which imposes GSO projections, truncating the existing spectrum, and adding new sectors and new states. The maximal moduli space of the $N = 4$ vacuum at the free fermionic point is the coset space $SO(6, 22)/(SO(6) \times SO(22))$. Applying the $Z_2 \times Z_2$ projections truncates the untwisted moduli space to $SO(2, 2)/(SO(2) \times SO(2))$, which correspond to three complex structure and three Kähler structure moduli. These moduli fields are always present in symmetric $Z_2 \times Z_2$ orbifolds. The realistic free fermionic models are constructed by adding additional boundary condition basis vectors, beyond the $Z_2 \times Z_2$ twistings. The additional vectors break the $SO(10)$ gauge symmetry down to a subgroup and reduce the number of generations to three. Their effect on the untwisted moduli space is extracted by focussing on the boundary conditions of the internal world-sheet fermions that correspond to the six dimensional compactified coordinates. The three generation free fermionic models give rise to the possibility of assigning asymmetric boundary conditions to the left and right-movers. These assignments are reflected in

the combinations of the real internal world-sheet fermions into complex fermions, or into Ising model world-sheet fermions. The second case corresponds to symmetric assignment of boundary conditions, whereas the first corresponds to asymmetric assignments, that distinguish between the left- and right-moving fermions. This possibility of assigning asymmetric boundary conditions has important phenomenological consequences. For example, for the problem of proton stability and the string doublet-triplet splitting mechanism ⁵.

By examining concrete three generation free fermionic models it is noted that some models employ boundary conditions that are fully symmetric ². The moduli space of such quasi-realistic models therefore contains the three complex and three Kähler structure moduli of the original $Z_2 \times Z_2$ orbifold. In these models the internal six dimensional manifold admit a classical geometrical interpretation. However, there also exist quasi-realistic free fermionic models that employ fully asymmetric boundary conditions. In these models all the six internal real coordinates have the asymmetric identifications

$$X_L + X_R \rightarrow X_L - X_R \tag{1}$$

As a consequence all the geometrical untwisted moduli fields are projected out in these models. The additional dimensions in these compactifications are therefore frozen at the enhanced symmetry point. These quasi-realistic string vacua therefore do not contain additional classical dimensions, which are therefore fictitious in these models. Namely the extra dimensions exist as organizing principle at some level in the string partition function, but are not realized physically in the low effective field theory. The situation is similar to the way in which gauge symmetries are broken in string theory by Wilson lines. Also in this case the models contain a GUT gauge symmetry at some level of the string partition function, which is broken by Wilson lines and is not an explicit symmetry of the low energy effective field theory.

It is of interest to note that in the quasi-realistic heterotic-string models discussed here the moduli that arise from the twisted sectors are projected out as well ². The reason is that the models correspond to (2,0) rather than (2,2) compactification. In the (2,2) models the sectors that complement the 16 representation of $SO(10)$ to 27 of E_6 , also at the same time produce the twisted moduli. In the (2,0) models these sectors give rise to vectorial 16 representations of the hidden $SO(16)$ gauge group and the moduli are projected out together with the 10+1 representations that are embedded in the 27 of E_6 . It should, however, be emphasized that the models may contain additional moduli. Additional moduli may arise from flat directions

of the superpotential and from charged moduli. What is noted here is that the moduli that are identified as coefficients of exactly marginal operators, and are therefore interpreted as geometrical moduli, are projected out from the massless spectrum. Hence the geometrical coordinates in these models are frozen at the enhanced symmetry point. In these models there is no apparent classical geometry that underlies the additional degrees of freedom that are required to restore the world-sheet reparameterisation invariance.

In the three generation free fermionic models with the fully asymmetric identification all the extra dimensions are frozen at the maximally enhanced symmetry point, which up to a rotation is the same as the self-dual point under T-duality ⁶. The attractive phenomenological structure of these models and the relation between the maximally enhanced symmetry point and the self-dual point under T-duality raises the intriguing possibility that the self-duality criteria is pivotal to the vacuum selection.

3. Phase-space self-duality and trivial selection

To illustrate further this possibility I discuss the association of a self-dual state with a “vacuum” state in a completely unrelated mathematical setting. Duality and self-duality also play a key role in the recent formulation of quantum mechanics from an equivalence postulate ⁷. An important facet of this formalism is the phase-space duality, which is manifested due to the involutive nature of the Legendre transformation. In the Hamilton-Jacobi formalism of classical mechanics the phase-space variables are related by Hamilton’s generating function $p = \partial_q S_0(q)$. One then obtains the dual Legendre transformations ⁷,

$$S_0 = p\partial_p T_0 - T_0$$

and

$$T_0 = q\partial_q S_0 - S_0,$$

where $T_0(p)$ is a new generating function defined by $q = \partial_p T_0$. Because of the undefinability of the Legendre transformation for linear functions, *i.e.* for physical systems with $S_0 = Aq + B$, the Legendre duality fails for the free system, and for the free system with vanishing energy. We can associate a second order differential equation with each Legendre transformation ⁷. There exist therefore a set of solutions, labelled by $pq = const$, which are simultaneous solutions of the two sets of differential equations. These are the self dual states under the phase-space duality.

