# Holographic Dark Energy and Present Cosmic Acceleration

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**Abstract.** We review the notion of holographic dark energy and assess its significance in the light of the well documented cosmic acceleration at the present time. We next propose a model of holographic dark energy in which the infrared cutoff is set by the Hubble scale. The model accounts for the aforesaid acceleration and, by construction, is free of the cosmic coincidence problem.

## INTRODUCTION

There is a growing conviction among cosmologists that the Universe is currently experiencing a stage of accelerated expansion not compatible with the up to now favored Einstein-de Sitter model [1]. According to the latter, the Universe should be now decelerating its expansion. This conviction is deeply rooted in observational grounds, mainly in the low brightness of high redshift supernovae type Ia which are fainter than allowed by the aforesaid model but consistent with accelerated models [2], as well as in other cosmological data. These include the position of the first acoustic peak of cosmic microwave background radiation (CMBR), which suggests that the Universe is spatially flat or nearly flat [3], combined with estimations of the amount of mass at cosmological scales -see e.g. [4]-, and correlations of the anisotropies of CMBR with large scale structures [5]. Overall, the data strongly hint at a Universe dominated by some form of energy -the so called, "dark energy"- that would contribute about 70 percent to the total energy density and nonrelativistic matter (dust) which would contribute the remaining 30 percent.

The trouble with dark energy is that we can only guess about its nature. To begin with, it must possess a huge negative pressure, at least high enough to violate the strong energy condition, something required (within general relativity) to drive accelerated expansion, and should cluster only at the highest accessible scales. The straightforward candidate is the cosmological constant, $\Lambda$, whose equation of state is simply $p_\Lambda = -\rho_\Lambda$, and whose energy is evenly distributed. Yet, it faces two serious drawbacks. On the one hand its quantum field theoretical value is about 123 orders of magnitude larger than observed; on the other hand, it entails the *coincidence problem*, namely: "Why are the vacuum and dust energy densities of precisely the same order today?" [6]. (Bear in mind that the energy density of dust redshifts with expansion as $a^{-3}$, where $a$ denotes the scale factor of the Robertson–Walker metric). This is why a large variety of candidates -quintessence and tachyon fields, Chaplygin gas, phantom fields, etc.-, of varying plausibility, have been proposed in the last years -see Ref. [7] for reviews. Unluckily, however, there is

not a clear winner in sight.

Recently, a new form of dark energy based on the holography notion and related to the existence of some or other cosmic horizon has been proposed [8]. Here, we present a specific model of holographic dark energy that accounts for the current stage of cosmic acceleration and is free from the coincidence problem that besets so many models of late acceleration [9]. The outline of this work is as follows. We first recall the notion of holography which is receiving growing attention and discuss possible choices for the infrared cutoff. Then we present our model of holographic dark energy. The last section is devoted to the conclusions and final remarks.

## HOLOGRAPHY

We begin by recalling the notion of holography as introduced by 't Hooft [10] and Susskind [11]. Consider the world as three-dimensional lattice of spin-like degrees of freedom and assume that the distance between every two neighboring sites is some small length $\ell$. Each spin can be in one of two sates. In a region of volume $L^3$ the number of quantum states will be $N(L^3) = 2^n$, with $n = (L/\ell)^3$ the number of sites in the volume, whence the entropy will be $S \propto (L/\ell)^3 \ln 2$. One would expect that if the energy density does not diverge, the maximum entropy varies as $L^3$, i.e., $S \sim L^3 \Lambda^3$, where $\Lambda \equiv \ell^{-1}$ is to be identified with the ultraviolet cutoff. However, the energy of most states described by this formula would be so large that they will collapse to a black hole of size in excess of $L^3$. Therefore, a reasonable guess is that in the quantum theory of gravity the maximum entropy should be proportional to the area, not the volume, of the system under consideration. (Bear in mind that the Bekenstein–Hawking entropy is $S_{BH} = A/(4 \ell_{Pl}^2)$, where $A$ is the area of the black hole horizon).

Consider now a system of volume $L^3$ of energy slightly below that of a black hole of the same size but with entropy larger than that of the black hole. By throwing in a very small amount of energy a black hole would result but with smaller entropy than the original system thus violating the second law of thermodynamics. As a consequence, Bekenstein proposed that the maximum entropy of the system should be proportional to its area rather than to its volume [12]. In keeping with this, 't Hooft conjectured that it should be possible to describe all phenomena within a volume by a set of degrees of freedom which reside on the surface bounding it. The number of degrees of freedom should be not larger than that of a two-dimensional lattice with about one binary degree of freedom per Planck area.

### Holographic energy interpreted as dark energy

Inspired by these ideas, Cohen *et al.* [8] argued that an effective field theory that saturates the inequality

$$L^3 \Lambda^3 \leq S_{BH},\qquad(1)$$

necessarily includes many states with $R_s > L$, where $R_s$ is the Schwarzschild radius of the system under consideration. Indeed, a conventional effective quantum field theory is expected to describe a system at temperature $T$ provided that $T \leq \Lambda$. So long as $T \gg L^{-1}$, the energy and entropy will correspond to those of radiation ($E \simeq L^3 T^4$, and $S \simeq L^3 T^3$). When (1) is saturated (by setting $T = \Lambda$ in (1)) at $T \simeq m_{Pl}^{2/3} L^{-1/3}$, the Schwarzschild radius becomes $R_s \sim m_{Pl}^{2/3} L^{5/3} \gg L$.

Therefore it appears reasonable to propose a stronger constraint on the infrared (IR) cutoff $L$ that excludes all states lying within $R_s$, namely:

$$L^3 \Lambda^4 \leq m_{Pl}^2 L \qquad (2)$$

(obviously, $\Lambda^4$ is the zero–point energy density associated to the short-distance cutoff). So, we conclude that $L \sim \Lambda^{-2}$ and $S_{max} \simeq S_{BH}^{3/4}$.

By saturating the inequality (2) -which is not compelling at all- and identifying $\Lambda^4$ with the holographic dark energy density we have

$$\rho_x = 3c^2 M_p^2 / L^2 \,, \qquad (3)$$

where $c^2$ is a dimensionless constant and $M_p^2 \equiv (8\pi G)^{-1}$.

## The infrared cutoff

Before building a cosmological model of late acceleration on the above ideas the IR cutoff must be specified. All the proposals in the literature identify $L$ with the radius of one or another cosmic horizon. The simplest (and most natural) choice is the Hubble radius, $H^{-1}$. However, as shown by Hsu [13], this faces the following difficulty. For an isotropic, homogeneous and spatially flat universe dominated by nonrelativistic matter and dark energy the Friedmann equation $\rho_m + \rho_x = 3M_p^2 H^2$ together with $\rho_m \propto a^{-3}$ implies that $\rho_x$ also redshifts as $a^{-3}$. In virtue of the conservation equation $\dot{\rho}_x + 3H(\rho_x + p_x) = 0$ it follows that $p_x$ vanishes, i.e., there is no acceleration. So, this first choice seems doomed.

Two other, not so natural choices, are:
($i$) $L = R_{ph}$ [14, 15], where

$$R_{ph} = a(t) \int_0^t \frac{dt'}{a(t')}$$

is the particle horizon. Yet, this option does not fare much better. Assuming the dark energy to dominate the expansion, Friedmann's equation reduces to $H R_{ph} = c$. Therefore, $H \propto a^{-(1+\frac{1}{c})}$ and consequently the equation of state parameter of the dark energy $w \equiv p_x / \rho_x = -(1/3) + (2/3c)$ is found to be larger than $-1/3$ whence this dark energy candidate does not violate the strong energy condition and cannot drive late acceleration either.

($ii$) $L = R_H$ [16], where

$$R_H = a(t) \int_t^\infty \frac{dt'}{a(t')}$$

is the radius of the future event horizon, i.e., the boundary of the volume a given observer may eventually see. Assuming again the dark energy to dominate the expansion it is found that $w = -(1/3) - (2/3c) < -1/3$. Thus, this choice is compatible with accelerated expansion.

## INTERACTING DARK ENERGY

This section focuses on our recent model of late acceleration based on three main assumptions, namely, ($i$) the dark energy density is given by Eq. (3), ($ii$) $L = H^{-1}$, and ($iii$) matter and holographic dark energy do not conserve separately but the latter decays into the former with rate $\Gamma$, i.e.,

$$\dot{\rho}_m \quad + \quad 3H\rho_m = \Gamma\rho_x, \tag{4}$$
$$\dot{\rho}_x \quad + \quad 3H(1+w)\rho_x = -\Gamma\rho_x. \tag{5}$$

As it can be checked, there is a relation connecting $w$ to the ratio between the energy densities, $r \equiv \rho_m/\rho_x$, and $\Gamma$, namely, $w = -(1+r)\Gamma/(3rH)$, such that any decay of the dark energy $\Gamma > 0$ into pressureless matter implies a negative equation of state parameter, $w < 0$. It also follows that the ratio of the energy densities is a constant, $r_0 = (1-c^2)/c^2$, whatever $\Gamma$ -see Ref. [9] for details.