The Legendre phase-space duality and its breakdown for the free system are intimately related to the equivalence postulate, which states that

all physical systems labelled by the function $W(q) = V(q) - E$, can be connected by a coordinate transformation, $q^a \rightarrow q^b = q^b(q^a)$, defined by $S_0^b(q^b) = S_0^a(q^a)$. This postulate implies that there always exist a coordinate transformation connecting any state to the state $W^0(q^0) = 0$. Inversely, this means that any physical state can be reached from the state $W^0(q^0)$ by a coordinate transformation. This postulate cannot be consistent with classical mechanics. The reason being that in Classical Mechanics (CM) the state $W^0(q^0) \equiv 0$ remains a fixed point under coordinate transformations. Thus, in CM it is not possible to generate all states by a coordinate transformation from the trivial state. From the Classical Hamilton–Jacobi Equation (CHJE) it is seen that $S_0 = Aq + B$ is the solution associated with $V(q) = 0$ & $E = \text{const}$, that is the state for which the Legendre duality breaks down. Consistency of the equivalence postulate therefore implies that $S_0(q)$ is not a solution of the CHJE, but rather a solution of the Quantum Stationary Hamilton–Jacobi Equation (QSHJE),

$$(1/2m) (\partial_q S_0)^2 + V(q) - E + (\hbar^2/4m)\{S_0, q\} = 0,$$

where $\{, \}$ denotes the Schwarzian derivative. The remarkable property of the QSHJE, which distinguishes it from the classical case, is that it admits a non-trivial solution also for the trivial state, $W(q) \equiv 0$. In fact the QSHJE implies that $S_0 = \text{constant}$ is not an allowed solution. The fundamental characteristic of quantum mechanics in this approach is that $S_0 \neq Aq + B$. Rather, the solution for the trivial state, with $V(q) = 0$ and $E = 0$, is given by

$$S_0 = i\hbar/2 \ln q,$$

up to Möbius transformations. Remarkably, this quantum trivial state solution coincides with the self-dual state of the Legendre phase-space transformation and its dual. We have that the quantum self-dual state plays a pivotal role in ensuring both the consistency of the equivalence postulate and definability of the Legendre phase-space duality for all physical states. Furthermore, it is noted that the self-dual state under phase-space duality is associated with the state with $V(q) = 0$ and $E = 0$. Hence providing another hint at the association between self-duality and trivial states in the space of all allowed states.

4. Conclusions

Existence of quasi-realistic string vacua in which all the untwisted and twisted sectors moduli are projected out was demonstrated. In such models

the extra dimensions are fictitious. This may indicate that extra dimensions are fictitious in phenomenologically viable string vacua. This is an appealing proposition. While string theory requires additional degrees of freedom, beyond the four space–time, the interpretation of those as extra physical dimensions is naive. Extra dimensions provide an organizing principle for the string symmetries, but are not realized as physical dimensions in the low energy effective field theory. It is the intrinsic left–right independence of the closed string modes, which allows for asymmetric boundary conditions, and results in the projection of all the Kähler and complex structure moduli. Thus, string theory, which needs the extra degrees of freedom for its consistency, also provides the intrinsic mechanism to fix the moduli. The mechanism afforded utilises the quantum nature of the extra dimensions, and therefore may indicate the limitation of the effective field theory analysis. It may also point to the possibility that dilaton fixation may have to await the quantum formulation of M–theory.

It is proposed further that phase–space duality is the guiding property in trying to formulate quantum gravity. In this respect T–duality is a key property of string theory. We can think of T–duality as a phase–space duality in the sense that we are exchanging momenta and winding modes in compact space. We can turn the table around and say that the key feature of string theory is that it preserves the phase–space duality in the compact space. Namely, prior to compactification the wave–function of a point particle $\Psi \sim \text{Exp}(iPX)$ is invariant under $p \leftrightarrow x$. However, in the ordinary Kaluza–Klein compactification this invariance is lost due to the quantization of the momentum modes. String theory restores this invariance by introducing the winding modes. It is further argued that the self–dual points under phase–space duality are intimately connected to the choice of the vacuum.

Acknowledgements

I would like to thank the Oxford Theoretical Physics Department for hospitality. This work is supported in part by the PPARC and by the Royal Society.

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