In the particular case that $\Gamma \propto H$ one has $\rho_m$, $\rho_x \propto a^{-3m}$ and $a \propto t^n$ with $m = (1+r_0+w)/(1+r_0)$ and $n = 2/(3m)$. Hence, there will be acceleration for $w < -(1+r_0)/3$. In consequence, the interaction is key to simultaneously solve the coincidence problem and have late acceleration. For $\Gamma = 0$ the choice $L = H^{-1}$ does not lead to acceleration. Before going any further, we wish to emphasize that models in which matter and ark energy interact with each other are well known in the literature -see [17] and references therein- and presently they are being contrasted with cosmological data [18].

Obviously, prior to the current epoch of accelerated expansion (during the radiation and matter dominated epochs) $r$ must not have been constant but decreasing toward its current value $r_0$, otherwise the standard picture of cosmic structure formation would be irremediably spoiled (as usual, a subindex zero means present time). To incorporate this we must allow the parameter $c^2$ to vary with time. Hence, we now have

$$\dot{\rho}_x = -3H \left[ 1 + \frac{w}{1+r} \right] \rho_x + \frac{(c^2)^{\cdot}}{c^2} \rho_x . \tag{6}$$

Combining it with the conservation equation (5) and contrasting the resulting expression

with the evolution equation for $r$, namely,

$$\dot{r} = 3Hr\left[w + \frac{1+r}{r}\frac{\Gamma}{3H}\right],\tag{7}$$

yields $\left(c^2\right)^{\cdot}/c^2 = -\dot{r}/(1+r)$, whose solution is

$$c^2(t) = \frac{1}{1+r(t)}.\tag{8}$$

At sufficiently long times, $r \to r_0$ whence $c^2 \to c_0^2$.

In this scenario $w$ depends also on $c^2$ according to

$$w = -\left(1 + \frac{1}{r}\right)\left[\frac{\Gamma}{3H} + \frac{\left(c^2\right)^{\cdot}}{3Hc^2}\right].\tag{9}$$

Since the holographic dark energy must fulfil the dominant energy condition (and therefore it is not compatible with "phantom energy") [19], the restriction $w \geq -1$ sets constraints on $\Gamma$ and $c^2$.

## DISCUSSION AND CONCLUSIONS

The holographic dark energy seems to be a simple, reasonable and elegant alternative (within general relativity) to account for the present state of cosmic accelerated expansion. It can solve the coincidence problem provided that matter and holographic energy do not conserve separately. In this connection, it was pointed out by Das *et al.* [20], that because the interaction modifies the dependence of matter density on the scale factor the observers who endeavor to fit the observational data under the assumption of noninteracting matter will likely infer an effective $w$ lower than $-1$. Therefore, most of the claims in favor of phantom energy may be considered as lending support to the dark energy–matter interaction. Yet, models of holographic dark energy must be further constrained by observations.

It should be noted that, contrary to what one may think, the infrared cutoff does not necessarily change when $c^2$ is varied. Indeed, the holographic bound can be expressed as $\rho_x \leq 3c^2 M_p^2/L^2$. Now, we first considered that it was saturated (i.e., the equality sign was assumed in the above expression) and that $L = H^{-1}$. Since the saturation of the bound is not at all compelling, and the "constant" $c^2(t)$ augments with expansion (as $r$ decreases) up to attaining the constant value $(1+r_0)^{-1}$, the expression $\rho_x = 3c^2(t)M_p^2H^2$, in reality, does not entail a modification of the infrared cutoff, which still is $L = H^{-1}$. What happens is that, as $c^2(t)$ grows, the bound gets progressively saturated up to full saturation when, asymptotically, $c^2$ becomes a constant. Put another way, the infrared cutoff stays $L = H^{-1}$ always, what changes is the degree of saturation of the holographic bound.

Before closing we would like to stress that there is no guarantee that the present accelerated epoch will not be followed by subsequent period of decelerated expansion.

Models to that effect, partly motivated by string theory demands, have been advanced, -see, e.g. [21]. If, indeed, the present epoch is followed by a decelerated one, then the future cosmic horizon will simply not exist and models of holographic dark energy based on that choice of the IR cutoff will be seen as essentially flawed.

# ACKNOWLEDGMENTS

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